# PHYSICS 105S

## Homework Assignment #5 — Due at 9am Tuesday, June 12, 2007

**NAME:** ___________________________________________

**DISCUSSION SECTION:**
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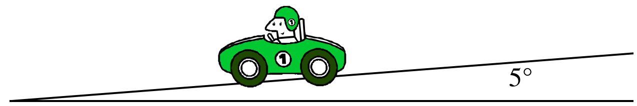
**INSTRUCTIONS:**

1. Please include appropriate units with all numerical answers.
2. **Please show all steps in your solutions!** If you need more space for calculations, use the back of the page preceding the question. For example, calculations for problem 3 should be done on the back of the page containing question 2. **You must show correct work to receive full credit. Support your answers with brief written explanations and/or arguments based on equations.**
3. **Indicate clearly** which part of your solution is the final answer.
4. Try answering these problems without a calculator.

| **Angle ( )** | **sin( )** | **cos( )** |
|---|---|---|
| $30^\circ$ | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ |
| $45^\circ$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{2}}{2}$ |
| $60^\circ$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ |

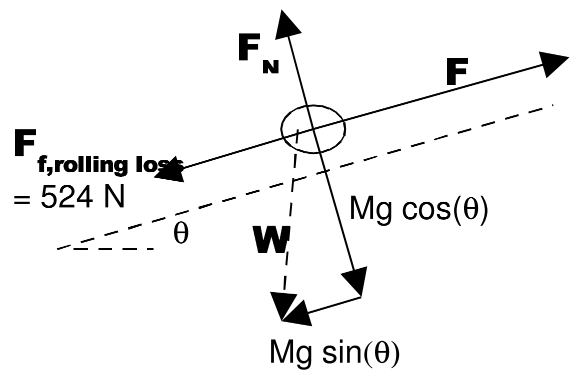
## Problem 1. Work for car going up hill [25 points]

A 1200 kg car is driven up a 5.0° slope that is 290 m long. Because of the internal mechanisms of the motor, the transmission, and the wheels in contact with the road, the car is propelled forward by a constant force **F** directed along the road. There is also gravity **W** directed downward, a normal force $\mathbf{F_N}$ directed perpendicular to the road, and a constant frictional force $\mathbf{F_f}$ = 524 N directed along the road associated with rolling friction in the tires. Neglect air resistance (which would not be independent of speed).




(a) [2 pts] The force **F** follows the rules for:
[ ] kinetic friction [ *X* ] static friction [ ] abnormal force [ ] antigravity [ ] nuclear fusion
***(The road exerts the force on the car through static friction on the car. Although the car is moving, the wheels are not slipping with respect to the road. Therefore it is NOT kinetic friction.)***

(b) [5 pts] Draw a free body diagram showing all the forces acting on the car.




c) [3 pts] Calculate the work done on the car by $\mathbf{F_N}$ from the bottom to the top of the hill.

***Zero work.* *The normal force is perpendicular to the road.***
***Work is any one of the following:***
**1) [*Magnitude of the force*] [*magnitude of the displacement*] [*cosine of angle between them*].**
**2) [*Component of force in the direction of the displacement*] [*the magnitude of the displacement*]**
**3) [*Magnitude of force*] [*component of the displacement in the direction of the force*].**
***Either 1) cos (90°) = 0 or by 2) or 3), neither force has a component in the direction of the other.***

(d) [5 pts] Calculate the work done on the car by **W** from the bottom to the top of the hill.
***1) [mg][d][cos(95°)] or 2) [–mg sin(5°)][d] or 3) [–mg][d sin(5°)].*** = $-3.03 \times 10^5$ J *[g =10]*

(e) [5 pts] Calculate the change of gravitational energy of the car from the bottom to the top of the hill.
$\Delta U = U_f - U_i = mgy_f - mgy_i = mg(y_f - y_i) = mg\,d\sin(5°) = +3.03 \times 10^5 J$ ***The fact that these are the same magnitude but opposite sign follows from the definition of potential energy.***

