PY 452 — Quantum Mechanics Syllabus–2018.v1

Professor Kenneth Lane

August 3, 2020

Physics 452 is the second semester of the upper-division course on quantum mechanics. The prerequisites are PY451 or equivalent (contact me ASAP if you have an equivalent course) and PY 355, of course. If you have not satisfied these prerequisites, you must have my permission to take the course.

Very unfortunately, it is not advisable for me to lecture to all of you, or even a portion of you, in person in the classroom. I would very much have to preferred to do that, but I've been advised not to. So, my lectures will be from home via Zoom. This may be somewhat rocky at first, but it is my sincere hope that we will learn to interact well in this new teaching model. Those of you who have had me in a course before know that I like to use the blackboard, to interact with the class by asking questions, and telling the occasional "joke".

The plan of my lectures for PY 452 is in the course outline below. This is the first time I've taught PY 452, so this plan is subject to change — depending on how it goes. Any changes will be distributed via email in a revised syllabus. *This is as good a time as any to urge you to check your email frequently — for updated syllabus + homework assignments, special announcements, etc.* We will be using the text *Quantum Mechanics*, by David McIntyre. This is the same book I used for PY 451 in Spring 2018. McIntyre's approach to quantum mechanics (QM) is to start with simple two-state systems: Stern-Gerlach apparatuses and spin-$\frac{1}{2}$ (Chap. 1). This immediately introduces you to the essential non-classical nature of QM and to a statement of the postulates of QM. It has the especially desirable feature of, from the very beginning, getting you into the very useful Dirac bra-ket notation of "state vectors", the central importance of *"amplitudes"* in quantum mechanics, and their relation to the *probabilities* of processes occurring, and then *"operators"*: the things that we measure, called *"observables"* (like momentum, energy, position, angular momentum, etc.) and the operations that we can do on quantum states (like spatially translate them in space or rotate them; hence, they are called *transformations* or, when appropriate, *rotations*). From there it's a short step to the single column or row "*representation*" of these ket and bra state vectors and the square matrix *representation* of operators (Chaps. 1 and 2). As stated in the preface of McIntyre's book, this approach is inspired by Dick Feynman's presentation in Vol. III of his famous Lectures on Physics — the 3rd volume of three books that every aspiring physicist should own and study.

Because you may have used another book (Griffiths, e.g.) for PY 451 or its equivalent, I have made available in a public site all of my notes from PY 451. It is at `http://physics.bu.edu/py452`. I'll also deposit there my daily lecture notes (hopefully with annotations as soon as I learn how to make them ;-)). I strongly recommend that you review the following Chapters in McIntyre (they will be the subject of the first two homework assignments and the first of our three (or possibly four — we'll see how it goes) take-home exams:

1.) Chapters 1–3, which spell out in clear detail the six postulates of quantum mechanics that we rely upon constantly. You'll find a lot more there too (and I hope to add some discussion of symmetries of the Hamiltonian in quantum mechanics and their deep connection to conservation laws).

2.) Chapter 9 notes on the quantum mechanics of harmonic oscillators, probably the most useful and important system in physics. And Chapters 7 and 8 on a more advanced approach to angular momentum and its spectrum (eigenstates and eigenvalues) that we will use in 452, and the hydrogen atom and hydrogen-like (one-electron) atoms, the other most important problem in your physics education. (This is the order I presented these important topics in PY 451.)

   If you have questions about these notes, please don't hesitate to ask; just send me an email: `lane@bu.edu`.

3.) A brief note on units: In the chapters on the hydrogen atom and its spectrum, the perturbations of its energy levels, and radiative transitions between its energy levels, we will be doing calculations with real experimental numbers. For this, McIntyre uses SI units, but we will use *cgs units*. The reason for this is simple: cgs units are not cluttered with unnecessary (and hard to understand, I've always found) constants such as ϵ_0 and μ_0. Even more important, $\boldsymbol{E}$ and $\boldsymbol{B}$-fields have the same dimensions, namely esu/cm^2 and the simpler units make dimensional analysis easier, helping everyone (including me!) avoid simple, but costly mistakes in calculations. This dimensional analysis is especially easy when discussing/calculating the H-atom energy levels and relativistic corrections to those energies. For our purposes, the conversion from SI to cgs is easy: Just put $4\pi\epsilon_0 = 1$, i.e., $\epsilon_0 = 1/4\pi$, and $\mu_0 \equiv 1/\epsilon_0 c^2 = 4\pi/c^2$ and take the unit of charge to be the esu. The charge on the electron is $-e = -4.803 \times 10^{-10}\,\mathrm{esu}$ where $1\,\mathrm{erg} \equiv 10^{-7}\,\mathrm{joule} = 1\,\mathrm{esu}^2/\mathrm{cm} = 1\,\mathrm{eV}/(1.602 \times 10^{-12}) \equiv 0.624\,\mathrm{TeV}$. In these units, the fine-structure constant is the dimensionless ratio $\alpha = e^2/\hbar c = 1/137.036$.

   After a brief review of this background from McIntyre's (and my) point of view, we will move on to the main topics of PY 452, starting with Perturbation Theory, Chapters 10–12 of McIntyre, then Identical particles (fermions and bosons) Chapter 13, and Time-Dependent Perturbation Theory, Chapter 14 (including, as time permits, electric and magnetic dipole radiation in the H-atom and positronium).

This is an ambitious plan: it depends on my being efficient and your keeping up with the material. The best advice I can give you for a successful semester is to read over the relevant material in the text **before** I lecture on it. You don't have to understand it all before my lecture, but getting a preview of what's coming will prevent its being a big mystery to you. *This is especially important in this unusual season of remote teaching.*

One last word: Try to be "in class", i.e., logged into the Zoom class meeting, **before** I start lecturing at 2:00 PM (2:30 PM for Discussion). Late arrivals disrupt the class and the lecture. So, please don't be late. Thanks!

Instructor: Kenneth Lane, `lane@bu.edu`

Class: Tuesday and Thursday, 2:00–3:15 PM Zoom meeting. You will receive an invitation link to join the meeting.

Discussion: Wednesday, 2:30–3:20 PM. Likewise, discussion sections will be a Zoom meeting.

Office hours: I will be available for Zoom Office Hours on Monday and Thursday, 4:00–5:15 PM. Again, there will be an invitation to join these meetings. To see me at other times, please email me to make an appointment. Please try to do this reasonably well in advance because this turns out to be a pretty busy semester for me. I can also answer questions via email if it does not require a long, involved reply.

Learning Assistant We are fortunate to have **TBA** as our Learning assistant (LA). **TBA** is available to answer questions and clarify QM issues for you. The LA's office hours, coordinates and email address are: **TBA**

Homework Grader: We are also happy to have **TBA** as our grader. **TBA**'s coordinates and email address are **TBA** If you have a question on how to solve a homework problem, ask me or the grader of the LA. For questions regarding grading of homework, please ask the grader first. If you are not satisfied, then ask me. **The procedure for submitting your homework will be announced soon.**

Attendance and Participation: Your attendance in class and discussion section and participation in class discussions are *strongly* recommended. As I said, our course plan is ambitious, which also means that it is tight! I may have to use discussion sections to cover things I did not have time to do in lecture. Of course, discussion is also for clarification of points that may be confusing or difficult.

The following QM texts are on reserve in the Science Library:

Principal Text:
Quantum Mechanics, by David H. McIntyre.

