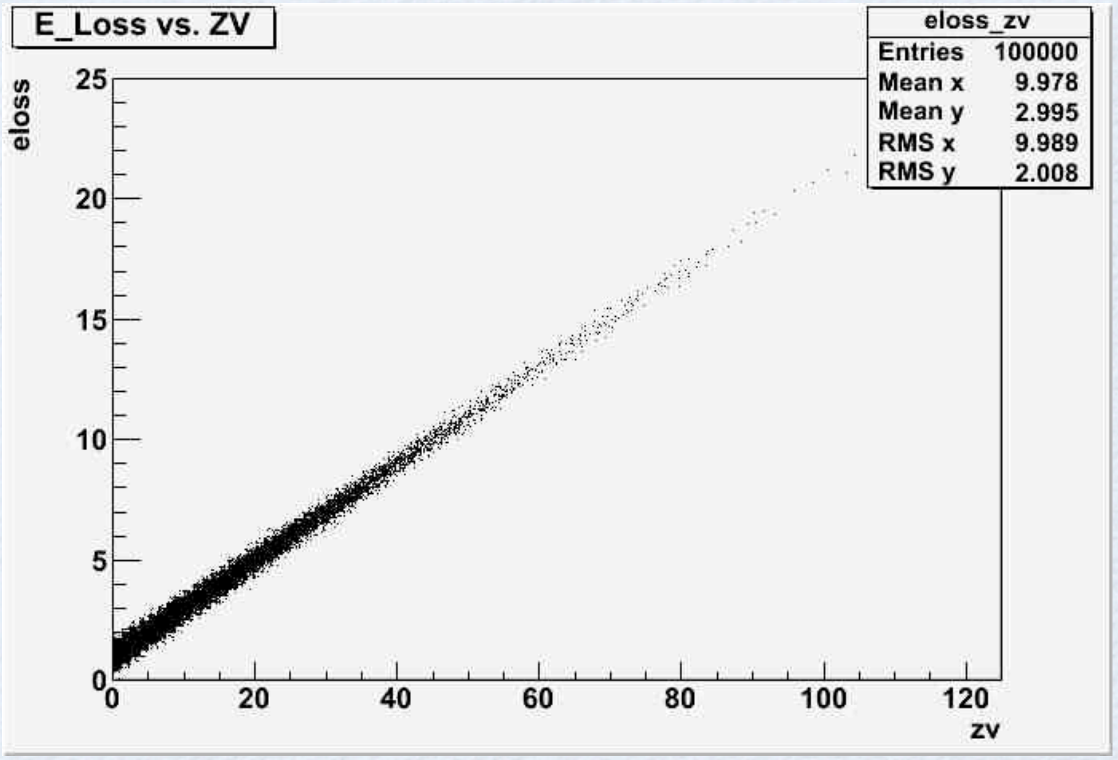
Event Reconstruction: Tracking

Few Points

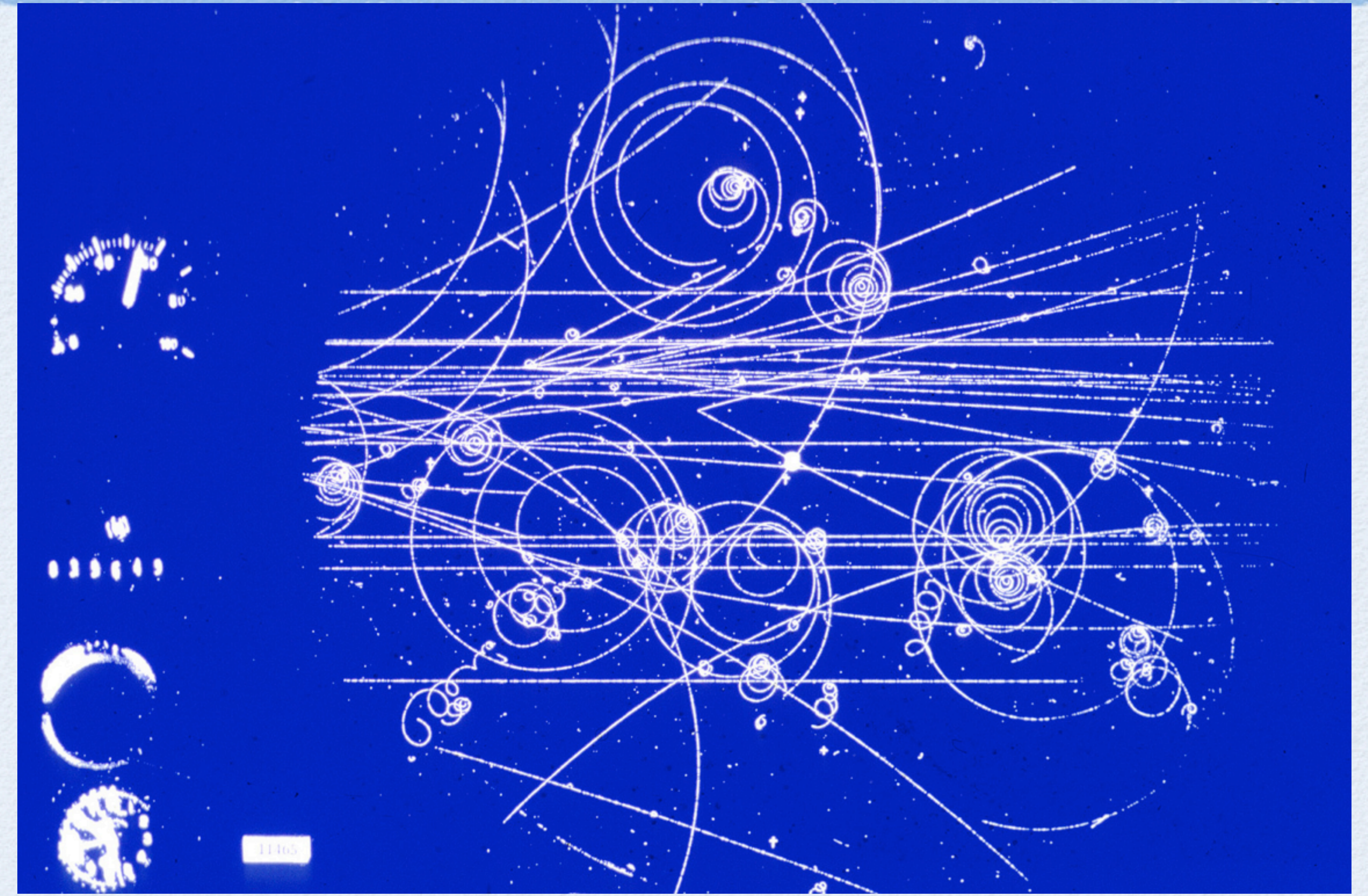
- Most everyone did not get the ‘randomness’ part of homework one correctly.
- If you want to generate a random number from a distribution you can just generate a random number within the range of that function uniformly and then apply the function to each random number
- Only 2 people have sent me requests about what topic they would like to report on
    - I will start a doodle poll about when to do the presentation (towards the end of the class for the last couple weeks of the course)

HOMEWORK PROBLEM

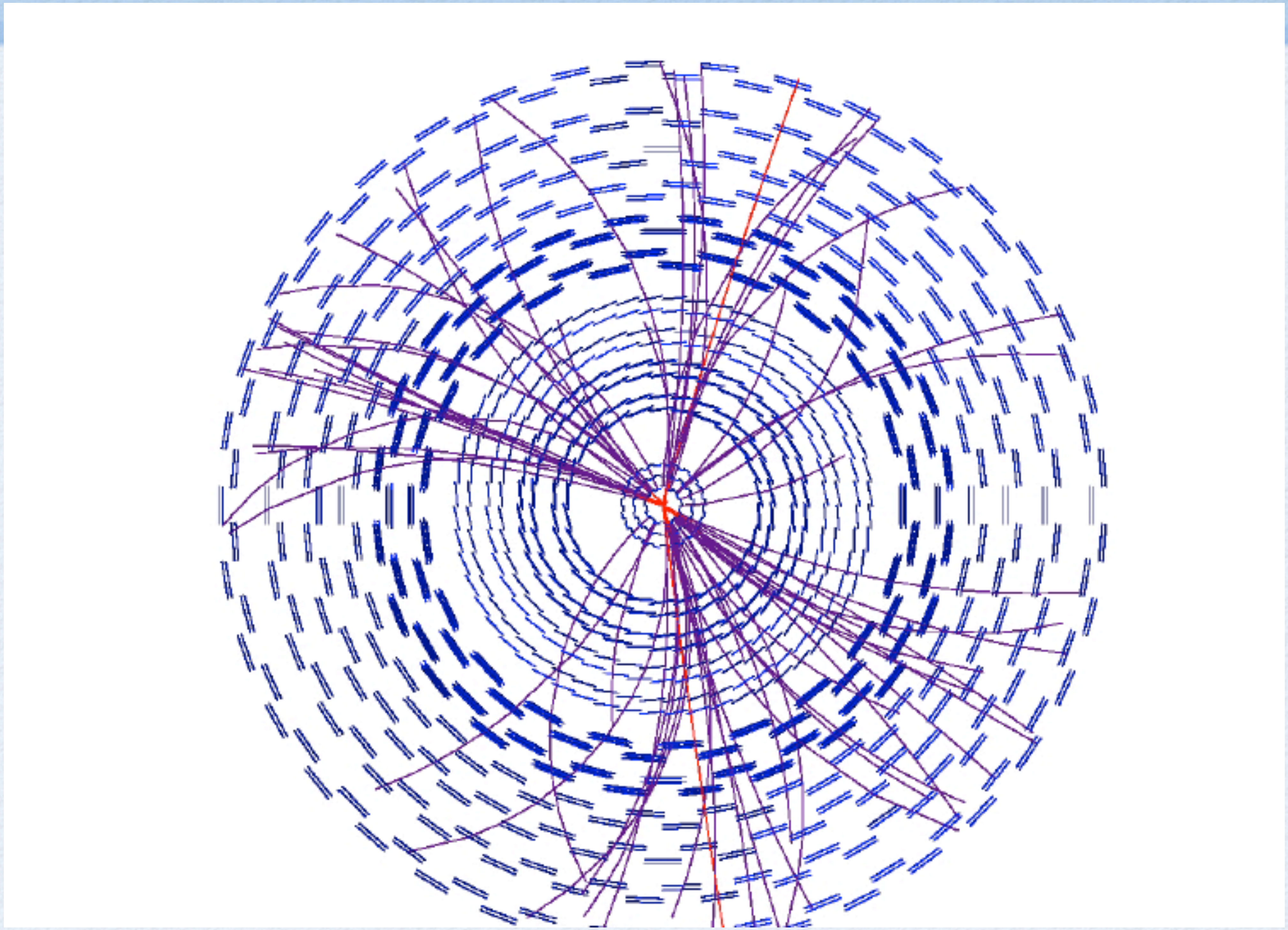
- Choose bins of zv with roughly equal number of events
- Average the eloss for each of those bins
- Make a new histogram or TGraph plotting these versus each other
- Fit the result to a line



