Physics Workshop | **Mechanics Tutorial** | **Number 9**

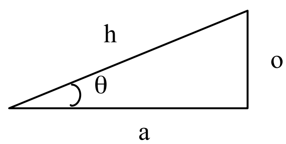
Rotational Dynamics

Equations and Relations:

- Average Speed = distance traveled / time elapsed
- Instantaneous Velocity = slope of tangent to x-t
- Instantaneous Acceleration = slope of tangent to v-t
- Average Velocity: $\bar{v} = \Delta x / \Delta t = (x_2 - x_1)/\Delta t$
- Average Acceleration: $\bar{a} = \Delta v / \Delta t = (v_2 - v_1)/\Delta t$

Trigonometry:

$\sin\theta = o/h$
$\cos\theta = a/h$
$\tan\theta = o/a$
$a^2 + o^2 = h^2$



Motion with Constant Acceleration:

Linear:	Rotational:
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$x = x_0 + v_0 t + \frac{1}{2}at^2$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a(x - x_0)$	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$
$g = 9.80 m/s^2 \approx 10 m/s^2$	$\tau = Fr_\perp$
	$\Sigma\tau = I\alpha$

Newton's Laws:

- 1st: An object will stay at rest or in motion with constant velocity unless acted on by a net force.
- 2nd: $\Sigma F_x = ma_x$, $\Sigma F_y = ma_y$, $\Sigma\tau = I\alpha$
- 3rd: Forces come in pairs. If A exerts a force on B, then B exerts a force on A with the same magnitude but in the opposite direction.

Friction: $f_s \le \mu_s F_N$, $f_k = \mu_k F_N$

Uniform Circular Motion: $a_c = v^2/r$

Momentum: $\vec{p} = m\vec{v}$, $\Sigma\vec{F} = \Delta\vec{p}/\Delta t$
$L = I\omega$ (angular momentum)

Work & Energy:

$W = F_{||}d = Fd_{||}$
$W_{net} = \Delta KE$
$W_{nc} = \Delta KE + \Delta PE$
$KE = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$
$PE_{grav} = mgy$

Springs: $F_{sp} = kx$, $PE_{sp} = \frac{1}{2}kx^2$

Angular and Linear Quantities:

$2\pi\ rad = 1\ rev$
$l = r\theta,\ v = r\omega,\ a_{\tan} = r\alpha$

Statics: $\Sigma F_x = 0, \Sigma F_y = 0, \Sigma\tau = 0$

Fluids: $p = \frac{F}{A}$, $\rho = \frac{M}{V}$
F_B = weight of displaced fluid
$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$

In your work in Tutorial, please remember to

1) Draw pictures and/or diagrams of the situation before beginning a solution.
2) Write down the known quantities and the desired unknown(s).
3) Start your solution by writing down a fundamental relation or equation that you think is relevant.

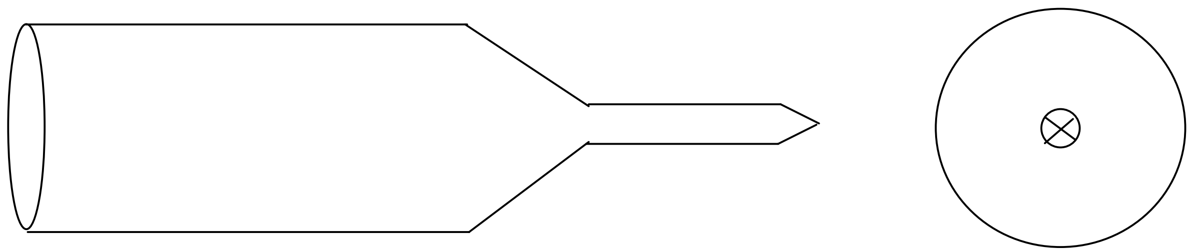
1. Discuss the following terms, and then write a clear definition of each *in words*:

   (a) Angular acceleration
   (b) Torque
   (c) Moment of inertia

Check Point

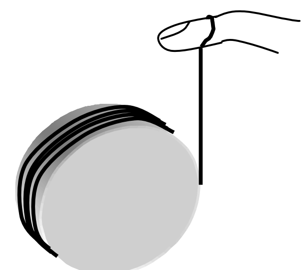
2. A Phillips' head screwdriver looks like this:

Side view: Head-on view:



Explain how the design of the screwdriver allows a relatively small force exerted by your hand to result in a much larger force exerted on a screw.