Supplemental Texts: (on reserve in the the Science Library):

Feynman Lectures on Physics, Vol. III, by R. P. Feynman, R. B. Leighton and M. Sands.
Quantum Mechanics, 3rd edition, by David J. Griffiths and Darrell F. Schroeter .
Quantum Physics, 3rd edition, by Stephen Gasiorowicz.
Mechanics—Berkeley Physics Course, Vol. 1 by Charles Kittel, et al. This is a useful resource on special relativity.
Electricity and Magnetism—Berkeley Physics Course, Vol. 2, by Edward M. Purcell.

Grading: Your final grade will be based on homework (25%) and three take-home "midterm" exams (worth 20%, 35% and 20%, respectively, because of the relative amount of material covered in each). Homework assignments and their due dates appear in the course outline below (the first five are listed now; later ones will be in updates of this syllabus). *You are encouraged to work together on homework, but your submitted homework solutions must be your own. Points will be deducted from copied homeworks.* Doing problems in addition to those assigned for homework provides good preparation for exams. Each take-home exam will have several multipart problems and will be distributed on a *Friday* and due to be returned to me via email on the following *Tuesday before noon*. **The exams will be on the Honor System: No collaboration is allowed. You will be allowed to use McIntyre's book, my notes and your own class notes, but no other sources. The Honor System will be strictly enforced.** There will be **NO** make-up exams without an adequate doctor's (or other) excuse.

Finally, required warnings: You are expected to be familiar with and adhere to the BU Academic Code of Conduct (posted at https://www.bu.edu/academics/policies/suspension-or-dismissal.) In particular, cheating on exams or other coursework will not be tolerated; suspected cases will be dealt with in accordance with BU Academic Conduct procedures.

The last day to drop classes (without a "W" grade) is 5 weeks from the start of the semester — Wednesday, October 7. The last day to withdraw (with a "W" grade) Friday, November 6. November 6 is also the last date on which you can designate your grade for this course as Pass/Fail. See http://www.bu.edu/reg/calendars/semester/.

PY 452 COURSE & LECTURE OUTLINE

Note Well: **All dates are preliminary. They will be finalized by August 20.**

1. Reviews, Part 1: The six postulates of Quantum Mechanics as spelled out in McIntyre Chapters 1–3 and, especially, my notes for those chapters. The first five "kinematical" postulates are developed using a spin-1/2 (two-state) system and Stern-Gerlach analyzers. The sixth "dynamical" postulate is made outside this kind of argument. I hope to include here a discussion of the symmetries of a Hamiltonian and their connection to conservation laws (parity/space inversion; rotational symmetry and angular momentum); an example: the precession of an electron's spin (really its magnetic moment) in a magnetic field: $\sim$ 3 lectures + discussion section. If it's not made here, this connection between symmetries and conservation laws will be in the discussion of Chapter 10 in the context of "degenerate-level perturbation theory".

   Homework Set 1, due Friday, September 11: Problems 1.10, 1.13, 1.16, 2.2, 2.6, 2.7, 2.12, 2.15, 2.22, 3.3, 3.4, 3.5, 3.6, 3.11, 3.13 (Hint for 3.13: Construct and use a unitary transformation that interchanges rows 2&3 and columns 2&3.)

2. Reviews, Part 2: The harmonic oscillator, the spectrum of angular momentum, and the hydrogen atom: the simplest realistic, non-relativistic 3-dimensional system that is exactly solvable. McIntyre Chapters 9,7,8 (the order in which they were covered in PY 451) and my notes: $\sim$ 3 lectures, possibly including a discussion section.

   Homework Set 2, due Thursday, September 24: Problems 8.6, 8.7, 8.8, 8.10, 8.11, 9.4, 9.9+add part (d): Verify Ehrenfest's theorem for the ground state of the oscillator by calculating $d(\langle 0|\hat{x}(t)|0\rangle)/dt$ and $d(\langle 0|\hat{p}(t)|0\rangle)/dt$ to show that these expectation values are the solutions to the classical equations of motion of a simple harmonic operator; add part (e): prove that the position and momentum *operators* at time t are given by

