# **Coulomb's Law**

#### Theory

Coulomb, using apparatus similar to what you have, measured the dependence of the electric force on the magnitude of two charges  $Q_1$  and  $Q_2$  and on the distance between the centers of charge, **r**. He found that the force varied directly with the product of the charge magnitudes,  $Q_1$  x  $Q_2$  and inversely with the square of the distance **r** between them. If one defines the unit of charge, the Coulomb, as the amount of charge delivered by a current of 1 ampere flowing for 1 second, then if the force between the charges is measured and expressed in Newtons, the proportionality constant  $\mathbf{k} = 9 \times 10^9$  can be found if  $Q_1$  and  $Q_2$  are measured. You will NOT be asked to find the value of **k** in this experiment (takes too long), but will be happy to measure something like an  $1/\mathbf{r}^2$  dependence and a force proportional to  $Q_1 \times Q_2$ .

#### **Principle of the Measurement**

Figure 1 shows the side view of the torsion balance. The spheres are charged and the force between them is measured as a function of their separation. The measurement of the force is obtained by twisting the wire which supports its charged sphere until sufficient torque is applied to hold that sphere still at the position  $\mathbf{r}$  cm away from the other charged sphere. The angle  $\boldsymbol{\theta}$  through which the wire is rotated is read off a dial. The force and the torque are proportional to each other. Torque  $\mathbf{T} = \mathbf{F} \times \mathbf{R}$  where  $\mathbf{R}$  is the length of the lever arm. For a twisted wire with twist constant  $\mathbf{K}$ ,  $\mathbf{T} = \mathbf{K}\boldsymbol{\theta}$ , so the force between the spheres is  $\mathbf{F} = \mathbf{K} \cdot \boldsymbol{\theta} / \mathbf{R}$ . Since we are not going to do an absolute measurement but only observe the dependence of  $\mathbf{F}$  on  $\mathbf{r}$  and  $\mathbf{Q}$ , it is enough to see that  $\mathbf{F}$  is directly proportional to  $\boldsymbol{\theta}$ .





# Set-Up

It is very easy to break the fine wire from which one of the charged spheres is suspended (torsion balance). We have set the balance up as best we can and you are not to try to alter the set-up. In fact, except for turning the knob on the torsion balance, moving the charged sphere to change  $\mathbf{r}$ , and touching the spheres with the high voltage probe to charge them, or with a grounded probe to discharge them, you will please not touch the apparatus at all. Check to be sure that the counterweight vane for the sphere on the torsion system is properly loaded with pennies so that it is level and does not scrape the damping magnet.

## **Operating Tips**

This experiment works best in the winter when the air is dry and charge will not leak rapidly from the spheres. It may help to put the apparatus on an insulating sheet of plastic. Keep the balance away from the walls or people which might be charged. Stand behind and away from the balance, and touch ground (water pipe) to lose any charge on yourself. After charging the spheres, turn off the switch on the supply. Keep your hands as far as possible from the sphere being charged. If the charge seems to be leaking away rapidly, clean the insulators with alcohol.

Note that the high voltage supply is turned all the way up to around 5000 volts. Do not touch the end of the high voltage probe or else you will receive a moderate shock. The shock is moderate because the high voltage supply has a very large resistor in series with the probe, limiting the current flow to a safe level.

## Part A. Testing the Inverse Square Law

### Procedure

1. **DISCHARGE** the spheres with the grounded probe. Move the sliding sphere as far away as possible. Set the torsion dial to **zero**. The index marks should nearly line up. If they do not, please do not try to adjust the bottom torsion wire retainer, but adjust the torsion dial until they do line up and add or subtract that offset from your measurements of the torsion balance angle  $\theta$  required to line up the index marks when the charged spheres are close together.

2. With the spheres still at maximum separation, charge both spheres to a potential of 5000 volts, using the charging probe. Turn off the HV.

