PY 211 Mid Term Exam I

February 13, 2007

The Formulae Sheet

Some Useful Formulae for Midterm #1

Gravitational Acceleration at the Earth's Surface: $g \quad 9.8\frac{m}{s}$

One Dimensional Motion:

Average Velocity: $\bar{v}_x \quad \frac{x}{t}$ Average Acceleration: $\bar{a}_x \quad \frac{v_x}{t} \quad \frac{v_{xf} \quad v_{xi}}{t}$

Instantaneous Velocity: $v_x \quad \lim_{t \quad 0}\frac{x}{t} \quad \frac{dx}{dt}$

Instantaneous Acceleration: $a_x \quad \lim_{t \quad 0}\frac{v_x}{t} \quad \frac{dv_x}{dt} \quad \frac{d^2x}{dt^2}$

Motion under constant acceleration in one dimension:

$v(t) \quad v_0 + at$ $\qquad v(t) \quad v_0 \quad gt$

$x(t) \quad x_0 \quad v_0 t \quad \frac{1}{2}at^2$ Free Fall: $x(t) \quad x_0 \quad v_0 t \quad \frac{1}{2}gt^2$

$v^2 \quad v_0^2 \quad 2a(x \quad x_0)$ $\qquad v^2 \quad v_0^2 \quad 2g(x \quad x_0)$

Two Dimensional Motion:

$r \quad x\hat{i} \quad y\hat{j}$ $\qquad v \quad \lim_{t \quad 0}\frac{r}{t} \quad \frac{dr}{dt}$ $\qquad a \quad \lim_{t \quad 0}\frac{v}{t} \quad \frac{dv}{dt}$

Motion under constant acceleration in two dimensions:

r_f [redacted] $\frac{1}{2}at^2$ $\qquad x_f \quad x_i \quad v_{xi}t \quad \frac{1}{2}a_x t^2$, $y_f \quad y_i \quad v_{yi}t \quad \frac{1}{2}a_y t^2$ $\qquad v_f \quad v_i \quad at$ $\qquad v_{xf} \quad v_{xi} \quad a_x t$, $v_{yf} \quad v_{yi} \quad a_y t$

Projectile Motion:

$r(t)$ [redacted] $\frac{1}{2}gt^2$ Range: $R \quad \frac{v_i^2}{g}\sin 2$ Maximum Height: $y_{max} \quad \frac{(v_i \sin \quad)^2}{2g}$

$v(t) \quad v_0 \quad gt$

Uniform Circular Motion:

Centripetal Acceleration: $a_c \quad \frac{v^2}{r}$ Period: $T \quad \frac{2 \quad r}{v}$ Frequency: $f \quad \frac{1}{T}$

Newton's Laws:

1st Law: If F 0, then v 0 and a 0.

2nd Law: $\sum_{i=1}^{n} F_i \quad ma$ Circular Motion: $\sum_{i=1}^{n} F_i \quad ma_c \quad m\frac{v^2}{r}\hat{r}$

3rd Law: $F_{12} = - F_{21}$

Forces: Gravity: $F = -mg\hat{j}$

Friction: Static: $f_s \quad {}_sN$ Kinetic: $f_k \quad {}_kN$

Resistive: Slow: $R \quad bv$ Fast: $|R| \quad \frac{1}{2}D\rho A v^2$

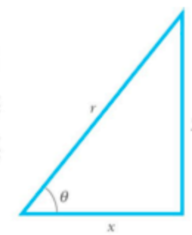
Right Triangle Relations:

$r \quad \sqrt{x^2 \quad y^2}$

$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x}$

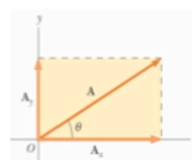


Quadratic Formula:

$ax^2 \quad bx \quad c \quad 0$

$x \quad \frac{b \quad \sqrt{b^2 \quad 4ac}}{2a}$

Vectors:



$A \quad A_x\hat{i} \quad A_y\hat{j}$

$A_x \quad A\cos$

$A_y \quad A\sin$

$A \quad \sqrt{A_x^2 \quad A_y^2}$

$\tan^{-1}\frac{A_y}{A_x}$

$R \quad A_x\hat{i} \quad A_y\hat{j} \quad A_z\hat{k} \quad B_x\hat{i} \quad B_y\hat{j} \quad B_z\hat{k}$