$$\hat{x}(t) = U^\dagger(t,0)\,\hat{x}(t=0)\,U(t,0) \equiv e^{iHt/\hbar}\,\hat{x}(0)\,e^{-iHt/\hbar} \tag{1}$$
$$\hat{p}(t) = U^\dagger(t,0)\,\hat{p}(t=0)\,U(t,0) \equiv e^{iHt/\hbar}\,\hat{p}(0)\,e^{-iHt/\hbar} \tag{2}$$

   where $\hat{x}(0) = \sqrt{\hbar/2m\omega}(a^\dagger + a)$ and $\hat{p}(0) = i\sqrt{\hbar\omega/2}(a^\dagger - a)$ are hermitian (where $a \equiv a(0)$ and $a^\dagger \equiv (a(0))^\dagger$). Then show that they also are the solutions to the classical equations of motion of a simple harmonic operator. (Hint: First derive the time dependence of $a(t) =$

$U^\dagger(t,0)aU(t,0)$ and $a^\dagger(t) = (a(t))^\dagger$. The simplest way to do this is to prove that and make use of $dU(t,0)/dt = (-iH/\hbar)U(t,0) = (-i/\hbar)U(t,0)H$. The proof is easy, because H is time-independent and $[H,H] = 0$.) What do you think is the lesson of parts (d) and (e)?

3. **Take-home Exam 1, distributed Friday, September 25 and due by noon (Eastern time!) on Tuesday, September 29.** The exam will cover the material in the reviews above: the QM postulates and their use for the harmonic oscillator, angular momentum and the hydrogen atom.

4. McIntyre Chapter 10, Perturbation Theory; This is Rayleigh-Schödinger perturbation theory, mostly for a *nondegenerate* energy eigenstate: We start with McIntyre's explicit example of a charged spin-1/2 particle in an external magnetic field, and then we solve the general two-state problem for an arbitrary 2×2 hermitian Hamiltonian. In both cases we exhibit the exact solution and the unitary matrix which diagonalizes the full Hamiltonian, the first and second-order corrections (in powers of a weak "coupling constant") to the energy levels, the first-order correction to the state vector (which does not come in until we carry out the expansion to *second* order!), and compare them to the exact results by expanding the exact results to second order. Then we present the general formalism for nondegenerate perturbation theory of any Hamiltonian with discrete energy levels (or even perturbation of a single level that is nondegenerate — often applied to the ground state). Next, we consider the important problem of the perturbation of a *degenerate* energy level; its connection to symmetries of physical systems and the breaking ("lifting") of those symmetries. This is illustrated by the Stark effect in hydrogen in which the *parity* symmetry of hydrogenic states is broken, so that states with different parities are mixed by the perturbation. There will be two problem sets for this chapter.

   Homework Set 3, due Tuesday, October 6: Problems 10.4, 10.5, 10.6, 10.10 (Note that the perturbation expansion must fail for this problem. Why? Hint: Draw the potential), 10.11.

   Homework Set 4, due Tuesday, October 13: Problems 10.12 (What is the symmetry reason for this result?), 10.13, 10.15, 10.16, 10.18, 10.21.