CERN Bubble Chamber 1960s



CMS TRACKING

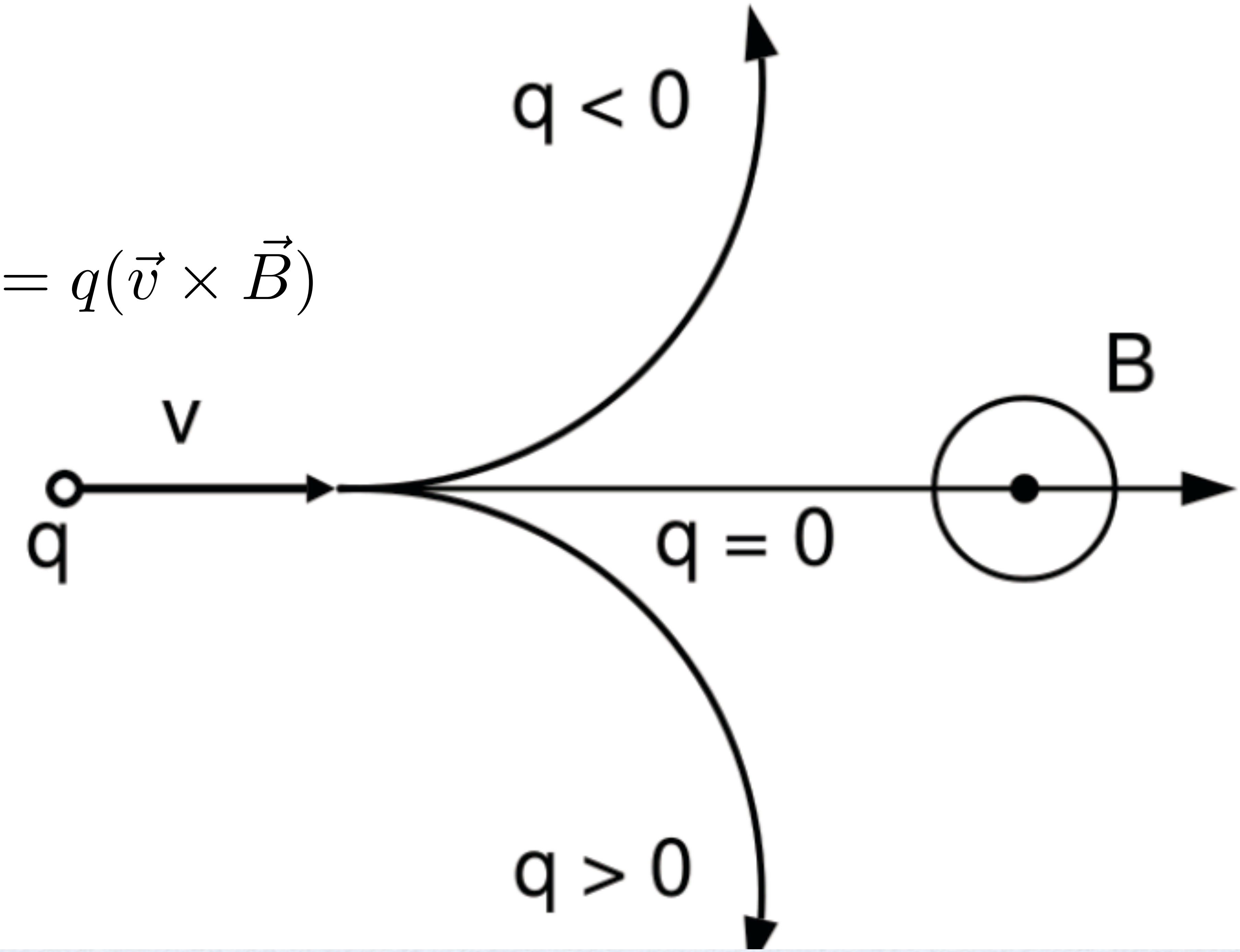


Tracking Basics

- When we talk about tracking we want to :
  - Measure the true path of the particle, which lets us know...
  - The momentum (3-momentum) if we know the field
  - The sign of the charge
  - The origin of the particle in space
  - without other information we cannot know the mass !

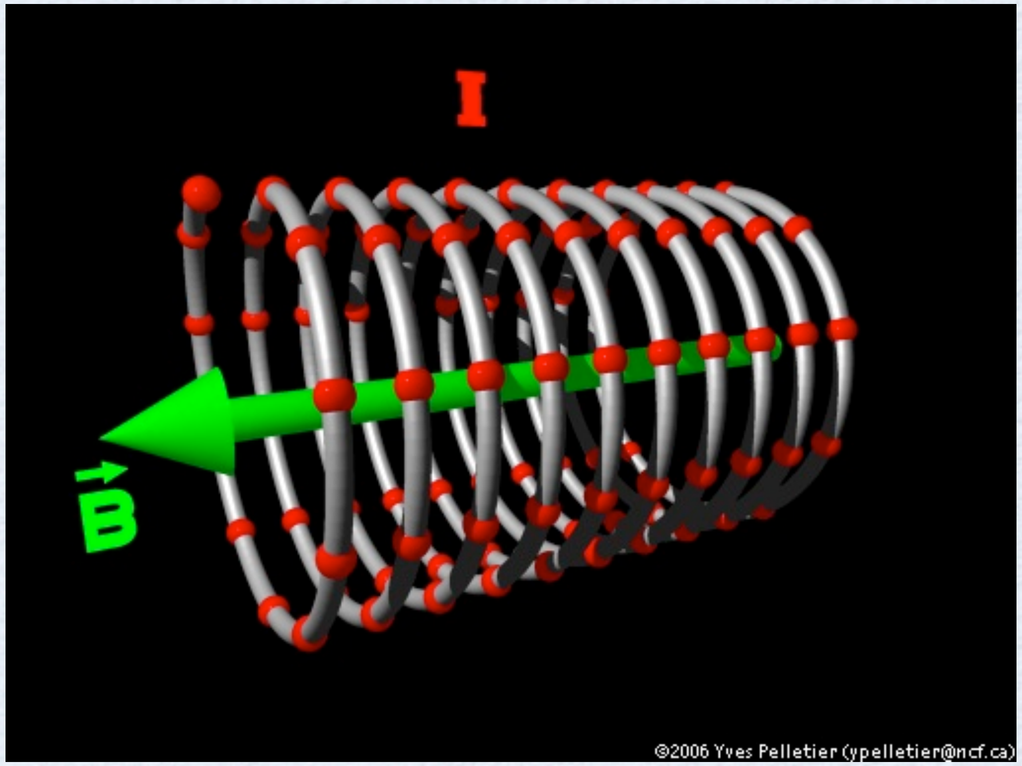
LORENTZ FORCE

$$F = q(\vec{v} \times \vec{B})$$



Particle Trajectory

- For a charged particle produced at the center of the solenoid how can we describe its trajectory?



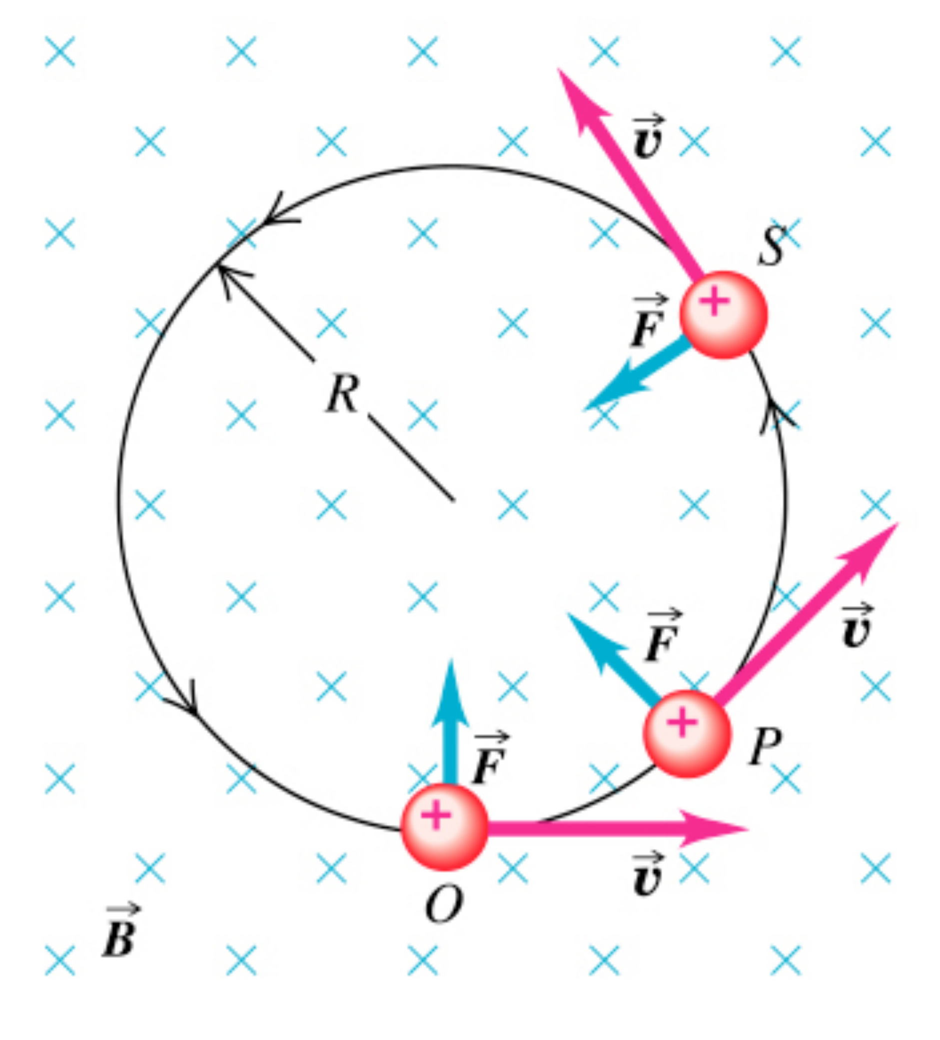
Aside

- Why is the inner magnet of a colliding beam experiment a solenoid?

IN 2-D

$$F = q(\vec{v} \times \vec{B})$$

- The force and hence the acceleration are always perpendicular to the velocity so..



