Specific Heat and Latent Heat

I. Background

In this lab we will use Newton's Law of Cooling to measure the specific heat and latent heat of a compound with potential uses for solar energy storage: sodium thiosulfate pentahydrate (STP). Solar energy researchers are interested in this material because it has a very large latent heat. Large areas of STP could be exposed to the sun during the daytime; melting the material. At night, the STP would gradually cool and solidify -- giving off its latent heat.

To measure the specific heat and latent heat of STP, we will make use of Newton's Law of Cooling. In particular, we will heat up a small quantity of STP in a test tube, insert the test tube into an ice bath at 0° C (which serves as a heat reservoir), and watch the rate at which the STP cools. The rate of heat conduction through a wall is:

$$\frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} = -\frac{kA}{d}(T - T_0) \tag{1}$$

Here k is the thermal conductivity of the test tube wall, d is its thickness, A is the area of the test tube wall, T is the instantaneous temperature of the STP, and T_0 is the temperature of the surrounding ice bath. We will assume that T_0 is exactly 0° C. Since the rate of heat loss dQ/dt is related to the rate of temperature change dT/dt by the specific heat C it follows that:

$$\frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} = mC\frac{dT}{dt} = mC\frac{\Delta T}{\Delta t} \tag{2}$$

then the differential equation describing the cooling of the STP is:

$$\frac{\Delta T}{\Delta t} = \frac{dT}{dt} = -\frac{kA}{mCd}T \tag{3}$$

The solution to this equation is a simple exponential decay of the STP temperature as the compound comes into thermal equilibrium with the water surrounding it:

$$T(t) = T_{initial}e^{-kAt/\,mCd} \tag{4}$$

This is Newton's Law of Cooling. We will make a semi-log plot of the STP temperature as a function of time. According to Newton's Law of Cooling, this should be a straight line. The slope of the semi-log plot tells us the characteristic time of the decay process. Equation 4 shows that the characteristic time of the cooling is mCd/kA. Thus if we measure the change in time Δt required for a 10-fold decrease in temperature T using the semi-log graph, we find that:

$$\frac{mCd}{kA} = \frac{\Delta t}{2.303} \tag{5}$$

Therefore if we know the parameters of the test tube -- k, d and A -- we can determine the specific heat. For pyrex glass, the thermal conductivity is k = 1.1 W/(m°K). The test tube wall thickness d can be measured with a caliper, but not directly; measure the outer diameter of the test tube, measure the inner diameter of the test tube, subtract outer diameter from inner diameter, and then divide the result by two. You should be able to estimate the area of the test tube through which the heat flows to within about 5%. Thus you can calculate the heat capacity C of the STP from Equation (5).

To calculate the latent heat L of the STP, we will take advantage of its ability to supercool, that is cool below the equilibrium crystallization temperature without nucleating crystals of solid STP. The reason that STP can be supercooled easily is that the crystal structure is complicated and it is not easy for the atoms in the liquid to find their correct crystal locations without some help. Therefore, after supercooling the liquid STP, we will put in a "seed" crystal of solid STP. The seed crystal acts as a template, bonding atoms in the liquid to its surface. Thus after the seed crystal is added, the liquid quickly crystallizes throughout. Because of the latent heat of crystallization, an amount of heat, $Q = mL$, is released. This raises the temperature of the STP by an amount ΔT given by the relation:

$$Q = mL = mC\Delta T \tag{6}$$

Since we know the heat capacity C of the STP, we can measure the latent heat L simply by measuring how much the compound heats up during crystallization.

II. Procedure

1. If there is not already STP in your test tube, fill the test tube about half full of crystals. Determine the mass of the STP (only the STP!) that is in your test tube.

2. Prepare an ice bath containing approximately 40% water and 60% ice. This will be used as a thermal reservoir to cool your test tube of STP. Check the temperature of the ice bath.

3. Heat the STP in a boiling water bath to approximately 95° C, and then immediately plunge it into the ice bath. The temperature will drop rapidly at this point so preparation and close coordination between lab partners is required. Lab partners should read temperature every ten seconds for a minute; record these temperatures in a lab notebook; and stir the ice bath to prevent the formation of a warm water jacket around the test tube -- all simultaneously!

4. After you have recorded one minute of data, let the STP cool to about 5° C. Remove the test tube from the ice bath and add a small seed crystal of STP. Observe the formation of crystals and watch the temperature increase carefully. Record the initial temperature and the maximum temperature -- the maximum should occur just as all of the STP has crystallized. The total temperature change ΔT is just the difference between these two temperatures.

5. Plot your measured temperatures for the cooling of liquid STP on a semi-log plot. Ideally, they should give a straight line in accord with Newton's Law of Cooling. From the slope of the line calculate the specific heat of STP using Equation (5).

6. From the measured rise in temperature during crystallization ΔT, calculate the latent heat L using Equation (6).

7. A typical house uses approximately 5 kW-hr of energy per day. If we made a solar collection system with STP, using the latent heat for energy storage, how much STP would we need to store enough energy to run a house for one day?