Check Point



3. Consider a yo-yo with its string wound up, ready for action. Suppose that we know nothing about this yo-yo except that it has the shape of a thin cylinder, with the string wrapped around the outside of the cylinder. (So this is even simpler than a regular yo-yo.) If you release the yo-yo from rest, the string will unwind and it will accelerate downward, but at what rate?

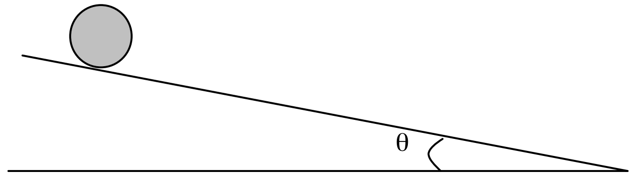
(a) Make a free-body diagram and choose a convenient coordinate system for the yo-yo as it falls.
(b) Apply Newton's 2nd Law ($\Sigma F = ma$) to your FBD. Assuming that the yo-yo's mass and radius are known quantities, can you determine the acceleration of the yo-yo in terms of known quantities from this equation? Why or why not?
(c) Now choose a rotational coordinate system and apply the torque equation $\Sigma\tau = I\alpha$ to the yo-yo (the moment of inertia of a disk is $\frac{1}{2}MR^2$).

Check Point

(d) What is the relationship between α (the angular acceleration of the yo-yo) and a (the linear acceleration of the yo-yo)?
(e) Using the results of (b), (c) and (d), determine the acceleration of the yo-yo. Does the answer depend on the mass or radius of the yo-yo? What *does* it depend on?
(f) What is the tension in the string as the yo-yo falls? Give your answer as some fraction or multiple of the weight of the yo-yo.

Check Point

4. A round ball (moment of inertia = I) is released from rest and rolls without slipping down a hill of steepness θ.



(a) Make a FBD for the ball, and choose both ordinary and rotational coordinate systems. (Note: friction can **not** be neglected in this problem.)
(b) Write out Newton's 2nd Law and the torque equation for the ball.
(c) Determine the acceleration of the ball down the hill.

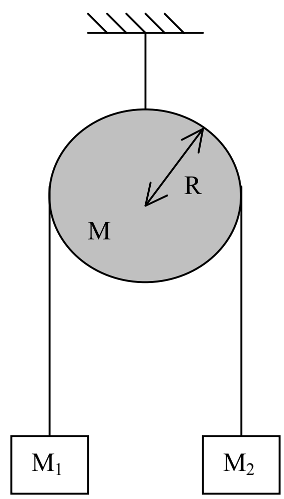
Check Point

(d) Use the results of part (c) to find the acceleration of the following objects down a hill:

- A cylinder ($I = \frac{1}{2}MR^2$)
- A sphere ($I = \frac{2}{5}MR^2$)
- A mass that slides without friction (what's the "effective" moment of inertia in this case?)

Check Point

5. Two masses are hung from a frictionless cylindrical pulley by a light string as shown in the sketch to the right. The string moves on the pulley without slipping, causing the pulley to rotate. Determine the acceleration of the masses if they are released and allowed to move.



Additional Questions

1. Is it possible to exert a net torque on an object without exerting a net force? Think of an example, or explain why it's not possible.
2. Is it possible to exert a net force on an object without exerting a net torque? Again, explain or provide an example.
3. Can a single object have more than one moment of inertia? Explain.