CHARGE TO MASS RATIO OF THE ELECTRON

In solving many physics problems, it is necessary to use the value of one or more physical constants. Examples are the velocity of light, c, and mass of the electron, m_e. Fundamental physical constants are universal since their values are believed to be the same regardless of the time or place in the Universe. Some constants can be measured individually, but usually experiments yield only the values of combinations; for example e/m_e, e/hc, or $m_e c^2$. The individual values are obtained by equating and canceling among the combinations. In this experiment, you will measure the combination e/m_e. The technique involves the motion of charged particles in a magnetic field. It should be noted that the value for e/m_e is very precisely known: $(1.7588047 \pm 0.0000049) \times 10^{11}$ coulomb/kg. The magnetic field used in this lab is far less precisely known, nevertheless, you should be able to obtain the first few decimal places.

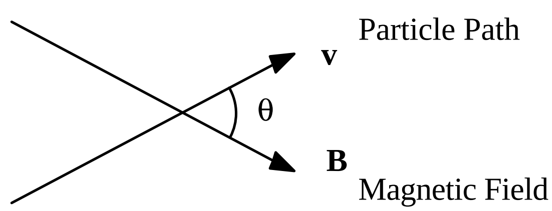
MOTION OF A CHARGED PARTICLE IN A MAGNETIC FIELD

A charged particle moving in a magnetic field experiences a force known as the Lorentz force, which in vector notation is given by

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B} \tag{1}$$

where q is the charge on the particle, $\mathbf{v}$ is the vector velocity, and $\mathbf{B}$ is the vector magnetic field. This vector cross product equation is a shorthand way of saying the following:

(a) The force is perpendicular to *both* the velocity of the particle $\mathbf{v}$ and the direction of the magnetic field $\mathbf{B}$. The direction of the force on a *positive* charge is given by a right-hand rule: $\mathbf{F}$ is in the direction of your thumb if the fingers of your right hand curl from $\mathbf{v}$ to $\mathbf{B}$ through the smaller angle. $\mathbf{F}$ is *into* the page in the sketch below if the charge is positive and *out of* the page if the charge is negative.



(b) The magnitude of the force varies as the sine of the angle, θ, between the direction of motion and the direction of the field.

(c) The magnitude of the force is proportional to q, v, and B. Hence,

$$\mathbf{F} = qvB \sin\theta \tag{2}$$

A corollary of the above is that the magnetic field cannot speed up or slow down a charged particle because the force is always perpendicular to its direction of motion. The Lorentz force will only cause the particle's path to curve. With this in mind, you should be able to convince yourself of the following:

(a) If the particle has its velocity vector parallel to the magnetic field direction, it will move in a straight line unaffected by the field. This is useful for finding the direction of a magnetic field.

(b) If the particle is launched perpendicular to the field direction, it will move in a circle with the Lorentz force at a maximum.

(c) If the particle is launched in any other direction, its path will be a helix.

In measuring the charge to mass ratio e/m_e in this experiment, the apparatus is arranged so that the force will be perpendicular to **v** and **B**. Then the charged particle will move in a curved path in a plane perpendicular to **B**. The velocity will change direction but not magnitude. In this case $\theta = 90°$ and from Equation (2)

$$F = qvB \tag{3}$$

Recall from mechanics that a mass which experiences a constant force perpendicular to its velocity will move in a circle such that the force is directed toward its center. From Newton's Second Law for uniform circular motion

$$\sum F_{radial} = \frac{mv^2}{R} \tag{4}$$

where R is the radius of the circular motion. In the situation where the only net force is the Lorentz force given by Equation (3),

$$qvB = \frac{mv^2}{R} \tag{5}$$

This expression can be simplified to give the charge to mass ratio as

$$\frac{q}{m} = \frac{v}{RB} \tag{6}$$

In the case of the electron, $q = e$ and m = mass of the electron, m_e. Thus, the objective of this experiment is to produce electrons of known velocity v moving in a known magnetic field, B, and to measure the radius, R, of their circular motion.