$R \quad A_x \quad B_x \quad \hat{i} \quad A_y \quad B_y \quad \hat{j} \quad A_z \quad B_z \quad \hat{k}$

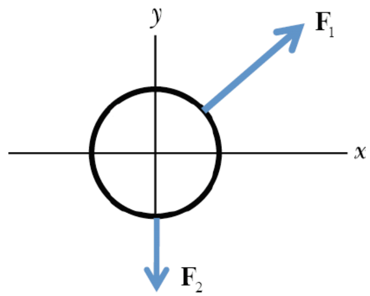
$R \quad R_x\hat{i} \quad R_y\hat{j} \quad R_z\hat{k}$

Problem 1

Three forces are applied to a circular ring in such a way that the ring remains in equilibrium. If the forces, in Newtons, are applied to the ring as shown below, find the magnitude and direction of $\mathbf{F}_3$ so that the ring remains in equilibrium.

$$\vec{F}_1 = 20\hat{i} + 10\hat{j}$$

$$\vec{F}_2 = -10\hat{j}$$



The Math

$$\sum \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$

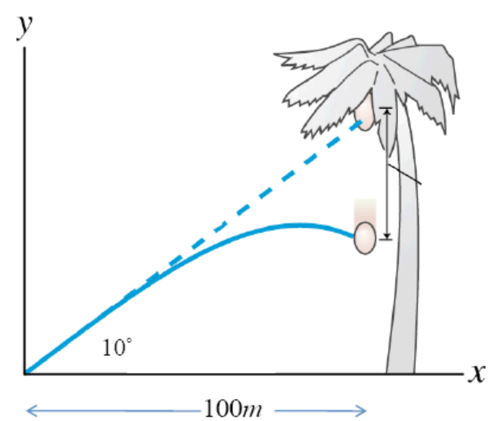
$$\sum \vec{F} = 20\hat{i} + 0\hat{j} + \vec{F}_3 = 0$$

$$\boxed{\vec{F}_3 = -20\hat{i}}$$

Questions?

Problem 2

2. A hunter aims his rifle at a 10 degree angle with respect to the horizontal at a coconut in the top of a palm tree. At the instant he pulls the trigger, a monkey drops the coconut which falls toward the ground. If the palm tree is 100 m from the hunter and the hunter's bullet hits the falling coconut 0.25 seconds later, what is the <u>*velocity*</u> *of the bullet as it leaves the barrel of the rifle*?



The Math

$$x_{coco}(t) = x_0 + v_{0,x}t + \frac{1}{2}a_x t^2$$

$$x_0 = x_{coco}$$

$$v_{0,x} = 0$$

$$a_x = 0$$

$$x_{coco}(t) = x_{coco}$$

$$y_{coco}(t) = y_0 + v_{0,y}t + \frac{1}{2}a_y t^2$$

$$y_0 = h$$

$$v_{0,y} = 0$$

$$a_y = -g$$

$$y_{coco}(t) = h - \frac{1}{2}gt^2$$

$$x_b(t) = x_0 + v_{0,x}t + \frac{1}{2}a_x t^2$$

$$x_0 = 0$$

$$v_{0,x} = v\cos\theta$$

$$a_x = 0$$

$$x_b(t) = v\cos(\theta)t$$

$$y_b(t) = y_0 + v_{0,y}t + \frac{1}{2}a_y t^2$$

$$y_0 = 0$$

$$v_{0,y} = v\sin\theta$$

$$a_y = -g$$

$$y_b(t) = v\sin(\theta)t - \frac{1}{2}gt^2$$

At t=0.25, both the bullet and the coconut are at the same place in space!

$$x_{coco} = x_b$$

$$100 = v\cos(\theta)t$$

$$v = \frac{100}{\cos(10°)(0.25)} = 407$$

$$\vec{v} = v\cos\theta\hat{i} + v\sin\theta\hat{j}$$

$$\vec{v} = 401\hat{i} + 71\hat{j}$$

Questions?

Problem 3

- A car makes a sharp turn to the left and it is observed that a mug of coffee initially at rest on the dashboard slide to the right until it is pressed against the frame of the car. Write a paragraph which explain how <u>each</u> of Newton's three laws of motion apply to this situation. You may uses sketches and equations if you wish, but your answer should be a paragraph.

