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EFFECTS OF BOUNDARIES IN AN ARRAY OF SUPERCONDUCTING WEAK LINKS*

George O. Zimmerman, Physics Department, Boston University, Boston, MA 02215, and C.J. Lobb and M.S. Rzchowski, Physics Department and Division of Applied Science, Harvard University, Cambridge, MA 02138, U.S.A.

We report the results of a computer simulation of the behavior of a finite array of superconducting weak links on a two dimensional square lattice. We find that the shape as well as the size of the array have an important influence on the behavior of quantized vortices in the array and the induction of those vortices into the array when a magnetic field is applied. In particular, we find it impossible to induce only one vortex in a regularly shaped array.

INTRODUCTION

Two dimensional arrays of superconducting islands coupled by Josephson junctions have been studied extensively [1]. Both theoretical and experimental investigations have been carried out, but most of the investigations concentrated on the case where the array was very large, or infinite.

Here we concentrate on the investigation of finite arrays, where the boundaries play a significant role. We find that the introduction of quantized vortices into the array depends on the shape of the boundaries, and the barrier for a vortex to move from one minimum energy position to another depends on the shape of the boundaries up to a 50×50 array. It also depends on the location of the vortex.

NUMERICAL SIMULATIONS

The simulations are carried out on an array of superconducting islands on a square lattice which are coupled to their nearest neighbors by Josephson junctions. As in reference 1, it was assumed that the Hamiltonian is the sum of the individual junction energies

$$H = E_J \sum_{ij} [1 - cos(\phi_i - \phi_j - \psi_{ij})] \qquad (1)$$

where ϕ_i is the phase of the superconducting order parameter at site i, and $\psi_{ij} = 2e/\hbar c \int_i^j \mathbf{A} \cdot d\mathbf{l}$ is the integral of the vector potential along the juction between sites i and j. E_j is the Josephson energy $\hbar i_c/2e$, where i_c is the isolated junction critical current.

Minimizing the energy with respect to the variation of the phases is equivalent to the requirement that the sum of the currents flowing into each island is zero[1]. The minimization is accomplished by means of an iterative procedure, as in reference 1. The boundaries are allowed to be free, and change in accord with the minimization conditions. The resulting state is not necessarily the absolute minimum, but may be a metastable configuration which depends on previous history.

The calculation is done at zero temperature and in the limit in which current induced magnetic fields are small compared with the externally imposed field normal to the plane. For convenience the units are chosen such that $E_J = i_c = 1$. f is the number of flux quanta enclosed by a unit cell.

RESULTS

The *first* result to be discussed is the size of the potential barrier a vortex has to overcome to move between two minimum potential sites, where the vortex center is in the middle of four islands. The vortex is at the array center. Reference 1 shows that this barrier approaches $0.199E_J$ in the limit of an infinite square array. We have carried out this calculation both for a square array and a rectangular one where the number of rows exceeded the number on columns by one. Thus, for instance, instead of an 8×8 square we had an 8×9 rectangle. The vortex was displaced along the long direction.

Fig. 1 shows the energy barrier as a function of the array size for both the square and the rectangle. One observes that the square approaches the infinite size limit from below, the rectangle approaches the same limit from above.

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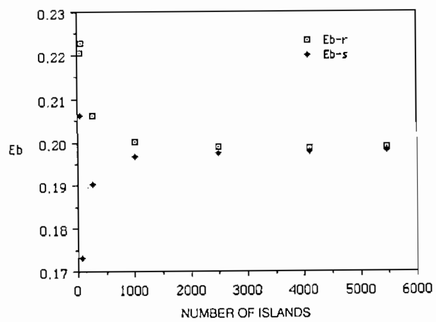

Figure 1

The *second* result of the calculation is the observation that it was impossible to introduce a single vortex into the array as one increased the magnetic field from zero. If one starts from zero field and increases the field gradually, using the previous configuration as the starting configuration for the next higher field, one encounteres shielding currents which raise the energy of a 16×16 array to over 100. Upon a small further increase in the field, four vortices enter the square array. Those vortices were situated symmetrically around the center as shown in Fig.2. More vortices enter as the field is increased. Those vortices are symmertrically arranged in low fields. The fact that one can induce no fewer than four vortices in a square array holds to the largest arrays investigated, which is 74×74.

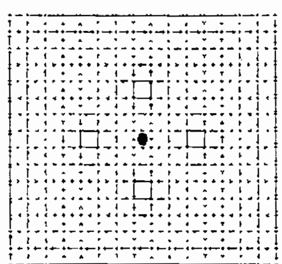

Figure 2

Fig.3 shows the energy of a 16×16 array with four vortices. The energy is given as a function of the vector potential fraction f. The point at E=100 is the energy before the vortices enter, and subsequently the field is reduced. The discontinuities in the curve occur when the vortices move away from the center by one lattice spacing. Each segment fits a quadratic in f.

We also tried an array which was circular to within the lattice constant. Fig.4 shows the result of the simulation.

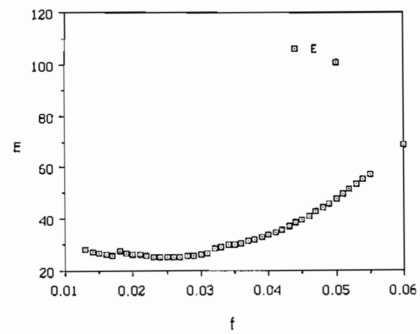

Figure 3

As one increases the field from zero, one sees that one can induce two vortices which tend to be distributed symmetrically about the center of the array. Again, it was impossible to induce only one vortex.

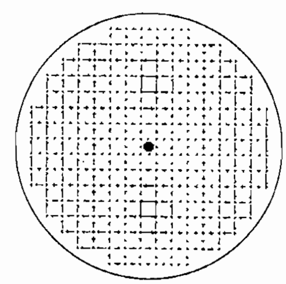

Figure 4

CONCLUSIONS

We find that the boundaries of our systems play a profound role on the behavior of vortices. We also find that it is impossible to induce *only one* vortex into a symmetric array with the application of a magnetic field. The vortices enter symmetrically. The metastable states obtained in this simulation may approximate the actual behavior af an array. Since the behavior of the arrays was used in the interpretation of the behavior of high T_c superconductors[2], this simulation may have a bearing on the behavior of small granules of high T_c materials.

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(1) C.J. Lobb, D.W. Abraham and M. Tinkham, Phys. Rev. B27, 150, (1983)

(2) M. Tinkham, Phys. Rev. Letters 61, 1658, (1988)