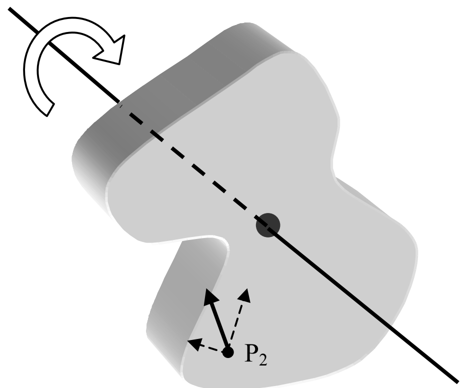
Physics Workshop **Rotational Kinematics** **Solutions**

1. (a) The two trajectories are both circles around the same central point, but P_2 moves in a circle with a larger radius.
   (b) P_2 moves at a greater speed in order to complete a larger circular trajectory in the same amount of time. Furthermore, the directions of the velocities will be different at any given instant of time.
   (c) Since the points P_1 and P_2 are moving in circular paths, they are accelerating. The acceleration is centripetal, and therefore its magnitude is given by v^2/r. Since v and r are both greater for P_2, it seems at first glance that the accelerations of P_1 and P_2 could be the same, but this does not turn out to be the case. In fact the centripetal acceleration grows linearly with increasing distance from the axis of rotation.
   (d) If the object speeds up in its rotation, the point will have both centripetal acceleration towards the center of their circular motion, and tangential acceleration parallel to their velocity vectors. A sketch of the acceleration vector for P_2 is shown below, where the dashed arrows represent the centripetal and tangential components of the vector.



   (e) Although the points P_1 and P_2 have different distances from the rotation axis, different velocities and different accelerations, they each rotate by the same amount in the same amount of time. In fact, they have all of their rotational quantities in common: angle of rotation, rotational speed and angular acceleration.

2. (a) One radian is about one sixth of a complete rotation (since 2π rad = 1 rev), or about 60°.
   (b) Twelve and a half radians per second is roughly two rotations per second.
   (c) Six radians per second squared means that an object rotates faster and faster by about one rotation per second each second. So if it started from rest, after one second it would be rotating at about one rotation per second, after two seconds it would be rotating at about two rotations per second, and so on.

3. This is a straightforward example of rotational kinematics with constant angular acceleration.

   (a) $$\begin{aligned}\Delta\theta &= \omega_0 t + \tfrac{1}{2}\alpha t^2 : \\ &= 0 + \left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right)(10)^2 \\ &= 25 rad \times \left(\frac{1 rot}{2\pi rad}\right) \\ &\approx 4 \text{ rotations}\end{aligned}$$

   (b) $$s = r\Delta\theta = (1m)(25rad) = 25m$$

   (c) $$\begin{aligned}\omega &= \omega_0 + \alpha t \\ &= (\tfrac{1}{2} rad/s^2)(10s) \\ &= 5 rad/s\end{aligned}$$

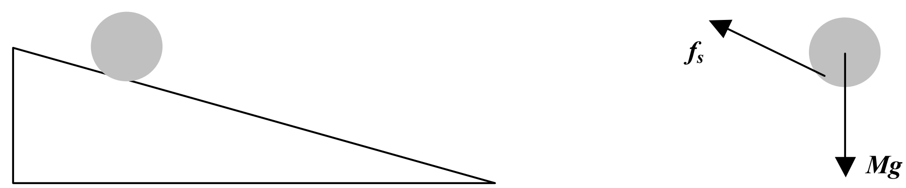
   (d) $$\begin{aligned}v &= \omega r \\ &= (5rad/s)(1m) \\ &= 5m/s\end{aligned}$$

   Since a sprinter runs at about 10 m/s, this seems somewhat realistic.

3. (a) This object will accelerate linearly, since there's a net force on it, but it won't rotate.
   (b) This object will obviously rotate, but it won't accelerate because the net force on it is zero.
   (c) This one will both rotate and accelerate.
   (d) Here the object won't move at all.
   (e) It's clear from the above that what makes something rotate is not as simple as a net force. In fact, what matters is that a force be applied in a direction that does not intersect the rotation axis of the object. This is called a torque, and a net torque is what makes something change its rotational velocity.

Additional Questions

1. A free-body diagram is shown below. It is clear from the diagram that the ball's weight does not make it rotate, but instead it's actually the force of friction that causes the rotation. An intuitive proof of this is to imagine placing the ball on a very slippery (frictionless) ramp, in which case it would just slide down and not roll.



2. We've seen that what causes something to rotate is a force exerted in a direction that passes some distance from the object's axis of rotation (a torque). In fact, the greater this distance, the greater the torque. So in this case, your goal is to maximize that distance. One good way to this would be to jam a metal rod or other similar object into the wheel so that you could exert your forces farther from the wheel's axis than the wheel itself allows: