Physics Workshop **Mechanics Tutorial** **Number 3**

Vectors & 2-D Kinematics

In your Tutorial work, please remember to

1) Draw pictures and/or diagrams of the situation before beginning a solution
2) Write down the known quantities and the desired unknown(s)
3) Start your solution by writing down a fundamental relation or equation that you think is relevant.

<u>Motion with Constant Acceleration:</u>

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

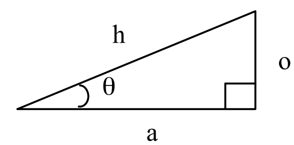
$$v^2 = v_0^{\,2} + 2a(x - x_0)$$

$$\bar{v} = (v + v_0)/2$$

$$g = 9.80 m/s^2 \approx 10 m/s^2$$

<u>Trigonometry:</u>

sin θ = o/h
cos θ = a/h
tan θ = o/a
$a^2 + o^2 = h^2$



1. A regulation-size billiards table has a playing area that is 100 inches long (y-direction) and 50 inches wide (x-direction). The target "pockets" are at the middle and ends of the long sides. [Use inches and inches per second.]
(a) At what point on the surface is(are) the farthest pocket(s) as close as possible?
(b) Take the origin of your coordinates to be the center of the playing area.
  i. With respect to this origin, what are the (x,y) position-coordinates of each of the six pockets?
  ii. What are the possible distances to the various pockets [without bouncing off a wall!] ?
(c) The ball is launched from the center with an initial speed v_i such that it reaches a pocket in 1.4 sec.
  i. What are the possible speeds (considering the distances to the various pockets) ?
  ii. What are the possible x-components of velocity [considering the various pockets] ?
  iii. What are the possible y-components of velocity [considering the various pockets] ?
  iv. What are the possible magnitudes of the velocity [considering the components in (ii) and (iii)] ?

2. A ballplayer in the dugout throws a ball for use in practice from ground level at an angle θ with respect to a level field. It hits the ground 8 meters away, 2 seconds later. Neglect air resistance.
(a) What was the horizontal component of velocity of the object?
(b) What was the initial vertical component of velocity of the object?
(c) What was the initial speed of the ball, and the angle θ?
(c) Suppose that for the next ball that he throws out, these two components are exchanged.
  i. What is the relationship of the new launch angle to the old one?
  ii. What is the hang time in the air before the ball hits?
  iii. How far away does the ball hit the ground?
(d) Finally, the last ball that he throws out has twice the velocity components of the first case.
  i. How much time is the ball in the air?
  ii. How far away does it hit the ground.?

3. Shawn and Maria are playing darts. Shawn is a beginner, and he starts with his first dart at exactly the same height as the bullseye, aims it *horizontally* (right at the bullseye) and throws it. He releases the dart with speed 10 m/s, a horizontal distance of 2 m from the dartboard. Use g = 10 m/s^2.

(a) Make a sketch of this situation, including a coordinate system. Then write down the known quantities in terms of your chosen coordinate system.
(b) Find the time that Shawn's dart is in the air before it hits the dartboard.
(c) How far below the bullseye does Shawn's dart hit?

(d) Now Maria throws. She also starts the dart level with the bullseye and gives it an initial horizontal velocity of 10 m/s, but in addition she gives the dart just enough initial vertical velocity so that she hits the bullseye perfectly. How much vertical velocity did she give it?
(e) Express the initial velocity of Maria's dart as a magnitude and direction.

4. Shawn and Maria are throwing darts at a bullseye attached to the wall of a "recreation center". They are feeling tipsy, because the rec center was built with its floor parallel to a 30º hillside. The wall and bullseye are perpendicular to the floor. The acceleration due to gravity is down, and therefore has components parallel to the floor and to the wall. The inner ears of Shawn and Maria tell them to stand upward, in the direction opposite to gravity, and they obey.

(a) Shawn stands on a table 2 m from the bull's eye, which puts his dart release at the same vertical height as the bulls-eye. He then does exactly what Maria did before, in problem 2. Draw a picture of the situation. Despite the tilting floor and wall, the initial position of the dart relative to the bullseye, the elements of Maria's strategy, and gravity are all still stated in terms of horizontal and vertical. What coordinate system is best to use? Without solving the problem (again), does the dart hit the bullseye?

(b) Maria suspects that she is somewhat incapacitated, because she feels strange and sees that she is leaning at a 30° angle to the floor. Therefore she carefully calculates and recalculates all the steps in her method of problem two. She releases the dart from a position at the same height above the floor as the bullseye, two meters from it. By dropping small stones from various heights and using a meter stick and watch (compensating correctly for reaction time) she determines that an object in free fall in this crazy rec center has an acceleration of 5.00 m/s^2 away from the wall, and 8.66 m/s^2 parallel to the wall.. Using the rec center coordinate system, with accelerations in both directions, she calculates the initial components of velocity v_{floor} and v_{wall} that will cause the dart to reach the bullseye in 0.2 seconds. At what speed and angle with respect to the floor should she throw the dart?

