Quantum Glassiness

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cond-mat/0404182

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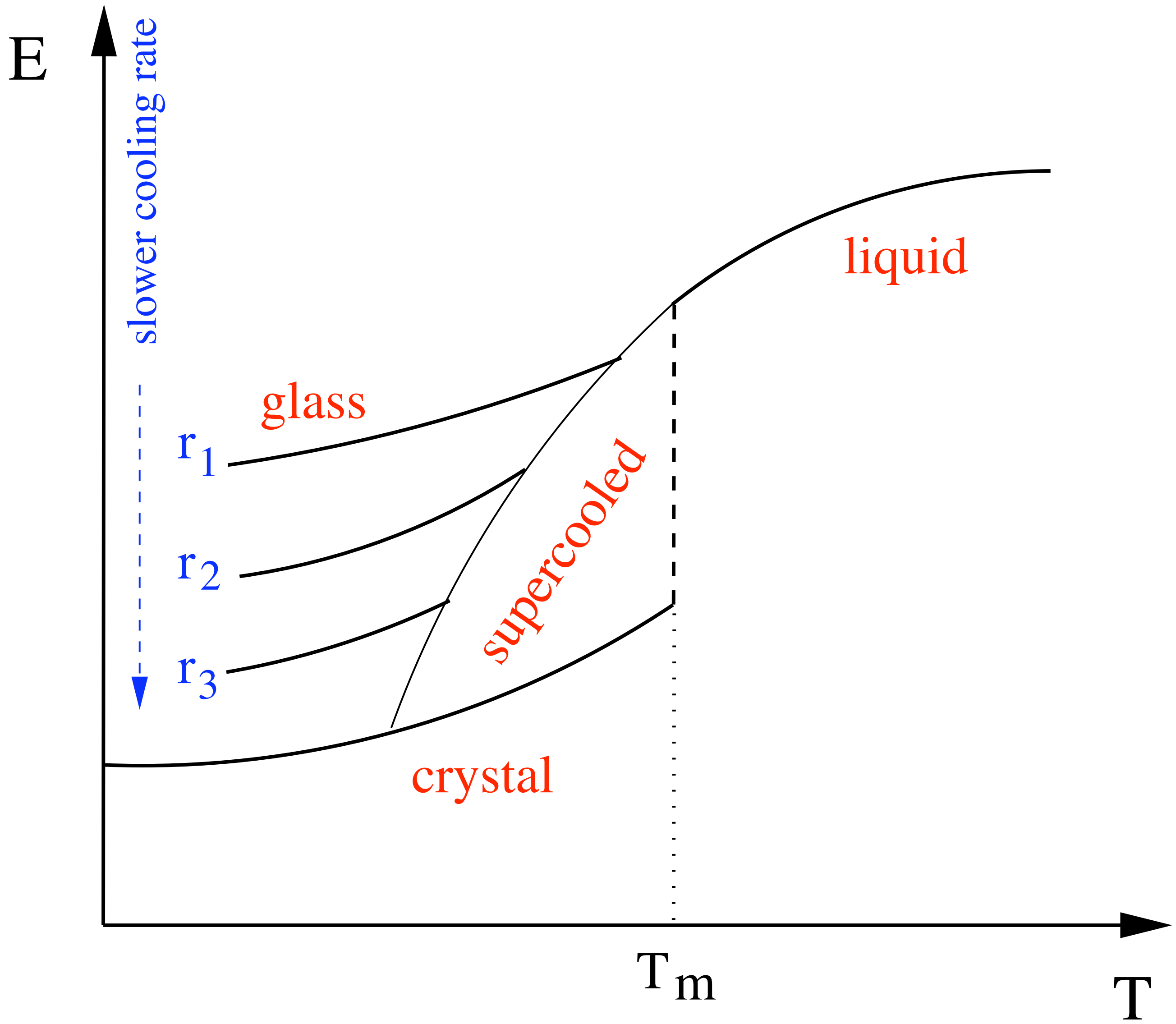
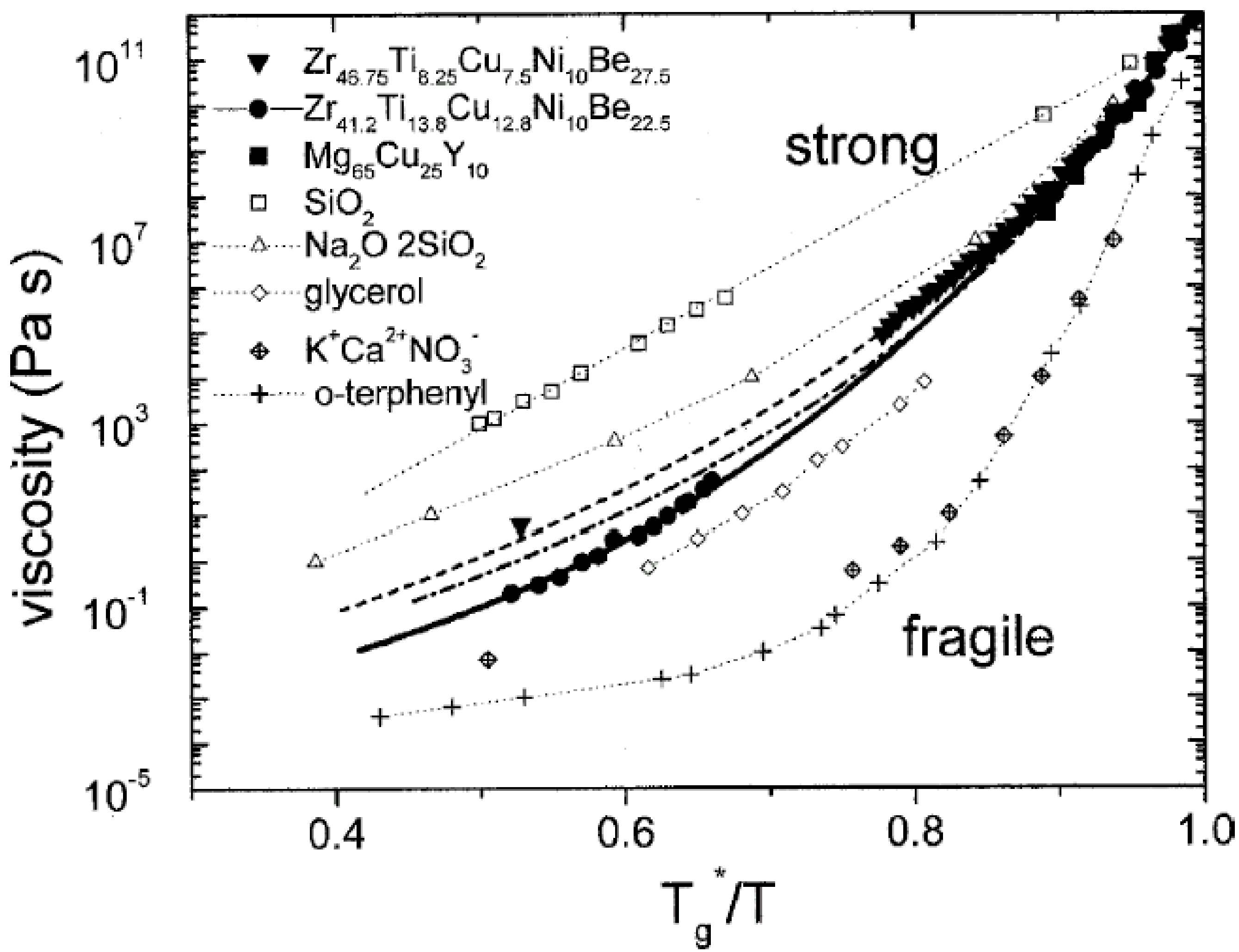
Classical glassiness