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In 3 Dimensions

- For a solenoid surrounding the beam pipe we typically take the beam direction to be z - which is where B points

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- This means that the distance it travels in z is proportional to the arclength s that the particle traces out in the x,y plane

Helical Track

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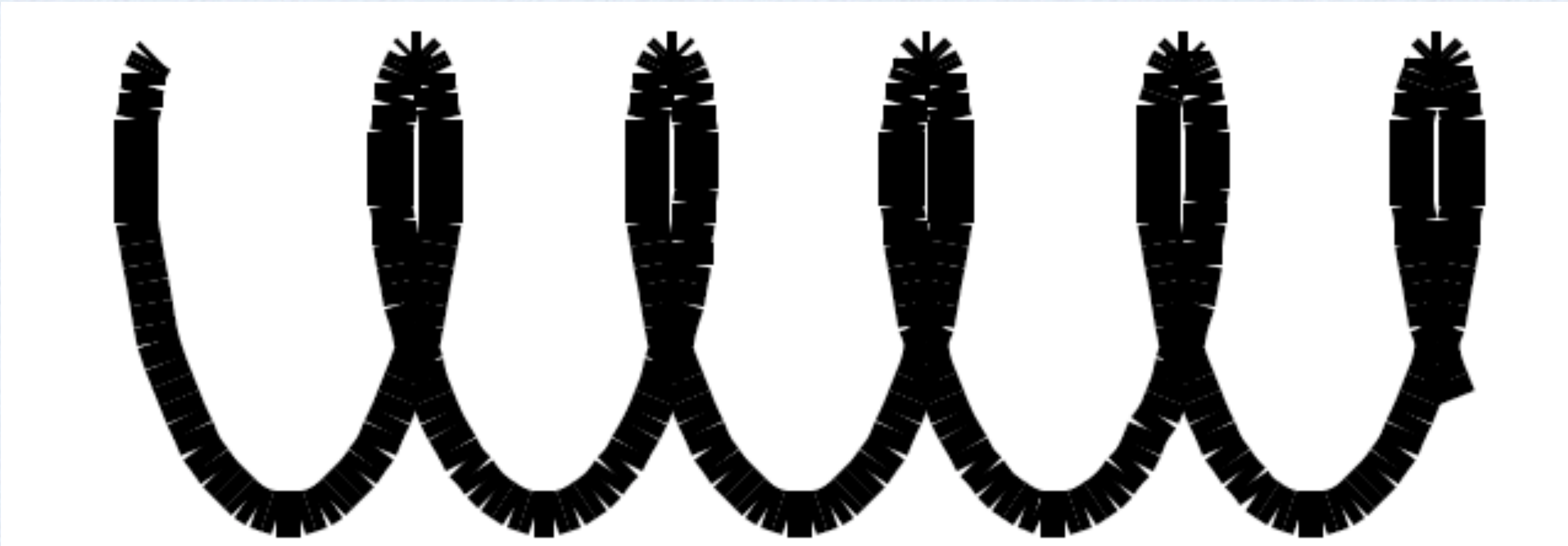
- The Lorentz force causes it to trace out a circle in the x,y plane but there is no force so it has constant velocity motion in the z direction
- This means that the distance it travels in z is proportional to the arclength s that the particle traces out in the x,y plane
- Think of the motion as a straight line in the s-z plane



Helix Parameters

- In spherical coordinates

$$p_x = p\ cos\phi sin\theta$$

$$p_y = p\ sin\phi sin\theta$$

$$p_z = p\ cos\theta$$

Helix Parameters

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- Different experiments use different conventions for the ranges of the angles

$$p_x = p\ cos\phi sin\theta$$
$$p_y = p\ sin\phi sin\theta$$
$$p_z = p\ cos\theta$$

$$\phi[-\pi, \pi]$$
$$\theta[0, \pi]$$

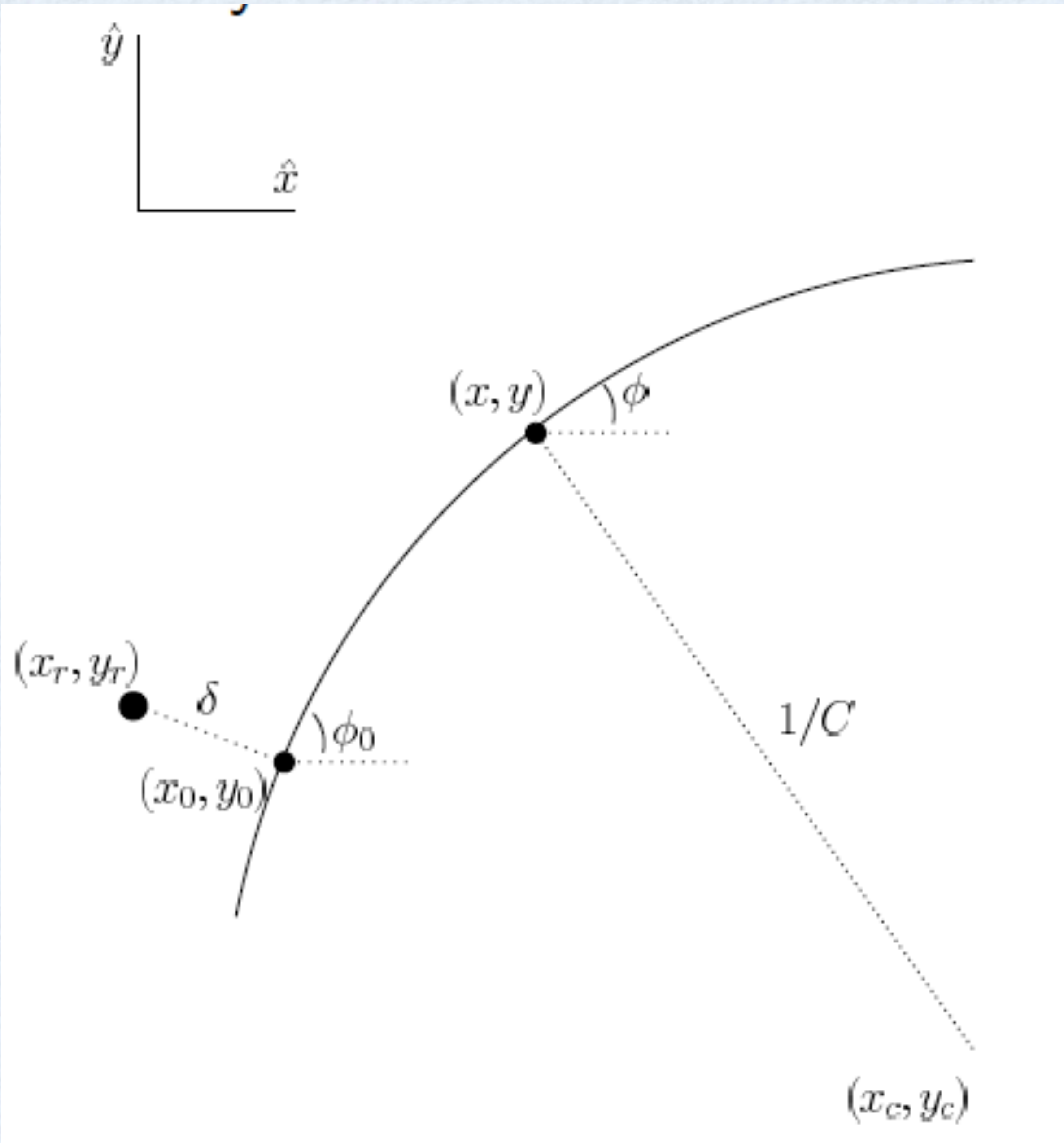
Helix Parameters

- In spherical coordinates
- Different experiments use different conventions for the ranges of the angles
- Also need a reference point for where the helix starts

$$p_x = p\ cos\phi sin\theta$$
$$p_y = p\ sin\phi sin\theta$$
$$p_z = p\ cos\theta$$

$$\phi[-\pi, \pi]$$
$$\theta[0, \pi]$$

Parameters



- C = curvature of the track signed with the charge
- ϕ_0 = phi of track at distance of closet approach
- δ: = distance of closet approach

