PHYSICS 105S

Assignment #2 **Due at 9am Wednesday, May 30, 2007**

NAME: Solutions

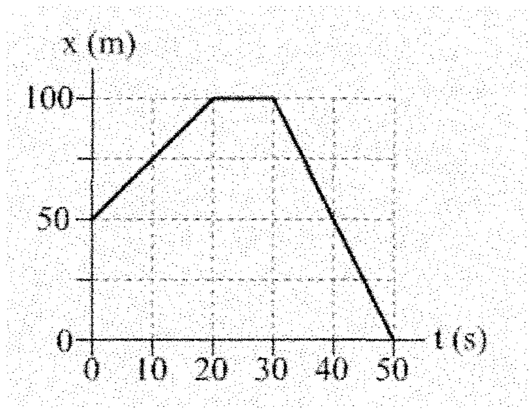
DISCUSSION SECTION: [ ] SA2 Tues Thurs Rachele PRB 150
[ ] SA3 Wed Fri Joel PRB 150
[ ] SA4 Tues Wed Chris PRB 146
[ ] SA5 Wed Fri Mark PRB 150

INSTRUCTIONS:

1. Please include appropriate units with all numerical answers.
2. **Please show all steps in your solutions!** If you need more space for calculations, use the back of the page preceding the question. For example, calculations for problem 3 should be done on the back of the page containing question 2. **You must show correct work to receive full credit. Support your answers with brief written explanations and/or arguments based on equations.**
3. **Indicate clearly** which part of your solution is the final answer.
4. Try answering these problems without a calculator.

Angle (θ)	**sin(θ)**	**cos(θ)**
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$

PROBLEM 1 – 20 points



The graph shows your position as a function of time as you move along a sidewalk.

[5 points] (a) At t = 10 s, what is your:

Position: 75 m Velocity: 2.5 m/s Acceleration: 0 m/s²

$$V = \frac{\Delta x}{\Delta t} = \frac{100m - 50m}{20s - 0s} = \frac{50m}{20s} = 2.5\ m/s$$

$a = \frac{\Delta V}{\Delta t}$ but V is constant between t = 0s and t = 20s

[5 points] (b) At t = 40 s, what is your:

Position: 50 m Velocity: – 5 m/s Acceleration: 0 m/s²

$$V = \frac{\Delta x}{\Delta t} = \frac{0m - 100m}{50s - 30s} = \frac{-100m}{20s} = -5\ m/s$$

$a = \frac{\Delta V}{\Delta t} = 0$

since V is constant also

[5 points] (c) What is your average velocity over the interval from t = 0 s to t = 40 s?

$$\bar{V} = \frac{\Delta x}{\Delta t} = \frac{50m - 50m}{40s - 0s} = \boxed{0\ m/s}$$

________/20

________/15

________/15

________/10

Total Score:

________/60

[5 points] (d) What is your average speed over the interval from t = 0 s to t = 40 s?

$$\text{avg speed} = \frac{\text{distance}}{\Delta t} = \frac{50m + 50m}{40s - 0s} = \frac{100m}{40s} = \boxed{2.5\ m/s}$$

PROBLEM 2– 15 points

Two balls are launched at the same time. Ball A is released from rest from the top of a tall building of height H. Ball B is fired straight up from the ground with an initial velocity such that it just reaches the top of the same building. Neglect air resistance.

[3 points] (a) Which ball has the largest magnitude acceleration at the point they pass one another?

[ ] Ball A [ ] Ball B [X] neither, they're equal

Briefly justify your answer: Both have an acceleration of g in the downward direction.

[3 points] (b) If ball A takes a time T to reach the ground, and ball B takes the same time T to reach the top of the building, which ball has the highest speed at time T/2?

$\left[V_{0A} = 0, \quad V_{0B} = gT \quad [V_B(T) = V_{0B} - gT = 0] \right]$

[ ] Ball A [ ] Ball B [X] neither, they're equal

Briefly justify your answer:

Both have same average speed, and since acceleration is constant, the average speed* is equal to the speed at half the total time.

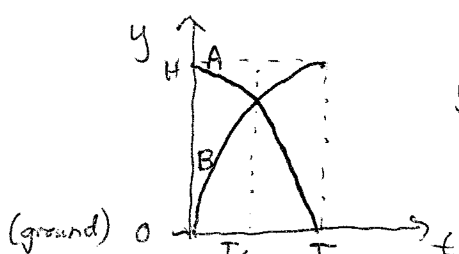
* $V_{avg} = V$ at half the time, and since directions never change, V_{avg} has the same magnitude as the avg speed.

or determine $V(\frac{T}{2})$

Ball A: $V_A(\frac{T}{2}) = V_{0A} - g(\frac{T}{2}) = 0 - g(\frac{T}{2})$

Ball B: $V_B(\frac{T}{2}) = V_{0B} - g(\frac{T}{2}) = gT - g\frac{T}{2} = +g\frac{T}{2}$

[4 points] How far from the ground are the two balls when they pass one another? Express your answer in terms of H.



