The Stefan Boltzmann Law

There is no more important law in environmentally relevant physics than the relationship between the power radiated by a dense hot body and the temperature:

$$P = e\ A\ \sigma\ T^4 \text{ watts} \qquad\qquad (1)$$

where **T** is the absolute temperature, **A** is the surface area of the radiator, and **e** is the emissivity, a function of emitted wave length. For a perfect black body **e** = 1. The Stefan Boltzmann Constant, σ, is equal to 5.67 x 10^{-8} W/(m^2 • K^4).

Note that this notation is widely used and hence is preferable, but Serway unfortunately refers to the emitted power (**P/A**) as **e**, and the emissivity **e** as **a**.

To investigate this **T^4** dependence we need a source of radiation and a detector of radiation. Our source is a light bulb with a tungsten filament. There is really nothing special about this light bulb -- in fact, it is a common automotive headlight bulb -- but it is set up to allow convenient measurement of the current passing through it and the voltage across it. We can vary the filament temperature by adjusting the voltage, and we can determine that temperature by calculating the resistance **R** = **V/I** at each voltage setting. The resistance of the filament is highly temperature dependent and serves as a thermometer.

Our detector is a device known as a thermopile. It has a very wide range, detecting with equal efficiency radiation from 0.5 microns to 25 microns in wavelength. This covers the visible range (0.5 to 0.8 microns) and a large stretch of the infrared region. At the temperatures at which the filament can operate (**T** < 3000°K) most of the radiation is in the infrared region.

Experimental Procedure

a. The temperature dependence

First make a careful determination of the room temperature resistance of the filament, **R$_{300}$**, using an ohmmeter. The bulb should be disconnected from everything else when this is done.
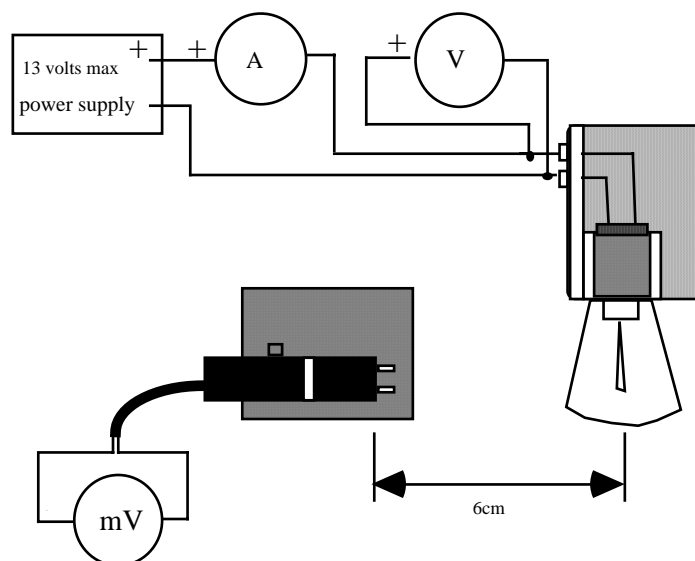
Figure 1: Thermopile and tungsten bulb source set-up

Set up the circuit as shown above. The voltmeter should be connected directly to the input terminals of the lamp. The sensor should be set at the same height as the filament and about 6 cm away. Note that the voltmeter connected to the thermopile sensor is set to the most sensitive range -- typical readings are in the range of tens of millivolts at most. There should be no objects close to the sensor other than the lamp.

Be sure that the voltage control is at the minimum, and then turn on the power supply. Vary the lamp voltage in steps of roughly one volt from about 1 to 12 volts, and record on the data sheet the voltage setting **V**, the lamp current **I**, and the sensor reading **S**. Note that it is not necessary to adjust the voltage precisely to integer values. **Never allow the lamp voltage to exceed 13 volts -- the filament will burn out if you do.**

Make each sensor reading quickly and place a sheet of insulating foam between the lamp and the sensor (silver side toward the lamp) between readings.   This procedure helps to keep the thermopile temperature constant which is important for accurate operation. Be careful not to move the detector between readings.

