PY 211 Mid Term Exam II

April 3, 2007

Formula Sheet, Useful Data, and Moments of Inertia

Useful Formulae – Midterm #2

Work done constant force: $W = F \quad r\cos$

Work done by varying force (1D): $W \quad \int_{x_i}^{x_f} F_x dx$ General Work Done: $W \quad \int_{r_i}^{r_f} \mathbf{F}\cdot d\mathbf{r}$

Spring Force: $F_s = -kx$

Kinetic Energy: $K = ½ mv^2$ Work Energy Theorem: $W_{12} \quad K$

Power: $P \quad \frac{dW}{dt} \quad F \quad \frac{dr}{dt} \quad F \quad v$ Units: 1 W = 1 J/s = 1 kg·m²/s³

Potential Energy (1D): $U_f(x) \quad \int_{x_i}^{x_f} F_x dx \quad U_i \qquad F_x \quad \frac{dU}{dx}$

PE(Earth): $U(r) \quad \frac{GM_E}{r}$ PE(Spring): $U(x) \quad \frac{1}{2}kx^2$

Total Mechanical Energy: $E_{mech} \quad K + U$

Linear Momentum: $\mathbf{p} \quad m\mathbf{v}$ **Newton's 2nd** Law: $\mathbf{F} \quad \frac{d\,m\mathbf{v}}{dt} \quad \frac{d\mathbf{p}}{dt}$

Impulse-Momentum Theorem: $\mathbf{I} \quad \int_{t_i}^{t_f} \mathbf{F}dt \quad \mathbf{p}$

Center of Mass: (discrete masses) $\mathbf{r}_{CM} \quad \frac{m_i \mathbf{r}_i}{M}$ (continuous) $\mathbf{r}_{CM} \quad \frac{1}{M} \quad \mathbf{r}\,dm$

Conservation of Linear Momentum: $\mathbf{p}_{1i} + \mathbf{p}_{2i} = \mathbf{p}_{1f} + \mathbf{p}_{2f}$

Motion of System: $\mathbf{F}_{ext} \quad M\mathbf{a}_{CM}$

Angular Speed: $\frac{d}{dt}$ Angular Acceleration: $\frac{d}{dt}$

Kinematics of Constant Angular Acceleration:

$f \quad i \quad t \, : \quad f \quad i \quad it \quad \frac{1}{2} \quad t^2 : \quad {}^2_f \quad {}^2_i \quad 2 \, (f \quad i)$

$f \quad i \quad \frac{1}{2} \quad i \quad f \quad t$

Rigid Object Rotating about Fixed Axis: $s = r \qquad v = r \qquad a_t = r$

Moment of Inertia of a System of Particles: $I \quad m_i r_i^2$

Moment of Inertia of Rigid Body: $I \quad r^2 dm$

Rotational Kinetic Energy: $K_R \quad \frac{1}{2} I^2$ Total Kinetic Energy: $K \quad \frac{1}{2} I_{CM}{}^2 \quad \frac{1}{2} Mv_{CM}^2$

Torque: $\boldsymbol{\tau} \quad \mathbf{r} \quad \mathbf{F}$ **Newton's 2nd** Law: $\tau \quad I\alpha$

Angular Momentum: $\mathbf{L} \quad \mathbf{r} \quad \mathbf{p} \qquad \boldsymbol{\tau} \quad \frac{d}{dt}\mathbf{L}$

z component of angular momentum-rigid object rotating about fixed *z* axis: $L_z = I$

Newton's Universal Gravitation: $F_g \quad G\frac{m_1 m_2}{r^2}$ G = 6.673 x 10⁻¹¹ N·m² / kg²

Kepler's Laws: 1st: Elliptical Orbits 2nd: $\frac{dA}{dt} \quad \frac{L}{2M_p}$ constant 3rd: $T^2 \quad \frac{4\ ^2}{GM_{Sun}} \quad a^3 \quad K_s a^3$

Gravitational Field: $\mathbf{g} \quad \frac{\mathbf{F}_g}{m}$

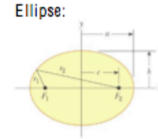
Gravitational Potential Energy of Earth: $U(r) \quad \frac{GM_E m}{r}$ where $U(\ ) \quad 0$

Total Energy of Satellite: $E \quad \frac{1}{2}mv^2 \quad G\frac{Mm}{r}$ Bound Circular: $E \quad \frac{GMm}{2r}$

Bound Elliptical: $E \quad \frac{GMm}{2a}$

Escape Velocity: $v_{escape} \quad \sqrt{\frac{2GM}{R}}$

Ellipse: Eccentricity: e = c/a



Area of Right Triangle: A = ½ x base x height

Scalar Product:

$\mathbf{A} \quad A_x\hat{\imath} \quad A_y\hat{\jmath} \quad A_z\hat{k}$

$\mathbf{B} \quad B_x\hat{\imath} \quad B_y\hat{\jmath} \quad B_z\hat{k}$

$\mathbf{A}\ \mathbf{B} \quad A_xB_x \quad A_yB_y \quad A_zB_z$

$\mathbf{A}\cdot\mathbf{B} = AB\cos$

Unit Vectors:

$\hat{\imath}\ \hat{\imath} = \hat{\jmath}\ \hat{\jmath} = \hat{k}\ \hat{k} = 1$

$\hat{\imath}\ \hat{\jmath} = \hat{\imath}\ \hat{k} = \hat{\jmath}\ \hat{k} = 0$

$\mathbf{R} \quad A_x\hat{\imath} \quad A_y\hat{\jmath} \quad A_z\hat{k} \qquad B_x\hat{\imath} \quad B_y\hat{\jmath} \quad B_z\hat{k}$