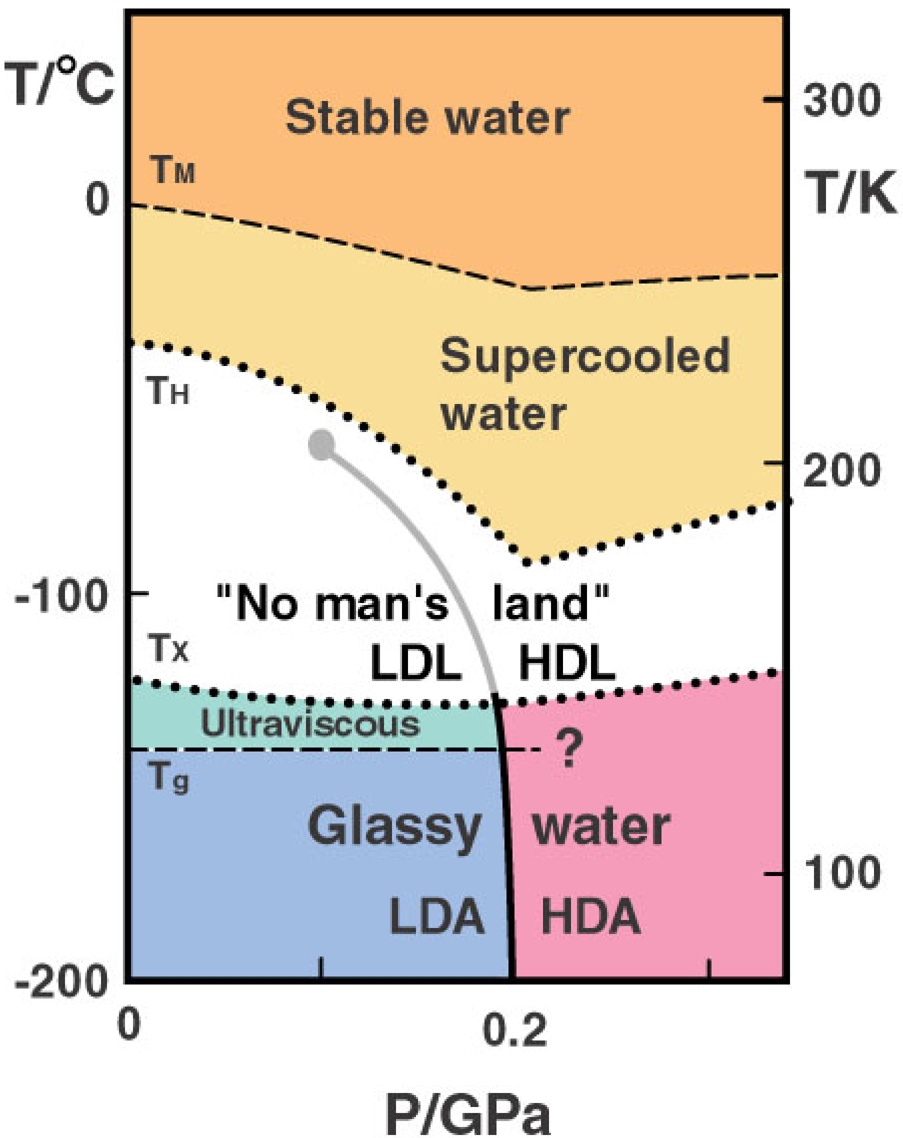
Classical glassiness

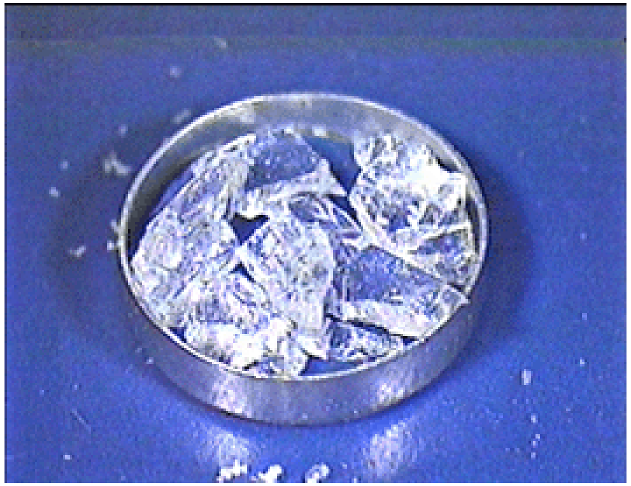
viscosity



Source: JOM, 52 (7) (2000)

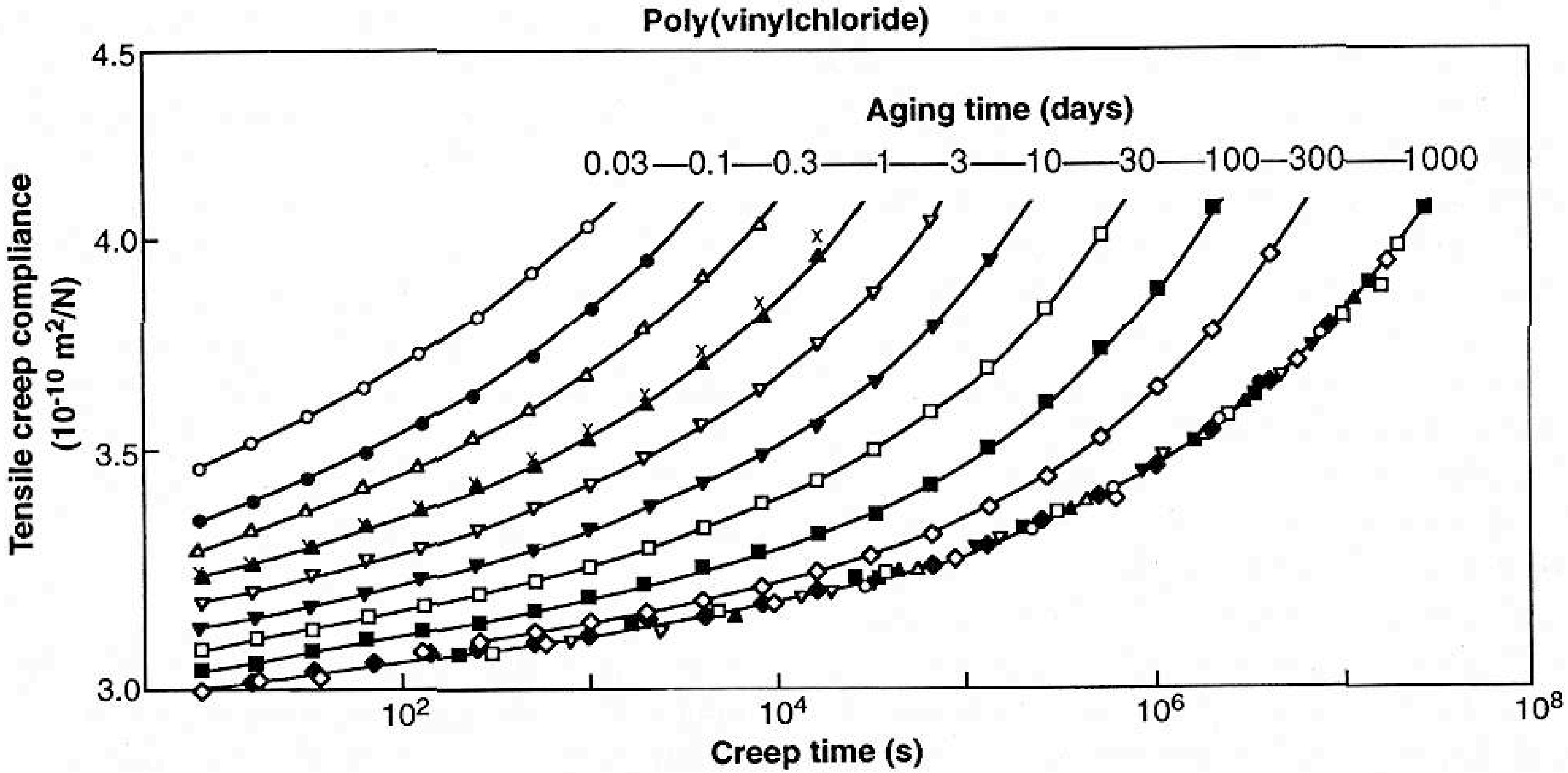
Glassy H_2O





Source: O. Mishima and H. E. Stanley's groups; Nature (1998)