The two balls are at the same height at $t = \frac{T}{2}$ (see graph)

$y_A = y_{0A} + V_{0A}t + \frac{1}{2}a_A t^2 = H + 0 - \frac{g}{2}\left(\frac{T}{2}\right)^2 = H - \frac{1}{4}\frac{gT^2}{2}$

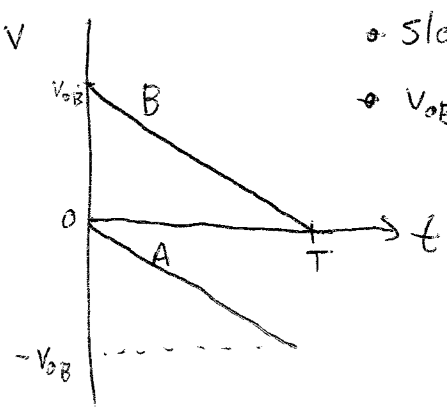
$y_B = y_{0B} + V_{0B}t + \frac{1}{2}a_B t^2 = 0 + gT\left(\frac{T}{2}\right) - \frac{g}{2}\left(\frac{T}{2}\right)^2 = \frac{gT^2}{2} - \frac{1}{4}\frac{gT^2}{2} = \frac{3}{4}\frac{gT^2}{2}$

set $y_A = y_B$: $H - \frac{1}{4}\frac{gT^2}{2} = \frac{3}{4}\frac{gT^2}{2} \Rightarrow H = \frac{gT^2}{2}$

determine either y_A or y_B

$y_B = \frac{3}{4}\left(\frac{gT^2}{2}\right) = \boxed{\frac{3}{4}H}$

[5 points] (d) Sketch a graph showing the velocity of ball A, and the velocity of ball B, as a function from the time over the interval from when the balls are launched until ball A reaches the ground.



- slopes same
- $V_{0B} = -V_{fA}$

PROBLEM 3 – 15 points

A tortoise and a hare are having a 100 m race. When the starting gun goes off the hare lies down for a nap. The tortoise moves forward with a constant acceleration, reaching a speed of 2.0 m/s when she is 20 m from the starting line. After this, the tortoise travels at a constant velocity of 2.0 m/s until crossing the finish line. After 45 seconds the hare wakes up from his nap, and covers the 100 m with a constant acceleration of 2.0 m/s^2.

[6 points] (a) Who wins the race? Clearly justify your answer. $\boxed{\text{Hare wins by } 5s}$

tortoise: Find a: $v^2 = v_0^2 + 2a(x - x_0)$

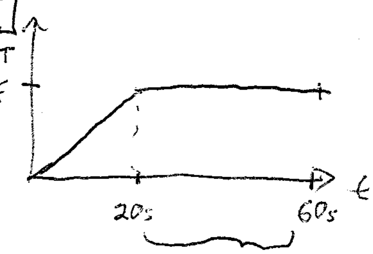
$4\frac{m^2}{s^2} = 0 + 2a(20m)$

$a = \frac{4\,m^2/s^2}{40\,m} = .1\,m/s^2$

Find time it reaches this point:

$v = v_0 + at$

$2\,m/s = (.1\,m/s^2)t \Rightarrow t = \frac{2\,m/s}{.1\,m/s^2} = 20s$



Δx = area under curve

$100m = 20m + (2\,m/s)(\Delta t)$

so $\Delta t = \frac{80m}{2\,m/s} = 40s$ $\Rightarrow$ tortoise takes 60

hare: $x = x_0 + v_0 t + \frac{1}{2}at^2$

$100m = \frac{1}{2}(2\,m/s^2)(\Delta t)^2 \Rightarrow \Delta t = \sqrt{\frac{2x}{a}} = \sqrt{100 s^2} = 10s$ so hare takes $45s + 10s = 55s$

[2 points] (b) How much time passes between the winner reaching the finish line and the other animal reaching the finish line?

See work in part (a) $\boxed{t = 5s}$

[2 points] (c) What is the distance between the animals when the winner crosses the finish line?

Hare crosses finish line at $t = 55s$.

At this time, the tortoise still needs to go $\Delta x = v(\Delta t)$

$= (2\,m/s)(5s)$

$= 10m$

so distance $= \boxed{10m}$

[5 points] (d) What is the distance between the animals at the only time (other than at the instant the starting gun is fired) they have the same velocity?

have same velocity when the hare has a velocity of $2\,m/s$

find when this happens:

$v_H = v_{0H} + a\Delta t$

$2\,m/s = 0 + (2\,m/s^2)\Delta t$

$\Rightarrow \Delta t = 1s$ (1s after starts moving)

so $t = 46s$

at this time, the tortoise's displacement is given by the area under the velocity curve

