Projectile Motion

i) Verify that the range, R, of a projectile launched from a height h (with respect to the ground), with speed v, and at an angle θ above the horizontal is given by:

$$R = \frac{v^2 \cos\theta}{g}\left(\sin\theta \pm \sqrt{\sin^2\theta + \frac{2gh}{v^2}}\right) \tag{1}$$

ii) Show that, in the limit $h \to 0$, this reduces to the familiar:

$$R = \frac{v^2 \sin 2\theta}{g} \tag{2}$$

iii) Show that the angle, θ, that maximizes R is, in general, given by:

$$\cos\theta = \sqrt{\frac{2gh + v^2}{2gh + 2v^2}} \tag{3}$$

Does this reduce to what you expect as $h \to 0$?