Physics Workshop **Rotational Dynamics** **Solutions**

1. (a) **<u>Angular acceleration</u>** is the rate of change of an object's rotational speed, measured in rad/s/s, or rad/s^2. So if an object is rotating at a constant rate, its angular acceleration is zero, but if its rotation either speeds up or slows down, then its angular acceleration is not equal to zero.

   (b) **<u>Torques</u>** are the causes of angular acceleration, just as forces are the causes of regular (linear) acceleration. Every force acting on an object has a related torque, which is equal to the force multiplied by the perpendicular distance between the line of the force and the object's axis of rotation (this distance is often called the lever arm of the force).

   (c) **<u>Moment of inertia</u>** is the property of an object that resists angular acceleration (rotation), just like mass is the property that resists linear acceleration.

The above can also be understood by analogy to linear motion. Consider Newton's 2nd Law alongside the torque equation:

$$\boxed{\Sigma F = m a}$$

Net force: causes linear acceleration

Mass: resists linear acceleration

Linear acceleration: produced by a net force

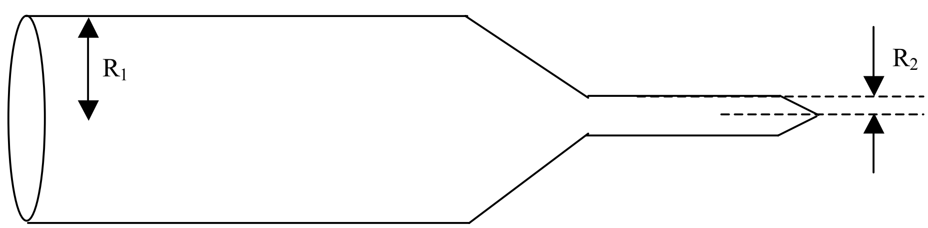
$$\boxed{\Sigma \tau = I \alpha}$$

Net torque: causes angular acceleration

Moment of inertia: resists angular acceleration

Angular acceleration: produced by a net torque

2. For simplicity, imagine turning a screw with the screwdriver at a constant angular speed (no angular acceleration). Then the net torque on the screwdriver must be zero. This torque is produced by two forces: the force of friction between your hand and the handle, and the force between the screwdriver threads and the head of the screw. But these two forces have very different lever arms:

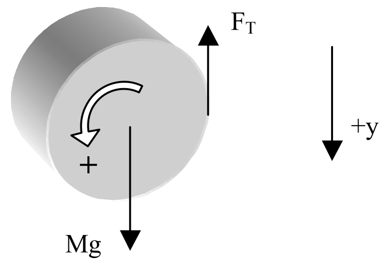


So the expression for the torque would look something like

$$\boxed{\begin{aligned}&\Sigma\tau = 0:\\ &F_{hand}R_1 - F_{screw}R_2 = 0\\ &\Rightarrow F_{screw} = F_{hand}\left(\frac{R_1}{R_2}\right)\end{aligned}}$$

Leading to a much larger force on the screw than that exerted by the hand.

3. (a) Here's a FBD for the yo-yo:



(b) $\boxed{\begin{aligned}\Sigma F_y &= ma_y :\\ Mg - F_T &= Ma\end{aligned}}$ We can't yet determine the acceleration of the yo-yo, because the tension is an unknown force in this equation.

(c) I've simply added a rotational coordinate system to my FBD; see above. Note that I chose my two coordinate systems (translational and rotational) so that if the yo-yo moves linearly in the positive direction, it also rotates in the positive direction. This consistency will help to avoid sign errors later in the problem.

$$\boxed{\begin{aligned}&\Sigma\tau = I\alpha :\\ &F_T R = \left(\tfrac{1}{2}MR^2\right)\alpha\end{aligned}}$$

(d) The angular and linear accelerations are related through the equation $a = R\alpha$. It's important to realize that if I had chosen a different coordinate system (for example, if I had chosen a clockwise rotation to be positive), then I would have to write $a = -R\alpha$, where the negative sign would come in because a positive rotation would correspond to a negative linear motion, and vice versa.