5. McIntyre Chapter 11, Hyperfine structure and the addition of Angular Momenta: The basic H-atom spectrum — $V(r) = -\alpha\hbar c/r \Longrightarrow 1/n^2$ energy-level spectrum — is a consequence of the H-atom being nonrelativistic, with $v_e/c = \mathcal{O}(\alpha) \ll 1$. This spectrum is perturbed by small relativistic corrections leading to what is called the fine-structure interaction (covered in Chapter 12) and the hyperfine interaction, discussed in this chapter. (In actuality, these perturbations come from the Dirac equation for hydrogen, but we won't be deriving them.) The hyperfine interaction in hydrogen is much weaker than the fine-structure interactions of Chapter 12. That is because the hyperfine interaction in hydrogen is suppressed by $m_e/M_p \simeq 1/1840$ relative to the fine-structure ones. The hyperfine interaction (necessarily!) conserves *total* angular momentum but not the individual spin and orbital angular momenta of the electron and proton. This leads to the coupling of spin and orbital angular momenta to form the total angular momentum $\boldsymbol{J} = \boldsymbol{S}_e + \boldsymbol{S}_p + \boldsymbol{L}$ where, as always, $\boldsymbol{L}$ is the angular momentum of the relative motion of the electron-proton bound state. [I do not like McIntyre's notation of $\boldsymbol{I}$ for the proton's spin operator and $\boldsymbol{F} = \boldsymbol{I} + \boldsymbol{S}$ for the total spin of the H-atom. I always use $\boldsymbol{S}$ (with a subscript if necessary) for a spin operator, $\boldsymbol{L}$ for orbital angular momentum, and $\boldsymbol{J}$ for total angular momentum or a generic angular momentum.] In adding these three

angular momenta, we can first add the two spin angular momenta, $\boldsymbol{S} = \boldsymbol{S}_e + \boldsymbol{S}_p$, then add total spin to $\boldsymbol{L}$. For our purposes, the formalism for this is that of the addition of two angular momenta, $\boldsymbol{J} = \boldsymbol{J}_1 + \boldsymbol{J}_2$, which we do with the famous Clebsch-Gordon (CG) coefficients, $\langle j_1 j_2 m_1 m_2 | JM \rangle$:

$$|JM\rangle = \sum_{m_1,m_2} |j_1 j_2 m_1 m_2\rangle\langle j_1 j_2 m_1 m_2|JM\rangle \qquad (M = m_1 + m_2); \tag{3}$$

$$|j_1 j_2 m_1 m_2\rangle = \sum_{J=|j_1-j_2|}^{j_1+j_2} |JM\rangle\langle JM|j_1 j_2 m_1 m_2\rangle \equiv \sum_{J=|j_1-j_2|}^{j_1+j_2} |JM\rangle\langle j_1 j_2 m_1 m_2|JM\rangle. \tag{4}$$

Note: I am also going to use my notation for the addition of these angular momenta and the CG's instead of McIntyre's unconventional and hard-to-remember notation. Note also that the CG coefficients are real. The simple ones are easy to calculate by hand; you can use Wolfram-Alpha for more complicated ones: `https://www.wolframalpha.com/input/?i=Clebsch-Gordan+calculator`. If I can find it, I will give you one-page that will allow you to calculate all low-order CG's even if you are stranded with nothing else on a desert island. If time permits, I will apply all this to positronium (e^+e^-), the hydrogen-like bound state of an electron e^- and its antiparticle, the positron e^+. In positronium, the so-called hyperfine interactions are not suppressed and, in fact, are of the same order of magnitude as the fine-structure interactions discussed in Chapter 12, namely, of $\mathcal{O}(\alpha^4 m_e c^2)$ compared to $E_n = \mathcal{O}(\alpha^2 m_e c^2/n^2)$.

Homework Set 5, due Tuesday, October 27: Problems 11.11, 11.12, 11.16, 11.17+add part (e): determine the effects of the hyperfine interaction on the $n = 2$ levels of positronium with $L = 0, 1$; show that the only angular momenta conserved by the total Hamiltonian are $\boldsymbol{J}^2$, $\boldsymbol{S}^2$, $\boldsymbol{L}^2$ and $J_z = L_z + S_z$ where $\boldsymbol{S} = \boldsymbol{S}_{e^+} + \boldsymbol{S}_{e^-}$; using this, you can construct the states $|nLSJM\rangle$ for $n = 2$ and determine their energies.

