QUANTUM STATISTICS AND THE MANY-BODY PROBLEM

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THERMAL BOUNDARY RESISTANCE BETWEEN SOLID ^3He AND CERIUM MAGNESIUM NITRATE

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We have measured the thermal boundary resistance between powdered cerium magnesium nitrate (CMN), (average particle diameter $\sim 50\mu$) and solid He^3 (23.9 cm^3/mole, $\sim$15ppm He^4) and liquid He^3 (S.V.P.) at several different applied magnetic field strengths in the temperature range between 45 mK and 250 mK. At temperatures below 70 mK and fields greater than $\sim$55G, the observed measurements of the magnetic Kapitza resistance are consistent with the T^2 dependence of that resistance predicted by R. A. Guyer[1] for the solid He^3 and the T dependence for the liquid observed by others at lower temperatures[2,3] and theoretically predicted[1,4,5]

The method used to measure the boundary resistance was similar to that of Ref. 2. The experimental chamber, shown in Fig. 1 was thermally tied to a dilution refrigerator and contained 0.5 gm of CMN and 2.7 cm^3 of He^3. These quantities were chosen so that the specific heat of the He^3 was always significantly greater than that of CMN. To insure that the CMN was coupled to the He^3 rather than directly to the dilution regrigerator, experiments were also carried out with no He^3 in the chamber, with only He^3 vapor and with liquid He^3. The temperature was measured by means of a resistor calibrated against the CMN susceptibility at zero magnetic field. As an added check, it was determined that at the temperatures of our measurement of the relation between $\chi(0)$, the susceptibility of CMN in zero

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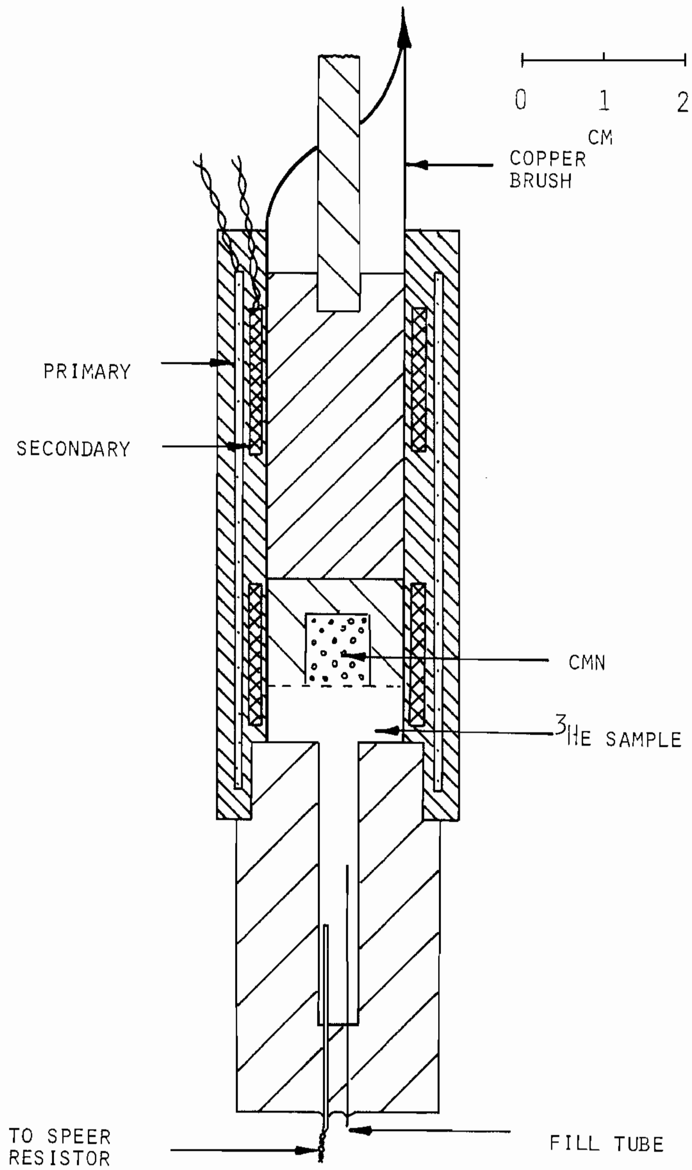

Figure 1

magnetic field, and $\chi(H)$, the susceptibility in a dc magnetic field, H, parallel to the susceptibility measuring field was given by[6]

$$\frac{\chi(0)}{\chi(H)} - 1 \propto H^2 \qquad (1)$$

to a high precision.

