212 Test 1 ____________Section ____Name(Last, others)
October 2, 2006

NO BOOKS, NOTES or CALCULATORS ARE PERMITTED

$$\epsilon_o \oint_S \vec{E}\cdot d\vec{A} = q_{enc} \qquad \oint_S \vec{B}\cdot d\vec{A} = 0 \qquad \oint_C \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt} \qquad \oint_C \vec{B}\cdot d\vec{s} = \mu_o I + \mu_o\epsilon_o \frac{d\Phi_E}{dt}$$

$$\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z} \quad \text{for spherical symmetry } \nabla = \hat{r}\frac{\partial}{\partial r}, \quad \vec{F} = -\nabla U$$

$\Phi_E = \int_S \vec{E}\cdot d\vec{s}$ several useful expressions are: $\frac{1}{2}Li^2$, $\frac{1}{2}\frac{q^2}{c}$, $\frac{1}{2}\epsilon_o E^2$ $\frac{1}{2}\frac{B^2}{\mu_o}$

$1 + \tan^2 x = \sec^2 x$ where $1/\cos x = \sec x$ $\quad e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...$ $\quad \sin(x \pm y) = \sin x\cos y \pm \cos x \sin y$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ...$ $\quad \cos(x \pm y) = \cos x\cos y \mp \sin x \sin y$

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + ...$ $\quad \cos 2x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$$\text{Binomial}: \quad (1+x)^n \simeq 1 + nx + \frac{n(n-1)x^2}{2!} + \sum_{j=4}^{\infty} c_j x^{(j-1)} \quad |x|<1; \quad c_j = \frac{n(n-1)(n-2)\cdots(n-(j-2))}{(j-1)!}$$

$$\text{Taylor}: \quad f(x) = f(a) + (x-a)f'(a) + \tfrac{1}{2!}(x-a)^2 f''(a) + \ldots\tfrac{1}{n!}(x-a)^n f^n(a)$$

$$\int x\cos ax dx = \frac{x}{a}\sin ax + \frac{1}{a^2}\cos ax; \qquad \int x\sin ax dx = \frac{1}{a^2}\sin ax - \frac{x}{a}\cos ax$$

$$\int \ln(ax+b)\,dx = \frac{ax+b}{a}\ln(ax+b) - x \qquad \int \sin^2(ax)\,dx = \frac{x}{2} - \frac{1}{4a}\sin(2ax)$$

$$\int \frac{xdx}{1-ax^2} = \frac{-1}{2a}\ln\left(1-ax^2\right) \qquad \int \sin^3(ax)\,dx = -\frac{1}{3a}\cos(ax)\left\{\sin^2(ax)+2\right\}$$

$$\int \frac{dv}{\left(1-\frac{v^2}{c^2}\right)} = c\ \tanh^{-1}\frac{v}{c} \qquad \text{where} \qquad \tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x}-1}{e^{2x}+1}$$

1 (20)	
2 (18)	
3 (30)	
4 (16)	
5 (16)	
total (100)	

Please do not write on the back of these pages, use the front!

There are 5 problems, with the points for each problem given, with 100 points total. You should first read all the problems, and then do them in an order which maximizes your point total. You must show how you get your answers. You will not get credit if you just write down an answer, unless you justify writing something.

1) (20 points: 4,4,4,4,4) Please give **concise** answers to the following questions. Do not feel compelled to fill up the space given!

1.a) What is the differential relationship between electric field $\vec{E}$ and the electric potential V, in three dimensions and in one dimension?

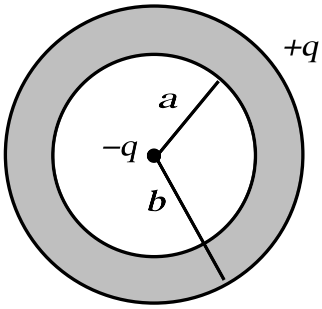
1.b) What is the (integral) definition of the potential difference between two points, a and b in a region filled with an electric field.

1.c) Give a simple example where the electric field can be zero, but the potential is not zero.

