Magnetic forces and potential energy - MBL

In this experiment you will determine how the force between two magnets varies with distance, and investigate the connection between force and potential energy. A computer will be used to collect, display, and help you analyze the data.

THEORY

The force experienced by an object can often be described by a function $F(x)$, where x is the distance from the object to whatever exerts the force. If the force is **only** a function of position, the force is **conservative**. Objects moving under the influence of conservative forces suffer no losses of energy by friction or radiation. When the object is set into motion, its total energy $E = K + U$ is conserved, where $K = mv^2/2$ is the kinetic energy and $U(x)$ is the potential energy. An example is a mass m near the Earth's surface, moving under the influence of the Earth's gravitational force only. $E = K + mgx$ is a constant where x is the height of the particle with respect to some convenient zero level. For this simple case $U(x) = mgx$. Force and potential energy are related by the equation:

$$F(x) = -\frac{dU}{dx} \tag{1}$$

a relationship which is easily seen to give the correct answer, $F = -mg$ for the case discussed above. However, the relationship is a more general one, as is the use of the potential energy concept.

When objects exert forces on one another without actually being in contact, the force between them is often inversely proportional to the distance r between them raised to some integer power. For instance, for two masses that interact via gravity the force goes as $1/r^2$. One goal of this experiment is to find the relationship between force and distance for magnets. The equation should be of the form:

$$F = k / x^n \tag{2}$$

You will determine the value of n, the integer exponent, and go on to investigate the relationship between force and potential energy, U. This relationship is given by equation (1), but is also:

$$\Delta U = -\int \mathbf{F} \cdot \mathbf{dx} \tag{3}$$

In this experiment the object is to study force and potential energy for a system consisting of two sets of magnets, one fixed in position, oriented so they repel one another. You will then use equation (3) to predict the equation for the potential energy associated with two interacting magnets. Part III of the experiment, where you verify your prediction, involves rolling the cart down the inclined track. If no energy is lost to friction, which may not be entirely true, the total mechanical energy is conserved. The cart is released from rest, so the total energy E is equal to the initial gravitational potential energy and also equal to the sum of the kinetic energy K, gravitational potential energy U_g, and magnetic potential energy U_m at any point along the track. U_m is therefore:

$$U_m = E - U_g - K \tag{4}$$

APPARATUS

- ❏ Computer and interface
- ❏ Cart with magnetic bumpers
- ❏ Motion sensor
- ❏ 2.2-meter track
- ❏ Force sensor with magnetic bumpers
- ❏ Ruler, and bar to tilt the track

PROCEDURE

NOTE: The magnets inside the cart and mounted on the force sensor have strong enough fields to destroy the magnetic memory of computer disks; **please keep the magnets away from any disks!**

Part I – Preparing to take measurements.

1. Initially, the track should be level. Check this by setting the cart on the track. If it rolls one way or the other, level the track using the adjustable feet on the track supports.

2. Make sure the switch on the force sensor is set to 10 N, and not 50 N.

3. You can use the ruler on the track to measure position. Choose a point 80 – 100 cm from the motion sensor as the zero position. This will be where the front of the magnets on the force sensor will be located, but first move the force sensor along the track away from the motion sensor and well beyond your zero point. Now position the cart so that the end furthest from the motion sensor is even with the zero point. Move it an additional 1 mm past the zero point away from the motion sensor. This accounts for the fact that the magnets in the cart are about 1 mm away from the front of the cart. From the Experiment menu, select Zero, and choose "Zero Distance". You should hear a series of clicks from the motion sensor as it measures the cart's position. Make sure the cart is stationary, and nothing is between the cart and the motion sensor, when the zeroing is going on.

4. Now move the cart well away from the zero point, and slide the force sensor back up the track. Position it carefully so the front of the magnets are even with the zero point. Make sure it does not move from this position for the rest of the experiment.

5. With the cart well away from the force sensor. From the Experiment menu, select Zero, and choose "Zero Force". After doing this make sure the fluctuations of the force readings are centered on zero.

Part II – Measuring force as a function of distance.

1. Place the cart on the track between the motion sensor and the force sensor, and position it so there is about a 25-cm gap between the cart and the force sensor. Hit the Collect button to begin collecting data, and then smoothly move the cart toward the force sensor. Hit the Stop button when the cart is **about 2 cm** from the sensor (the computer is set to record for 10 seconds, so it may not be necessary to stop it recording). Repeat this until you get a reasonably smooth curve showing force as a function of distance.

2. Check the data carefully. When the cart is far from the force sensor the force readings may be small positive or negative values. Because you zeroed the force sensor previously, this should surprise you – the readings should fluctuate around zero. The reason they don't is that there is some interference when both the motion sensor and force sensor are recording simultaneously.