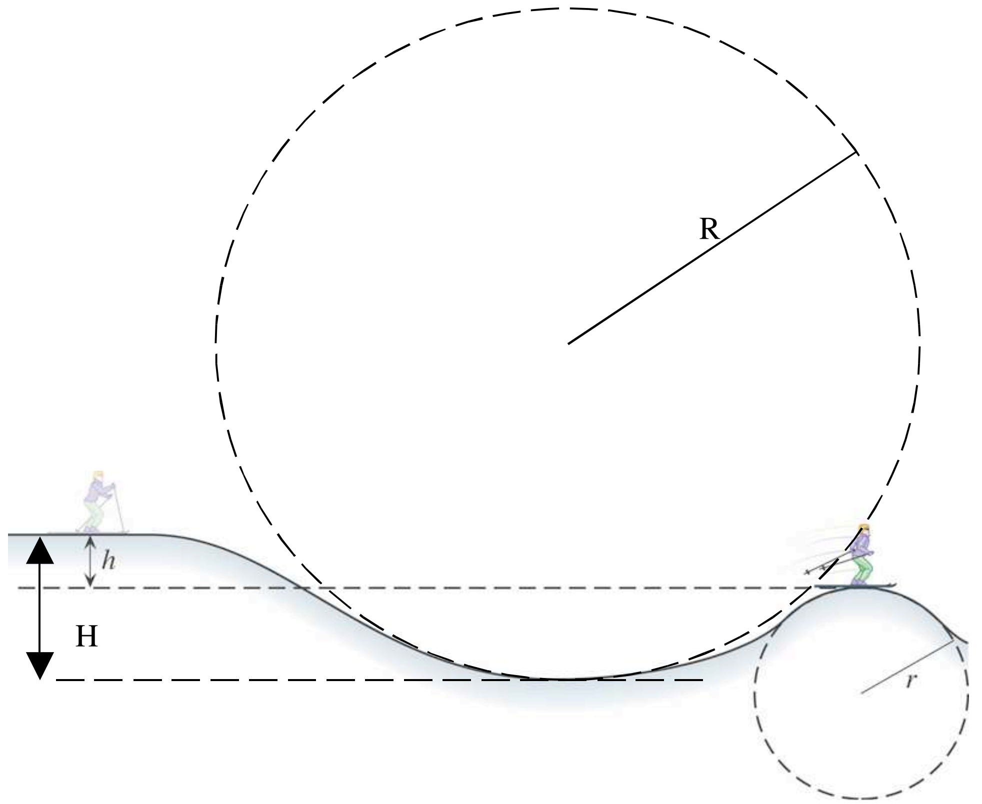
(e) [5 pts] If the kinetic energy of the car increased by +150 kJ from the bottom to the top of the hill, how much work was done by **F**, and what was the magnitude of **F**? ***Use either gravitational potential energy or work done by gravity, not both. The work done by rolling friction is –(524N)(290 m)= –1.52 x 10$^5$ J and by F is $W_F$ = +F(290 m), but you don't know F, and you are just asked for $W_F$ .***

***Using All work = Change of kinetic energy:*** $W_F + 0 - 1.52 \times 10^5 J - 3.03 \times 10^5 J = 1.50 \times 10^5 J$
***Using Nonconservative work = Change of (K+U):*** $W_F - 1.52 \times 10^5 J = 3.03 \times 10^5 J + 1.50 \times 10^5 J$

Either way, $W_F = 6.05 \times 10^5 J$

## Problem 2. Skiing on ice [30 pt]

The valley is a distance ***H* = 6m** below the starting point and has a radius of curvature ***R* = 20m**. The next hill is only a distance ***h* = 2m** below the original height, and has a radius of curvature ***r* = 5m**. The skier of mass ***m* = 60 kg** pushes off against a rock and starts with $v_o$ **= 3 m/s**. Take $g$ **= 10 m/s²**.




a) [6 pts] If there is no friction, calculate the speed of the skier at the bottom of the valley.

$K_i + U_i = K_f + U_f$

$\frac{1}{2} m v_o^2 + mgh_i = \frac{1}{2} m v_f^2 + mgh_f$

***Cancel m's, multiply thru by 2 etc***

$v_{valley}^2 = v_o^2 + 2g(h_i - h_f) = 3x3+2(10)(6-0)=129$, so $v_{valley} = (129)^{1/2}$ = **11.36 m/s**

b) [4 pts] If there is no friction, calculate the apparent weight of the skier at the bottom of the valley.