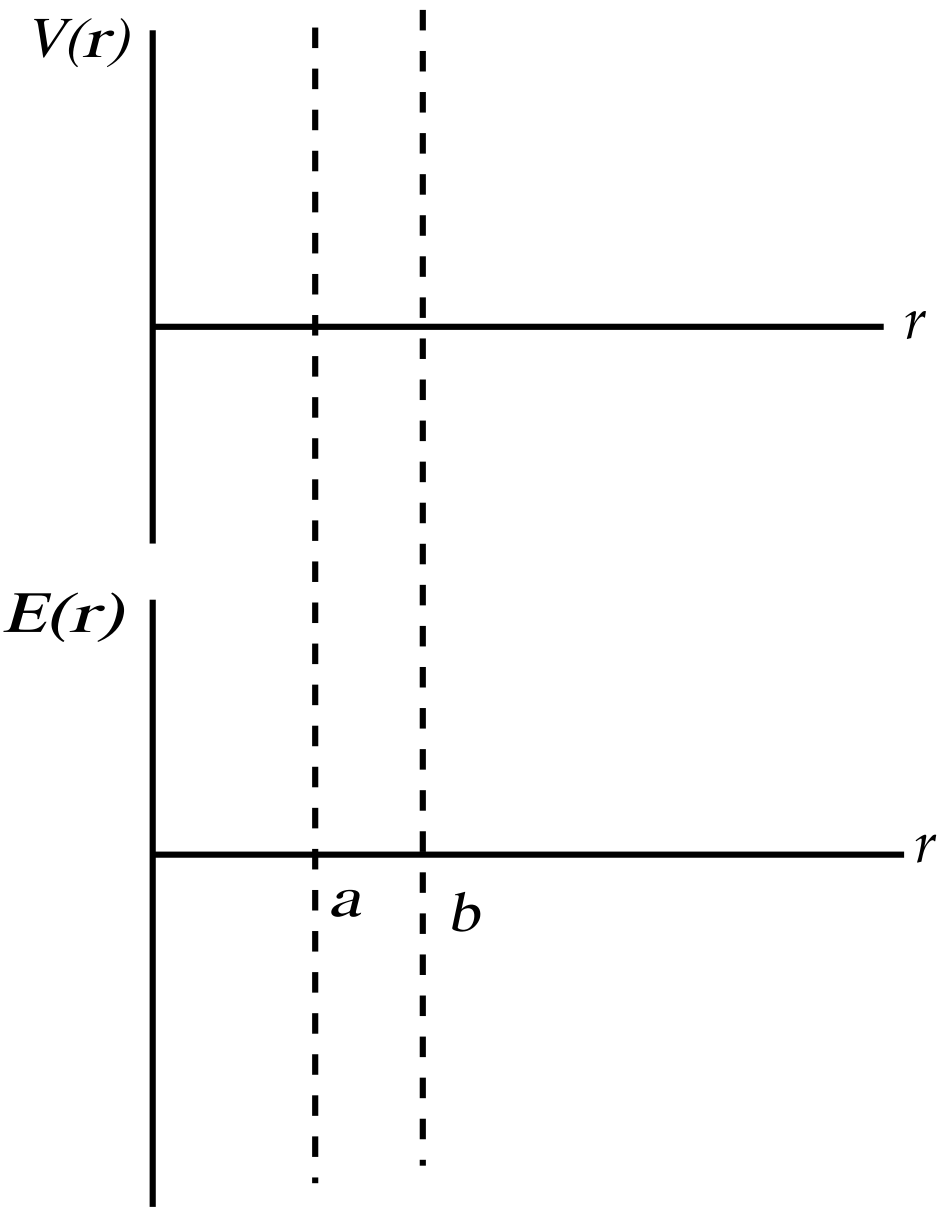
1.d) Explain why we conclude that the electric field is zero inside of a conductor in equilibrium.

1.e) Explain why we conclude that at equilibrium, the electric field at the surface of a conductor is perpendicular to the surface.