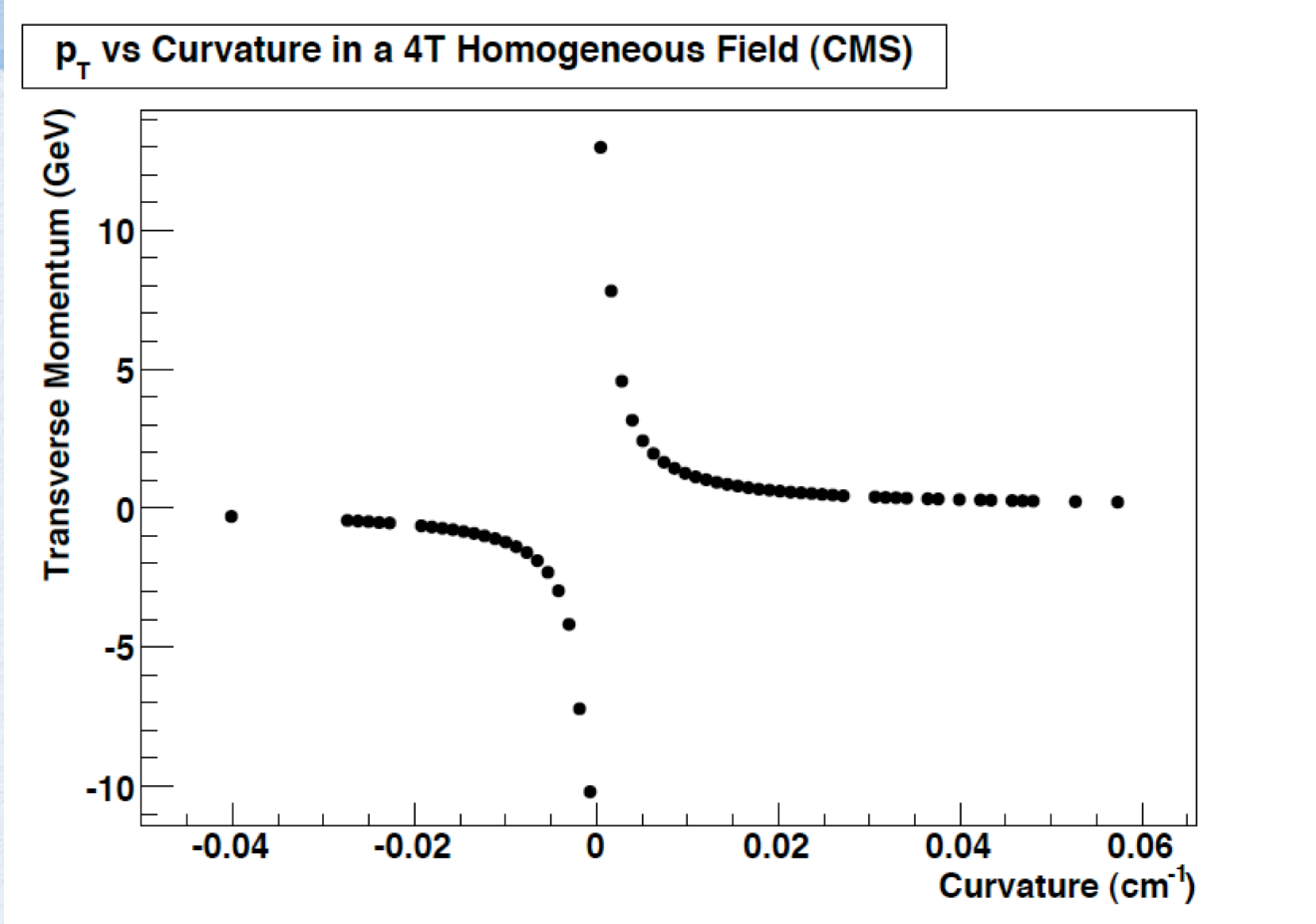
Transverse Momentum

- The component of momentum in the plane transverse to the beam line

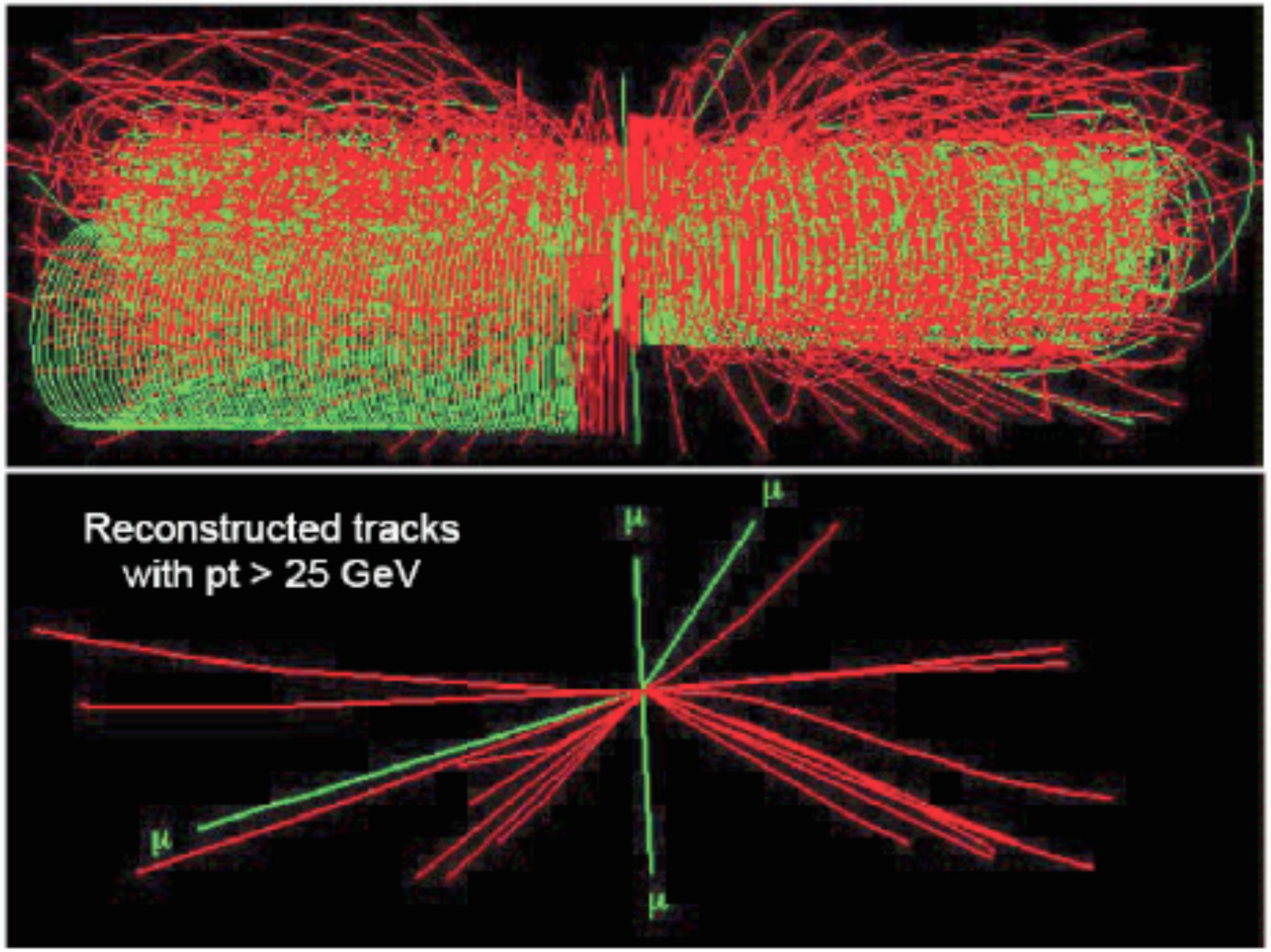
$$p_\perp[\,\mathrm{GeV}] = \frac{B[\mathrm{kG}]\; c[\,\mathrm{mm/s}]\; 10^{-10}}{C[\,\mathrm{mm}^{-1}]} = \frac{B[\mathrm{T}]\; c[\,\mathrm{cm/s}]\; 10^{-13}}{C[\,\mathrm{cm}^{-1}]}$$

$$p_\perp = p \sin\theta$$

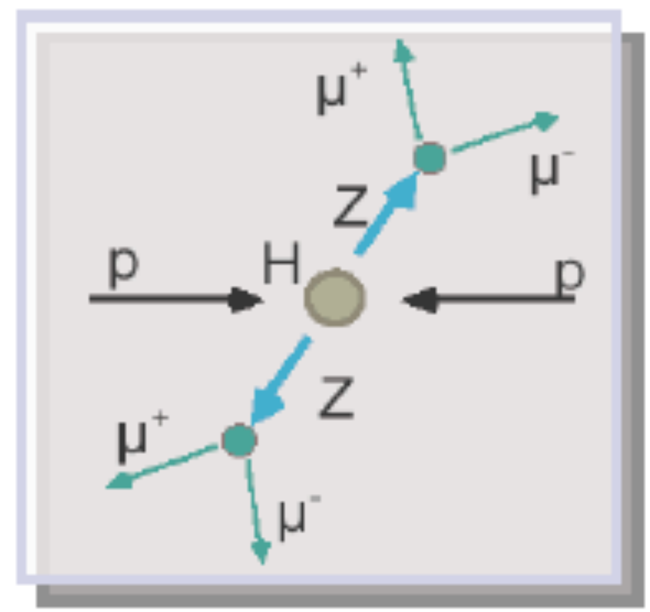
EXAMPLE



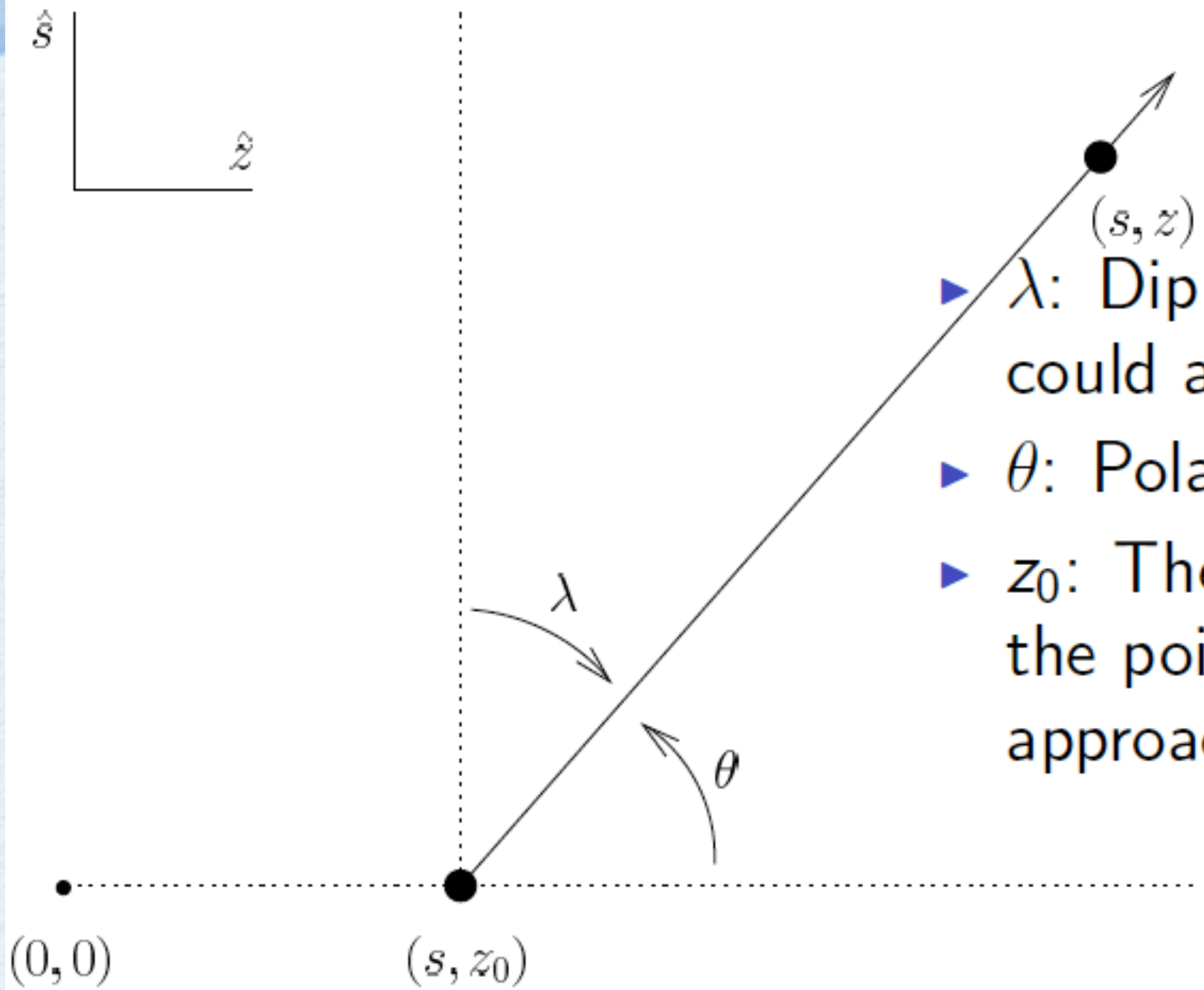
p_T vs Curvature in a 4T Homogeneous Field (CMS)
Transverse Momentum (GeV)
10
5
0
-5
-10
-0.04
-0.02
0
0.02
0.04
0.06
Curvature (cm^{-1})