Newton’s Laws

- Objects in motion stay in motion and at constant velocity, lest a force act upon them.
- The vector sum of the forces equals to the product of mass and acceleration.
- For every action there is an equal and opposite reaction.

Solution

An observer in an inertial frame notes that, from Newton I, the cup is moving in a straight line and will continue to do so unless acted on by a non-zero net force. When the car turns, there would need to be a force acting on the cup such that $F = ma$ (Newton II). The friction force is not sufficient to provide this, so the cup does not turn with the car, but slides out tangentially until it hits the frame. The cup then exerts a contact force against the frame equal (in magnitude) to the force exerted by the frame on the cup. This force then provides the centripetal acceleration such that the cup undergoes circular motion.

Problem 4

4. An object moves along the x-axis according to the equation, $x(t) = (3t^3 - 2t + 6)$ meters. Find the <u>average speed</u> of the object between $t = 2\ s$ and $t = 3\ s$. Find the <u>instantaneous speed</u> and <u>instantaneous acceleration</u> at $t = 2\ s$. (3 answers required)

$$\bar{v} = \frac{x(t_2) - x(t_1)}{t_2 - t_1}$$

$$x(3) = 3(3)^3 - 2(3) + 6 = 81$$

$$x(2) = 3(2)^3 - 2(2) + 6 = 26$$

$$\Delta x = 55$$

$$\Delta t = 1$$

$$\bar{v} = \frac{\Delta x}{\Delta t} = 55$$

$$v(t) = \frac{dx(t)}{dt} = 9t^2 - 2$$

$$v(2) = 36 - 2 = 34$$

$$a(t) = \frac{d^2x(t)}{dt^2} = \frac{dv(t)}{dt} = 18t$$

$$a(2) = 18(2) = 36$$

Questions?

Problem 5

5. A spaceship with its rocket engine turned off initially is traveling through space with a constant velocity of $100\ m/s$ relative to an observer on a nearby planet (fixed reference system). A special rocket engine on the spaceship is turned on at time $t = 0$, providing an increasing rate of acceleration, $a(t) = 10t\ m/s^3$, where t is the time since the engine was turned on. What is the velocity of the rocket 10 seconds after turning on the engine? How far did the rocket travel in those 10 seconds?

The Math

Caution! The acceleration is NOT constant so the kinematic equations will NOT work!

$$v(t) - v(0) = \int_{t=0}^{t} a(t')dt'$$

$$a(t) = 10t$$

$$v(t) = \int_{0}^{t} 10t'\,dt' = 5t^2 + 100$$

$$v(10) = 5(10)^2 + 100 = 600$$

$$x(t) - x(0) = \int_{t=0}^{t} v(t')dt'$$

$$v(t) = 5t^2 + 100$$

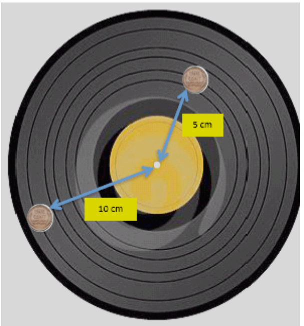
$$x(t) = \int_{0}^{t} 5t'^2\,dt' = \frac{5}{3}t^3 + 100t + x_0$$

$$x(10) - x_0 = \frac{5}{3}(10)^3 + 100(10) = \frac{5{,}000}{3} + 1{,}000 \approx 2{,}670$$

Questions?

Problem 6

6. Two pennies are resting on the turntable of a record player, with one of them *5.0 cm* from the center and the other *10.0 cm* from the center. The coefficient of static friction between a penny and the surface of the turntable is $\mu_s = 0.5$. If the turntable makes one complete revolution every *0.5 s*, what happens to each of the pennies? Support your answer with the appropriate calculation.