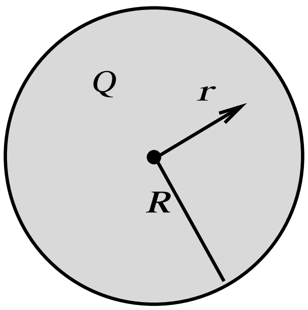
2 (18 points: 4,4,4,6) Consider a point charge $-q$ surrounded by a spherical conducting shell, inner radius a, outer radius b. The shell has a net charge of $+q$. Please answer the following questions. **PLEASE LABEL EACH PART OF THE PROBLEM AND BOX YOUR ANSWERS**



2.a) Find the Electric field and the potential for $r \geq b$.
2.b) Find the Electric field and the potential for $a \leq r \leq b$.
2.c) Find the Electric field and the potential for $r < a$.
2.d) Sketch the field and potential on the axes given below. This means indicate the functional dependence and any key values of the function on the sketch.



3. 30 points - (5,5,5,4,5,6) Consider a uniform non-conducting spherical charge distribution of radius R, and total charge $+Q$. **PLEASE LABEL EACH PART OF THE PROBLEM AND BOX YOUR ANSWERS**



3.a) Find $E(r)$ and $V(r)$ outside of the sphere. You may write it down if you justify why you can write it down.

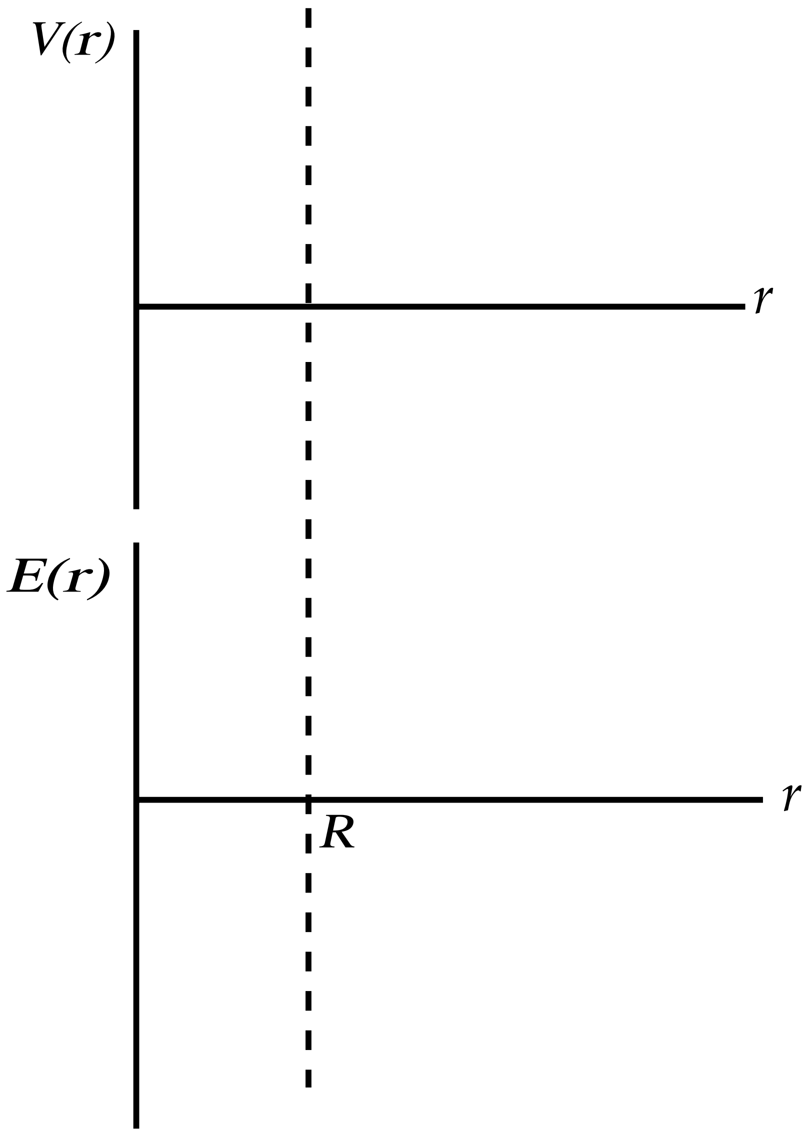
3.b) For $r \leq R$, find ρ, the charge per unit volume, and also find $q(r)$ the charge enclosed in a sphere at a radius r.