Higgs event
+
~25 minimum bias events



$z = z_0 + s \tan\lambda$



- λ: Dip angle of track, or could also use
- θ: Polar angle of track
- z_0: The z of the track at the point of closest approach in $x - y$

Energy Loss

- If a charged particle moves through material it can lose energy and slightly change direction

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- If a charged particle moves through material it can lose energy and slightly change direction
- As a particle bends in a magnetic field it can emit radiation and lose energy
  - Our model of the track trajectory must take these into account

Tracking Detector

- Should have the least amount of material as possible

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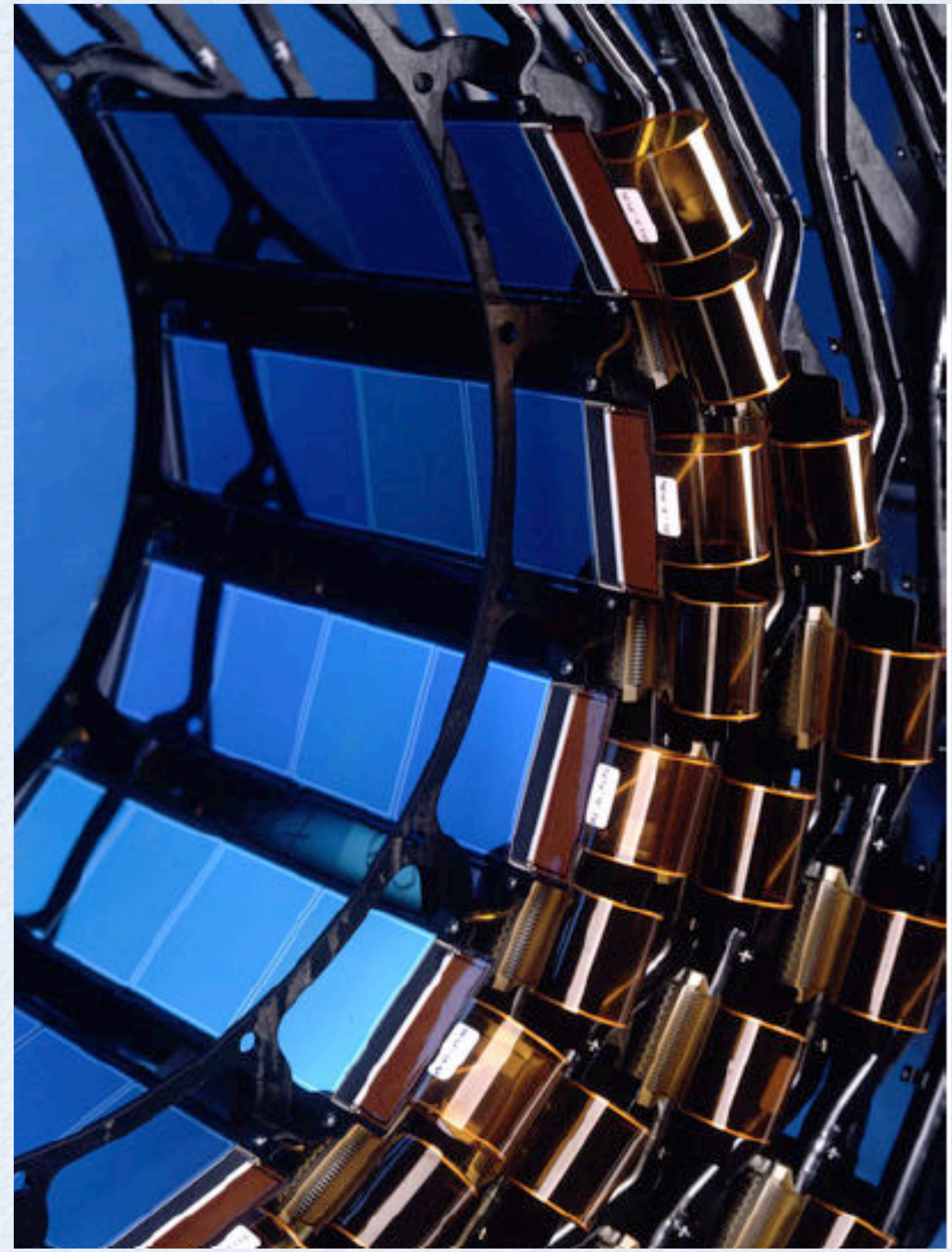
- Should have as long a lever arm as possible
- Should have as large a magnetic field as possible
- Should be as cheap as possible
- Note - these are conflicting goals!!!

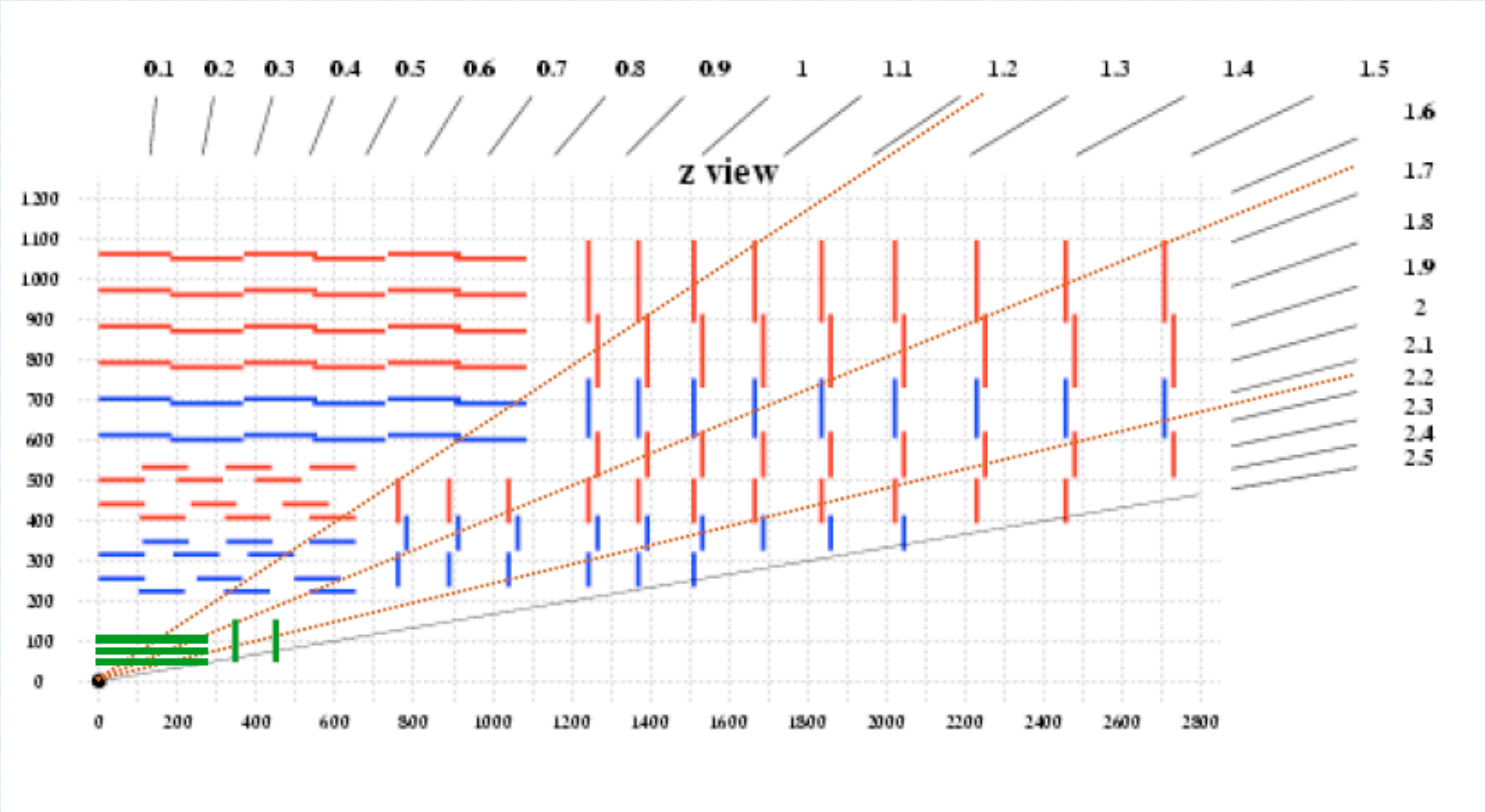
Some technologies

- Particle interacts with detector and convert energy loss into signal
    - Gas and wires: ions in gas drift towards wire under influence of electric field
    - Scintillating Fibers
    - Silicon (reverse biased diode)