METHOD

The apparatus consists of a cathode ray tube (CRT) mounted between a set of Helmholtz coils which produce the magnetic field. The CRT contains an “electron gun” which shoots out a stream of electrons along the axis of the CRT as shown in Figure 1. The electrons are accelerated inside the gun by a potential difference, V, which is applied to the CRT by a power supply. The kinetic energy, $\frac{1}{2} m_e v^2$, gained by an electron (charge = e) passing through a potential difference, V, is equal to eV. Hence,

$$eV = \frac{1}{2} m_e v^2 \qquad or \qquad v^2 = \frac{2Ve}{m_e} \tag{7}$$

The stream of electrons passes a phosphorescent screen which makes a luminous line along the path of the electrons. The magnetic field of the Helmholtz coils is perpendicular to the screen and causes the path of the electrons to be an arc of a circle. The (x,y) grid markings on the screen are used to determine the radius of the electron path.

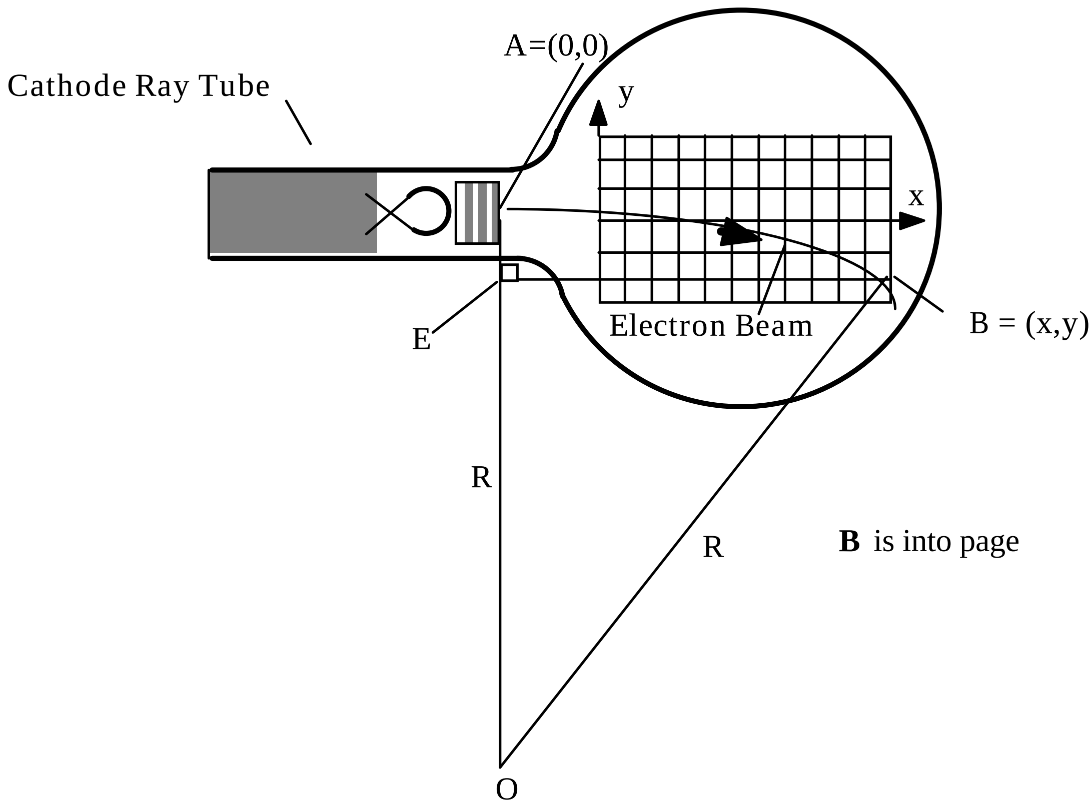


Figure 1. Schematic of Cathode Ray Tube (CRT).