By applying a magnetic field and then reducing it to a desired field value, we quickly demagnetized the CMN powder, thereby lowering its temperature ($\frac{\Delta T}{T} \sim 3\%$) and then observed the subsequent equilibrium time constant as it warmed back to the ambient temperature of the He^3. These time constants are shown in Fig. 2 for the solid and Fig. 3 for the liquid at different final magnetic fields.

The time constants at H=0, shown in Fig. 4 (H is the applied magnetic field) fit a straight line described by

$$\tau_{P.B.} = 7.4 \times 10^{-3} T^{-2} \text{ sec} \tag{2}$$

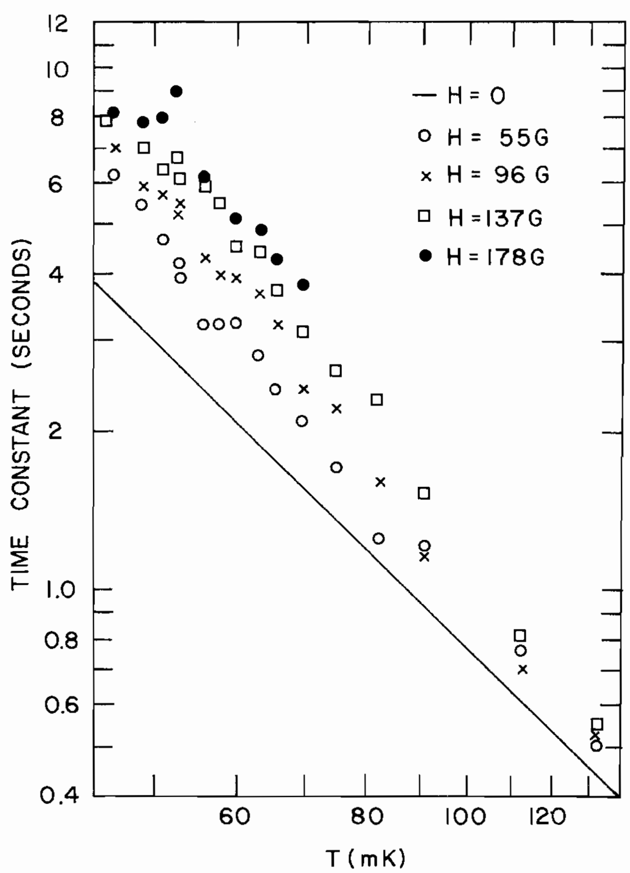

Figure 2

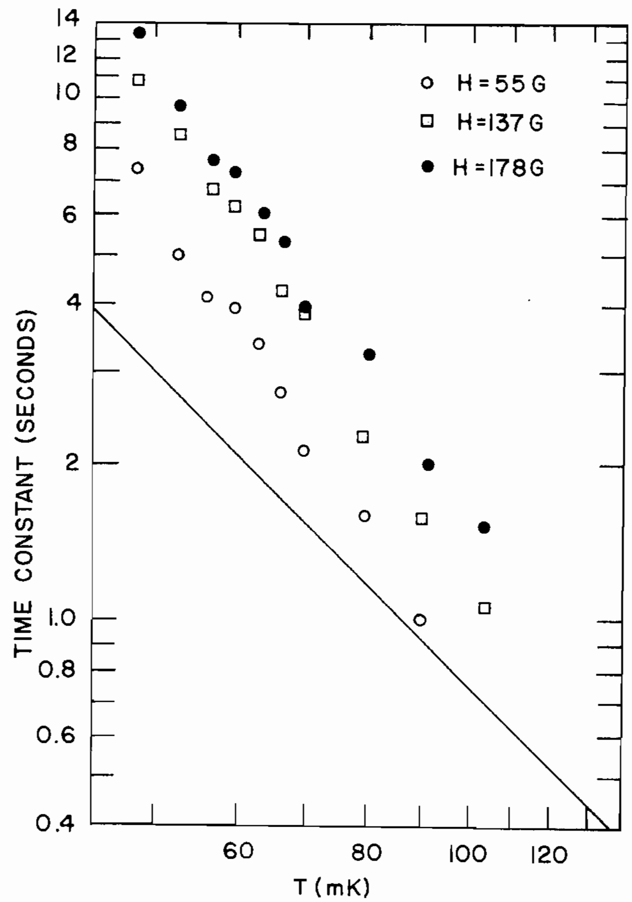

Figure 3

which is the expected "phonon bottleneck" observed in CMN at these temperatures.[7] It has been shown[8] that the phonon bottleneck time constant in CMN, $\tau_{P.B.}$ is field independent in the high temperature approximation. Thus, if one assumes that the phonon bottleneck is in series with the total thermal boundary resistivity ρ_B, which is associated with the time constant, τ_B, through the relationship