CMS TRACKER

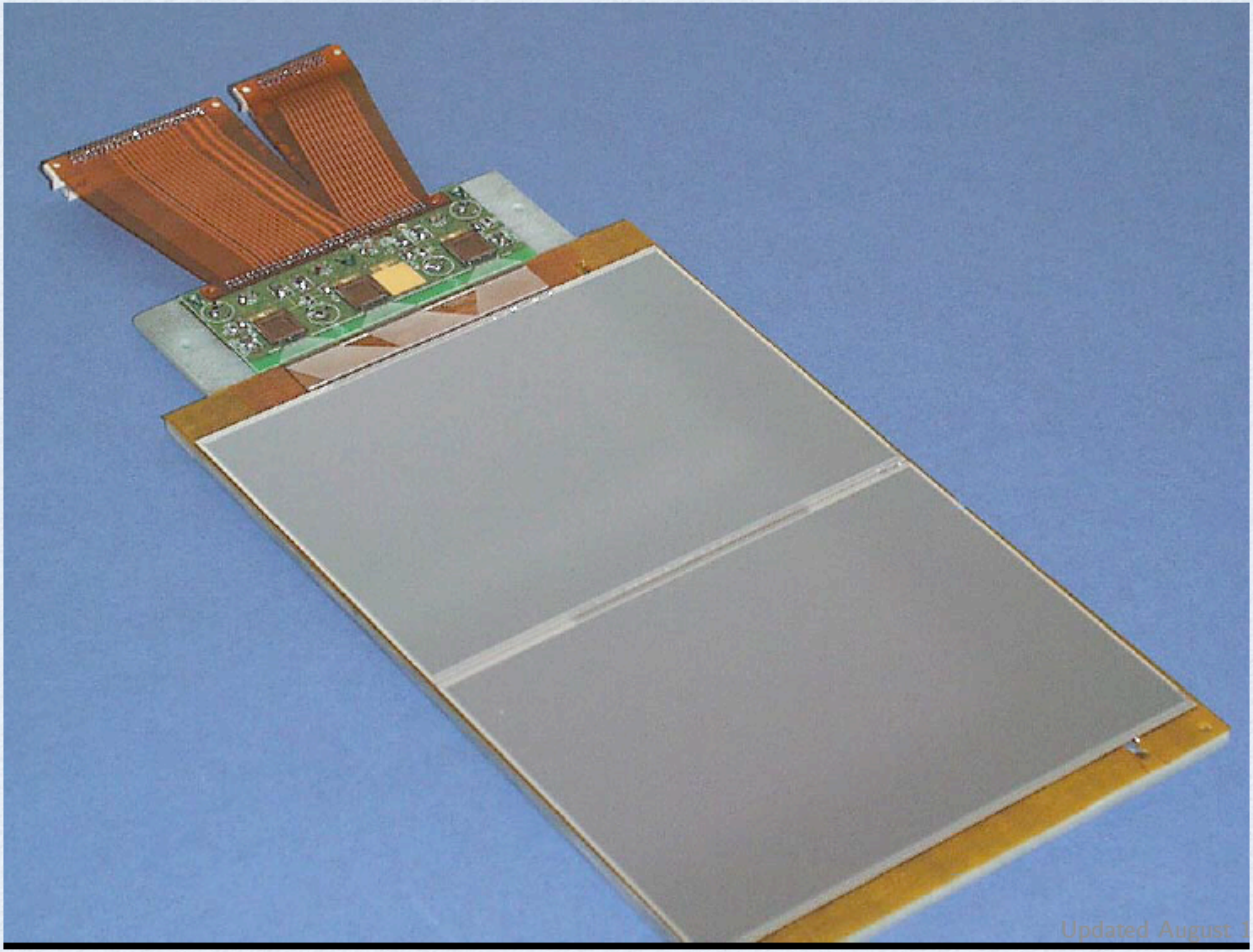


CMS Layout

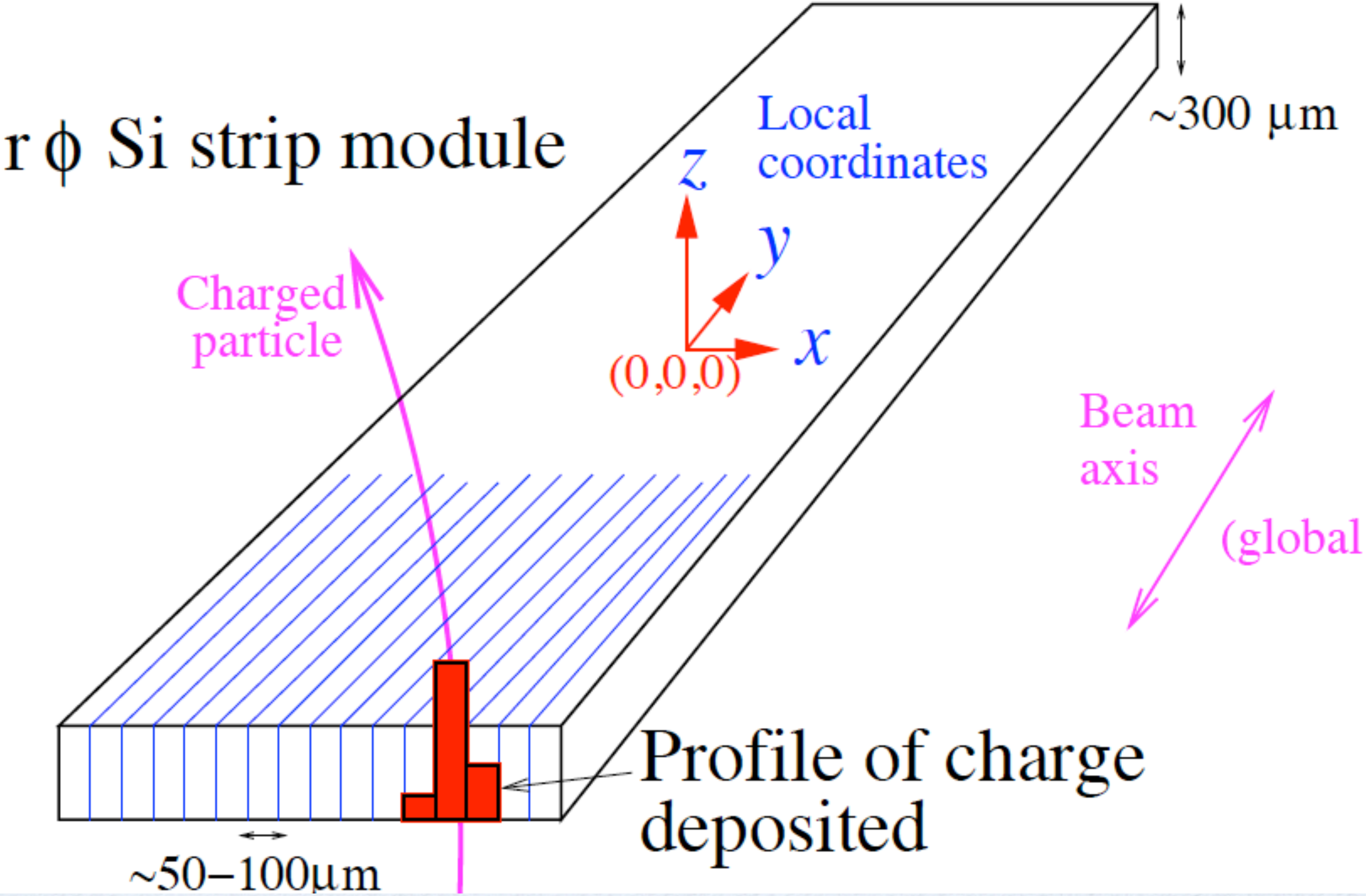


0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2
2.1
2.2
2.3
2.4
2.5
z view
1200
1100
1000
900
800
700
600
500
400
300
200
100
0
0
200
400
600
800
1000
1200
1400
1600
1800
2000
2200
2400
2600
2800