5. In a projectile problem the acceleration components are usually $a_x = 0$ and $a_y = g$. There are four final values at time t: $x(t)$, $v_x(t)$, $y(t)$, and $v_y(t)$. From these, other quantities such as distance, speed and direction can be determined (or vice versa). There are also four initial values corresponding to these quantities at t = 0. All these quantities are related by four independent equations. Thus if four are specified, then the other four can be found. Translate the following words into a value for one or more of the quantities.

(a) The ball reaches its maximum height:

(b) The ball is at the same height it was released at:

(c) The bat hits the ball at a height of 1 m.

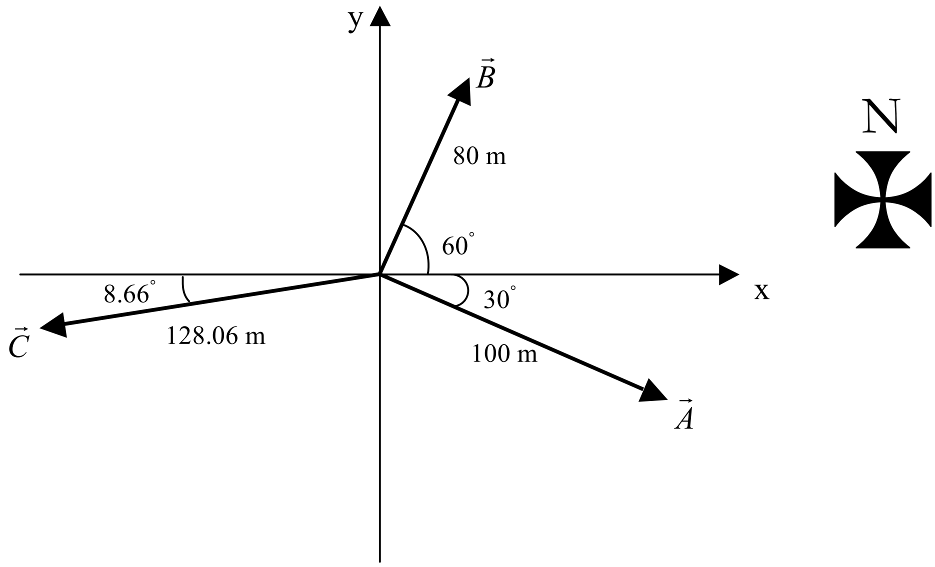
(d) The ball clears a 10 m fence 120 m away from home plate.

(e) The ball was hit at a speed of 24 m/s and at an angle of 37° above the horizontal.

(f) The horizontal range of the hit was 50 m.

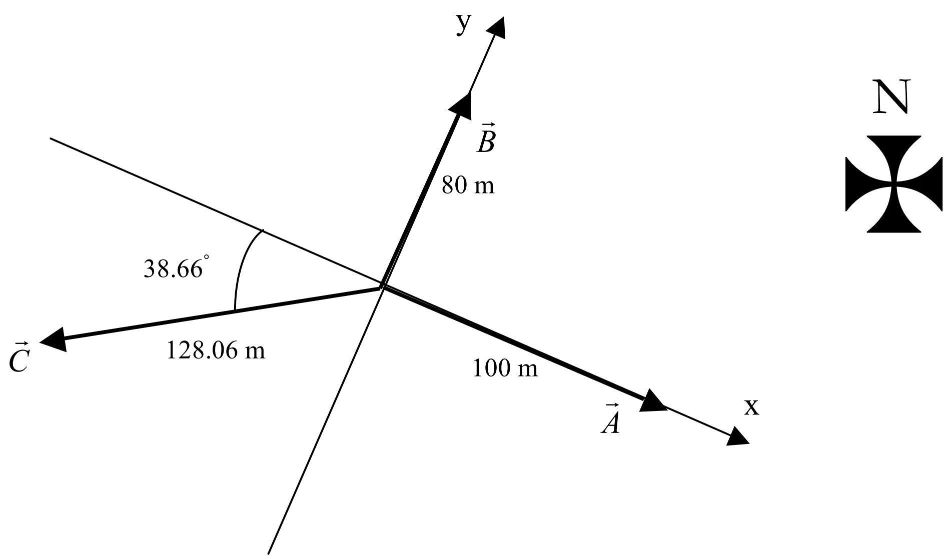
(g) A basket ball player raises his center of mass by less than two meters during a stuff-it-in-the-basket jump.

4. A treasure map drawn by a physicist is shown below. With the map is a note that states the following: "*Start at the big oak tree, walk along the vector sum* $\vec{A} + \vec{B} + \vec{C}$ *and dig*."



(a) Determine where to dig, using both graphical and algebraic methods.

(b) Imagine that the author of the map had used a different coordinate system, rotated clockwise by 30° relative to the first one, as shown below. Add the vectors algebraically using this new map. Are your results any different? Was one map easier to use than the other?



Check Point

Additional Questions

1) Describe the features of a convenient coordinate system for a given physics problem.
2) How do you determine whether one particular component of a vector is positive or negative in a given coordinate system? Describe a method in words that someone with no mathematical background could successfully use.