The Math

$$v = r\omega = 2\pi r f = \frac{2\pi r}{T}$$

$$F_c = \frac{mv^2}{r} = \frac{m(4\pi^2 r^2)}{rT^2} = \frac{4\pi^2 mr}{T^2}$$

$$F_f^{\max} = \mu F_N = \frac{1}{2}mg$$

$$\frac{1}{2}mg > ? < \frac{4\pi^2 mr}{T^2}$$

$$\frac{1}{2}mg = \frac{4\pi^2 mr}{T^2}$$

$$r = \frac{gT^2}{8\pi^2}$$

$$r = \frac{(9.8)(0.5)^2}{8\pi^2} \approx 0.031m$$

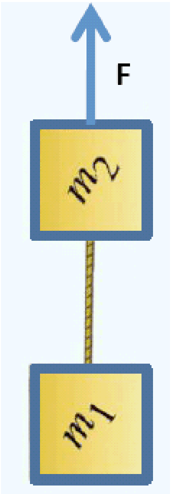
$$r \approx 3.1cm$$

So, they both slide off the turn table!

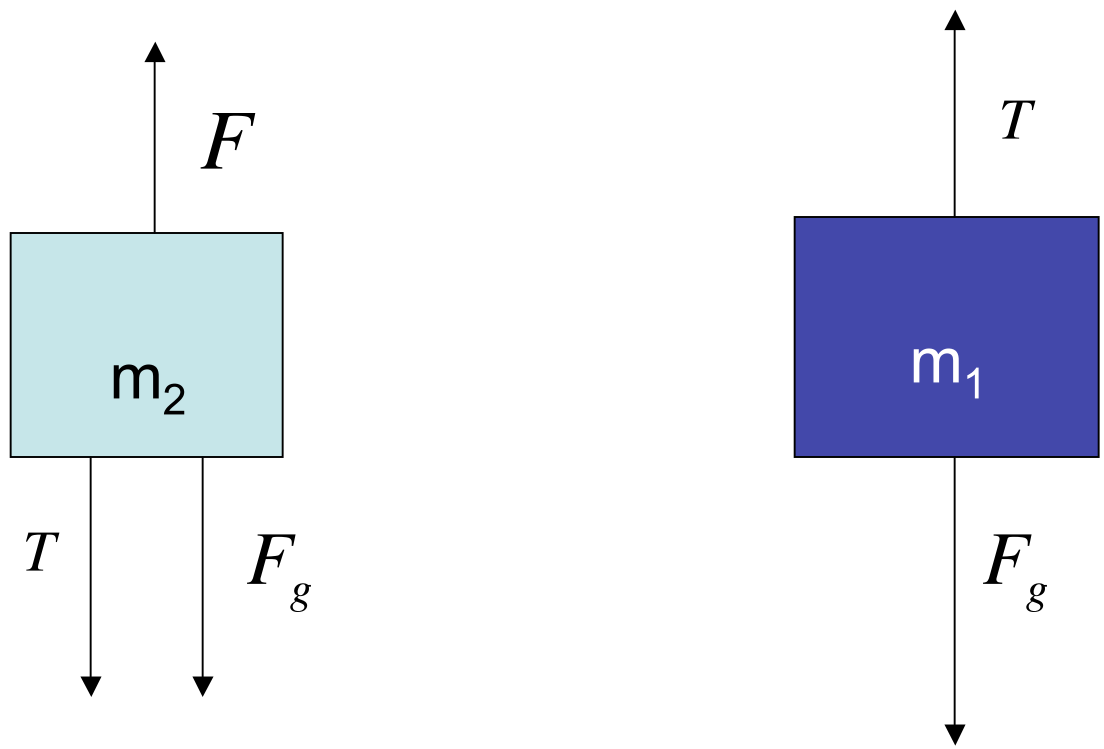
Questions?

Problem 7

7. Two masses (m_1 and m_2) are joined by a massless rope as shown in the figure below. A vertical force, $\boldsymbol{F} = 30\ N$ is applied to the upper mass which gives the system of masses an upward acceleration of $3.2\ m/s^2$. If the tension in the rope joining the masses is $18\ N$, what are the values of the two masses in kilograms?



