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PHYSICA B

Vortex formation on Josephson junction lattices

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Using relaxation techniques, the authors have simulated the flow of current vortices caused by a magnetic field applied perpendicular to a two dimensional lattice whose sites are connected by Josephson junctions. The inception of vortex flow coincides in each case with a reduction in the energy of the lattice. Different boundary conditions give rise to a different initial number of vortices, but energies per junction remain approximately the same regardless of the boundary conditions. In each case, the threshold magnetic field decreases inversely with the number of junctions in the lattice. A square "honeycomb" lattice, i.e. with each Josephson junction connected to nearest three instead of four neighbors, displays the similar behavior in an applied external magnetic field as an ordinary square lattice.

1. INTRODUCTION

A lattice whose points are connected by Josephson junctions has the Hamiltonian

$$H = E_J \sum_{ij} [1 - \cos(\varphi_i - \varphi_j - \psi_{ij})] \qquad (1)$$

where the sum runs over all nearest neighbors i and j, φ_i is the phase of each lattice site and $\psi_{ij} = 2e/\hbar c \int_i^j \mathbf{A} \cdot d\mathbf{l}$ is the line integral of the vector potential along the junction connecting sites i and j. E_J is the Josephson energy $\hbar i_c / 2e$, where i_c is the junction critical current. Minimization of the Hamiltonian with respect to the phase is equivalent to the condition that the sum of the currents flowing through each site is zero. Units are chosen such that $E_J = 1$. Using an iterative procedure, the phase is relaxed until a metastable minimum is achieved.[1,2,3]

2. RESULTS

We have carried out the above relaxation procedure for three different lattice structures. First, the relaxation was carried out on an isosceles right angled triangular lattice (type I), comprising half the square lattice (Figure 1). As the vector potential is increased, the energy per junction of the lattice rises quadratically up to a point at which it abruptly decreased, conforming to earlier results for a square lattice.[3] At this point, two current vortices were introduced into the lattice. As the vector potential was increased further, the pattern repeated itself, and a pair of current vortices was introduced at each discontinuous jump in energy . As the size of the lattice was increased, the first critical vector potential point, f_c, decreased inversely as the total number of junctions, n_j (Figure 2). Furthermore, the spacing between subsequent f_c 's decreased.

Second, the relaxation was carried out for an right angled isosceles triangle (type II) comprising one fourth of the square lattice (Figure 3). It exhibited the same energy characteristics as the type I triangle except that a single vortex was introduced at each energy discontinuity.

Third, the relaxation was carried out for a square lattice with each lattice site connected to the

Figure 1. A 16x16 triangle (type I).

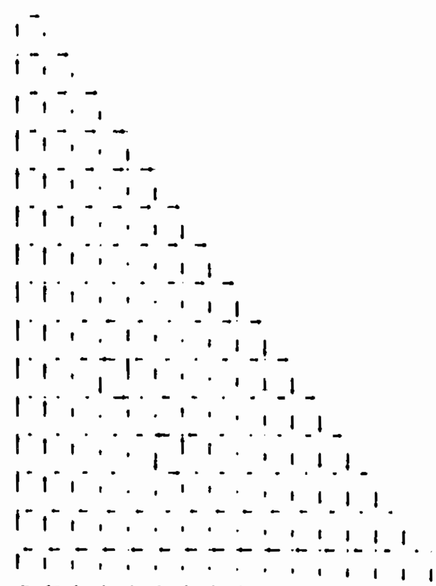

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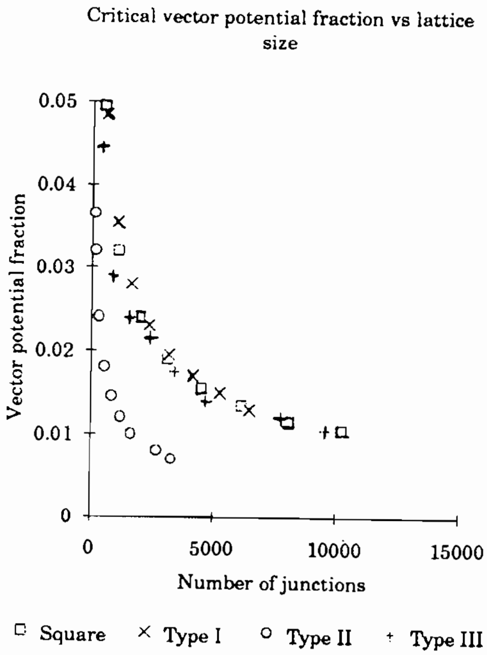
Figure 2.



nearest three neighbors instead of nearest four (type III) as previously investigated by G.O.Z.. The geometry of such a lattice is equivalent to that of a honeycomb lattice which has been continuously deformed so as to make all angles 90°. We find that

Figure 3. A 16x16 triangle (type II).

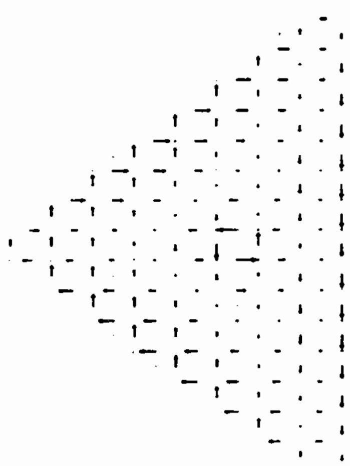
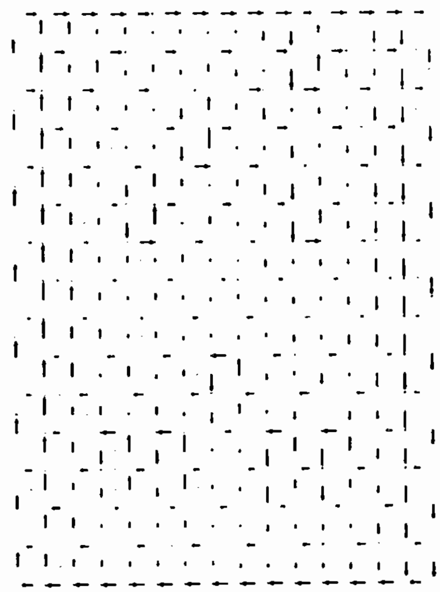


Figure 4. A 16x16 "honeycomb" lattice (type III).



such a lattice exhibits similar energy and current features as a regular square lattice except that the initial number of vortices introduced is one instead of four.[3] Again, f_c decreases inversely with the number of junctions. Such a lattice with six current vortices is shown in Figure 4.

3. CONCLUSION

We conclude that boundary conditions and lattice structure play a significant role in determining the number of current vortices resulting from the application of a magnetic field on a two dimensional Josephson junction lattice. Furthermore, the critical vector potential fraction at which the vortices are introduced decreases inversely with the number of junctions on the lattice.

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