Physical aging



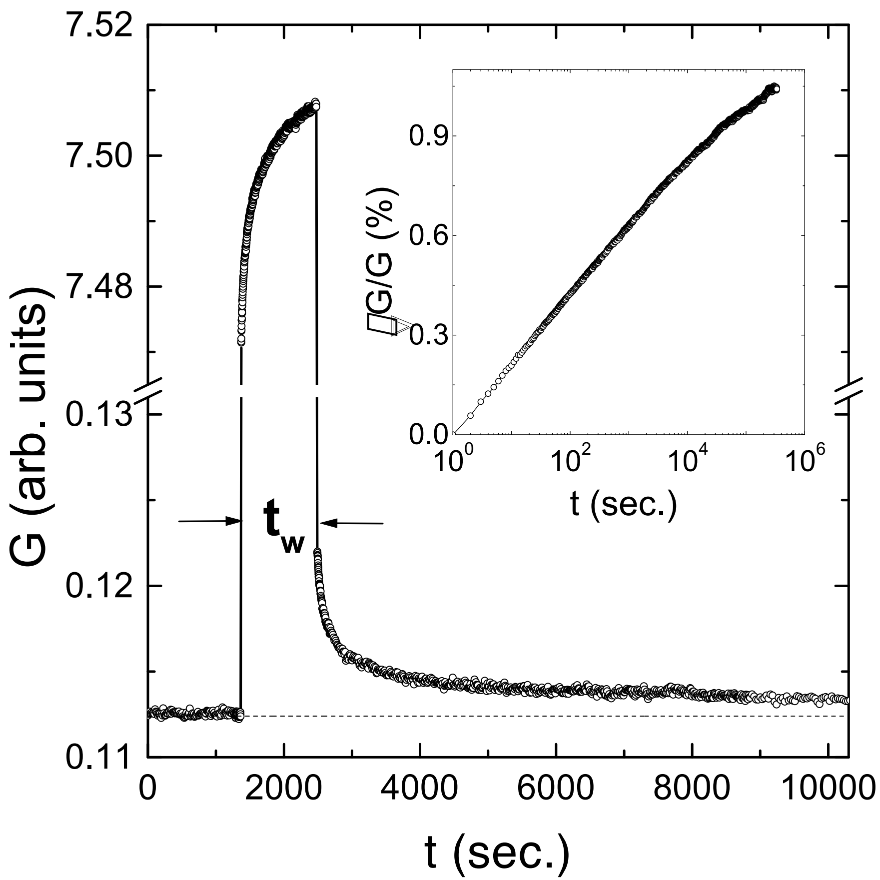
Source: L.C.E. Struik, *Physical aging in amorphous polymers and other materials*, Elsevier, Amsterdam (1978)

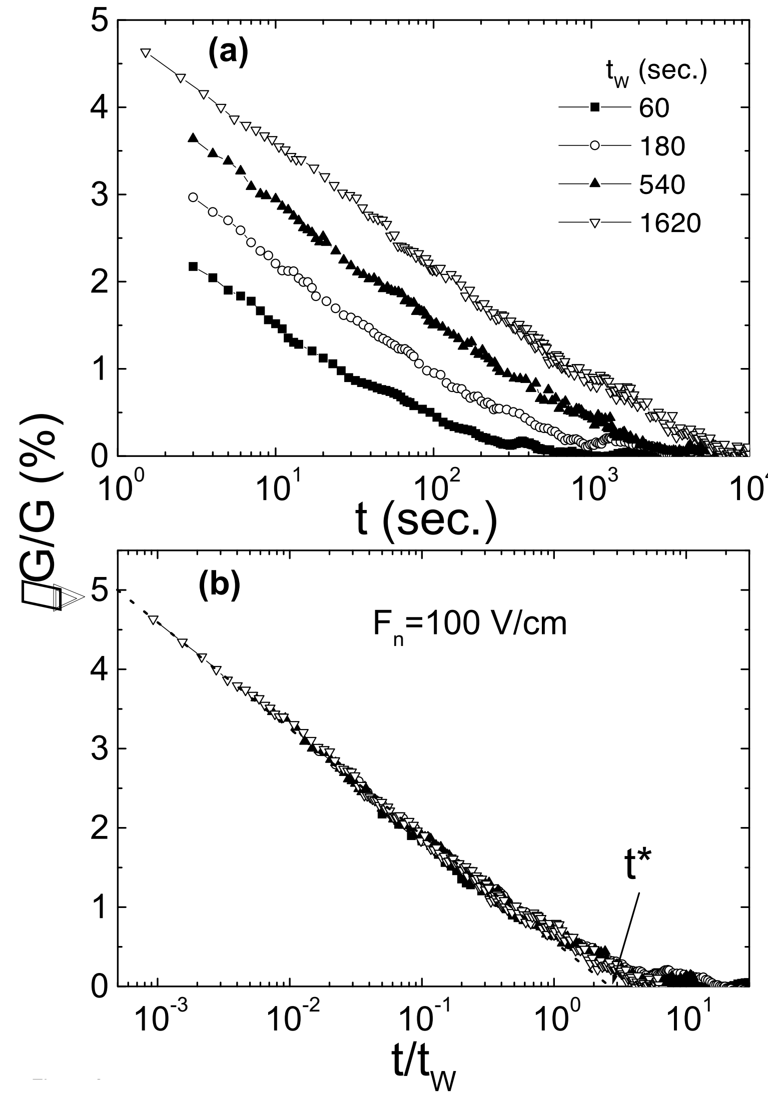
Physical aging II

2D electron glass

Orlyanchik & Ovadyahu, PRL (2004)

2D thin films of crystalline In_2O_{3-x}





Source: Orlyanchik & Ovadyahu, PRL (2004)

Quantum glassy systems

disordered systems
eg. quantum spin glasses

Bray & Moore, J. Phys. C (1980)
Read, Sachdev, and Ye, PRB (1995)

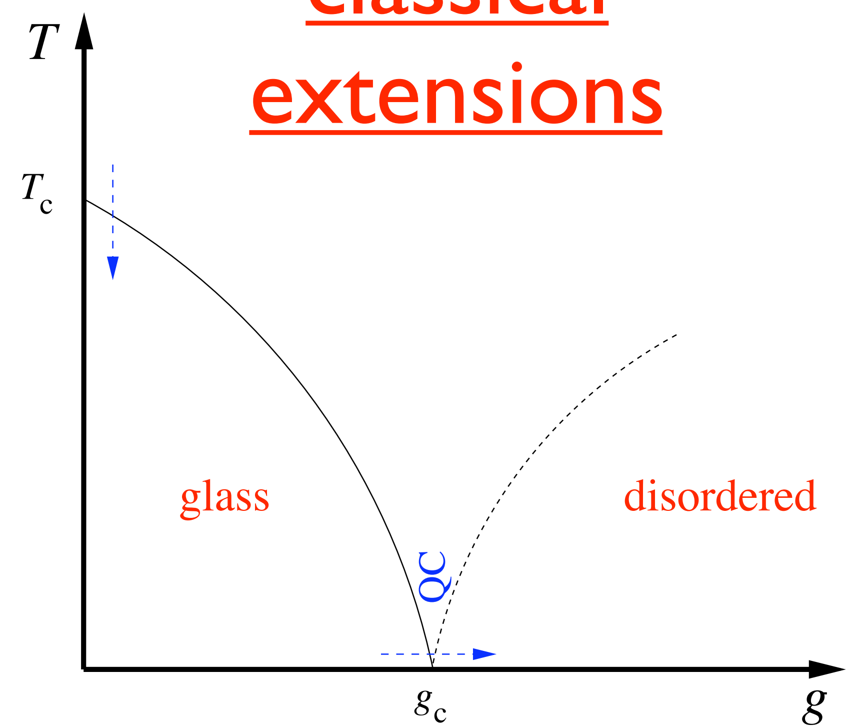
frustrated systems
eg. I frustrated Josephson junctions with long-range interactions

Kagan, Feigel'man, and Ioffe, ZETF/JETP (1999)

eg. II self-generated mean-field glasses

Westfahl, Schmalian, and Wolynes, PRB (2003)

classical extensions



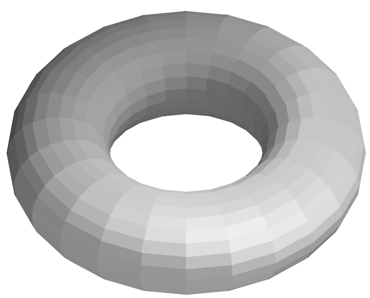
Pure breed quantum glasses

strongly correlated systems with
topological order and fractionalization

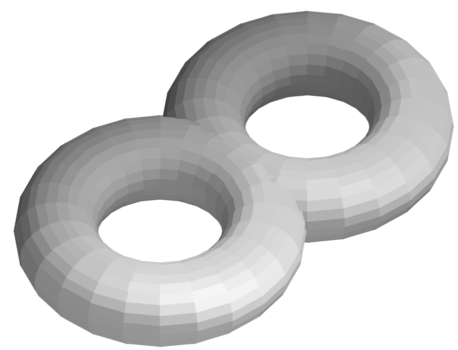
What is topological order?
eg. fractional quantum Hall effect