6. McIntyre Chapter 12, The Fine Structure Interactions in Hydrogen. These interactions are also relativistic effects, but suppressed relative to the hydrogenic energies $E_n = -\alpha^2 m_e c^2/2n^2$ by $v_e^2/c^2 \simeq \alpha^2$, not by $v_e^2/c^2(m_e/M_p)$. There are two interactions: A rotationally-invariant one coming from the $\mathcal{O}(p^4/(mc)^4)$ term in the expansion of $E/mc^2 = \sqrt{p^2c^2 + m^2c^4}/mc^2$ and the "Larmor interaction" of the electron's magnetic moment with the magnetic field it experiences because of its motion around the proton. If time permits, we shall also take up the Zeeman effect, which explains the splitting of spin-up and spin-down states in the Stern-Gerlach apparatuses of Chapter 1.

   Homework Set 6, **TBA**.

7. **Take-home Exam 2, distributed Friday, October TBA and due by noon on Tuesday, October TBA.** The exam will cover McIntyre Chapters 10, 11, 12, with emphasis on the hyperfine and fine structure in the H-atom and, possibly, positronium. Recall that it counts as 35% of your final grade.

8. McIntyre Chapter 13, Identical Particles: Fermions (half-integer spin particles) and Bosons (integer-spin particles). The system of two identical spin-1/2 particles (electrons, e.g.), one with spin-up, the other with spin-down <u>or</u> with their spins aligned; the exchange operator,

and the *symmetrization postulate* of quantum mechanics. What it means to have a state of N identical particles that is "overall symmetric" or "overall antisymmetric". I will complement McIntyre's development with some explicit examples: (1) 3-D bound states of two identical fermions with spin-1/2 (e.g., the helium atom) versus 3-D bound states of two spin-1 bosons; (2) The ground state of helium (also covered by McIntyre); (3) "Isotopic spin$\times$Angular momentum$\times$parity" and the (strong-interaction) decays of a spin-1 ρ-meson to two spin-0 π-mesons (pions) and (3) the (weak-interaction) decays of a spin-0 K-meson to two pions.

Homework Set 7 **TBA**.

9. McIntyre Chapter 14, Time-Dependent Perturbation Theory (also see Feynman, Volume III, Chapters 8&9). As I stated in my PY 451 notes, The total energy of an *isolated* system is conserved — constant in time — and so, therefore, is its Hamiltonian. But sometimes it is convenient (easier) to treat two parts of a total isolated quantum system separately, so that energy is not conserved in each part separately. Then, the total Hamiltonian in such an approximation is *not* time independent and we cannot solve the Schrödinger equation by the simple time-evolution unitary operator as in

$$|\psi(t)\rangle = e^{-iHt/\hbar}|\psi(0)\rangle. \tag{5}$$

A "classic" example of this is the treatment of radiative EM transitions between the "stationary-state" energy levels of an atom. Here, there are two types of transitions: *induced* ones brought about by the atom's being "bathed" in an external EM field and consisting of absorption and emission of photons; and *spontaneous* emission of photons which occur even when there is no external EM field because, in quantum mechanics, there are always *virtual* photons that couple to the electric charge of the atomic electrons. In such situations, we treat the EM-field part of the Hamiltonian as being time-dependent. Of course the *total* energy of the system — here, that of the atom and the photons — is conserved and the total Hamiltonian is time-independent and its eigenstates are stationary states, but that requires a more sophisticated (and actually not much more illuminating) treatment.

In this final chapter of PY 452, we will study: The first-order perturbation formalism for the probability of transition between energy levels (see also Feynman): The Rabi formula; "Fermi's Golden Rule" (actually due to Dirac, I have heard); application to electric dipole (E1) transitions in the H-atom and positronium with the *angular distribution* of E1 radiation (compare to the E1 radiation angular distribution in PY 406); the *selection rules* of E1 transitions. If time permits, magnetic dipole (M1) transition rates and angular distribution in the H-atom and positronium; and, finally, the Einstein A and B coefficients.

Homework Set 8 **TBA**.

10. **Take-home Exam 3, TBA**. The exam will cover McIntyre Chapters 13 and 14.