Analysis

The data table is accompanied by a calculation table. For each voltage setting, calculate the filament resistance. The ratio of that resistance to room temperature resistance (R_{300}) is the indicator of filament temperature. The calibration of this thermometer depends on the well known temperature dependence of tungsten resistivity, and is supplied in convenient graphical form in the lab. Once you determine the ratio R/R_{300} you can simply read off the value of **T**. Fill in the table with **T** and T^4 for each filament voltage setting.

To complete the analysis, plot **S**, the sensor reading in millivolts as a function of $\mathbf{T^4}$; that is, plot $\mathbf{T^4}$ on the x axis, **S** on the y axis. Alternatively, you can plot the graph of **S** versus **T** on log-log paper and find the slope of the resulting curve.

Questions

1. Can you conclude from your data that the $\mathbf{T^4}$ law holds over the entire range of temperatures considered in this experiment?

2. The glass bulb of the lamp absorbs infrared radiation far more effectively than it absorbs visible radiation. What effect might this absorption have on your results? Does this absorption affect your data more at lower or at higher temperatures?

b. Power emitted by the bulb

The measurements made so far are relative measurements -- all that is needed to investigate the $\mathbf{T^4}$ dependence or radiated power. However, the calibration of the sensors is known, at least to a few percent accuracy, so the actual power emitted by the bulb can be determined. The procedure to accomplish this follows.

Position the sensor about 0.15 meters from the filament. This distance should be determined as accurately as possible. Set the lamp voltage at 12 volts, and read the corresponding value of **S**.

You can convert **S** millivolts to an equivalent **P** watts using the manufacturer's indicated sensitivity of 22 mV/mW, which indicates that 1 milliwatt of power incident on the detector gives a reading of 22 millivolts, or:

$$P \;=\; (S/22) \cdot 10^{-3} \text{ watts } (S \text{ in millivolts}) \tag{2}$$

The sensitive sensor area **A** is 6.3 x 10^{-6} m^2, so the intensity **Int** (watts/m^2) at the detector is easily calculated to be:

$$\text{Int} \;=\; P/A \;=\; 7.2\, S \text{ watts} / m^2 \;\; (S \text{ in millivolts}) \tag{3}$$

Finally, the total power emitted by the lamp is given by:

$$P\,(\text{total}) \;=\; (\text{Int}) \cdot (\pi\, r^2)\, \text{watts} \tag{4}$$

where r is the distance from the source to the sensor, and the source is assumed to emit light with equal intensity in all directions.

More Questions

3. Justify the expression given above for **P(total)**.

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4. Determine **P(total)** and compare it with the input power to the lamp (**IV**). Can you account for the difference between input power and light output power?

5. You have determined the temperature of the filament, and the total radiated power. It is also well known that the emissivity for tungsten is about 0.33. Put this all together to estimate the area of the filament. Then examine the filament carefully and comment: is your answer reasonable?

Optional: a filter is available in the laboratory which only passes radiation with wavelengths 0.8 microns or greater, so virtually all visible light is blocked. Take a reading at 12 volts with the filter in and out, and use that to estimate the fraction of power radiated by the filament in the visible region. Is this roughly what you expect for **T** = 2000°K? A blackbody spectrum for **T** = 2000°K will also be available.

Additional Questions:

Review the derivation of Stefan's Law (Serway-Moses-Moyer, Example 2.4). Study the function $u(x) = x^3 / [e^x - 1]$ numerically:

6. At what value $x = x_{max}$ does this function's maximum occur (to three significant figures)?

7. (a) At what values $x = x_{10}$ does this function fall to 10% of its maximum (to two significant figures)? (b) at what (large) value $x = x_1$ does this function fall to 1% of its maximum (to one significant figure)?

Carefully plot this function on a scale where you can estimate the areas under the curve for different values of **x**.

8. What fraction of the emitted energy occurs for photon energies corresponding to less than x_{max}?

9. What values of $x = hf/kT$ correspond to 800 nm wavelength and 400 nm wavelength at a typical light bulb temperature of 2500°K? What fraction of the emitted energy occurs between these values of **x**?

10. Answer the questions in 9 for the solar temperature of 5800°K.

11. Compare the "efficiency" of the sun vs. tungsten light bulbs at delivering visible light.

12. Discuss why indoor pictures, taken using "daylight" film and tungsten light bulb illumination, without flash, look yellowish. Refer quantitatively to the function $u(x)$.