(e) Substituting $\alpha = a/R$ into the equation from part (c), we have

$$\boxed{\begin{aligned}&\Sigma\tau = I\alpha :\\ &F_T R = \left(\tfrac{1}{2}MR^2\right)\frac{a}{R}\\ &F_T = \tfrac{1}{2}Ma\end{aligned}}$$

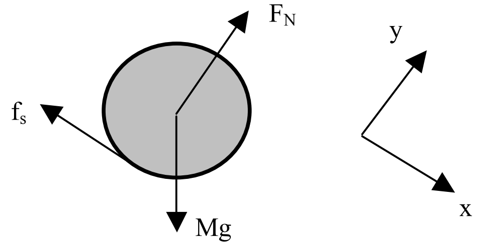
and then substituting this result into the equation from part (b):

$$\boxed{\begin{aligned}&\Sigma F_y = ma_y :\\ &Mg - F_T = Ma\\ &Mg - \tfrac{1}{2}Ma = Ma\\ &\tfrac{3}{2}Ma = Mg\\ &\Rightarrow a = \tfrac{2}{3}g\end{aligned}}$$

So the effect of the string was to slow down the acceleration of the yo-yo by a factor of one third.

(f) Substituting the acceleration back into our expression for the tension, we find $\boxed{F_T = \tfrac{1}{3}Mg}$.

4. (a) FBD:



(b)

$$\boxed{\begin{array}{l}\Sigma F_x = ma_x : \\ Mg\sin\theta - f_s = Ma\end{array}}$$

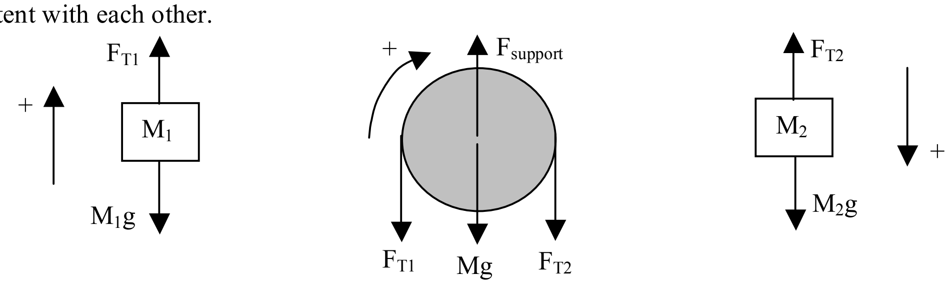
$$\boxed{\begin{array}{l}\Sigma\tau = I\alpha : \\ f_s R = I\dfrac{a}{R}\end{array}}$$

(c) Solving the second equation for f_s and substituting the result into the first equation:

$$\boxed{\begin{array}{l}\Sigma F_x = ma_x : \\ Mg\sin\theta - f_s = Ma \\ Mg\sin\theta - I\dfrac{a}{R^2} = Ma \\ \Rightarrow a = \dfrac{g\sin\theta}{1 + I/MR^2}\end{array}}$$

(d) For a cylinder ($I = \frac{1}{2}MR^2$), we find $a = \frac{2}{3}g\sin\theta$ (note that this is the same as for the yo-yo). For a sphere ($I = \frac{2}{5}MR^2$), we find $a = \frac{5}{7}g\sin\theta$. A sliding object doesn't rotate, so it effectively has zero moment of inertia. The result of part (c) then reduces to $a = g\sin\theta$, which you can verify is what we would get from Newton's 2nd Law if the force of friction is taken to be zero.

5. Below are the FBDs and for each part of the pulley system. Note that the coordinate systems are again consistent with each other.



$$\boxed{\begin{array}{l}\Sigma F = ma : \\ F_{T1} - M_1 g = M_1 a\end{array}}$$

$$\boxed{\begin{array}{l}\Sigma\tau = I\alpha : \\ F_{T2}R - F_{T1}R = \frac{1}{2}MR^2\left(\dfrac{a}{R}\right)\end{array}}$$

$$\boxed{\begin{array}{l}\Sigma F = ma : \\ M_2 g - F_{T2} = M_2 a\end{array}}$$

After a little algebra, you should find that the acceleration is

$$\boxed{a = g\left(\frac{M_2 - M_1}{M_1 + M_2 + \frac{1}{2}M}\right)}$$

Additional Questions

1. Sure, just twirl your pencil between your fingers.
2. Again, yes. Drop your pencil to the ground without rotating it.
3. Yes, because the moment of inertia depends on the axis of rotation. Rotating your pencil along its own axis requires a different torque than rotating it around an axis perpendicular to its length. Try it!