$\Delta x_T = 20m + (2\,m/s)(26s) = 72m$

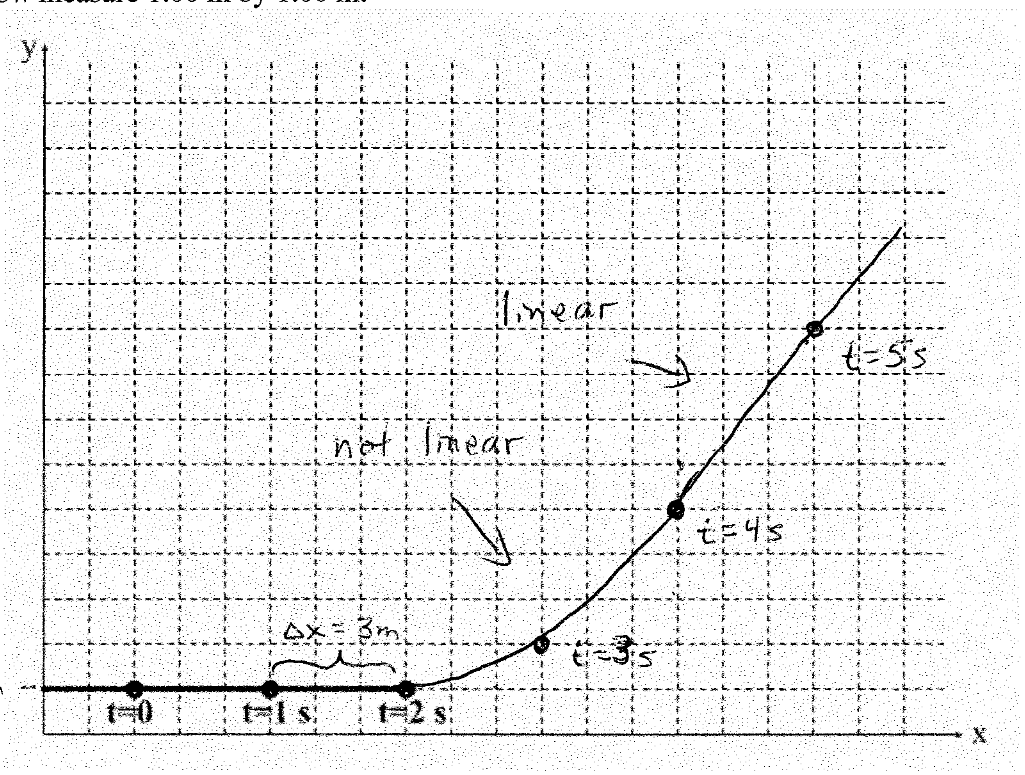
so distance $= 72m - 1m = \boxed{71m}$

and for the hare: $\Delta x_H = \frac{1}{2}(2\,m/s^2)(1s) = 1m$

PROBLEM 4 – 10 points

(This is a two-dimensional problem. The x-motion and the y-motion are independent of each other. You need to think about each of them in order to know where to plot a point (x,y) at a particular time t.)

A spaceship is drifting at constant velocity through outer space, unaffected by any gravitational interactions. The figure below shows the trajectory followed by the spaceship in a particular x-y coordinate system during a 2.00 second interval. At t = 2.00 seconds the spaceship fires its engine, producing an acceleration of 2 m/s^2 in the +y direction. **The engine is turned off again after 2.00 seconds, at t = 4.00 seconds.** The square boxes in the figure below measure 1.00 m by 1.00 m.



(a) [2 points] A t = 2.00 s, what are the components v_{ox} and v_{oy} of the initial velocity needed for calculations regarding the next two second interval (constant acceleration only in the y direction)

$V_{ox} = \frac{\Delta x}{\Delta t} = \frac{3m}{1s} = 3\,m/s \qquad V_{oy} = \frac{\Delta y}{\Delta t} = \frac{0m}{\Delta t} = 0\,m/s$

(b) [~~6~~ 5 points] On the figure above carefully plot the trajectory followed by the spaceship after t = 2.00 seconds. Note in particular where the spaceship is at t = 3.00 s, t = 4.00 s, and t = 5.00 s. The trajectory beyond t= 4 s is a new calculation, taking (x,y) and (v_x , v_y) at 4 seconds as the starting values.

let $t = 2s$ be defined as $t' = 0s$

$x(t') = x_0 + V_{ox} t'$ (each second goes to the right 3m)

$y(t') = y_0 + V_{oy} t' + \frac{1}{2} a_y t'^2$

$= 1m + 0 + \frac{1}{2}(2\,m/s^2)t'^2$ for $t = 3s$ and $t = 4s$ →

$y(t=4s) = 1m + (1\,m/s^2)(1s)^2$
$= 2m$
$y(t=4s) = 1m + (1\,m/s^2)(2s)^2$
$= 1m + 4m$
$= 5m$

and $V_y(4s) = a_y \Delta t$
$= (2\,m/s^2)($
$= 4\,m/s$

(b) [~~4~~ 3 points] What is the speed of the spaceship at t = 5.00 seconds?

speed $= \sqrt{V_x^2 + V_y^2}$

$V_x(5s) = 3\,m/s$

so speed $= 5\,m/s$

speed V_y (3-4-5 triangle)

after $t = 4s$,

$y(t) = y(4s) + V_y(4s)(\Delta t)$