Refer to Figure 1 to follow the geometry used to convert (x,y) values along the electron's path into its radius. Assume the electron enters the magnetic field at point A with coordinates (0,0) and exits at point B with coordinates (x,y). From the figure, the length of line OE = $R - y$ and the length of line EB = x. Hence for the right triangle OEB, $R^2 = x^2 + (R - y)^2$ which reduces to

$$R = \frac{x^2}{2y} + \frac{y}{2} \tag{8}$$

Combining Equations (6), (7) and (8) yields

$$\frac{e}{m_e} = \frac{2V}{B^2}\left(\frac{x^2}{2y} + \frac{y}{2}\right)^{-2} \tag{9}$$

where B is the magnetic field of the Helmholtz coils, e and m_e are the electron's charge and mass, V is the potential applied to the CRT, and (x,y) are selected coordinates of the electron's path on the CRT grid. In SI units, B will be in tesla, x and y in meters, and V in volts.

A schematic of the Helmholtz coil geometry is shown in Figure 2. As current passes through the Helmholtz coils, a magnetic field is generated. The field in a region near the center is uniform if the separation of the coils is equal to half of their diameter. Such an arrangement is named after its inventor, Helmholtz. Although the magnetic field strength decreases with distance along the axis from one coil, the sum of the fields from the two coils is nearly constant in the region between them. Variation in the field off the axis may produce some error (perhaps 10 - 30%) in your results. The magnetic field in tesla at the center of these particular Helmholtz coils is given by

$$B = 4.23 \times 10^{-3} \times I \tag{10}$$

where I is current through the coils in amperes. For an electron path measured near the center of the coils, this value is sufficiently accurate. The magnetic field strength can be changed by adjusting the current passing through the coils. The magnetic field direction can be reversed by reversing the direction of the current.

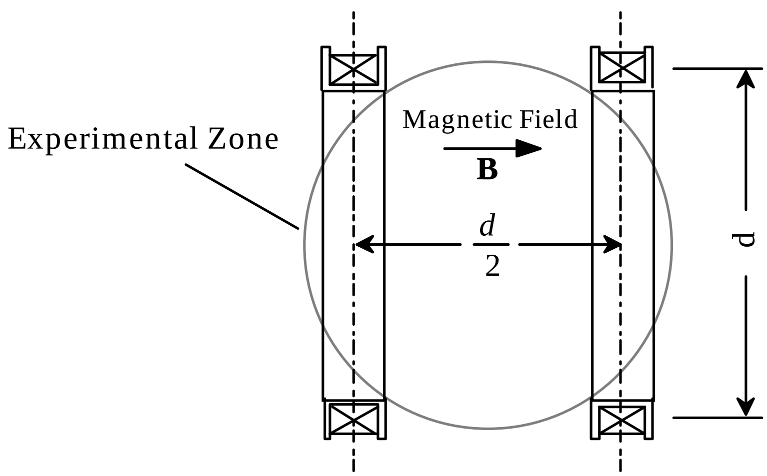


Figure 2. The Helmholtz Geometry (end view of coils).

APPARATUS

- ❑ Helmholtz coils
- ❑ e/m tube
- ❑ High voltage power supply
- ❑ 35V power supply
- ❑ DPDT switch
- ❑ Multimeter

PROCEDURE

1. Examine your apparatus and check the Helmholtz geometry. Are the coils aligned parallel with each other? Do they have the correct separation, i.e. 1/2 the coil diameter? If not, loosen the plastic knobs at the coil bases, align, and re-tighten the knobs to hold coils in position.

2. Connect the Helmholtz coils, ammeter, current direction switch, and the TEL 800 power supply (select dc) as shown in Figure 3. Be sure the switch is wired as shown. Since you want the current in both coils to be in the same direction, make sure that the "A" and "Z" connections are wired properly. The Keithley multimeter (not power supply) should be used as the ammeter in this circuit to obtain accurate current readings .