$\mathbf{R} \quad A_x \quad B_x \ \hat{\imath} \quad A_y \quad B_y \ \hat{\jmath} \quad A_z \quad B_z \ \hat{k}$

$\mathbf{R} \quad R_x\hat{\imath} \quad R_y\hat{\jmath} \quad R_z\hat{k}$

Vector Product: $\hat{\imath} \quad \hat{\imath} \quad \hat{\jmath} \quad \hat{\jmath} \quad \hat{k} \quad \hat{k} \quad 0$

$\hat{\imath} \quad \hat{\jmath} \qquad \hat{\jmath} \quad \hat{\imath} \quad \hat{k}$

$\hat{\jmath} \quad \hat{k} \qquad \hat{k} \quad \hat{\jmath} \quad \hat{\imath}$

$\hat{k} \quad \hat{\imath} \qquad \hat{\imath} \quad \hat{k} \quad \hat{\jmath}$

$$\mathbf{A} \quad \mathbf{B} \quad \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \quad \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix}\hat{\imath} \quad \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix}\hat{\jmath} \quad \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}\hat{k}$$

$$\mathbf{A} \quad \mathbf{B} \quad A_yB_z \quad A_zB_y \ \hat{\imath} \quad A_zB_x \quad A_xB_z \ \hat{\jmath} \quad A_xB_y \quad A_yB_x \ \hat{k}$$

$\mathbf{C} = \mathbf{A} \quad \mathbf{B}$ then its magnitude is $C \quad AB\sin$

Moments of Inertia of Homogeneous Rigid Objects:

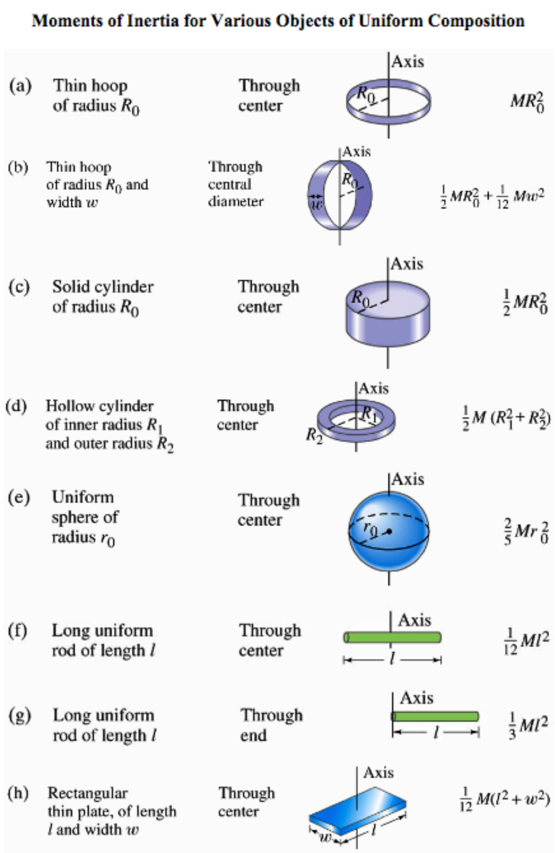
Thin Hoop: $I_{CM} \quad MR^2$ Solid Cylinder: $I_{CM} \quad \frac{1}{2}MR^2$ Hollow Cylinder: $I_{CM} \quad \frac{1}{2}M(R_1^2 \quad R_2^2)$

Solid Sphere: $I_{CM} \quad \frac{2}{5}MR^2$ Thin Spherical Shell: $I_{CM} \quad \frac{2}{3}MR^2$

Thin Rod Center Rotation: $I_{CM} \quad \frac{1}{12}ML^2$ Thin Rod End Rotation: $I_{CM} \quad \frac{1}{3}ML^2$

Rectangular Plate: $I_{CM} \quad \frac{1}{12}M(a^2 \quad b^2)$ Parallel Axis Theorem: $I \quad I_{CM} \quad MD^2$

Formula Sheet, Useful Data, and Moments of Inertia (Cont.)



Moments of Inertia for Various Objects of Uniform Composition

	Object	Location of axis	Moment of inertia
(a)	Thin hoop of radius R_0	Through center	MR_0^2
(b)	Thin hoop of radius R_0 and width w	Through central diameter	$\frac{1}{2}MR_0^2 + \frac{1}{12}Mw^2$
(c)	Solid cylinder of radius R_0	Through center	$\frac{1}{2}MR_0^2$
(d)	Hollow cylinder of inner radius R_1 and outer radius R_2	Through center	$\frac{1}{2}M(R_1^2 + R_2^2)$
(e)	Uniform sphere of radius r_0	Through center	$\frac{2}{5}Mr_0^2$
(f)	Long uniform rod of length l	Through center	$\frac{1}{12}Ml^2$
(g)	Long uniform rod of length l	Through end	$\frac{1}{3}Ml^2$
(h)	Rectangular thin plate, of length l and width w	Through center	$\frac{1}{12}M(l^2 + w^2)$

Useful Numbers:

$G = 6.67 \times 10^{-11}$ Nm^2/kg^2
$g = 9.8$ m/s^2
$R_{Earth} = 6.38 \times 10^3$ km
$M_{Earth} = 5.97 \times 10^{24}$ kg
$R_{Moon} = 1.74 \times 10^3$ km
$M_{Moon} = 7.35 \times 10^{22}$ kg
Earth-Moon Distance = 384×10^3 km

Problem 1 (Multiple Choice)

B
______

Two objects, a spherical ball with mass M and rectangular block with mass $3M$, start from rest on top of a hill. The spherical ball rolls down the hill without slipping. The rectangular block slides down the hill without friction. Which has the higher linear velocity of its center-of-mass at the bottom of the hill?