$W_{app} = m(g+a) = m(g+v^2/R) = 60(10+129/20) =$ ***987 N The acceleration a is toward the center of the circle which is upward, positive. g is the usual +10 Apparent weight is NOT the force of gravity. It is the magnitude of the force that supports the object (a normal force or whatever).***

c) [6 pts] If there is no friction, calculate the speed of the skier at the top of the next hill.

***You can start back at the beginning and go to the end. As in a),*** $v_{hill}^2 = 3x3+2(10)(6-4) = 49$ ***so*** $v_{hill}$ ***= 7 m/s***

d) [4 pts] If there is no friction, calculate the apparent weight of the skier at the top of the next hill.

***As in b), but noting that the center of the circle is downward,*** $W_{app} = m(g+a) = m(g-v^2/R) = 60(10-49/20) =$ ***453 N***

e) [6 pts] Assume that a snow surface would have a certain amount of friction, such that the skier just comes to rest at the top of the next hill. Calculate the work done by friction.

$$K_i + U_i + W_{nc} = K_f + U_f$$

$$\frac{1}{2} m v_o^2 + mgh_i + W_{friction} = \frac{1}{2} m v_f^2 + mgh_f$$

$$\frac{1}{2}(60)(3x3) + (60)(10)(6) + W_{friction} = 0 + (60)(10)(4)$$

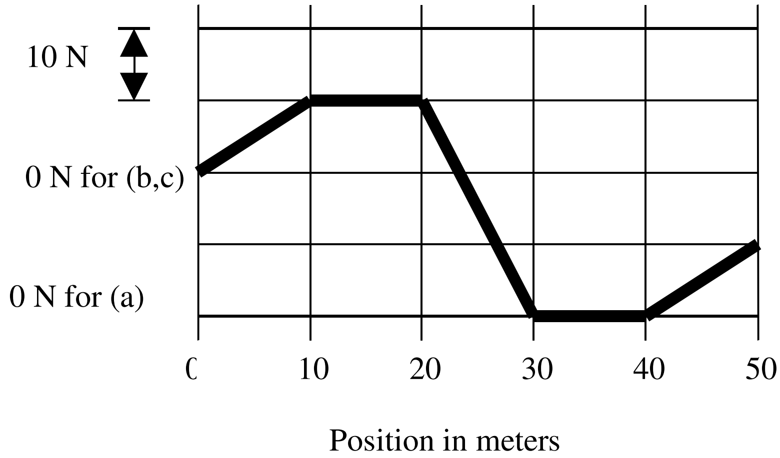
$$W_{friction} = 2400 - 2700 - 3600 = \mathbf{-3900\ J}$$

f) [4 pts] If the total distance traveled is D = 40 m, and the average normal force is something like 0.9 mg, estimate the coefficient of friction of the snow.

***Locally the force of kinetic friction is opposite to the displacement so the work is the total distance times the average force. The magnitude of the normal force varies throughout the motion so I estimate its average value for you. This lets you calculate the coefficient*** $\mu_k$.

$-3900\ J = W_{friction} = -\mu_k F_{N,avg} D = -\mu_k (0.9)(60)(10)(40)$ ***so*** $\mu_k = (3900\ J)/(21600\ J) =$ ***0.18***

**Problem 3. Force versus position graphs [20 points]**




Remember that the area under a graph of Force vs. Position gives the work done over that interval of position.

Consider three cases, and give the kinetic energy at each position grid line for each case in the table below.

Case (a): [10 pt] Assume that the bottom grid line is zero newtons, so that the force is always positive, and the scale goes from 0 N to 50 N. The particle starts from rest at position 0 meters.

Case (b) and (c): [10 pt total] Assume that the bottom grid line is –20 N so that the scale goes from –20 N to + 20 N.

