Centripetal Motion and Friction

In your work in Tutorial, please remember to

1) Draw pictures and/or diagrams of the situation before beginning a solution
2) Write down the known quantities and the desired unknown(s)
3) Start your solution by writing down a fundamental relation or equation that you think is relevant.

Equations and Relations:

Motion with Constant Acceleration:

$v = v_0 + at$

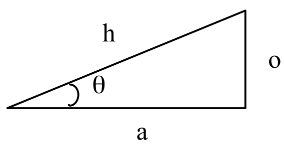
$x = x_0 + v_0 t + \frac{1}{2}at^2$

$v^2 = v_0^2 + 2a(x - x_0)$

$g = 9.80 m/s^2 \approx 10 m/s^2$

Trigonometry:

sin θ = o/h
cos θ = a/h
tan θ = o/a
$a^2 + o^2 = h^2$



Newton's Laws:

- 1st: An object will stay at rest or in motion with constant velocity unless acted on by a net force.
- 2nd: $\Sigma F_x = ma_x$, $\Sigma F_y = ma_y$
- 3rd: Forces come in pairs. If A exerts a force on B, then B exerts a force on A with the same magnitude but in the opposite direction.

Weight: $W = mg$

Friction: $f_s \le \mu_s N$, $f_k = \mu_k N$

Circular Motion: $a_c = v^2 / r$

1. In each case below make a sketch, indicating with arrows the direction of the ***acceleration*** and the ***velocity*** of the object. Then make a separate free-body diagram (FBD) for the object. Ignore air resistance.

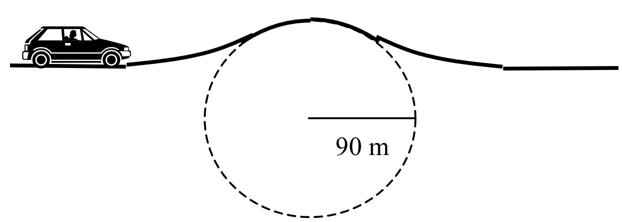
   a) A beer glass is on top of a record turntable, near the edge. The turntable is rotating, and the glass is moving along with the rotating turntable at *constant speed*, without slipping.

   b) A car is moving at a constant speed around a curve on a horizontal road.

   c) The Earth is orbiting the Sun in an approximately circular orbit with constant speed.

Check Point

2. A car of mass 1000 kg is driving along the highway at a constant speed of 30 m/s when it encounters a small hill in the road. The hill has a radius of curvature of 90 meters, as shown in the sketch to the right. The car maintains a constant speed of 30 m/s as it drives over the bump. [Reminder: solve this problem using symbols first, and only put in numbers as your last step.]



   a) Make a FBD for the car when it is at the exact top of the hill.
   b) Write out Newton's 2nd Law in the vertical direction. (Hint: At the top of the hill, what is the car's acceleration?) Then determine the normal force acting on the car when it is at the top of the hill, assuming that $g = 10\ m/s^2$. Describe in words what has happened to this car.
   c) Is it safe to assume that the normal force on an object is always equal to its weight? What is the best way to determine the normal force acting on something?
   d) Now make a FBD for the car at some arbitrary point along its motion on the curved hill. Using Newton's 2nd Law, determine the conditions under which the car will leave the road. Is the top of the hill the most likely place for this to happen?

Check Point

3. You are driving your car (of mass M) at constant speed around a curve on a horizontal road.

   a) Make a FBD for the car, and then write out Newton's 2nd Law in both the horizontal and the vertical direction.
   b) Determine the normal force acting on the car, in terms of symbols.
   c) If the radius of curvature of the road is 40 meters and the road is dry concrete, find the maximum speed the car can go without skidding. You can take $g = 10\ m/s^2$. (Hint: the coefficient of static friction between rubber and dry concrete is approximately 1.0.)
   d) Repeat part d) but assume that the road is wet concrete. The coefficient of static friction between rubber and wet concrete is approximately 0.7.

Check Point

4. A ball of mass M is attached to a string, and you swing the ball in a *vertical circle* of radius R at constant speed v.

   a) Where in the motion of the ball do you expect to feel the greatest tension in the string?
   b) Find the tension in the string at 3 different places: the top of the circle, the bottom of the circle, and the side of the circle. Your answers should be symbolic.
   c) After you've solved the problem symbolically, you can substitute M = ½ kg, R = 1m and v = 6 m/s to get numerical answers.

Check Point

5. A boy runs and grabs a rope swing, and then swings out over the water.

   a) Make a FBD for the boy at some arbitrary point in his motion (neither the beginning nor the end of his swinging motion).
   b) Use Newton's 2^{nd} Law to determine an algebraic expression for the tension in the rope.
   c) Where in the boy's swing will the tension in the rope be the greatest?

Check Point

Additional Questions

1) When you go around a curve in your car, you feel "thrown" to the outside of the car. What is causing this feeling?

2) A washing machine uses a spin cycle to rapidly remove large quantities of water from wet clothes. Give a physics description (i.e. in terms of forces, acceleration and Newton's Laws) of how this works.

3) Is it possible to whirl a ball (attached to a string) in a circle in the air with the string *horizontal*? If you're not sure, try making a FBD.