A) The rolling ball
B) The rectangular block
C) Both have the same velocity
D) Not enough information

$$E_i = E_f$$

$$E_i = mgh \Rightarrow E_f = mgh$$

Rolling:

$$E_f = KE_{CM} + KE_{Rolling}$$

Sliding:

$$E_f = KE_{CM}$$

Problem 1 (Multiple Choice) (Cont.)

A

The moons of Mars are called Phobos and Deimos and have orbital radii of 9,378 km and 23,459 km respectively. What is the ratio of the period of revolution of Phobos to that of Deimos?

A) 0.253
B) 0.400
C) 1.58
D) 2.53
E) 4.00
F) 0.158

$$\mathrm{T} = \frac{2\pi r}{v}$$

$$F_G = \frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{GM}{r}}$$

$$\mathrm{T} = 2\pi\sqrt{\frac{r^3}{GM}} \propto r^{3/2}$$

$$\frac{\mathrm{T}_1}{\mathrm{T}_2} = \left(\frac{r_1}{r_2}\right)^{3/2}$$

$$\frac{\mathrm{T}_{Phobos}}{\mathrm{T}_{Deimos}} = \left(\frac{r_{Phobos}}{r_{Deimos}}\right)^{3/2} = \left(\frac{9,378\ \mathrm{km}}{23,497\ \mathrm{km}}\right)^{3/2} \approx 0.253$$

Problem 1 (Multiple Choice) (Cont.)

A

Compare a ball bouncing elastically off of a wall to a piece of putty of the same mass and initial velocity hitting the wall and sticking to it. In which case is the *impulse* to the wall the largest?

A) The ball hitting the wall
B) The putty hitting the wall
C) They give the same impulse to the wall
D) Not enough information is given

$$I \equiv \int F \cdot dt = \Delta p$$

Bounce	Stick
$\Delta p = 2p_i$	$\Delta p = p_i$

Problem 1 (Multiple Choice) (Cont.)

C A projectile is launched from Earth's surface straight up with an initial speed equal to the *escape velocity* from Earth, v_{esc}. At an altitude of twice the radius of the Earth ($2R_E$), neglecting air resistance, what is the projectile's speed?

A) $2v_{esc}$
B) $\sqrt{2}v_{esc}$
C) $v_{esc}/\sqrt{2}$
D) $v_{esc}/4$
E) $v_{esc}/2$
F) v_{esc}

$$E_i = E_f$$

$$E_i = \frac{mv_{esc}^2}{2} - \frac{GMm}{r_E}$$

$$E_f = \frac{mv^2}{2} - \frac{GMm}{2r_E}$$

$$\frac{mv_{esc}^2}{2} - \frac{GMm}{r_E} = \frac{mv^2}{2} - \frac{GMm}{2r_E}$$

$$v^2 = v_{esc}^2 + \frac{GM}{r_E}(1-2) = v_{esc}^2 - \frac{GM}{r_E}$$

$$v_{esc}^2 = \frac{2GM}{r_E}$$

$$v^2 = v_{esc}^2 - \frac{1}{2}v_{esc}^2$$

$$v = v_{esc}/\sqrt{2}$$

Problem 1 (Multiple Choice) (Cont.)

D

A figure skater stands on one spot of the ice and spins with her arms extended. As she pulls her arms in close to her body, her angular velocity changes. Compared to having her arms extended outwards, with arms pulled in, her new angular momentum:

A) Equals zero
B) Increases
C) Decreases
D) Remains unchanged
E) Not enough information

$$\sum \vec{\tau} = 0 \Rightarrow \vec{L} \text{ is conserved!}$$

Problem 1 (Multiple Choice) (Cont.)

B
I build a children's slide to be *3 m* high. I can build the slide so that it is either straight or has curves and bumps. Neglecting friction, how does the shape of the slide affect the time that it takes to slide down it and the final speed at the bottom?

A) the time required to go down the slide and the final speed are both independent of the shape
B) the time required to go down the slide depends on the shape, but the final speed does not
C) the time required to go down the slide and the final speed both depend on the shape
D) the final speed required to go down the slide depends on the shape, but the time does not
E) Not enough information

$$E_f = E_i$$

$$mgy = \frac{mv^2}{2} \Rightarrow v \text{ is unchanged by the shape!}$$

But, the time it takes to slide down IS affected by the shape!

Problem 1 (Multiple Choice) (Cont.)

D

A spring-loaded dart gun shoots a dart straight up into the air and the dart reaches a maximum height of *24 m*. The same dart is shot up a second time with the spring compressed half as far. How far does the dart go up this time?