$$\tau_B = (V/A)\ c_V(H)\rho_B \tag{3}$$

where $c_V(H)$ is the heat capacity of CMN per unit volume[9] and equals

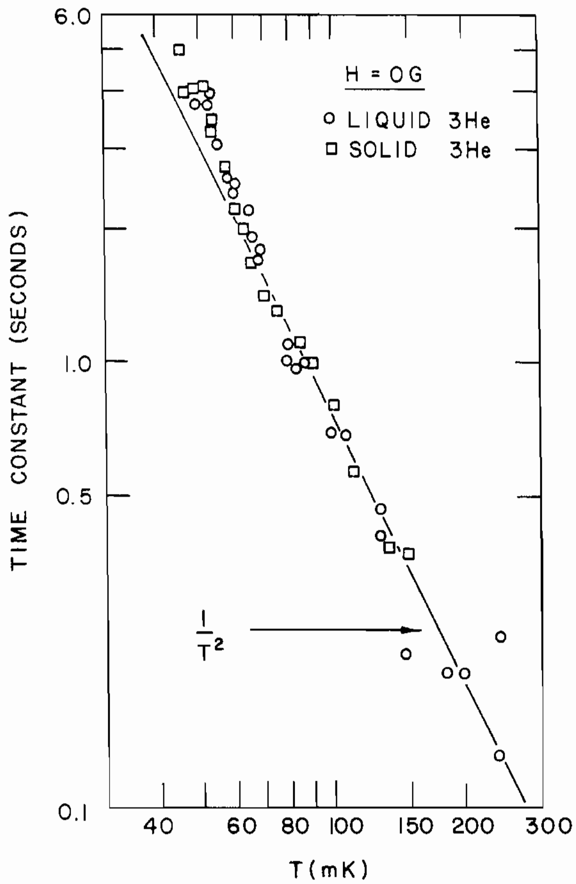

Figure 4

$c_V(0) + CH^2/T^2$ where C is the Curie constant, V is the CMN volume and A is its surface area,[10] $\tau_B(H,T)$ can be determined by measuring the total relaxation time, $\tau(H,T)$ in a magnetic field at a temperature T, and subtracting the value of the field independent $\tau_{P.B.}(T)$, given by Eq. (2). These results for ρ_B are shown for the solid in Fig. 5 and the liquid in Fig. 6.

The value of V/A for the powdered CMN sample was estimated according to the method used by Bishop, Cutter, Mota and Wheatley.[3] However, the reliability of this estimate is in question and might possibly be smaller by an order of magnitude because the particles are in contact with each other and have anisotropic heat conductivity. Thus the absolute value of ρ_B is only approximate and the significant result is the dependence of ρ_B on T.