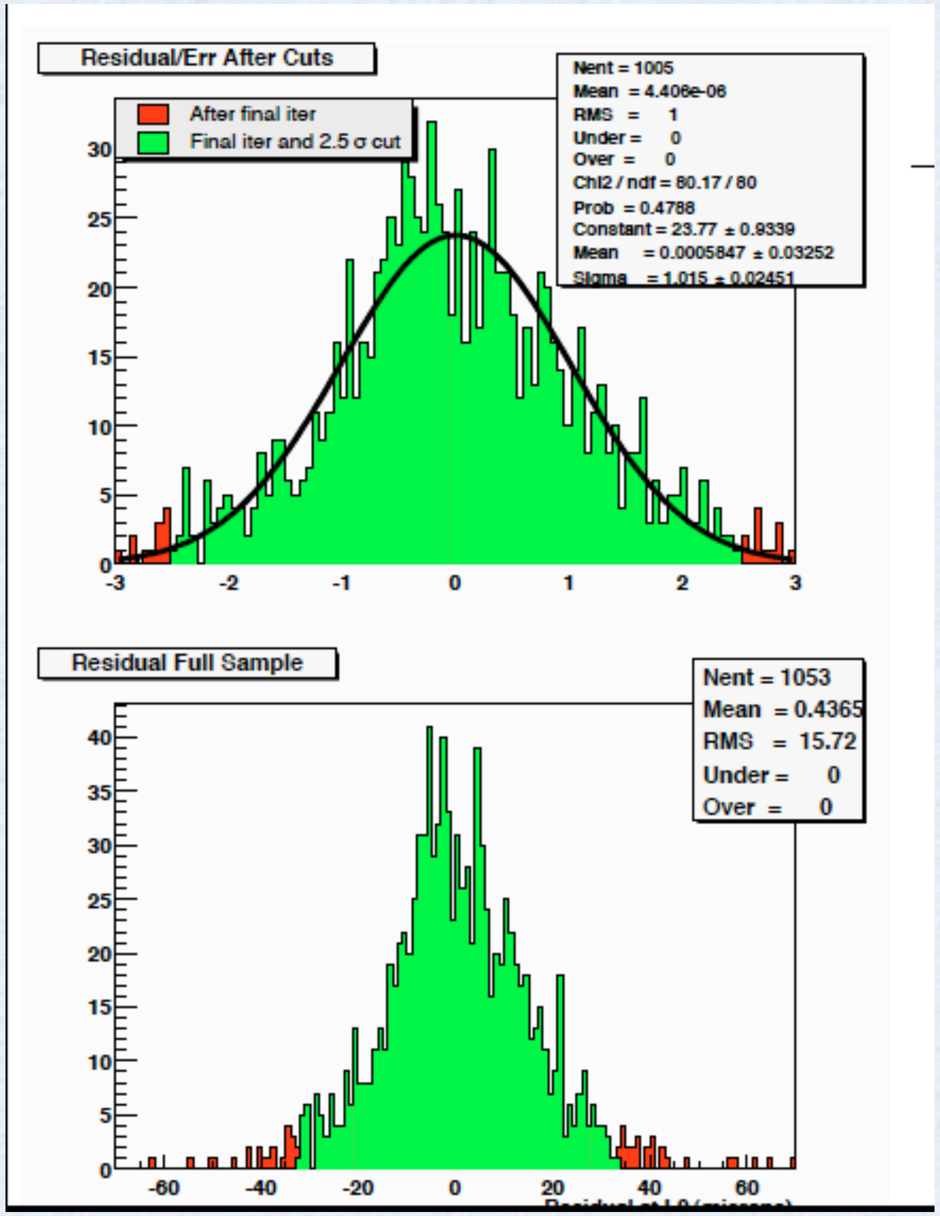
Strip Module



2D Point ($r\phi$ or rz): $\bar{x} = (\sum i * q_i) / \sum q_i$

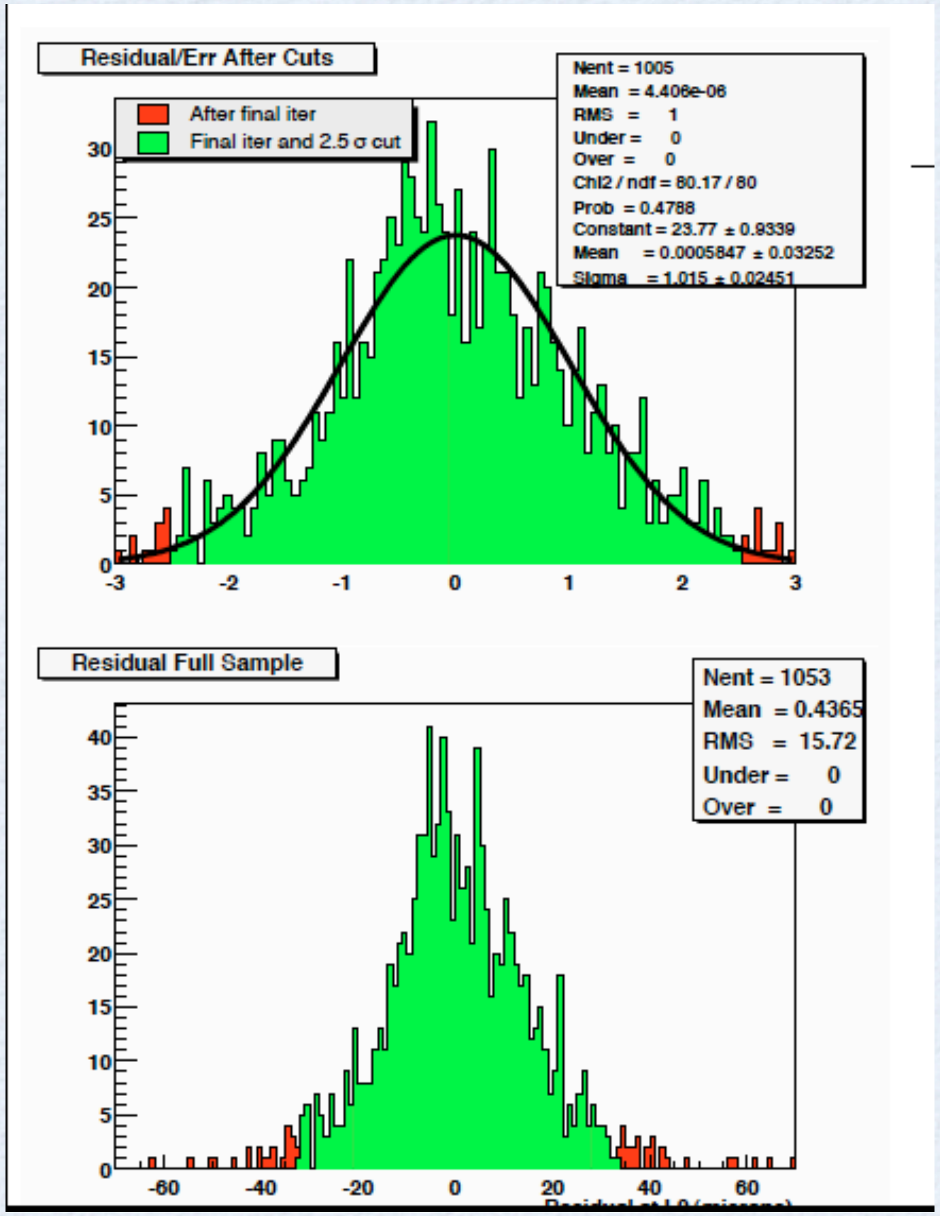


Precision of Silicon with 50 Micron Pitch



Cluster Width	Resolution
1	12 μm
2	9 μm
3	14 μm
4+	22 μm

Precision of Silicon with ~ 50 micron pitch



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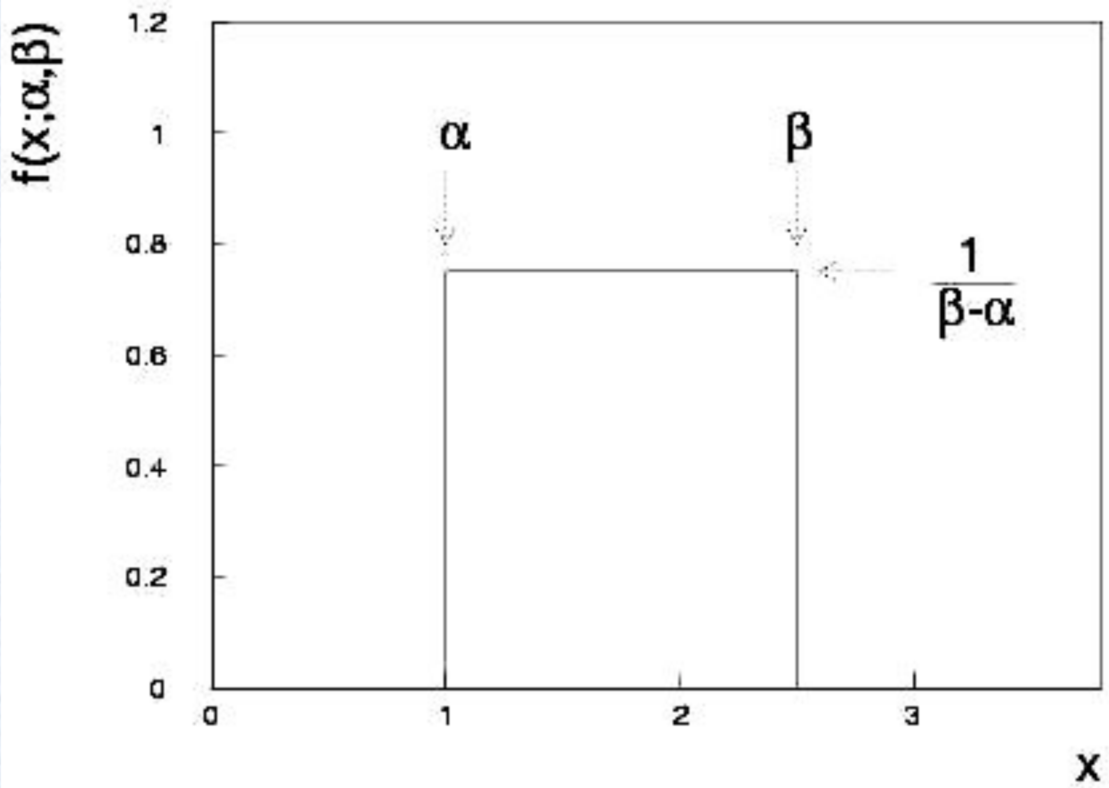
Why is the 2 strip cluster the most precise?

Aside : Uniform distribution

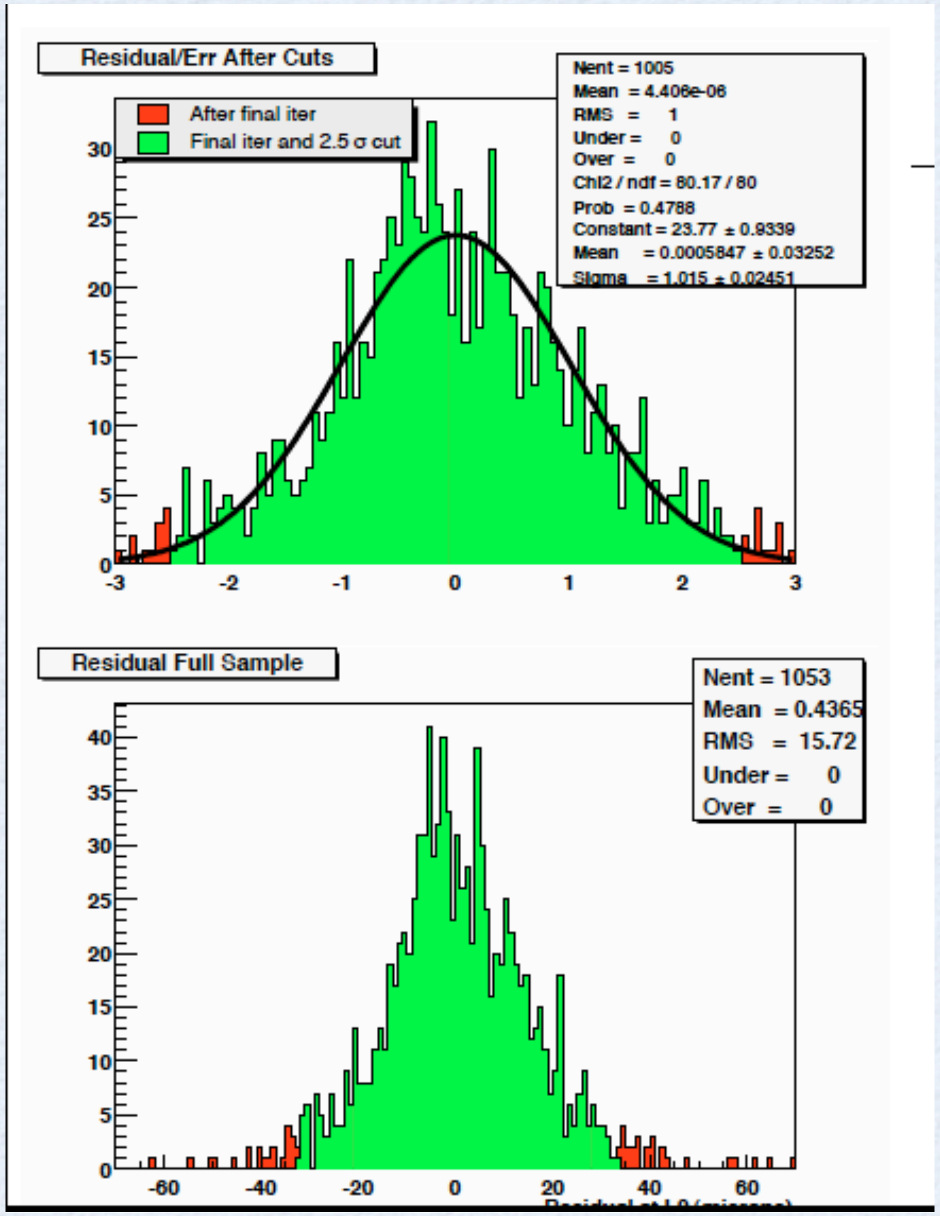
- Consider the uniform distribution which is constant between a lower and upper limit
- Not surprisingly the expectation value is just in the exact middle
- Perhaps surprisingly the standard deviation is the range/sqrt(12)
    - prove this for the homework!