Wen, Int. J. Mod. Phys. B (1991), Adv. Phys. (1995)

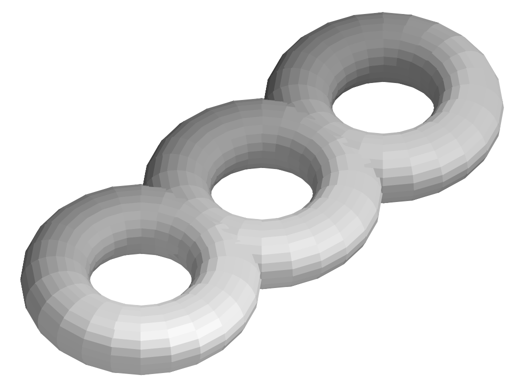
$$\nu = 1/3 \Rightarrow N_{\rm GS} = 3^g$$



$N_{\rm GS} = 3^1$



$N_{\rm GS} = 3^2$



$N_{\rm GS} = 3^3$

Interestingly enough, strong correlations that can lead to these exotic quantum spectral properties can in some instances also impose kinetic constraints, similar to those studied in the context of classical glass formers.

Why solvable examples are useful?

Classical glasses can be efficiently simulated in a computer; but real time simulation of a quantum system is doomed by oscillating phases (as bad as, if not worse, than the fermion sign problem)!

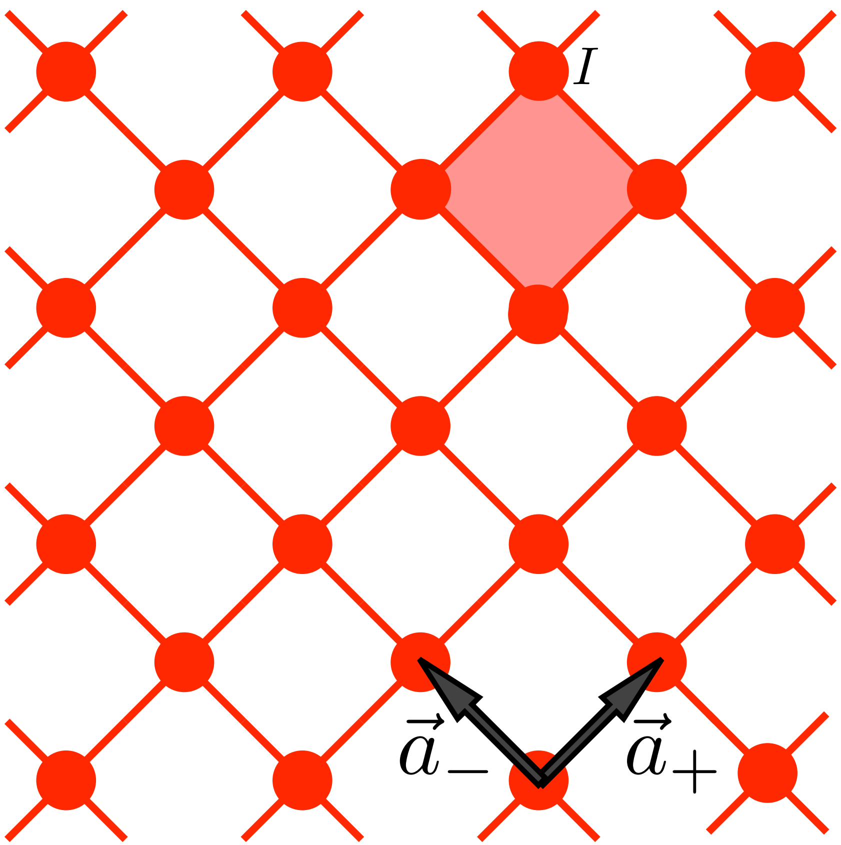
Even a quantum computer does not help; quantum computers are good for unitary evolution. One needs a "quantum supercomputer", with many qubits dedicated to simulate the bath.

Solvable toy model can show unambiguously and without arbitrary or questionable approximations that there are quantum many body systems without disorder and with only local interactions that are incapable of reaching their quantum ground states.

2D example
(not glassy yet)

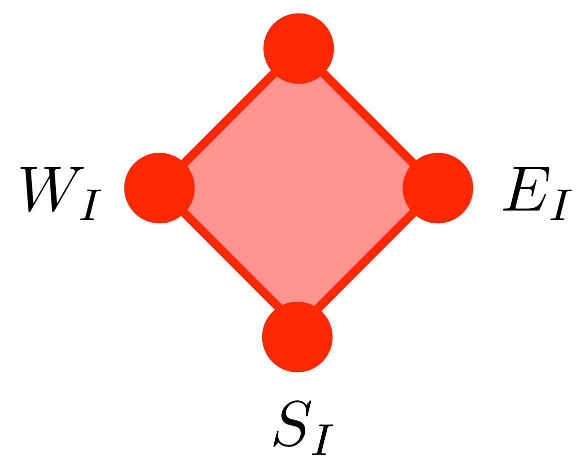
topological order for quantum computing

Kitaev, Ann. Phys. (2003) - quant-phys/97
Wen, PRL (2003)

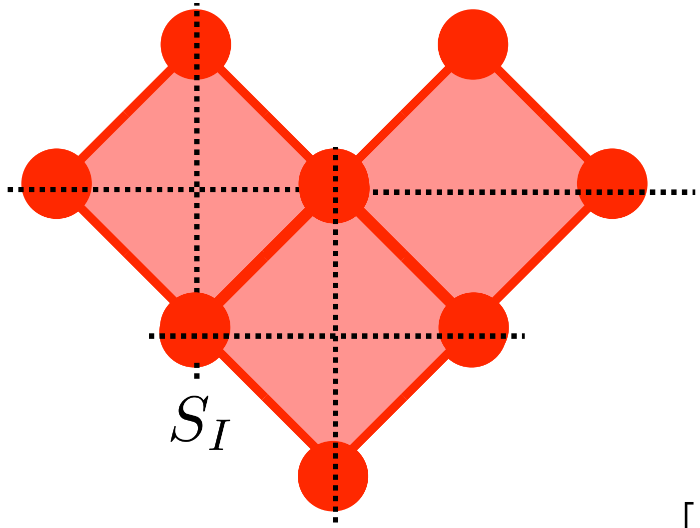


$$\vec{R} = i\vec{a}_+ + j\vec{a}_- \qquad I \equiv (i,j)$$

$$N_I = I$$



$$H = -\frac{h}{2}\sum_I \sigma^y_{N_I}\sigma^x_{W_I}\sigma^y_{S_I}\sigma^x_{E_I}$$



$$\hat{P}_I = \sigma^y_{N_I} \sigma^x_{W_I} \sigma^y_{S_I} \sigma^x_{E_I}$$

$$H = -\frac{h}{2} \sum_I \hat{P}_I$$

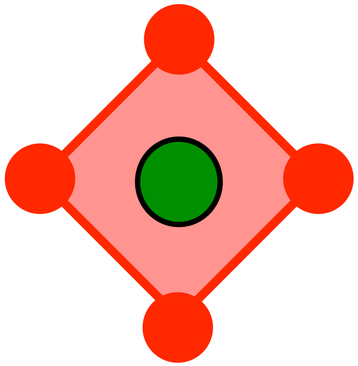
$$[\hat{P}_I, \hat{P}_{I'}] = 0$$

$$\hat{P}_I^2 = 1 \Rightarrow \text{eigenvalues} \;\; P_I = \pm 1$$

Same spectrum as free spins in a magnetic field

However, $\langle \sigma^x_I \rangle = \langle \sigma^y_I \rangle = \langle \sigma^z_I \rangle = 0$

Plaquettes with $P_I = -1$: defects



Ground state degeneracy

ground state: $P_I = +1 \text{ for all } I$

on a torus:

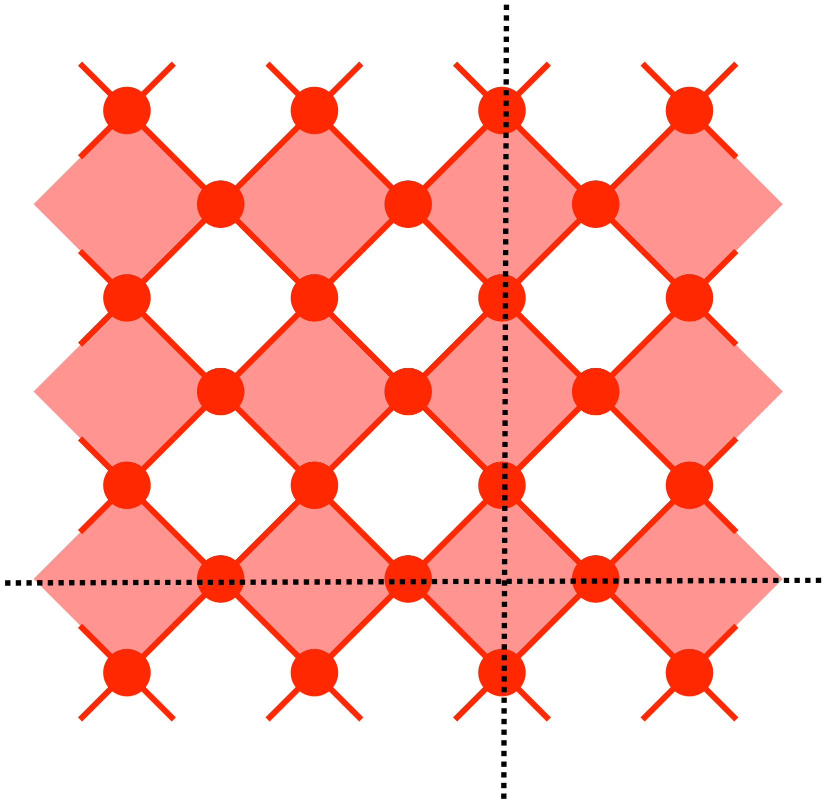
$\mathcal{N}$ spins $\Rightarrow 2^{\mathcal{N}}$ states

$\mathcal{N}$ plaquettes $\Rightarrow 2^{\mathcal{N}}$ states? ***NO***

two constraints:

$$\prod_{I\in A} \hat{P}_I = \prod_{I\in B} \hat{P}_I = 1$$

$$\hat{T}_x = \prod_{I\in\mathcal{L}_x} \sigma_I^y$$



$$\hat{T}_y = \prod_{I\in\mathcal{L}_y} \sigma_I^x$$

$\mathcal{N}$ plaquettes $\Rightarrow 2^{\mathcal{N}-2} \times 2^2$ states

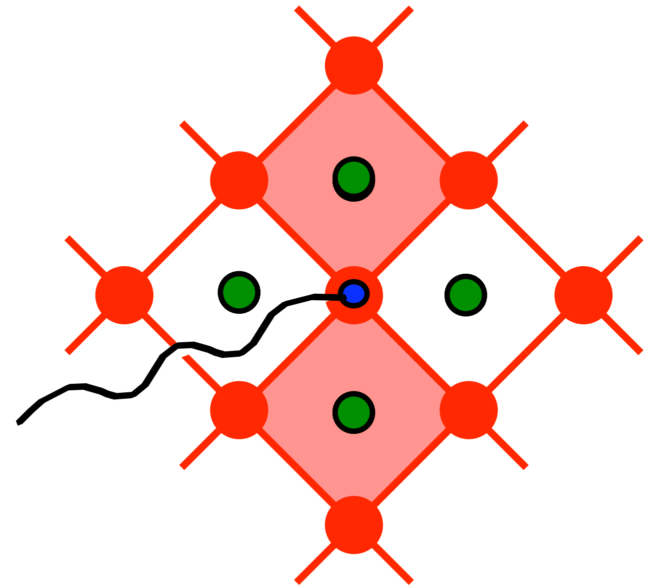
4 ground states

$$T_x = \pm 1, T_y = \pm 1$$

Is the ground state reached?

bath of quantum oscillators;
acts on *physical* degrees of freedom

Caldeira & Leggett, Ann. Phys. (1983)
Feynman & Vernon, Ann. Phys. (1963)

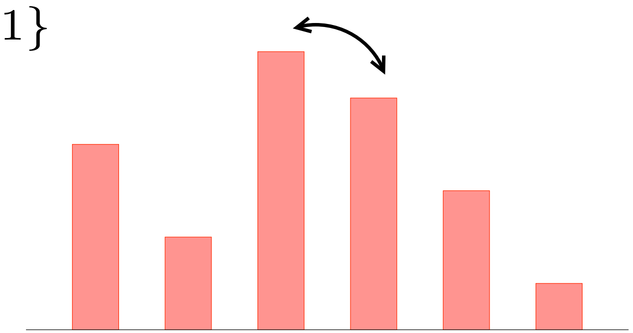


$$\hat{\mathcal{H}} = \hat{H} + \hat{H}_{\text{bath}} + \hat{H}_{\text{spin/bath}}$$

$$\hat{H}_{\text{spin/bath}} = \sum_{I,\alpha} g_\alpha \sum_{\lambda} \left(a^{\alpha}_{\lambda,I} + {a^{\alpha}_{\lambda,I}}^{\dagger} \right)$$

defects cannot simply be annihilated; plaquettes are flipped in multiplets

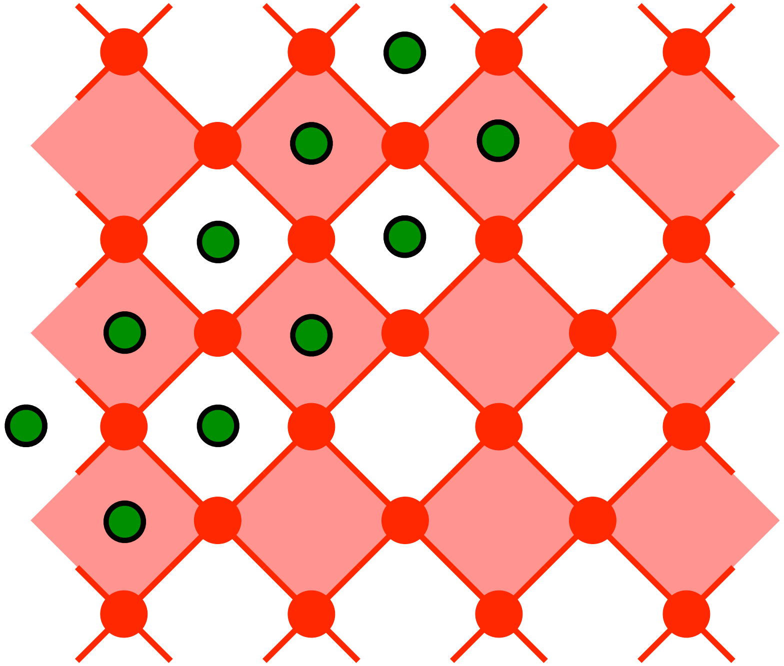
$$|\Psi(t)\rangle = \sum_{\{T_a, O_I = \pm 1\}} \Gamma_{\{T_a,O_I\}}(t)\ |\{T_a, O_I\}\rangle \otimes |\Upsilon_{\{T_a,O_I\}}(t)\rangle$$



Is the ground state reached?