Free Body Diagrams



The Math

$$\sum F = ma$$
$$F - T - F_g = m_2 a$$
$$F - T - m_2 g = m_2 a$$

$$F - T = m_2(a + g)$$
$$m_2 = \frac{F - T}{a + g}$$

$$\boxed{m_2 = \frac{30 - 18}{3.2 + 9.8} = \frac{12}{13} \approx 0.923}$$

$$\sum F = ma$$
$$T - F_g = m_1 a$$
$$T - m_1 g = m_1 a$$

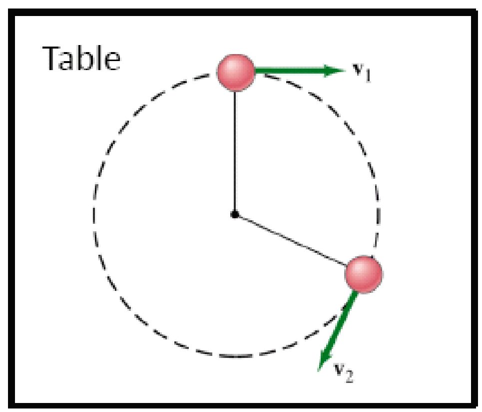
$$T = m_1(a + g)$$
$$m_1 = \frac{T}{a + g}$$

$$\boxed{m_1 = \frac{30}{3.2 + 9.8} = \frac{30}{13} \approx 2.31}$$

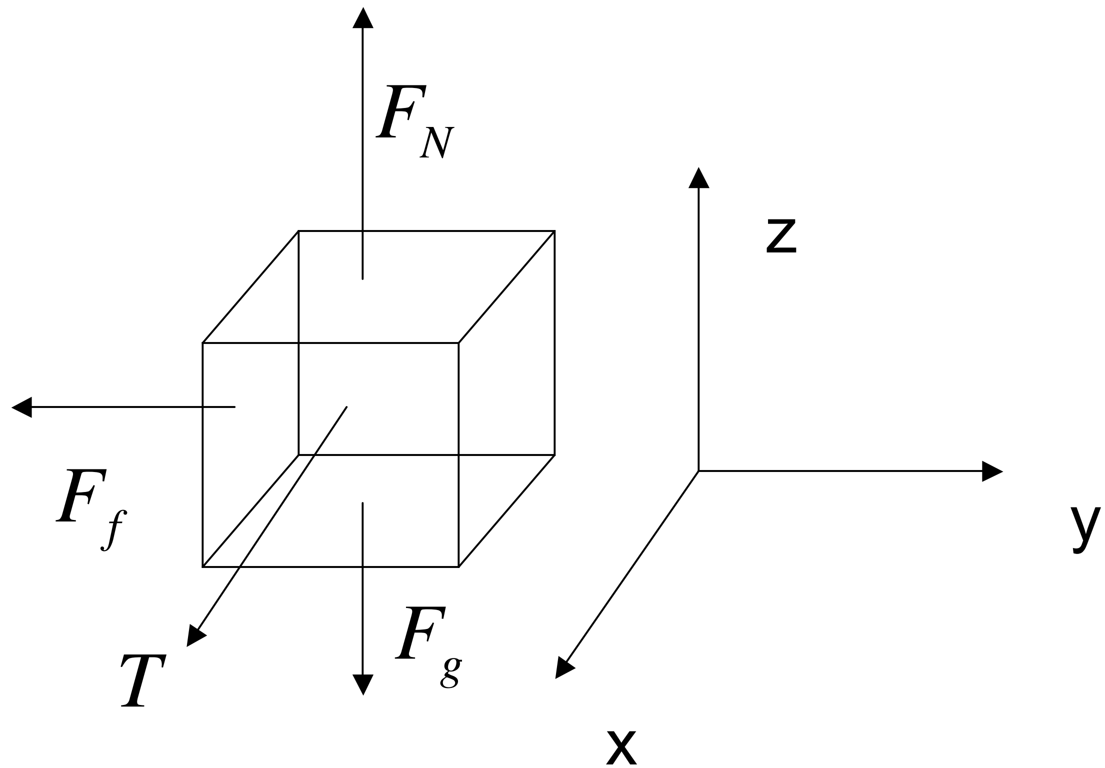
Questions?

Problem 8

8. An object of mass $m = 3\ kg$ is attached to a string $0.5\ m$ long. The other end of the string is looped over a nail in the center of a flat table as shown below. The object is given an initial velocity of $v_1 = 2.5\ m/s$ causing it to slide in a circular motion on the table top. The coefficient of kinetic friction between the object and the table is $\mu_k = 0.03$. What is the tension in the string after 2 seconds?