A) 96 m
B) 24 m
C) 12 m
D) 6 m
E) Not enough information

$$E_i = E_f$$

$$E_i = \frac{kx^2}{2}$$

$$E_f = mgy$$

$$y = \frac{kx^2}{2mg}$$

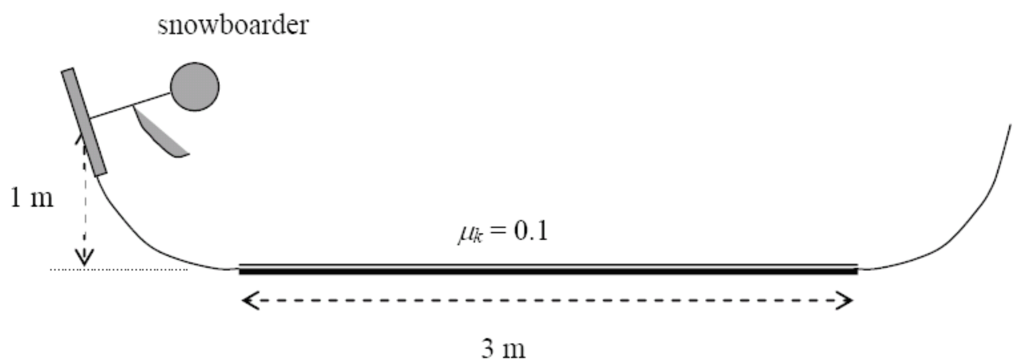
$$x \rightarrow \frac{1}{2}x$$

$$y \rightarrow \frac{1}{4}y$$

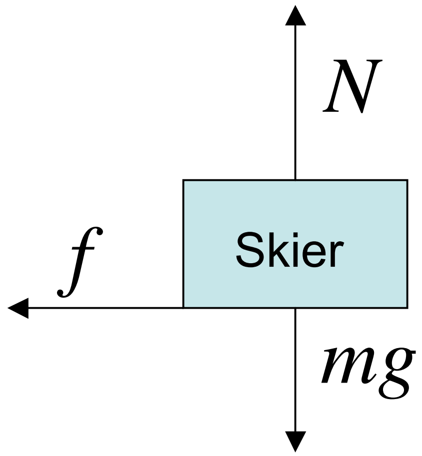
$$y = \frac{24\ \text{m}}{4} = 6\ \text{m}$$

Problem 2

2. A *40 kg* snowboarder is at the top of an icy *1 m* high curved ramp as shown below. The ramp has no friction. At the bottom of the ramp is a flat region of snow with a coefficient of kinetic friction, $\mu_k = 0.1$. Three meters away from the bottom of the first ramp is a second icy ramp that the snowboarder can go up. This ramp also has no friction. The snowboarder goes back and forth until eventually coming to a stop. Find the amount of work done by friction each time the snowboarder goes completely across the snow to the opposite ramp? How many times does the snowboarder cross the flat region? Where does the snowboarder finally stop?

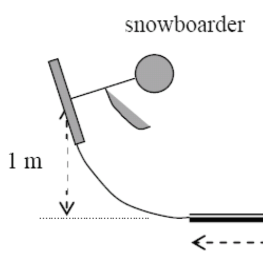


The Math



$$N - mg = 0$$
$$M = mg$$
$$f = \mu N = \mu mg$$

$$W = \vec{F} \cdot \vec{d} = \mu mg \Delta x$$
$$W = (0.1)(40\ \text{kg})(9.8\ \text{m/s}^2)(3\ \text{m})$$
$$W = 117.6\ \text{J}$$



$$E_i = E_f$$
$$E_i = mgy$$
$$E_i = (40\ \text{kg})(9.8\ \text{m/s}^2)(1\ \text{m})$$
$$E_i = 392\ \text{J}$$

Let n be the number of trips taken by the skier. Then:

$$E_i = nW$$
$$n = \frac{E_i}{W} = 3.33$$

Thus the skier makes 3 *complete* trips before stopping.

The Math (Cont.)

After the 3rd trip, the skier has:

$$E = E_i - 3W = 39.2 \text{ J}$$

$$\sum W = \Delta KE$$

$$39.2 \text{ J} = \mu m g \Delta x'$$

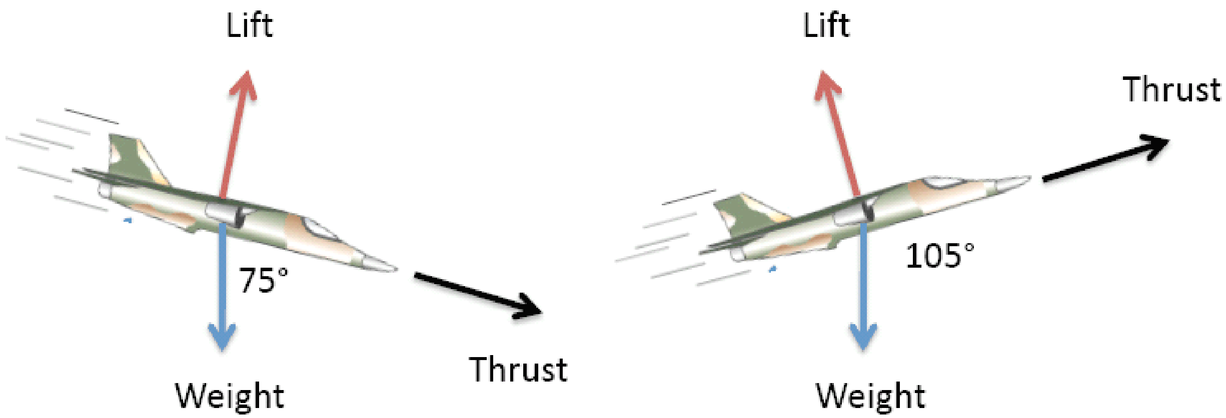
$$\Delta x' = \frac{39.2 \text{ J}}{(0.1)(40 \text{ kg})(9.8 \text{ m/s}^2)}$$

$$\Delta x' = 1 \text{ m}$$

Thus, the skier stops 1 m from the right edge of the snowy surface.