defects must go away
equilibrium concentration: $c \approx e^{-h/T}$

defects cannot be annihilated;
must be recombined

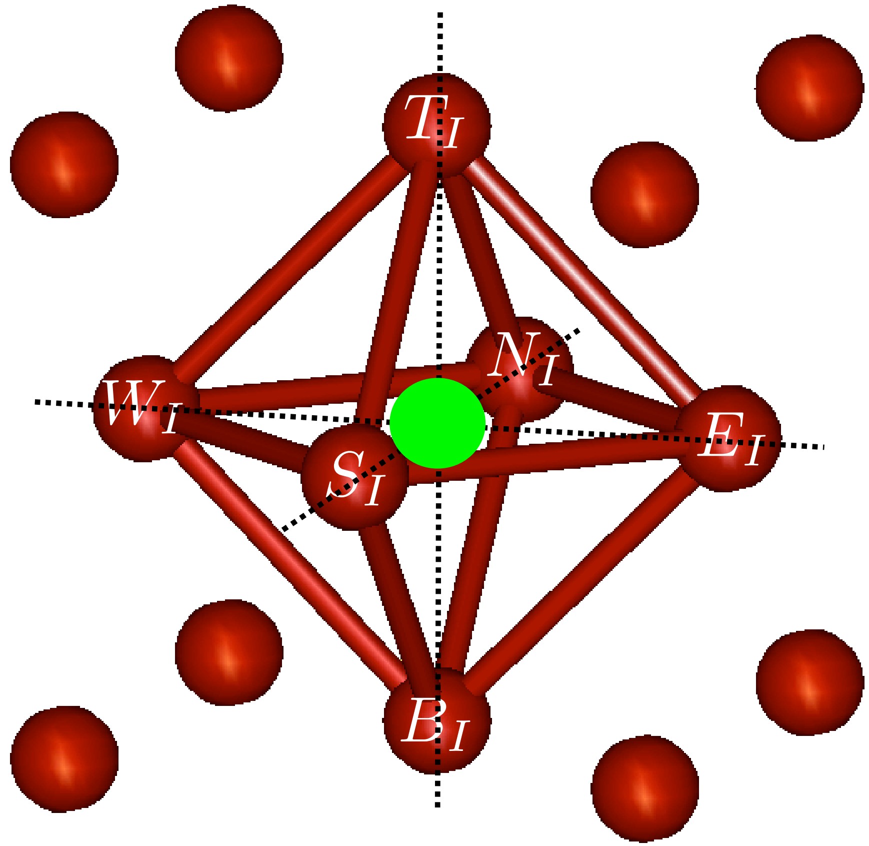


$\sigma_I^{x,y} \Rightarrow$ simple defect diffusion (escapes glassiness)

$\sigma_I^z \Rightarrow$ activated diffusion $\quad t_{\text{seq.}} \sim \tau_0 \exp(2h/T)$ (Arrhenius law)

equivalent to classical glass model by Garrahan & Chandler, PNAS (2003)
Buhot & Garrahan, PRL (2002)

3D strong glass model

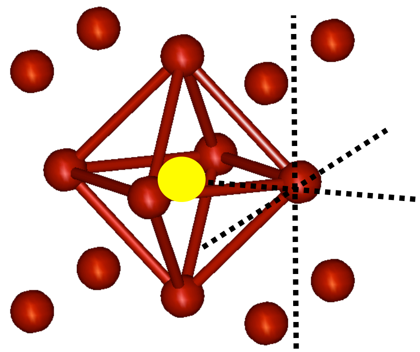


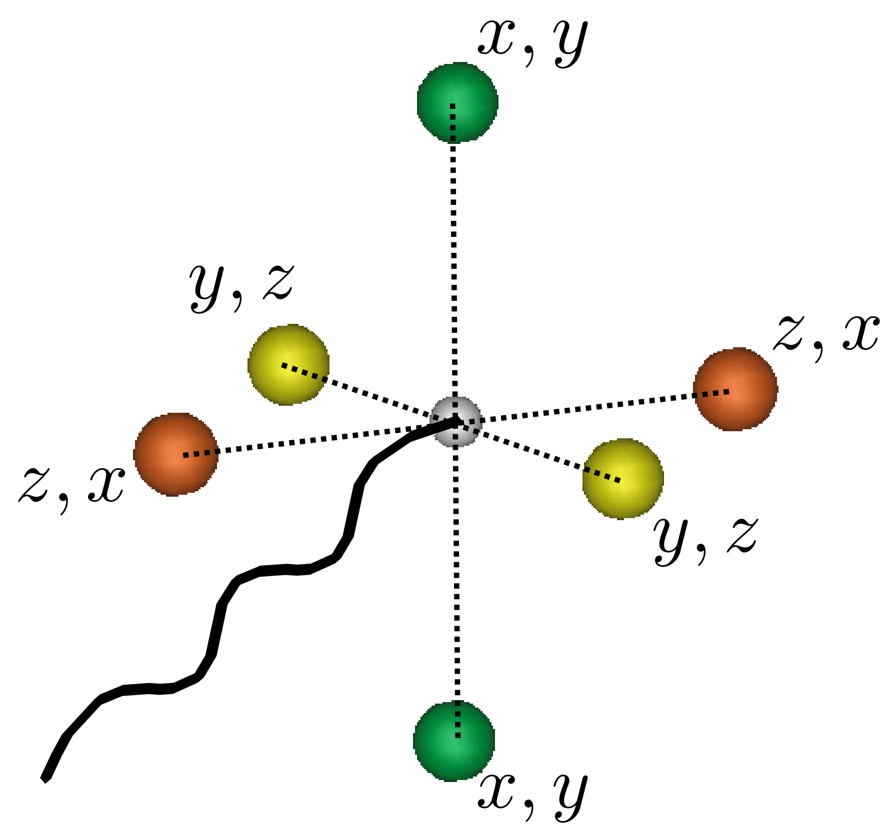
$$\hat{O}_I = \sigma^z_{T_I}\sigma^y_{N_I}\sigma^x_{W_I}\sigma^z_{B_I}\sigma^y_{S_I}\sigma^x_{E_I}$$

$$H = -\frac{h}{2}\sum_I \hat{O}_I$$

ground state degeneracy $g = 2^4$

3D strong glass model





$$\hat{H}_{\text{spin/bath}} = \sum_{I,\alpha} g_\alpha \, \sigma_I^\alpha \sum_\lambda \left(a_{\lambda,I}^\alpha + {a_{\lambda,I}^\alpha}^\dagger \right)$$

always flip 4 octahedra: never simple defect diffusion

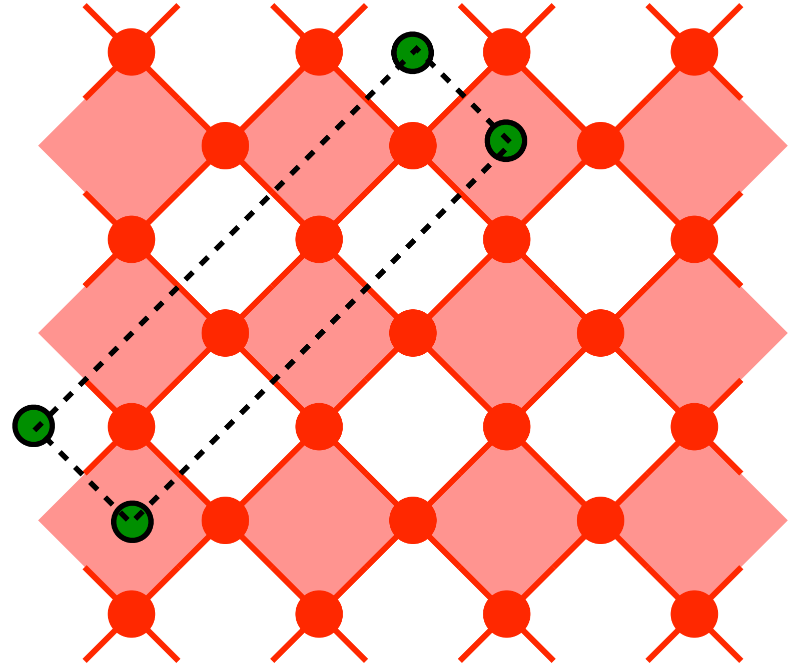
$t_{\text{seq.}} \sim \tau_0 \exp(2h/T)$ (Arrhenius law)

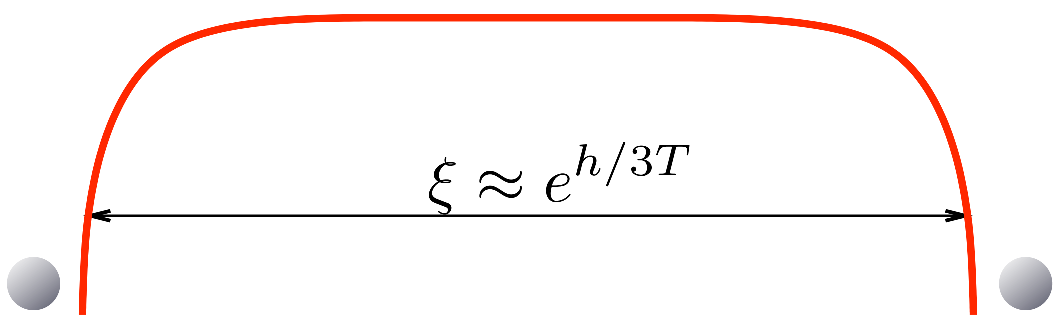
What about quantum tunneling?

