Physics Workshop
Rotational Dynamics
Suggested Checkpoint Questions with Answers

- Checkpoint following 1(c):

1. Can you describe the analogy between Newton's 2nd Law ($\Sigma F = ma$) and the torque equation ($\Sigma \tau = I\alpha$)?
2. Why is it harder to do sit-ups with your hands behind your head than with them crossed over your chest?

- Checkpoint following 2:

1. Can you think of other examples where a mechanical advantage is gained using the same principle (a varying lever arm)?
2. Does the *length* of a screwdriver affect the force it exerts on a screw?

- Checkpoint following 3(c):

1. What would be different about your work so far if this were a real yo-yo, where the string was wrapped around an inner radius that differs from the outer radius?
2. How do you determine the moment of inertia of an odd-shaped object like a real yo-yo?

- Checkpoint following 3(f):

1. Again, how would the problem differ for a real yo-yo?
2. Without repeating all of your work, can you predict the downward acceleration of a ball of yarn if it is held at one end and allowed to unravel?

- Checkpoint following 4(c):

1. Do your linear and angular coordinate systems "agree"? Is it possible to solve the problem correctly if they don't agree?

- Checkpoint following 5:

1. Show that your results make sense in the limiting cases where $M \rightarrow \infty$, $M_1 \rightarrow \infty$ and $M_2 \rightarrow \infty$.
2. Is the tension in the string the same on the two sides of the pulley? Why or why not?

Answers to checkpoint questions:

- Checkpoint following 1(c):

1. See tutorial solutions.
2. In this situation you are attempting to use your stomach muscles to exert a torque around the axis through your hips. The farther your hands are from the axis, the greater your moment of inertia relative to that axis, and the more force your muscles must exert to produce the desired rotation.

- Checkpoint following 2:

1. There are many ways to gain a mechanical advantage. One that comes to mind is a simple lever, where again it's the disparity between lever arms that allows the forces on the two ends of the lever to differ. Other related examples are pulley systems and hydraulic levers.
2. No, as we've seen it's just the radius of each end that determines the ratio of forces. However, screwdrivers come in many lengths as a matter of convenience.

- Checkpoint following 3(c):

1. Two things would be different. The lever arm of the force of tension would now be the inner radius of the yo-yo (where the string is attached), and the moment of inertia of the yo-yo would be different since it would no longer be a cylinder.
2. The fundamental way to find a moment of inertia is of course by doing an integral, but in this case you could estimate the total rotational inertia by treating the yo-yo as three cylinders sandwiched together. The only missing information is the mass of the central axle, but if this were estimated the total moment of inertia would just be the sum of that for each cylindrical part.

- Checkpoint following 3(f):

1. Most importantly, the relationship between linear and rotational accelerations would now be $a = \alpha r_{axle}$, where r_{axle} means the inner radius of the yo-yo.
2. The only difference would be the moment of inertia, which for a sphere is $\frac{2}{5} MR^2$. By inspection, the answer becomes $a = \frac{5}{7} g$.

- Checkpoint following 4(c):

1. Suppose, for example, that we chose down the ramp to be the +x direction, but counterclockwise to be the positive direction for rotations. This would seem to lead to a sign error in the answer, but in fact it does not. The reason is that if these coordinates are used, then instead of $\alpha = a / r$, the correct relationship would be $\alpha = -a / r$ due to the fact that a positive rotation corresponds to a negative linear acceleration and vice versa. In fact, the answer turns out to be the same regardless of which coordinate systems are chosen, as it must.

- Checkpoint following 5:

1. Show that your results make sense in the limiting cases where $M \rightarrow \infty$, $M_1 \rightarrow \infty$ and $M_2 \rightarrow \infty$. The solution to the problem is:

$$a = g\left(\frac{M_2 - M_1}{M_1 + M_2 + \frac{1}{2}M}\right)$$

If $M \rightarrow \infty$, the acceleration becomes zero. This makes sense since the pulley would then have infinite inertia and could not be made to rotate. If $M_1 \rightarrow \infty$, the acceleration becomes $-g$. This also makes sense because the inertia of the pulley and the other mass would then be irrelevant, so that M_1 would act as though it were in freefall. Similarly, the acceleration becomes $+g$ if $M_2 \rightarrow \infty$.

2. The tension in the string is not the same on the two sides of the pulley (this can be verified by substituting the acceleration back into Newton's 2nd Law and explicitly finding the two forces of tension). In fact if it were, the pulley would never feel a net torque and would therefore stay at rest. So the tension in a light (massless) string is only constant as long as the string doesn't encounter any inertial objects (the massive pulley in this case).