(b) The particle starts with 500 J of kinetic energy at 0 m, headed in the positive direction.
(c) The particle starts with 100 J of kinetic energy at 0 m, headed in the positive direction.

| Case \ @ | 0 m | 10 m | 20 m | 30 m | 40 m | 50 m |
|---|---|---|---|---|---|---|
| (a) [2 ea] | 0 J | 250 J | 550 J | 700 J | 700 J | 750 J |
| (b) [1 ea] | 500 J | 550 J | 650 J | 600 J | 400 J | 250 J |
| (c) [1 ea] | 100 J | 150 J | 250 J | 200 J | 0 J | Never gets here |

***The area under a segment of force vs. position curve is the Work done by the force during that segment of motion. By the work energy theorem, it increases or decreases the kinetic energy that you had at the beginning of the segment. The area of one box under the curve is 10 newtons times 10 meters = 100 joules. If the force is negative, that is negative work. The hardest area to determine is cases b) and c) where the force goes from +10N to –20N in 10 meters. The little triangles at the top and bottom cancel, and the box in the middle is divided in half.***

***In case a) the force is always positive, so the work is always positive, and the kinetic energy always increases (except over the segment where F is zero).***

***In case b) there is enough initial kinetic energy to keep going all the way to the end***

***In case c) there is just enough initial kinetic energy to reach 40 m, but kinetic energy can't go negative since it involves the square of the speed. At that point it is at rest, and there is a negative force acting. Hence that is the turning point where it stops and heads back toward the beginning. It never gets beyond 40 m.***

Problem 4. Power – How fast, etc? [25 points]

MINI Cooper Coupe, 2546 lbs curb weight, maximum power 118 hp, 40 / 32 miles per gallon highway / city

Ferrari 599 GTB Fiorano, 3722 lbs curb weight, 620 hp, 15 / 11 miles per gallon highway / city

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(a) [6 pts] Convert the maximum powers to kW.

***(118 hp)(746 W/ 1 hp) = 88 kW***

***(620 hp)(746 W/ 1 hp) = 463 kW***

(b) [5 pts] The Ferrari has roughly 5 times the maximum power. At high speeds, air resistance is the most important limiting force. The force of air resistance F is proportional to $v^2$ and therefore the power required to overcome air resistance (F v) is proportional to $v^3$. Assume that the form factors of the cars are similar, so that they have the same constants of proportionality. Based on this information, estimate how many times faster a Ferrari can go than a MINI.

***For each car, $P = cv^3$ where the constant c is assumed to be about the same. As a result $(P_2 / P_1) = (v_2 / v_1)^3$ or $(P_2 / P_1)^{1/3} = (v_2 / v_1) = (620/118)^{1/3} =$ 1.74 times as fast***

(c) [5 pts] In the mountains with twisty unbanked turns and cliffs at the edge of the road, relying only on friction, which car would have the larger safe speed on the tight turns? Explain your reasoning.

***They should have similar safe speeds. Friction is proportional to normal force which is proportional to mass. Similarly "ma" is proportional to mass. Thus F = ma reduces to $v^2/R = \mu_k g$. The coefficient of friction of rubber on dry roads doesn't depend much on tire design, so the speeds for a given turn should be comparable. The Ferrari has the advantage of a lower center of gravity and wider base of support, so it is more stable against rollover, but you didn't have the physics of static equilibrium yet.***

(c) To establish the EPA miles per gallon ratings, both cars are tested the same way, based on programs of speed appropriate for highway driving and for city driving. Compared to one Ferrari, approximately how many MINI's does it take to put the same amount of $CO_2$ (a greenhouse gas indicted in global warming) into the atmosphere **… $CO_2$ is proportional to gasoline used. Because the question involves miles per gallon, we should be thinking of both cars driving the same distance. In a) and b) they have to have the same speed so they also go the same time. They don't use their full power, so that is irrelevant. Their "measured" miles per gallon ratio is inverted to get gallons/mile ratio, which shows that the Ferrari uses more gallons per mile. That ratio is the number of MINI's , approximately. In c) they have very different speeds and use their full power, which is assumed to be proportional to gasoline consumption. Gallons per hour is proportional to power, but to go a fixed distance the Ferrari gets there in less time, so you divide by the velocity.**

i) [3 pts] driving down Commonwealth Avenue at legal speeds? ***Use city values of mpg.***

***Ferrari gpm/ MINI gpm = 32/11 = 2.9*** *approx 3 MINI's*

ii) [3 pts] driving between Boston and San Francisco at legal speeds? ***Use highway values of mpg.***

***Ferrari gpm/ MINI gpm = 40/15 = 2.7 approx 3 MINI's***

iii) [4 pts] driving across the desert at top speed? Explain your reasoning.

***(Ferrari max power / MINI max power)/(Ferrari max speed / MINI max speed) = (620/118)/1.74 = 3.02 = approx 3 MINI's***