defect separation: $\xi \approx c^{-1/3} \approx e^{h/3T}$

virtual process: $\mathcal{O}[(g/h)^{\xi}]$

$$t_{\text{tun.}} \sim \tau_0 \exp\left[\ln(h/g)\ e^{h/3T}\right]$$





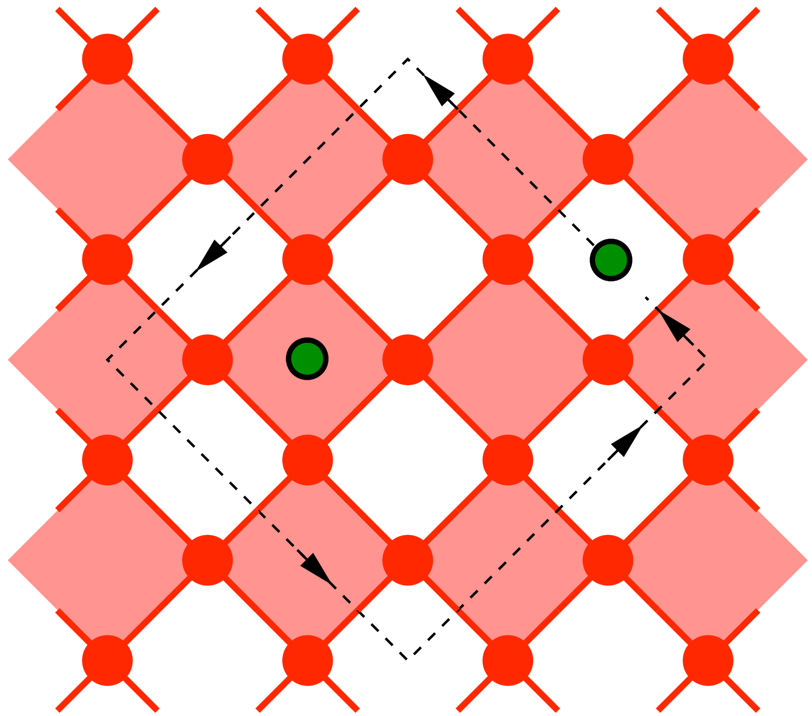
topological quantum protection ⇒ quantum <u>OVER</u> protection

From another angle... fractionalization

2D cut:

defects have fractional statistics

$$\theta = \pi/2$$



3D:

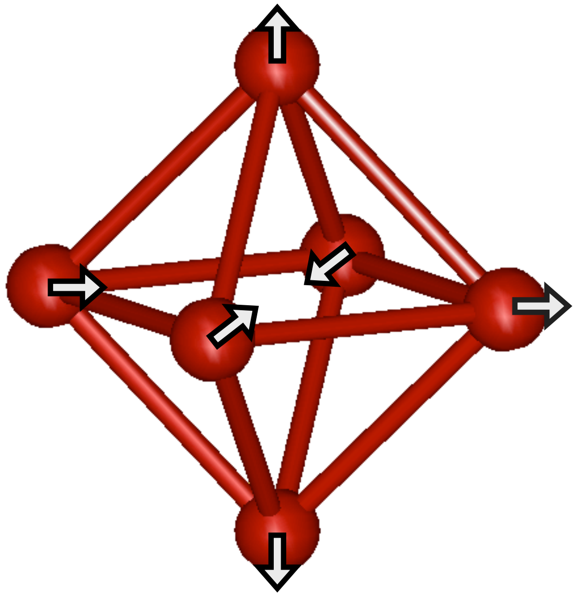
no fractional statistics

octahedra flipped not in pairs but in quadruplets

⇩

rigidity and no winding

Beyond the toy model...



$$\hat{O}_I = \sigma^z_{T_I}\sigma^y_{N_I}\sigma^x_{W_I}\sigma^z_{B_I}\sigma^y_{S_I}\sigma^x_{E_I}$$

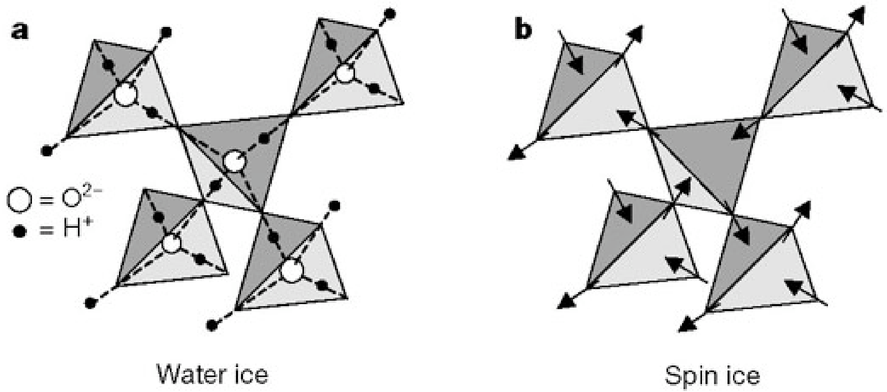
$$H = -\frac{h}{2}\sum_I \hat{O}_I \qquad O_I = 1 \Rightarrow$$ ice rules mod 4

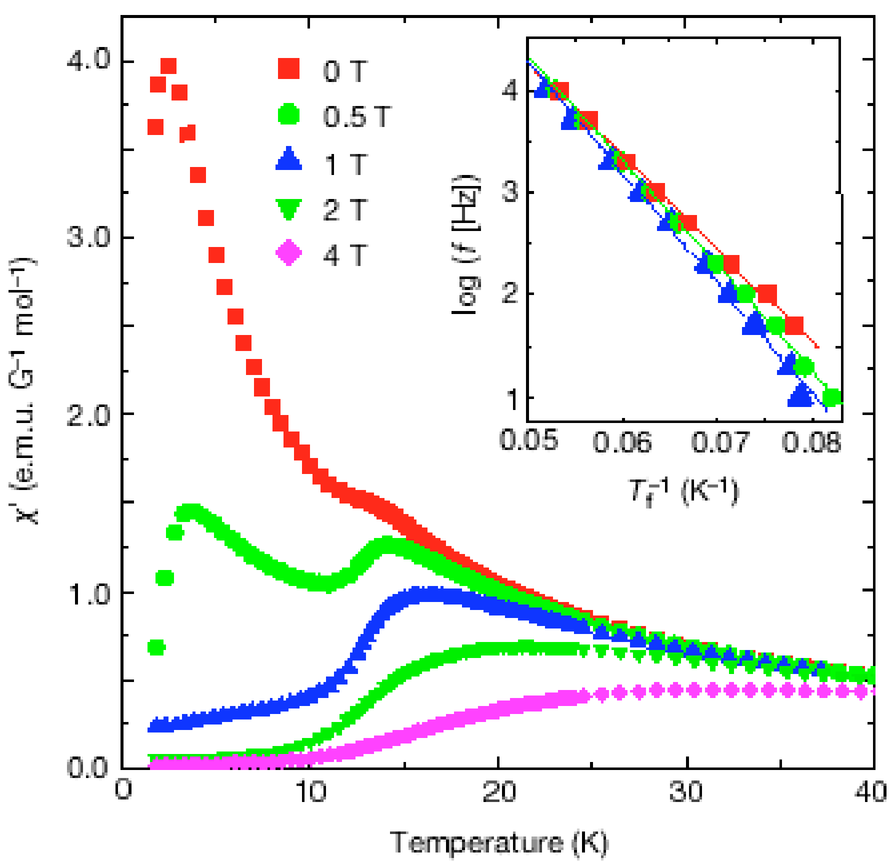
Hard constrained systems are good candidates for glasses