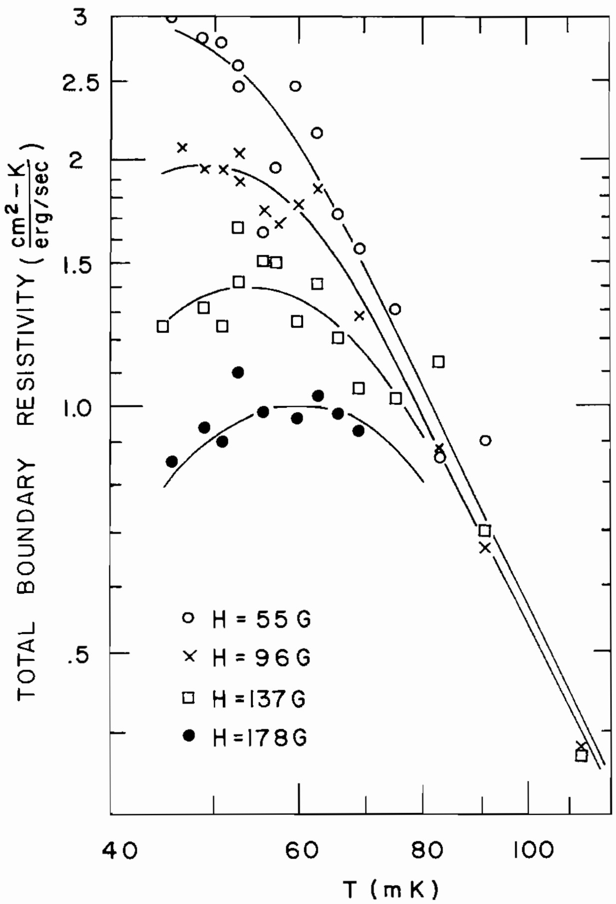

Figure 5

The total thermal boundary resistivity, consists of a parallel combination of phonon resistivity, ρ_P, and magnetic dipolar resistivity, $\rho_m(H)$.[3] We can write

$$\rho_B(H) = \rho_P \rho_m(H) \;/\; (\rho_P + \rho_m(H)) \tag{4}$$

By assuming the field independent ρ_P takes the form $\rho_P \alpha T^{-3}$, we calculated from the data in Fig. 5 and Fig. 6 that

$$\text{(Solid)} \quad \rho_P = 5.7 \times 10^{-4}((K^4\text{-}cm^2)/(erg/sec))T^{-3}.$$

$$\text{(Liquid)} \quad \rho_P = 5.5 \times 10^{-4}((K^4\text{-}cm^2)/(erg/sec))T^{-3}. \tag{5}$$

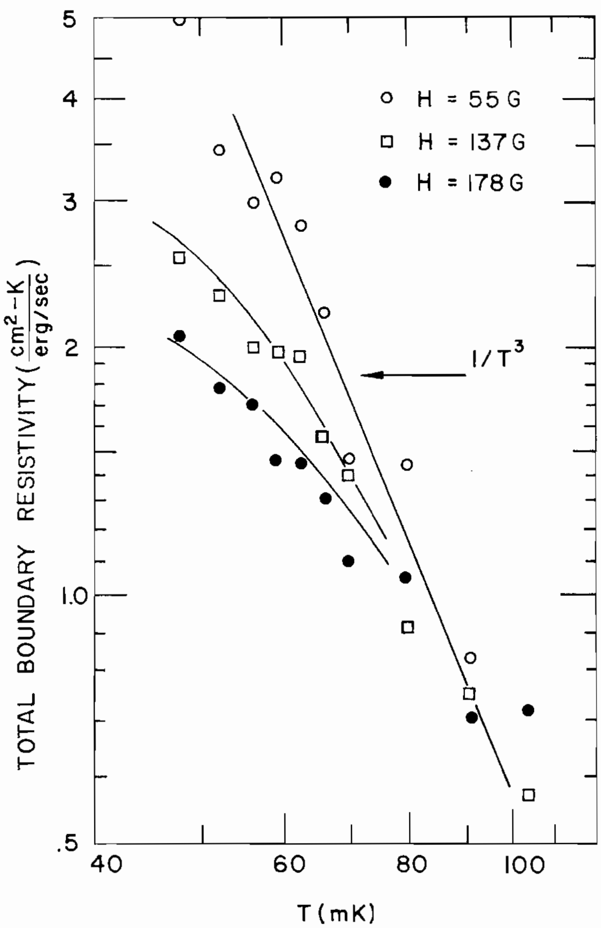

Figure 6

From this, the values of $\rho_B(H,T)$, and Eq. 4 we obtain $\rho_m(H,T)$. In order to compare our data with theoretical prediction, it was useful to relate $\rho_m(H,T)$ to $\rho_m(0,T)$. Using a Redfield-like theory,[11] J. Bishop derives the relation

$$\rho_m(H,T) = (H_d^{\,2}/(H_d^{\,2} + \tfrac{1}{2} H^2))\ \rho_m(0,T) \qquad (6)$$

where H_d is the CMN mean square local dipolar field and H is the applied field. Eq. 6 is valid for the condition of H, $H_d \ll k_B T/\gamma h$ (γ is the CMN gyromagnetic ratio) which holds in our measurements, and implies that one improves thermal contact by increasing the applied field.

If we assume H_d = 40G and apply Eq. 6 to the obtained $\rho_m(H,T)$ values we obtain the data shown in Fig. 7 for the solid. It is seen here that our data are in agreement with the predicted form for the solid[1]

$$\rho_m(0,T) = \text{const.} \times T^2 \quad \text{(solid)} \qquad (7)$$

with a mean square deviation of ∿15%. The constant, (which, as stated before is not reliable) is equal to 5×10^3 cm^2-sec/erg-K.