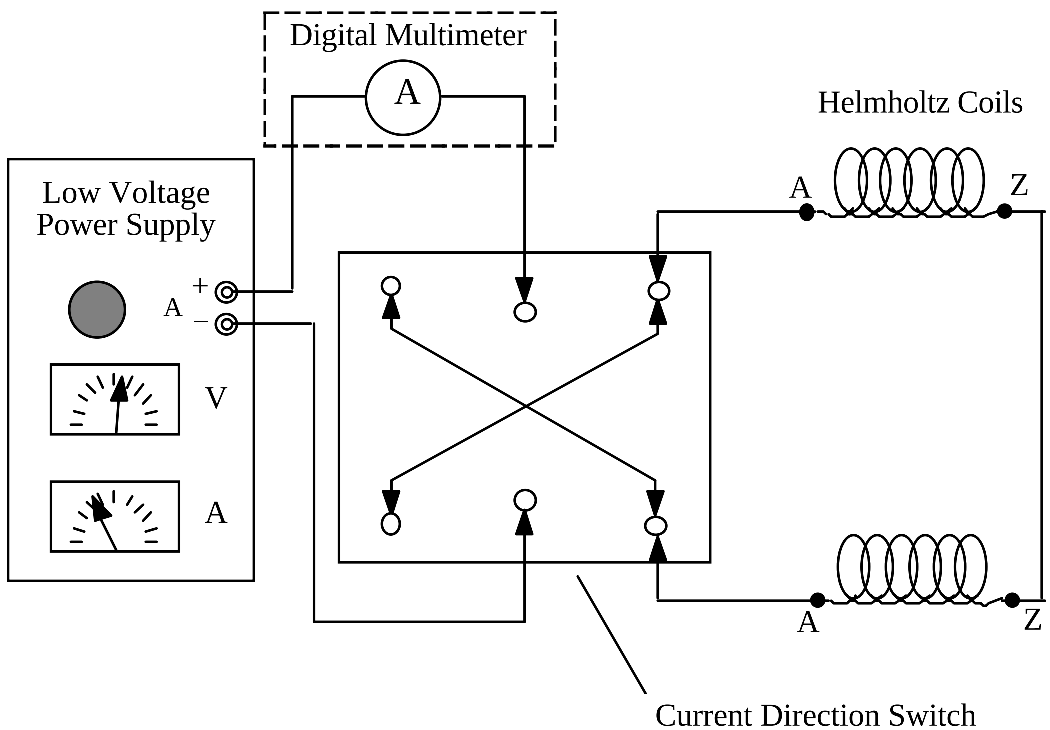


Figure 3. Wiring Diagram for Helmholtz Coils

3. Connect the CRT and TEL 813 high voltage power supply as shown in Figure 4. The 6.3 V connection provides the heat required for electron emission from the tube filament. Be sure to use a high voltage wire (white) in connecting the (+) HV terminal to the CRT. Do not use the Center Tap (CT) terminal. The two (—) terminals (-HV and -6.3 V) of the TEL 813 power supply should be connected together *and* to the (—) side of the CRT

filament. In applying high voltage to the CRT, always start low and increase slowly until the electron beam becomes visible. Watch the meter on the supply and *do not exceed 3.5 kV* or you may damage the CRT. Keep the high voltage set at minimum except when making measurements.

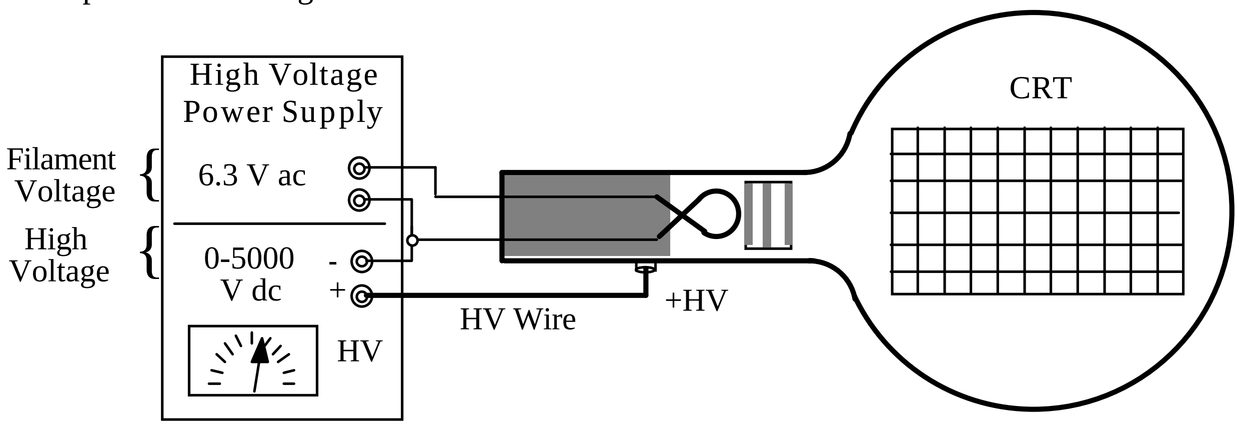


Figure 4. CRT-Power Supply Connections.