Free Body Diagram



The Math

$$\sum F_c = ma_c = m\frac{v^2}{R}$$

$$T = \frac{mv^2}{R}$$

$$\sum F_{\tan} = ma_{\tan} = -F_f$$

$$a_{\tan} = \frac{-F_f}{m}$$

$$\sum F_z = F_N - mg = 0$$

$$F_N = mg$$

$$F_f = \mu F_N$$

$$F_f = \mu mg$$

$$a_{\tan} = -\mu g$$

$$v(t) = v_0 + at$$

$$v(t) = v_0 - \mu gt$$

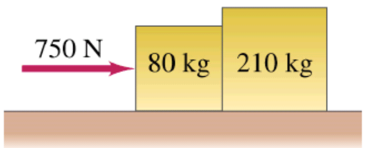
$$T(t) = \frac{mv(t)^2}{R} = \frac{m(v_0 - \mu gt)^2}{R}$$

$$T(2) = \frac{3[2.5 - (0.03)(9.8)(2)]^2}{0.5} \approx 22$$

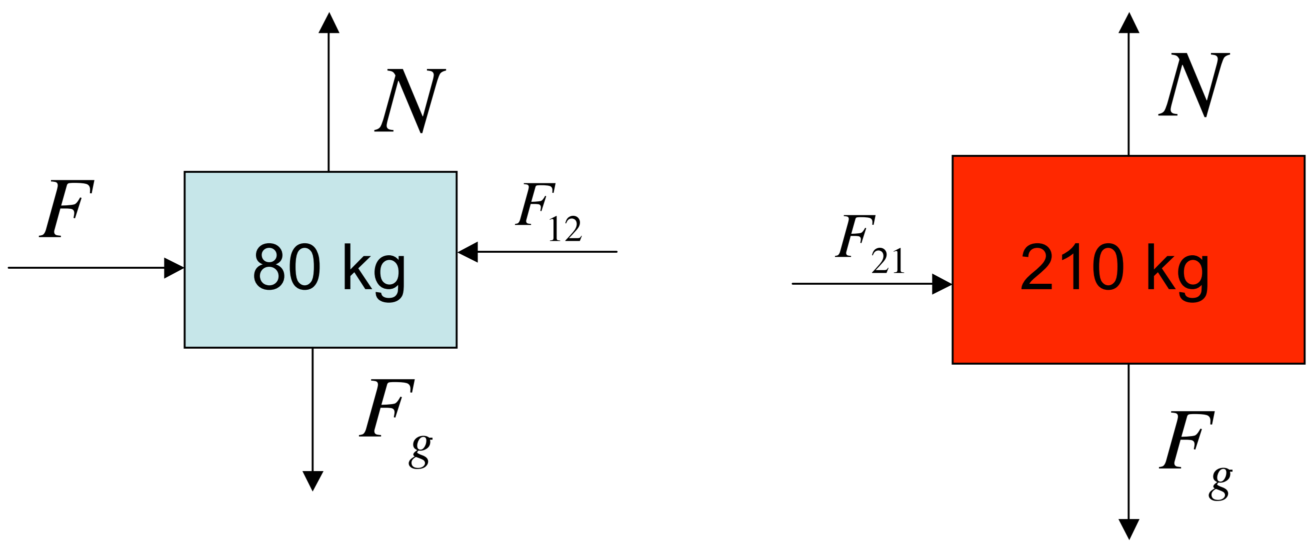
Questions?

Problem 9

9.) The two masses below are sliding on a *frictionless* table surface due to the 750 N applied force as shown below. Find the acceleration of the system of masses. Identify and label *all of the action – reaction* force pairs in this situation.

