A Vibrating Molecule

A good approximation to the potential energy between two atoms is given by the so-called Leonard Jones potential. In dimensionless units, the potential is given by

$$U(r) = A\left(\left(\frac{\sigma}{r}\right)^6 - \left(\frac{\sigma}{r}\right)^{12}\right), \tag{1}$$

where A is the strength of the interaction, σ is a sort-of length scale for the interaction, and r is the distance between atoms. The potential is a familiar one from chemistry as shown in Figure 1. What is the separation distance

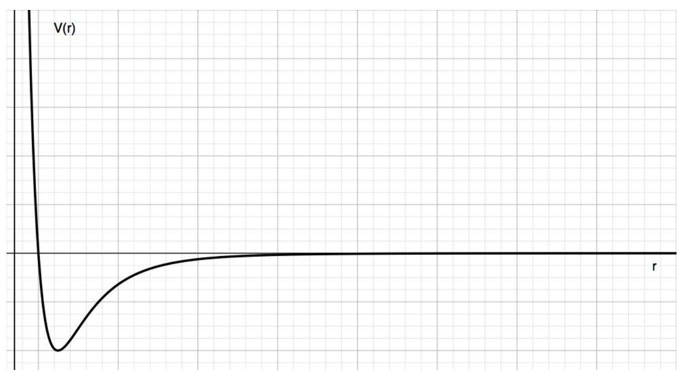

Figure 1: The Leonard Jones Potential

that minimizes the energy of the system (this is where the system is most likely to be found)? What is the frequency of oscillation about this minimum?

Faster Route to China?

Suppose man drilled a hole through the Earth from the US to China as a method of transcontinental travel. Follow the steps below to show that the motion on a trip through a hole drilled through the earth is simple harmonic motion. Find the period for this trip and determine how fast you would need to travel to circumnavigate the earth in the same amount of time. Figure 2 should prove useful.

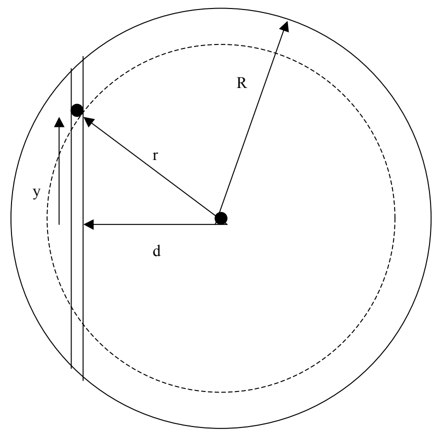

Figure 2: Diagram with Relevant Variables

Note that it can be shown (c.f. Gauss) that a particle at location r experiences a gravitational force due *only to the mass within the circle of radius* r.

i. Using the universal gravitation law, write down the force felt by the object at point r assuming the mass within the sphere or radius r is given by some function $M(r)$ (which we find later).

ii. Since the motion is periodic in y, write r in terms of y.

iii. Since the object is confined to the tunnel, we only want the y component of force. Using trig, find the y-component of this force.

iv. We want to learn $M(y)$. Assuming the density of Earth is constant and equal to ρ, write down $M(y)$ (remember $\rho = m/V$ where V is volume...another function of r).

v. Replace ρ with M_{Earth}/V_{Earth} and write V_{Earth} as a function of the radius of Earth, R.

vi. Put it all together and find the y-component of force as a function of y.

vii. Write $F = ma = m\ddot{y}$. Does this look familiar?

viii. Deduce ω.

ix. From ω determine the period, T, and determine how long the round trip takes.

x. IF $v = s/T$ and s is the distance *around* the world, determine v as a function of d. Pick some reasonable values of d and see what you get!

A Glancing Blow

Suppose NASA puts a satellite in orbit $3R$, where R is the radius of the Earth, above the surface of the earth. Suppose further, it is a spy satellite that Generic Rogue Terrorist Group wants to knock out of orbit. Due to miscalculations, only a glancing blow is received and the deviation from the original orbit, δ is small compared to R. Show that for $\delta/R \ll 1$, the motion about the original orbit is simple harmonic motion. Determine the period of this motion. Compare it with the period of rotation about the earth if the orbit was / is very nearly circular. Remember the universal gravitation law:

$$F(r) = \frac{GMm}{r^2} = ma = m\ddot{r}. \tag{2}$$

Peter Griffin Annexes Joe's Pool

In *E Peterus Unum*, an episode of *Family Guy*, Peter learns his property is not part of the US and takes the opportunity to declare his own sovereign state, *Petoria*. Wanting a pool, he decides to invade his next-door neighbor, Joe, and seize his pool. This aggravates the USA who responds with the full might of its military. While floating in the pool, (Peter floats as fat is less dense than water) the US plans a military strategy. First, assuming Peter is 300 lbs and about 6 ft $\times$ 2 ft $\times$ 3 ft, calculate his density. Determine what percent of Peter is below the water and what percent of him is above. Suppose, the military fires a warning shot over Peter's belly and that this causes a small piece of debris of 20 lbs to land on him. Find his new equilibrium position (model Peter as a solid rectangular box for sake of ease). Annoyed by its presence, Peter flicks the debris at Meg and off of himself. Show Peter under goes approximate simple harmonic motion as $20/300 \ll 1$. Determine the period of Peter's bobbing.