Problem 3

3. A plane is diving toward the ground and then climbs back upwards as shown below. A **lift** force acts perpendicular to the plane's displacement, $s = 1.7 \times 10^{3}$ *m*, which should be taken to be the same for each case. The engines of the plane exert a **thrust** as shown which has the same magnitude during the dive and the climb. The plane's **weight** is 5.9×10^{4} *N*. Is the *total net work* done by all the forces *greater* during the dive or the climb? <u>Explain your answer</u>. Find the <u>difference</u> in the *total net work* done between the dive and the climb.



The Math

$$W = \sum \vec{F} \cdot \vec{d} = \sum F d \cos\vartheta$$

The Dive

$$W = \vec{F}_{lift} \cdot \vec{s} + \vec{F}_{weight} \cdot \vec{s} + \vec{F}_{thrust} \cdot \vec{s}$$

$$W = 0 + \mathrm{F}_{\mathrm{weight}}\mathrm{s}\cos(75^{\circ}) + \mathrm{F}_{\mathrm{thrust}}\mathrm{s}$$

$$W = (5.9 \times 10^{4}\ \mathrm{N})(1.7 \times 10^{3}\ \mathrm{m})\cos(75^{\circ}) + \mathrm{F}_{\mathrm{thrust}}\mathrm{s}$$

$$W = 2.6 \times 10^{7}\ \mathrm{J} + \mathrm{F}_{\mathrm{thrust}}\mathrm{s}$$

The Climb

$$W = \vec{F}_{lift} \cdot \vec{s} + \vec{F}_{weight} \cdot \vec{s} + \vec{F}_{thrust} \cdot \vec{s}$$

$$W = 0 + \mathrm{F}_{\mathrm{weight}}\mathrm{s}\cos(105^{\circ}) + \mathrm{F}_{\mathrm{thrust}}\mathrm{s}$$

$$W = (5.9 \times 10^{4}\ \mathrm{N})(1.7 \times 10^{3}\ \mathrm{m})\cos(105^{\circ}) + \mathrm{F}_{\mathrm{thrust}}\mathrm{s}$$

$$W = -2.6 \times 10^{7}\ \mathrm{J} + \mathrm{F}_{\mathrm{thrust}}\mathrm{s}$$

More work is done during the fall than the climb. The difference in the work is 5.2 x 10^{7} J (the thrust cancels!)

Problem 4

4. A *75.0 kg* stunt man horizontally out of a circus canon with a velocity of 10 m/s. He collides with a pile of mattresses which are compressed *1.00 m* before he is brought to rest. Find the *average force*, $\overline{F}$, exerted by the mattresses on the stuntman?

$$\sum W = \Delta KE$$

$$\sum W = \langle F \rangle \Delta x$$

$$\Delta KE = \frac{mv^2}{2}$$

$$\langle F \rangle = \frac{mv^2}{2\Delta x} = \frac{(75\text{ kg})(10\text{ m/s})^2}{2(1.0\text{ m})}$$

$$\langle F \rangle = 3{,}750\text{ N}$$

Problem 5

5. A billiard ball moving at *5.00 m/s* collides elastically with a stationary ball of the same mass. After the collision the first ball moves at *4.33 m/s at an angle of 30.0 degrees* with respect to the original line of motion. Ignoring friction and any possible rotational motion, find the struck ball's velocity (direction and magnitude) after the collision.

