A Heuristic Derivation of the Ideal Gas Law

C. A. Serino

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Pressure, P, is defined as:

$$P = \frac{F}{A} \tag{1}$$

where F is the force exerted on the material and A is the cross-sectional area of that material. Thus the average pressure $\bar{P}$ is given by:

$$\bar{P} = \frac{\bar{F}}{A} \tag{2}$$

The average force is given by:

$$\bar{F} = \frac{1}{\Delta t}\int_{t}^{t+\Delta t} F(t')dt' = \frac{I}{\Delta t} \tag{3}$$

where I is the impulse. From the impulse-momentum theorem we know:

$$I = \Delta p \tag{4}$$

and so:

$$\bar{F} = \frac{mv^2}{\ell} \tag{5}$$

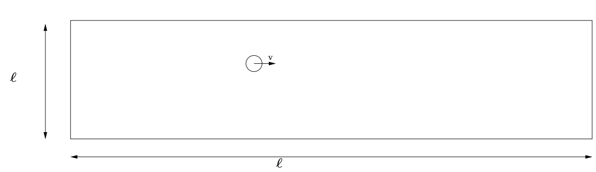

Figure 1: A particle of mass m and velocity v, traveling in a "box" of length ℓ.

where we used:

$$\Delta p = 2mv \tag{6}$$

From dimensional analysis, or, if you like, the kinetic theory of gasses we know:

$$v^2 = k_B T \tag{7}$$

where T is the temperature and k_B is Boltzmann's constant. Putting it all together we have:

$$\begin{aligned} PV &= F\ell && (8)\\ &= mv^2 && (9)\\ &= k_B T && (10) \end{aligned}$$

Finally, if we have N particles rather than just the one, and if we write N in terms of Avogadro's Number and the number of moles (n) we find:

$$PV = nRT \tag{11}$$

where R is the so-called universal gas constant.