3.c) Find the electric field inside of the charge distribution, (for $r < R$) as a function of radius r.

3.d) What is the potential at the surface of the charge distribution?

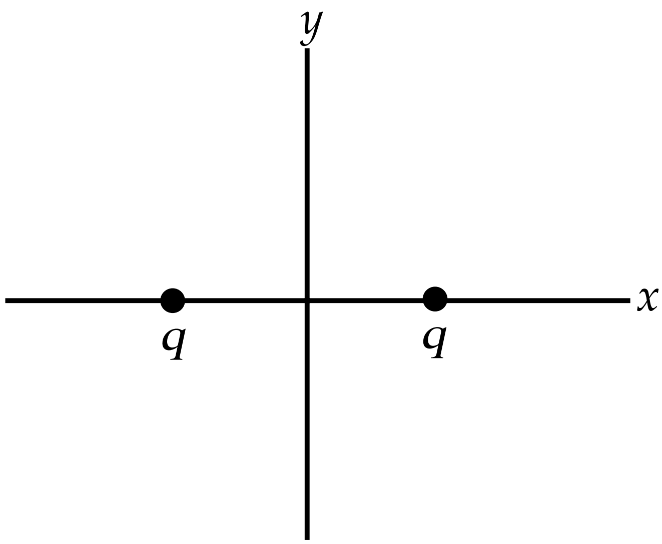
3.e) Find the potential inside of the sphere.

3.f) Sketch $E(r)$ and $V(r)$ inside of the sphere.This means indicate the functional dependence and any key values of the function on the sketch.

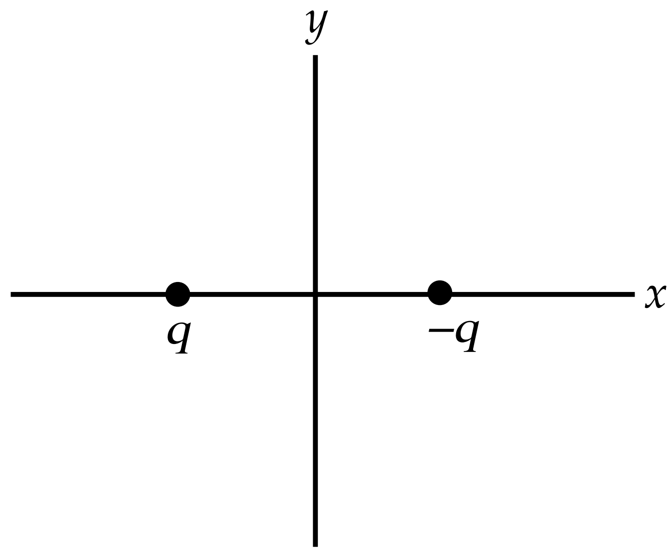

V(r)
r
E(r)
r
R

4. 16 points - (8,4,4) Consider two like charges q on the x-axis as shown in the fig. (a), or two equal magnitude but opposite sign charges as shown in fig. (b).

4.a) Sketch the electric field lines and equipotential lines for each case. Make sure you label which is which!



(a)



(b)

4.b) Where is the Electric field zero for (a) and (b)?

4.c) Using the usual definitions, where is the potential zero for (a) and for (b)?

5 16 points (5,5,6) Three charges are brought together into the configuration shown below. You may leave any answer in terms of a simple square root, meaning for example the square root of 5, 3, or 2, including in the denominator of a fraction. So your answer may have the product of a simple number times this square root. Even if you remember that $\sqrt{2} \simeq 1.414...$ you don't need to use it. a is an unspecified length, so your answers will be in terms of a. (Extra Hint: You can write down the sin or cos of any angle you need from these numbers, and it will be a simple fraction.

5.a) What is the work needed to bring them together? Simplify your answer so that you can see what the sign is. What does the sign mean?

5.b) What is the potential V at point P where the coordinates of P are $(4a, 0)$ Simplify this answer to a simple fraction.

5.c) Find $\vec{E}$ at point P. (see hint above)