$$E_i = \frac{mv^2}{2} \qquad p_{x,i} = mv \qquad p_{y,i} = 0$$

$$=$$

$$E_f = \frac{mv_1^2 + mv_2^2}{2}$$

$$p_{x,f} = mv_{1,f}\cos\vartheta + mv_{2,f}\cos\varphi$$

$$p_{y,f} = mv_{1,f}\sin\vartheta - mv_{2,f}\sin\varphi$$

$$\Longrightarrow$$

$$v_{1,f}\sin\vartheta = v_{2,f}\sin\varphi$$

$$v^2 = v_1^2 + v_2^2$$

$$v = v_{1,f}\cos\vartheta + v_{2,f}\cos\varphi$$

$$2.17\ \text{m/s} = v_{2,f}\sin\varphi$$

$$25\ \text{m}^2/\text{s}^2 = 18.75\ \text{m}^2/\text{s}^2 + v_{2,f}^2$$

$$v_{2,f} = \sqrt{25\ \text{m}^2/\text{s}^2 - 18.75\ \text{m}^2/\text{s}^2}$$

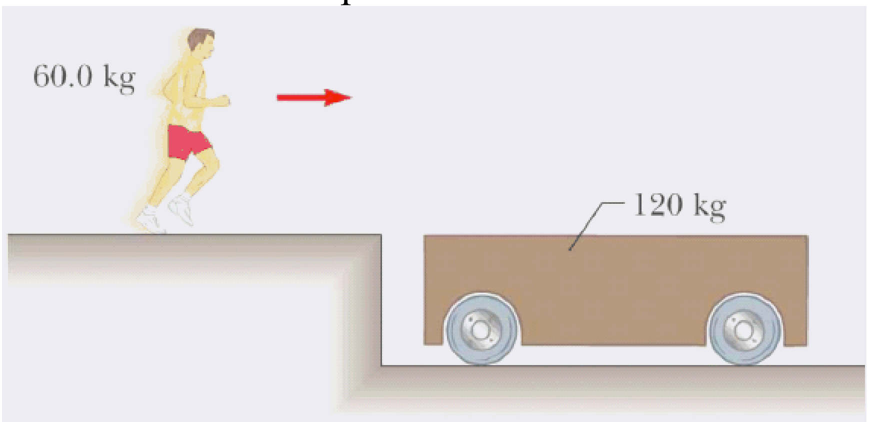
$$v_{2,f} = 2.5\ \text{m/s}$$

$$\sin\varphi = \frac{2.17\ \text{m/s}}{2.5\ \text{m/s}}$$

$$\varphi = 60^\circ$$

Problem 6

6. A *60.0 kg* person running at an initial speed of *3.25 m/s* jumps onto a *120 kg* cart that is initially at rest as shown below. The person slides on the cart's top surface and finally comes to rest relative to the cart. The coefficient of kinetic friction between the person and the cart is *0.375*. Friction between the cart and the ground can be neglected. (a) Find the *final speed* of the person and cart relative to the ground. (b) Find the friction force acting on the person as he is sliding across the top surface of the cart. (c) Find how long the friction force acts on the person.



The Math

All forces are *internal*, thus:

$$p_f = p_i$$

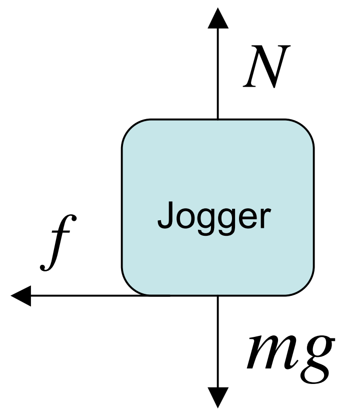
$$p_i = mv_i$$

$$p_f = (m+M)v$$

$$v = \frac{m}{m+M} v_i$$

$$v = \frac{60 \text{ kg}}{60 \text{ kg} + 120 \text{ kg}} 3.25 \text{ m/s}$$

$$\text{v} = 1.08 \text{ m/s}$$



$$N - mg = 0$$

$$N = mg$$

$$f = \mu N = \mu mg$$

$$f = (0.375)(60 \text{ kg})(9.8 \text{ m/s}^2)$$

$$f = 221 \text{ N}$$

$$\Delta p_{jogger} = F_{jogger} \Delta t$$

$$\Delta t = \frac{\Delta p_{jogger}}{F_{jogger}}$$

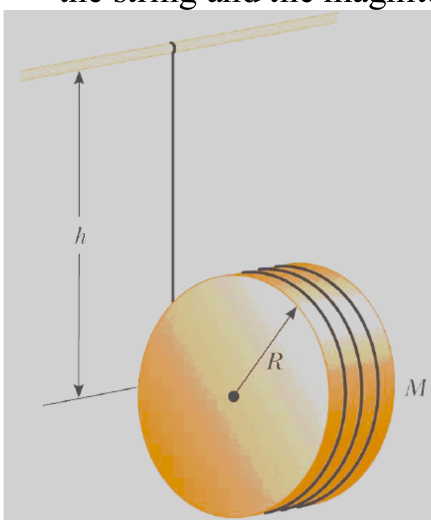
$$\Delta t = \frac{mv_i - mv}{\mu mg} = \frac{v_i - v}{\mu g}$$

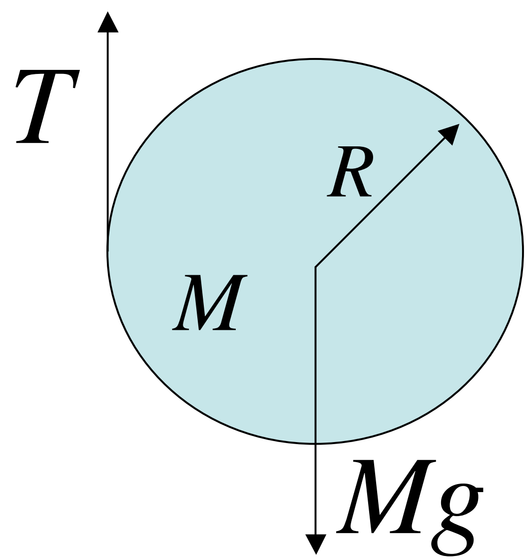
$$\Delta t = \frac{3.0 \text{ m/s} - 1.08 \text{ m/s}}{(0.375)(9.8 \text{ m/s}^2)}$$

$$\Delta t = 0.33 \text{ sec.}$$

Problem 7

7. A string is wound around a uniform disk of radius R and mass $M = 2\ kg$. The disk is released from rest with the string vertical and its top end tied to a fixed bar as shown below. Find the *tension* in the string and the magnitude of the *acceleration of the center-of-mass* of the disk.