The data for $\rho_m(0,T)$ in the liquid case are presented in Fig. 8. They show agreement with the equation

$$\rho_m(0,T) = 760 \frac{cm^2}{erg/sec} T \qquad \text{(liquid)} \qquad (8)$$

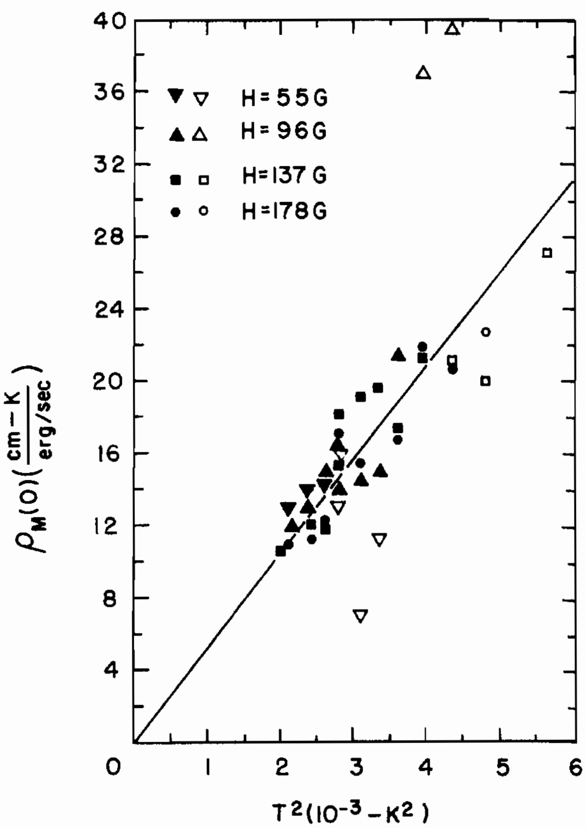

Figure 7

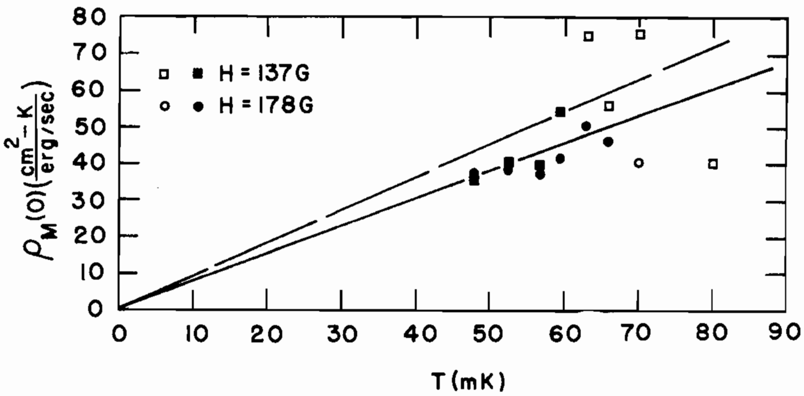

Figure 8

with a 20% mean square deviation in the temperature region between 50 mK and 70 mK. For comparison the coefficient of T in Eq. 8 is given as 900 in (BCMW)[3] and as 1400 in (BMWBB).[3] The precision of the liquid measurement was smaller than that of the solid because at the lowest temperatures, about 50 mK, $\rho_m(0,T)$ of the liquid is a factor of 5 greater than that of the solid. The observations of $\rho_m(0,T)$ in the liquid at these temperatures seems to contradict the interpretation by Harrison and Pendrys[12] of the data of Bishop et al. in terms of only phonon Kapitza boundary resistance and the phonon bottleneck.

To our knowledge, this is the first experimental observation of the T^2 dependence of ρ_m between CMN and solid He^3. Although the precision of the data is not sufficient to establish the T^2 dependence precisely in the solid He^3 case, the mean square deviation of the data from this law is 15% and thus it definitely rules out the T dependence obeyed between CMN and liquid He^3. This result is also significant because of its implication on paramagnetic cooling of He^3 solid by CMN where the limiting resistance will now become the phonon bottleneck. $\tau_{P.B.}$, however, depends on the particle size[12] and could thus be made small. Although CMN undergoes a paramagnetic to antiferromagnetic transition at 1.6 mK, samples have been cooled down to 0.4 mK[13] by adiabatic demagnetization, and thus CMN should be able to cool solid He^3 to temperatures below 1mK.

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