4. Have the T.F. check your circuit wiring before proceeding. Turn on the TEL 813 high voltage supply and increase the voltage (CRT potential, V) until the electron beam is visible. The Helmholtz coils should be off for now. Is the beam exactly horizontal? Why or why not? If there is some magnetic field causing a deviation, what is its approximate direction relative to the electron beam direction? Make a sketch. Do not change the orientation of the CRT on the lab table from this point onward.

5. You will need to compensate for any deviation of the beam from horizontal by taking two measurements with the Helmholtz coil magnetic field reversed. Turn on the TEL 800 power supply connected to the Helmholtz coils. Increase the current in the coils until the electron beam passes exactly through a convenient point (x,y) on the CRT grid. A coordinate around (10, 2) works well, but you may find a different point more convenient. Record the CRT potential, V; the current in the Helmholtz coils, I, measured with the Keithley meter; and your selected (x,y) coordinate.

   Now change the direction of the current in the Helmholtz coils and thus the magnetic field direction by reversing the *current direction switch*. Do not change the CRT potential. Does the electron beam pass exactly through the corresponding point, e.g. $(x,-y)$, on the opposite half of the grid? If not adjust the current until it does. Record this current. Use the average of your 2 currents above for the determination of the Helmholtz coil magnetic field given in Equation 10.

6. Change the CRT potential (without exceeding 3.5 kV). Adjust the Helmholtz coil current until the electron beam passes exactly through the same (x,y) point used above. Record the current, CRT potential, and (x,y). Now reverse the current and use the same procedure as in Step 5 to determine the average current for use in Equation (10) for the magnetic field. Make additional measurements until you have values for 5 different CRT potentials.

7. For a given CRT potential (e.g., 2.0 kV), adjust the Helmholtz coil current so that the electron beam passes through some (x,y) point. Vary the current until you have 4 or more values for (x,y). Reverse the current direction as before for each of your (x,y) values. For each case, calculate the average current and the corresponding values of the magnetic field. Make a plot of B vs. $1/R$.

ANALYSIS AND QUESTIONS

Use your data to calculate the values of the electron path radius, R; the magnetic field strength, B; and the electron charge to mass ratio, e/m_e. Fill in the table on the data sheet. Determine your average value for e/m_e and compare it to the "accepted value". How well did you do? Discuss the possible sources of systematic error in your measurement of e/m_e.

Does reversing the current and averaging the values compensate for the effect of the Earth's magnetic field? Refer to your sketch from Step 4 and make a diagram to explain. Assume the Earth's field is constant.

Make a plot of B vs. $1/R$ from the data obtained in Step 7 above. What is the functional dependence of the curve? What does it represent?

If the electron's velocity were relativistic, ($> 10^8$ m/s), the equations derived for this experiment would not be valid. Using the highest CRT potential that you used, calculate the electron's velocity from Equation 7. Is it relativistic?

How would you arrange the CRT and Helmholtz coils to cause the electron beam to undergo a helical (or spiral) motion? Draw a sketch of your proposed arrangement showing the directions of **v**, **B**, and **F**.

DATA SHEET — e/m_e TA [ ]

NAME: ____________________________INSTRUCTOR:____________________________

PARTNER: ________________________SECTION: __________DATE:_______________

Data for a fixed (x,y) or (x,-y) location:

x = ______ y = ______ R = ______

CRT Potential	Coil Current	Reversed Current	Average Current	Magnetic Field	e/m_e

Average value for e/m_e = ________

Data for a fixed CRT Potential:

V = ____________________

Coil Current	Reversed Current	Average Current	x	y	R	Magnetic Field	e/m_e

Average value for e/m_e = __

Make plots and sketches on separate pages.