3. Slide the moveable charged sphere to the position  $\mathbf{r} = 20$  cm. Adjust the torsion knob to line up the index marks. You will find that there is very little deflection since the force (and hence the torque) is so weak. Write down the angle  $\boldsymbol{\theta}$  you read from the dial, recording  $\mathbf{r}$  as well in your table.

Now check for leakage. Note how long it takes you to do a single measurement, maybe a minute. Now put the charged spheres fairly close together (6 cm), and see how many degrees you need to change the torsion balance to keep the index marks lined up over a longer period of time, say over four minutes. If the air is very dry the leakage rate will be low. Suppose that you

find a very low leakage rate. In that case you need not bother to recharge between each measurement, thus avoiding possible problems with jarring the torsion balance with the probe, and speeding up the data acquisition rate. Do what seems best.

4. Repeat 3 twice. You may find that it is hard to get reproducible angles at 1 degree accuracy. This exercise will give you a pretty good idea of the error in the zero setting of step 3 above. Put another way, it can help you find a better zero value when you analyze your data.

5. Make measurements for separations of 14, 10, 9, 7, 6, 5 cm. Repeat the sequence of measurements three times so that you have three data sets. In order to minimize the effect of charge leakage, you may need to recharge the spheres between each measurement. If the data look fairly reproducible you can average the results. If they do not look reproducible, then you need to plot them as three separate runs. What could have changed? For example maybe one set of data was taken with more or less charge than the other two.

## Analysis

Correction for the finite size of the charges. The force law is for point charges. You are using conducting spheres of finite radius. As you bring the spheres closer together, the charge is repelled to the far side of each sphere, increasing the effective distance between the centers of the charge distributions. The theoretical correction factor for this effect is:

$$B = 1 - 4 a^3 / r^3$$

where **a** is the radius of the sphere. To correct your  $\theta$  reading, divide it by **B**, and record the corrected values in your table.

1. Plot  $\log \theta$  vs  $\log \mathbf{r}$ . If  $\theta = \mathbf{b} \times \mathbf{r}^{\mathbf{n}}$  where **b** and **n** are unknown constants, then  $\log \theta = \mathbf{n} \log \mathbf{r} + \log \mathbf{b}$ . The slope of the graph of  $\log \theta$  vs  $\log \mathbf{r}$  is a straight line, with slope equal to **n** and **y** intercept equal to  $\log \mathbf{b}$ . Use  $\log$ -log paper, plot your data and find **n**.

2. If the angle of the torsion balance is proportional to the force, and the force is inversely proportional to  $\mathbf{r}^2$ , the product of the angle  $\boldsymbol{\theta}$  times  $\mathbf{r}^2$  should be a constant. Put these in the table. Plot that product versus  $\mathbf{r}$ .

## Part B Force vs Charge

## Procedure

With the sphere separation held at a constant **r** of about 8 cm, charge the sphere on the wire to 5 kV. Then charge the moveable sphere to 2, 3, 4 and 5 kilovolts and measure  $\theta$ . As before, charge the moveable sphere at its maximum distance from the other sphere and move it in for the measurement. If charge leakage is a problem, you will have to renew the fixed charge each time you change charges.

Plot  $\theta$  vs kilovolts. You can assume that the charge on the spheres is proportional to the voltage.

### **Discussion of Results**

In your report, present your results (the tables and graphs). Be sure to have an Instructor's signature on the data. Comment on the success of your attempt to find an inverse square law in Part A. How can you be sure that you have seen a  $1/r^2$  law instead of a 1/r law?

In part B, comment on the success of your attempt to show that the force increases linearly with  $Q_2$ . Plot the curve of what the data would look like if the force varied as the product of the square root of  $Q_2$  (Hint: plot  $\theta$  vs the square root of  $Q_2$  and see if it is fit by a straight line).

Remember: You take data together with your lab partner, but you analyze and write up the lab by yourself.