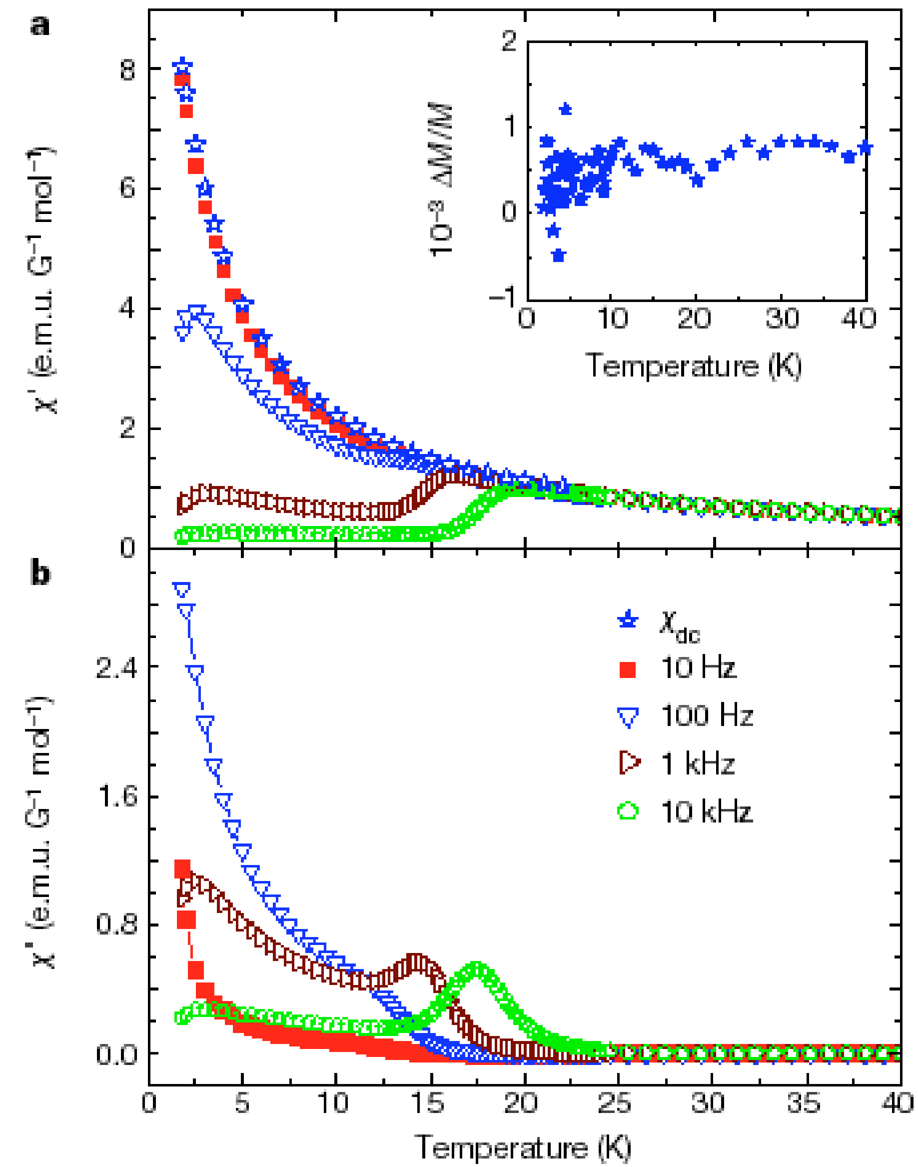
Spin Ice

$Dy_2Ti_2O_7$ ($Ho_2Ti_2O_7$)

Snyder *et al*, Nature (2001)



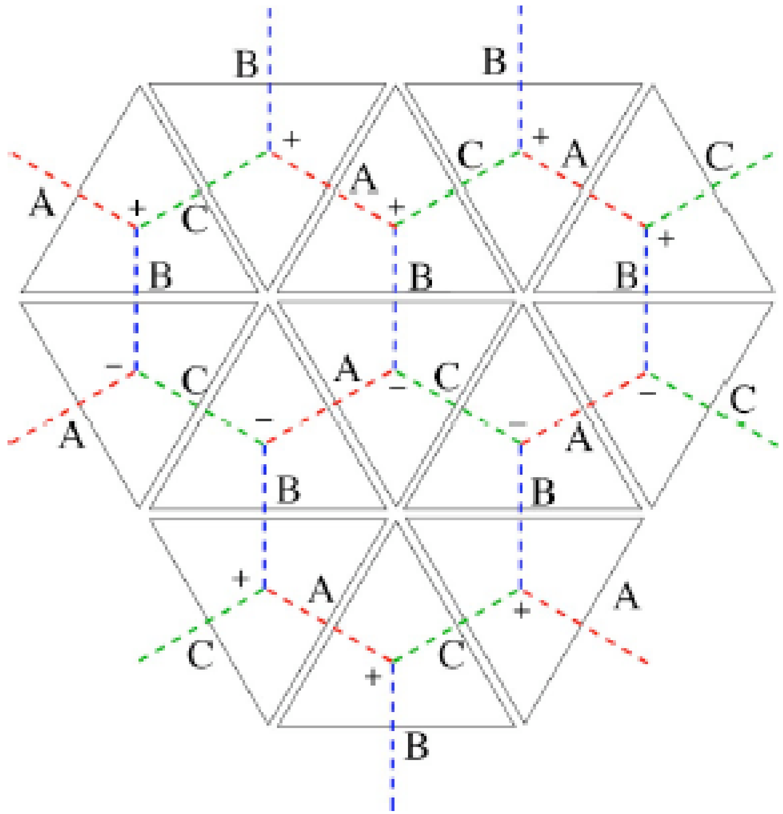


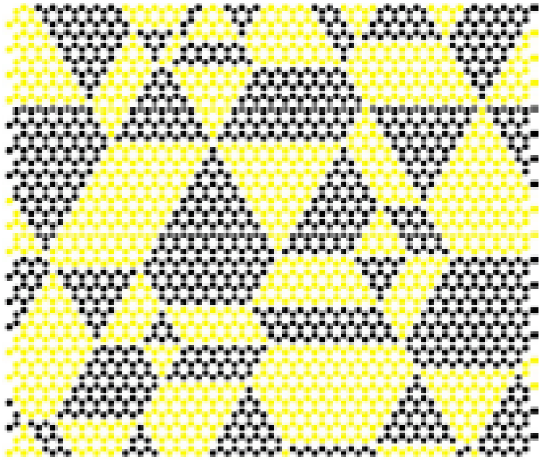


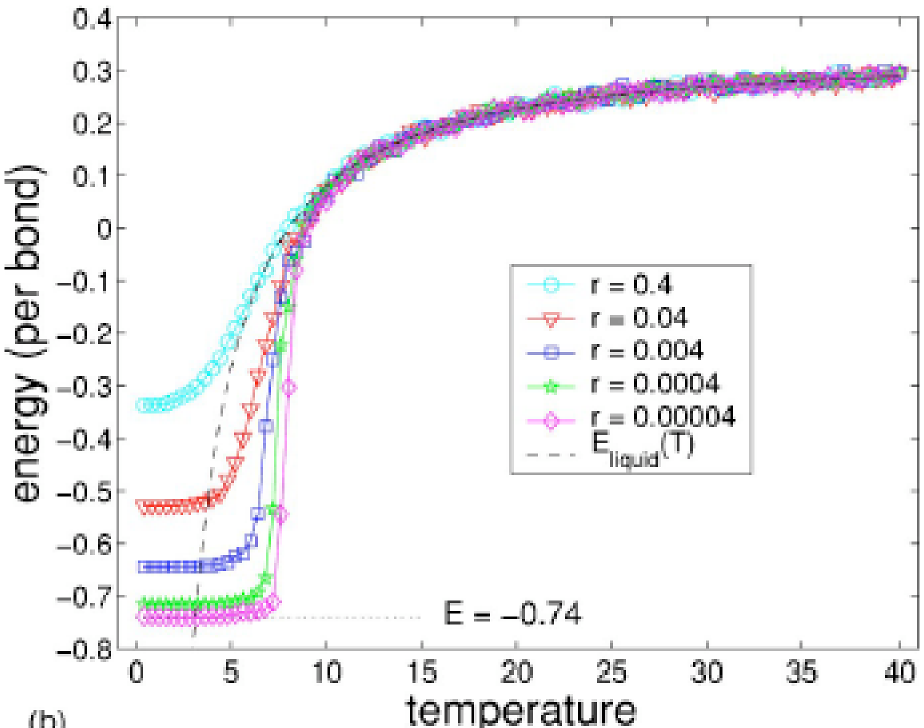
Source: Snyder *et al*, Nature (2001)

Josephson junction arrays of T-breaking superconductors

Sr_2RuO_4 $\quad p_x \pm ip_y$







Moore & Lee, cond-mat/0309717
Castelnovo, Pujol, and Chamon, PRB (2004)

Conclusion

Presented solvable examples of quantum many-body Hamiltonians of systems with exotic spectral properties (topological order and fractionalization) that are unable to reach their ground states as the environment temperature is lowered to absolute zero - systems remain in a mixed state down to T=0.

New constraint for topological quantum computing: that the ground state degeneracy is protected while the system is still able to reach the ground states.

Out-of-equilibrium strongly correlated quantum systems is an open frontier!