   The small offset you observe is a good example of a systematic error – all the force readings are consistently high or low. You should correct this by creating a new "Corrected force" column. From the Data menu select "New Column" and then "Formula". Enter the name of the new column, an abbreviated version of the name, and the units, and then select the Definition tab. Set up the equation by choosing "Force" from the Variable pull-down menu, and then subtracting off the average value of the force readings you obtained when the cart was far from the force sensor. For example, if your force data fluctuated from -0.011 N to -0.017 N, your formula for Corrected force should read "Force" + 0.014

Question 1: Another thing you should notice about the force readings is that they are quantized, meaning that they can take on particular values, all separated by a small increment. What is the smallest increment separating the readings? The uncertainty in each measurement is half of this increment. What is the uncertainty in the force readings?

3. Fit a curve to the data to determine the equation giving force as a function of distance. To do this, first select a region of the graph by clicking-and-dragging with the mouse. Then click the Curve Fit button, . and select the "Nth Inverse". Enter your initial guess at what the exponent is in the "Degree/Exponent" box (it must be an integer) and hit the Try Fit button. Keep doing this until you get the best possible fit to the data. Hit "OK" and then record the values of A, the exponent, and B, which together will give you the equation relating force and distance.

Question 2: B is the constant term in the equation. What should its value be? If it is not what you expect, what is a possible explanation?

4. Repeat steps 1-4 above a few times, each time recording the equation giving force as a function of distance. The exponent in the equation should be the same each time.

5. Using your several trials, arrive at an equation giving force as a function of distance for the magnets. Estimate the uncertainty in the coefficient A based on the several values you obtained in your trials.

6. Now that you have an equation for force, use the relationship between force and potential energy to arrive at a prediction of the equation giving potential energy as a function of distance for the magnets. What is the uncertainty in the coefficient?

Part III – Verifying your magnetic potential energy prediction.

You should use energy conservation to check your prediction of how the magnetic potential energy varies with distance. A suggested method is described below, but you are free to come up with your own method if you like. Bonus points if your method is either better than the suggested method, or if your method is easier than the suggested method but gives comparable results.

1. No matter which method you try, first set up a new column that uses your predicted equation to create a graph of magnetic potential energy as a function of distance. Remove the "Corrected force" graph and display only the magnetic potential energy graph. Do this by clicking on the y-axis label, checking what you want to display and un-checking what you don't want.

2. Following the suggested method you will need to set up several additional columns, each showing different aspects of the energy of the system. Before doing this, however, place the bar under one set of legs of the track, and hold the cart at rest about 20 cm from the force sensor. Hit the Collect button to start recording data and then release the cart. The track should be tilted so the cart rolls toward the force sensor! Stop the cart when it changes direction (if you don't do this the graphs you display later could be rather confusing).

3. Create a column for the gravitational potential energy of the cart. The expression should be in terms of the distance measured along the track, with zero corresponding to the zero point, where the magnets on the force sensor are located. The simplest way to do this is to use $h = X\sin(\theta)$, where X is the position measured by the force sensor, and $\sin(\theta)$ is stated as the ratio of two lengths you measure from your apparatus.

4. Create a column for the kinetic energy of the cart. The graph you obtain from this will probably be rather noisy. This is because the speed is obtained by differentiating the position, and that introduces some error. In addition, squaring the speed then amplifies these errors. Note that the speed can be entered in your equation as Derivative("Position"). Select Derivative from the Functions pull-down menu, and Position from the Variables pull-down menu. The program assumes the derivative is with respect to time.

5. Create a column showing the sum of the cart's kinetic energy and its gravitational potential energy. Look carefully at the result.

Question 3 –When you look at the sum of the kinetic energy and the gravitational potential energy, what is the general behavior when the cart is far from the force sensor? What happens as the cart gets close to the force sensor? Where is the energy going?

6. Create a new column showing the total initial energy minus the sum of the kinetic and gravitational potential energies (see equation 4). Note that the total initial energy is the gravitational potential energy at the point where you release the cart – you could even read that off the graph and put it in the equation as a number.

Question 4 – When you look at the graph of this new column, it should agree reasonably well with the graph of the magnetic potential energy you created in step 1. Are they matched closely enough for you to conclude that the theoretical relationship between force and potential energy, which you used to predict the magnetic potential energy, is valid?

Question 5 – Some of the cart's energy will be lost to friction, which we have not accounted for. If you do account for friction, will the agreement between your two graphs (magnetic potential energy, and total energy minus all energies except magnetic potential energy) be better or worse? Feel free to set up a new column estimating the energy lost to friction to see if you are correct.