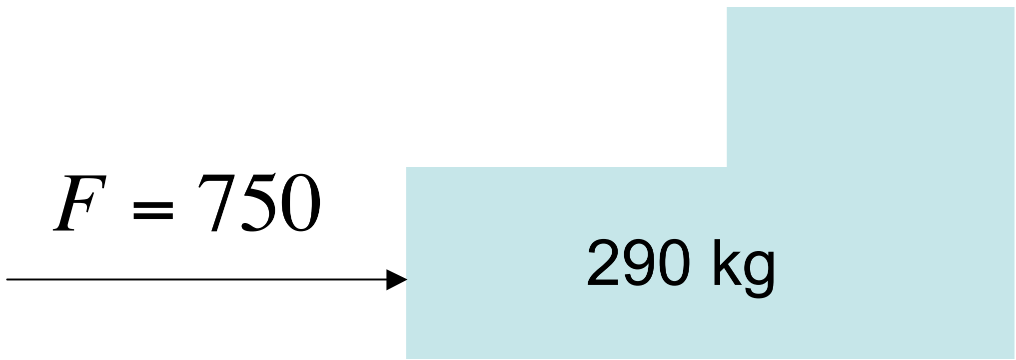


Free Body Diagrams



F_{12} and F_{21} are the action reaction pair.

Free Body Diagram and Math



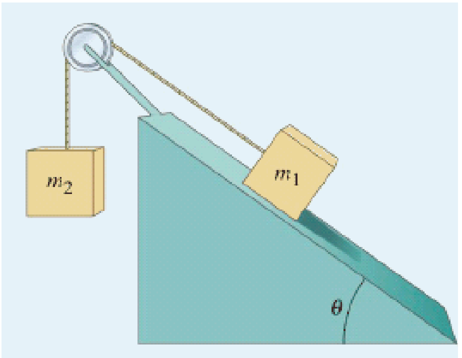
$$\sum F = M_{TOT}a$$

$$a = \frac{750}{290} \approx 2.59$$

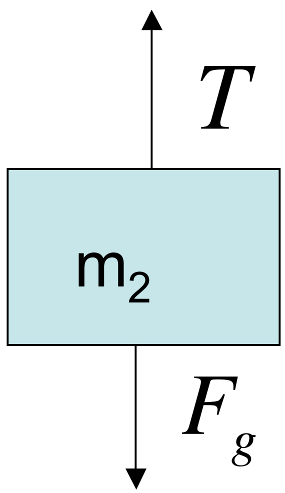
Questions?

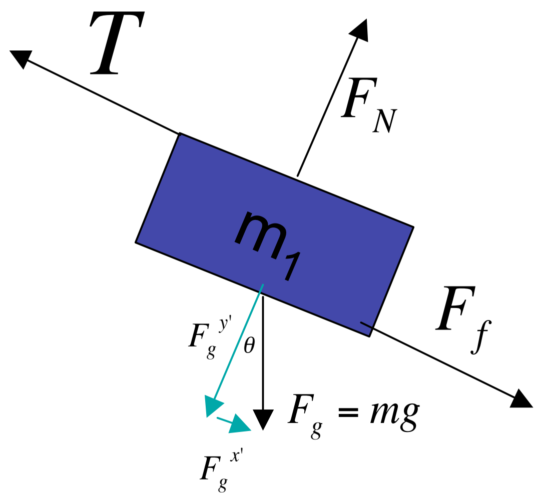
Problem 10

10. Two masses *(m_1 = 2 kg)* and *(m_2 = 3 kg)* are connected by a rope looped over a frictionless pulley as shown below. Mass m_1 rests on an inclined plane which makes an angle $\theta = 37°$ with respect to the horizontal. The coefficient of friction between mass m_1 and the incline is $\mu_k = 0.25$. Find the magnitude and direction of the *acceleration* of the system of masses.



Free Body Diagrams





We know friction points opposite tension because, in the absence of friction, the mass on the inclined plane would slide up the plane as $m_2 > m_1$.

The Math

$$\sum F_y = m_2 a$$
$$m_2 g - T = m_2 a$$

$$\sum F_{x'} = m_1 a = T - F_f - m_1 g \sin\theta$$
$$\sum F_{y'} = m_2 a = F_N - mg\cos\theta = 0$$
$$F_N = m_1 g \cos\theta$$
$$F_f = \mu F_N = \mu m_1 g \cos\theta$$