The Math

$$\sum \vec{\tau} = I\alpha = RT$$

$$\alpha = \frac{a_{cm}}{R}$$

$$\sum F_y = ma_{cm} = mg - T$$

$$I = \frac{mR^2}{2}$$

$$T = \frac{ma_{cm}}{2}$$

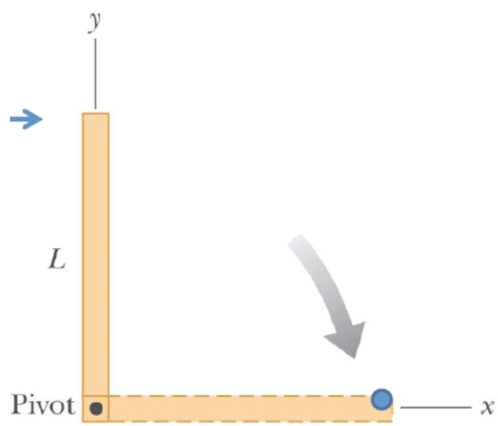
$$ma_{cm} = mg - \frac{ma_{cm}}{2}$$

$$a_{cm} = \frac{2}{3}g$$

$$T = \frac{m\frac{2}{3}g}{2} = \frac{mg}{3}$$

Problem 8

8. A $1\ kg$ point-like mass moves to the right with velocity, $v = 2.0\ m/s$, and strikes a long thin vertical rod of length $L = 1\ m$ and mass $M = 3\ kg$ pivoted on a frictionless pin at one end as shown. The point-like mass sticks to the rod and the combination rotates clockwise. Find the *angular velocity* of the combination just after impact of the small mass. (For 5 bonus points, find the *angular velocity* of the combination after it has rotated 90°as shown.)



The Math

$$L_i = L_f$$
$$L_i = \ell m v$$
$$L_f = I\omega$$

$$I = m\ell^2 + \frac{M\ell^2}{3} = \left(m + \frac{M}{3}\right)\ell^2$$

$$\ell m v = \left(m + \frac{M}{3}\right)\ell^2\omega$$

$$\omega = \frac{mv}{\left(m + \frac{M}{3}\right)\ell}$$

Bonus:

$$E_i = \frac{(m + M/3)\ell^2\omega^2}{2} + Mg\frac{\ell}{2} + mg\ell$$

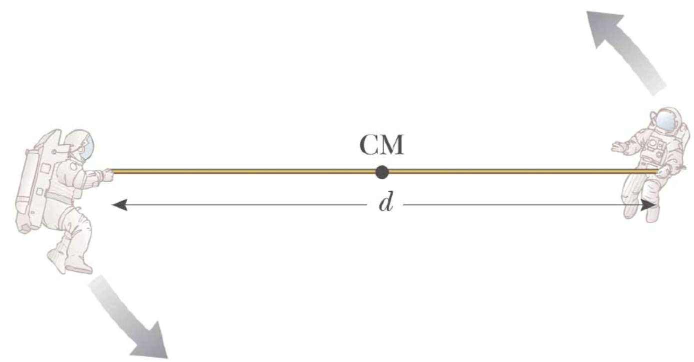
$$E_f = \frac{(m + M/3)\ell^2\omega'^2}{2}$$

$$\omega' = \sqrt{\frac{2}{(m + M/3)\ell^2}\left(\frac{(m + M/3)\ell^2\omega^2}{2} + Mg\frac{\ell}{2} + mg\ell\right)}$$

$$\omega' = \sqrt{\omega^2 + \frac{(2m + M)}{(m + M/3)}\frac{g}{\ell}} > \omega$$

Problem 9

9. Two astronauts, each having a mass of *75.0 kg*, are connected by a *10.0 m* rope of negligible mass. They are isolated in space, orbiting their center-of-mass with tangential speeds of *5.0 m/s*. Treating the astronauts as particles, what are the initial *magnitude and direction of the angular momentum* of the system? By pulling on the rope, one of the astronauts shortens the distance between them to *5.0 m*. What are the astronauts' new tangential speeds?



$$L_i = I\omega = (2m\left(\frac{d}{2}\right)^2)\frac{v}{d/2} = dmv$$

$$L_i = (10 \text{ m})(75 \text{ kg})(5 \text{ m/s}) = 3{,}7500 \text{ J}\cdot\text{s}$$

The RHR tells us $\vec{L}$ points out of the page.

$$L_i = L_f$$

$$L_f = d'mv'$$

$$v' = \frac{L_i}{d'm}$$

$$v' = \frac{3{,}500 \text{ J}\cdot\text{s}}{(5 \text{ m})(75 \text{ kg})} = 10 \text{ m/s}$$

Problem 10

10. Consider the following table of planetary data. Using these data, find the *distance from the center*, *R*, of the planet for a satellite in a synchronous orbit above the equator of Mars. Note that the Earth's mass is $M_{Earth} = 5.97 \times 10^{24}\ kg$.

Object	Mean Distance from Sun (millions of km)	Period of Revolution	Period of Rotation	Eccentricity of Orbit	Equatorial Diameter (km)	Mass (Earth = 1)
Sun	—	—	27 days	—	1,392,000	333,000.00
Mercury	57.9	88 days	59 days	0.206	4,880	0.553
Venus	108.2	224.7 days	243 days	0.007	12,104	0.815
Earth	149.6	365.26 days	23 hr 56 min 4 sec	0.017	12,756	1.00
Mars	227.9	687 days	24 hr 37 min 23 sec	0.093	6,787	0.1074
Jupiter	778.3	11.86 years	9 hr 50 min 30 sec	0.048	142,800	317.896
Saturn	1,427	29.46 years	10 hr 14 min	0.056	120,000	95.185
Uranus	2,869	84.0 years	17 hr 14 min	0.047	51,800	14.537
Neptune	4,496	164.8 years	16 hr	0.009	49,500	17.151
Pluto	5,900	247.7 years	6 days 9 hr	0.250	2,300	0.0025
Earth's Moon	149.6 (0.386 from Earth)	27.3 days	27 days 8 hr	0.055	3,476	0.0123