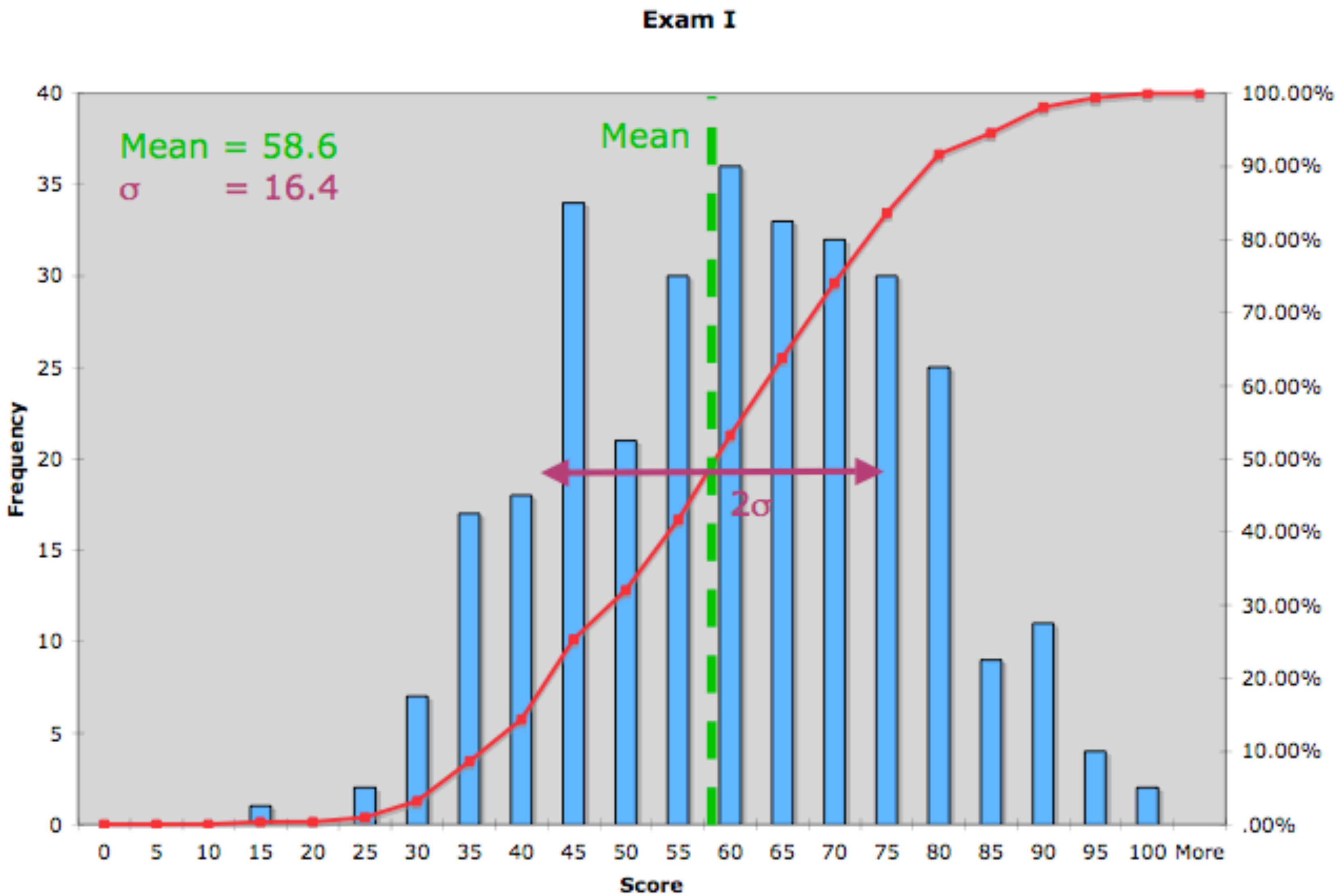
$$m_1 a = T - mg(\mu\cos\theta - \sin\theta)$$
$$T = m_2(g - a)$$
$$m_1 a = m_2 g - m_2 a - m_1 g(\mu\cos\theta - \sin\theta)$$
$$a(m_1 + m_2) = g(m_2 - m_1\mu\cos\theta + m_1\sin\theta)$$

$$a = \frac{g(m_2 - m_1\mu\cos\theta + m_1\sin\theta)}{(m_1 + m_2)} \approx 2.74$$

The block slides up the incline.

Questions?

Statistics



Exam I
Mean = 58.6
σ = 16.4
Mean
2σ
Frequency
Score
0
5
10
15
20
25
30
35
40
0
5
10
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70.00%
80.00%
90.00%
100.00%

Fin!