The Math

To achieve a geosynchronous orbit, the satellite must make one revolution about Mars in the same amount of time it take Mars to rotate once about its own axis.

$$\mathrm{T} = \frac{2\pi r}{v}$$

$$F = \frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$v = \sqrt{GM/r}$$

$$\mathrm{T} = 2\pi\sqrt{\frac{r^3}{GM}}$$

$$r = \left(GM\left(\frac{\mathrm{T}}{2\pi}\right)^2\right)^{1/3}$$

$$M = M_{Mars}$$

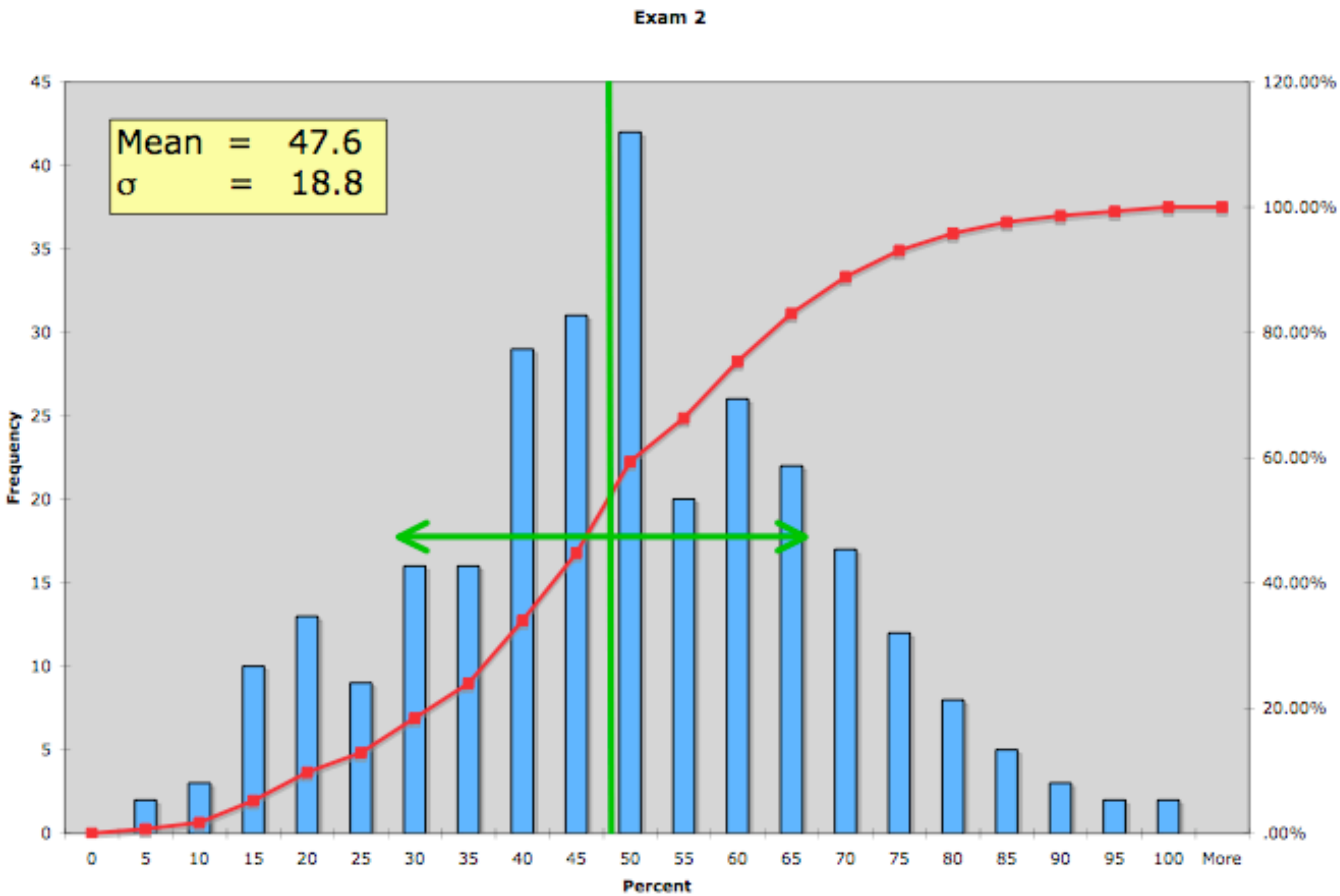
$$\mathrm{T} = \text{Period of Mars Rotation}$$

$$r = \left((6.67\times10^{-11}\ \mathrm{N\cdot m^2\cdot kg^{-2}})(0.1074)(5.97\times10^{24}\ \mathrm{kg})\left(\frac{24\ \text{hr.}\ (3600\ \text{sec./1 hr.}) + 37\ \text{min.}\ (60\ \text{secd/1 min.}) + 23\ \text{sec.}}{2\pi}\right)^2\right)^{1/3}$$

$$r \approx 20{,}417\ \text{km}$$

Fin!

Statistics


Exam 2
Mean = 47.6
σ = 18.8
Frequency
Percent
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100
More
120.00%
100.00%
80.00%
60.00%
40.00%
20.00%
.00%

Statistics (Cont.)