$$E[x] = \frac{1}{2}(\alpha + \beta)$$

$$V[x] = \frac{1}{12}(\beta - \alpha)^2$$

Precision of Silicon with ~ 50 micron pitch



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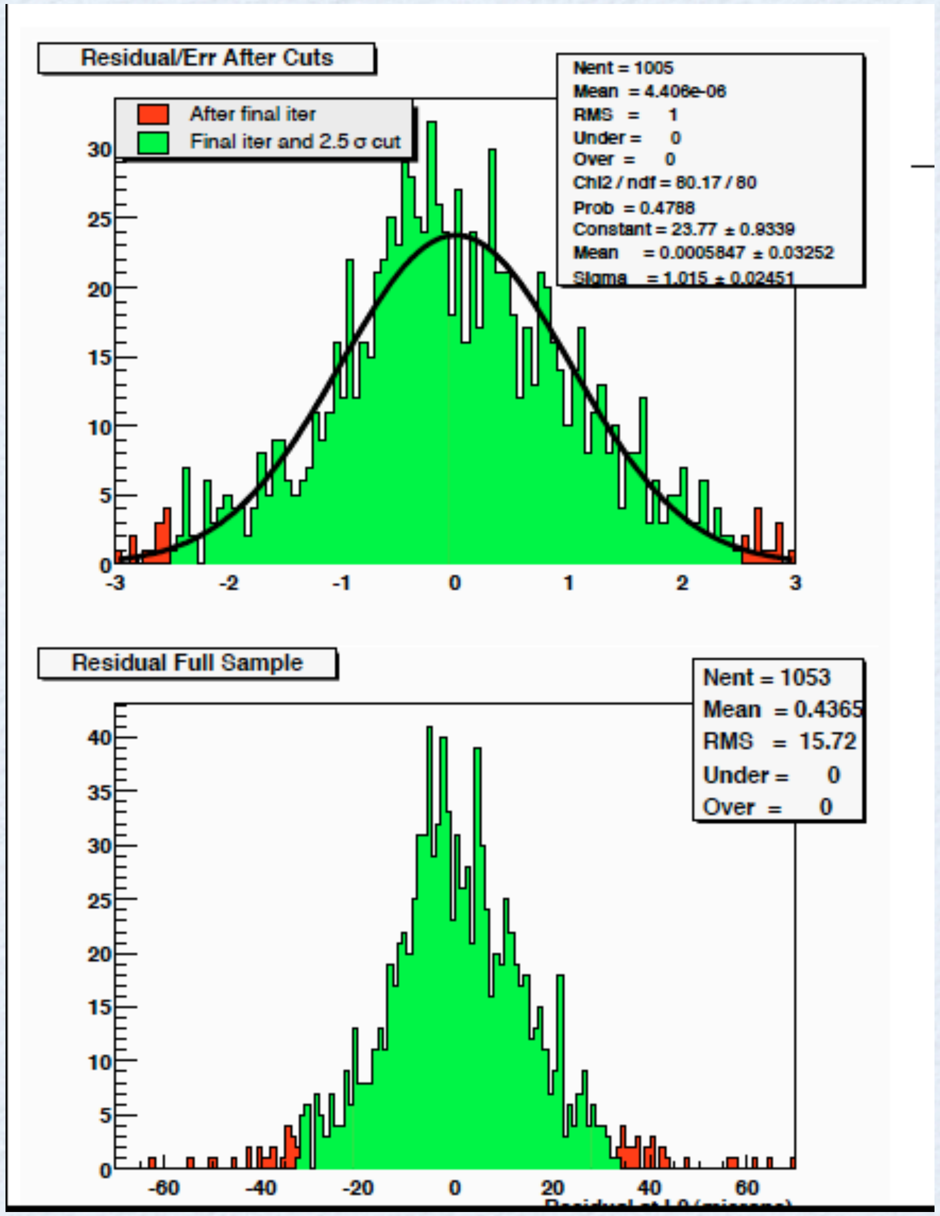

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1 strip cluster: somewhere within that strip (~pitch/sqrt(12))

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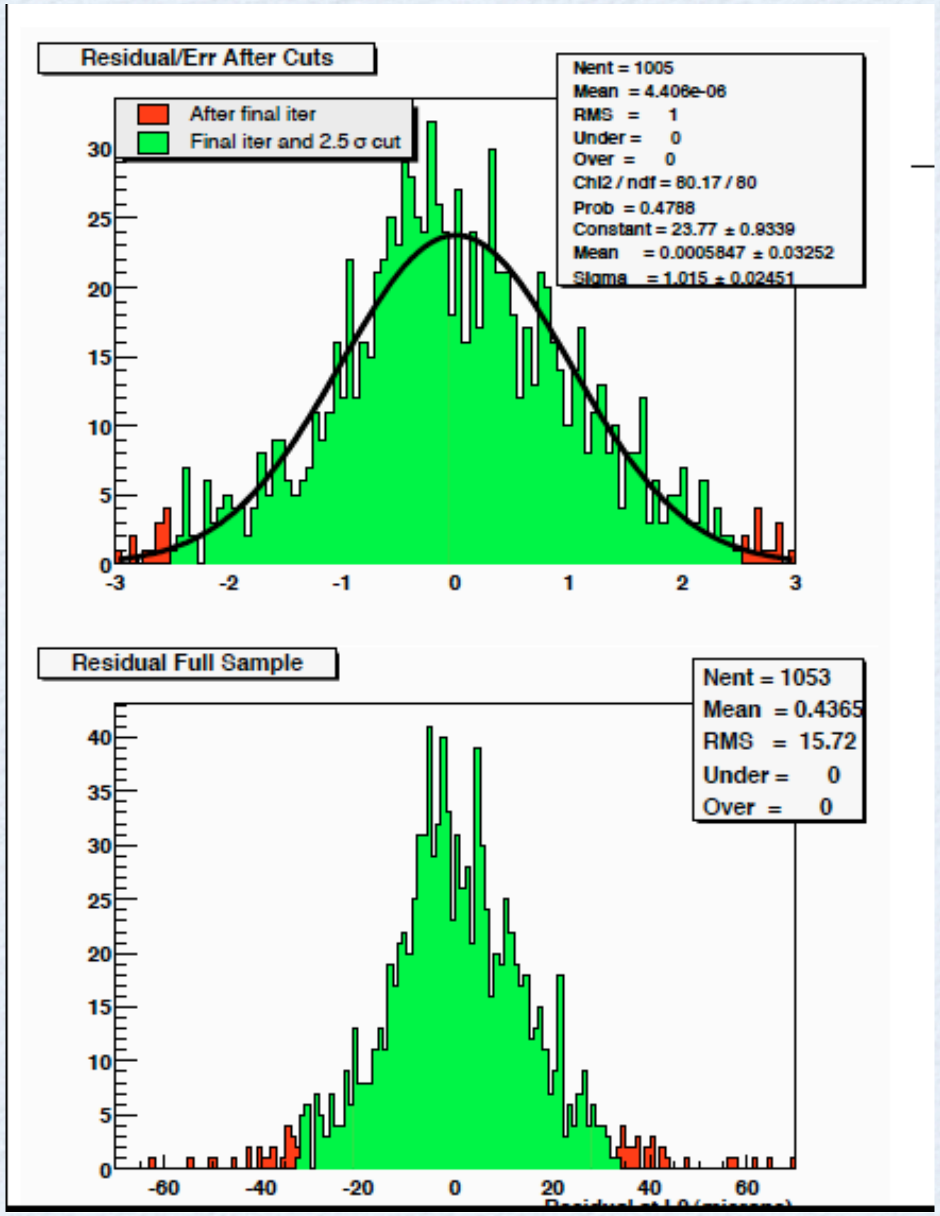

Cluster Width	Resolution
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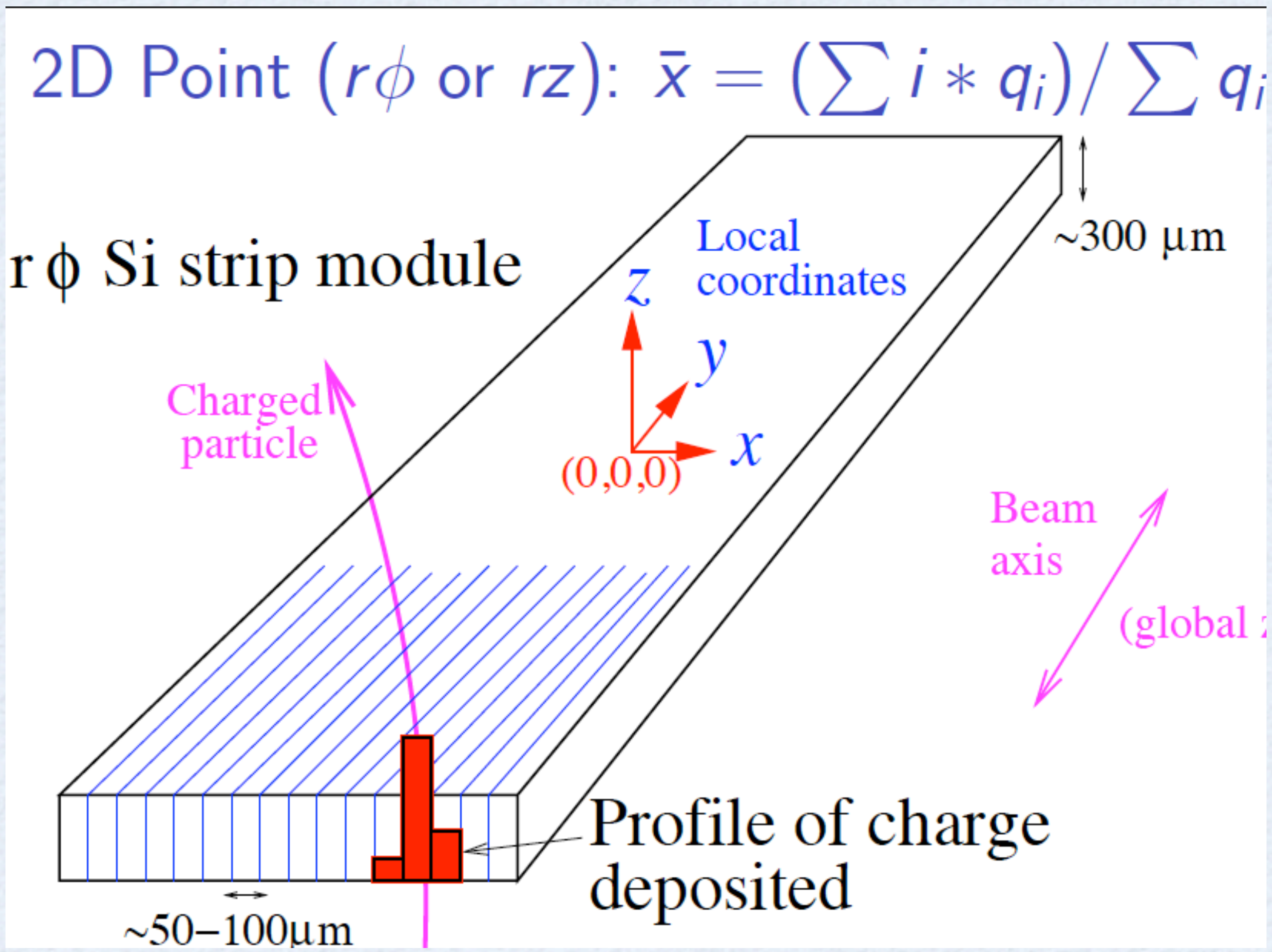
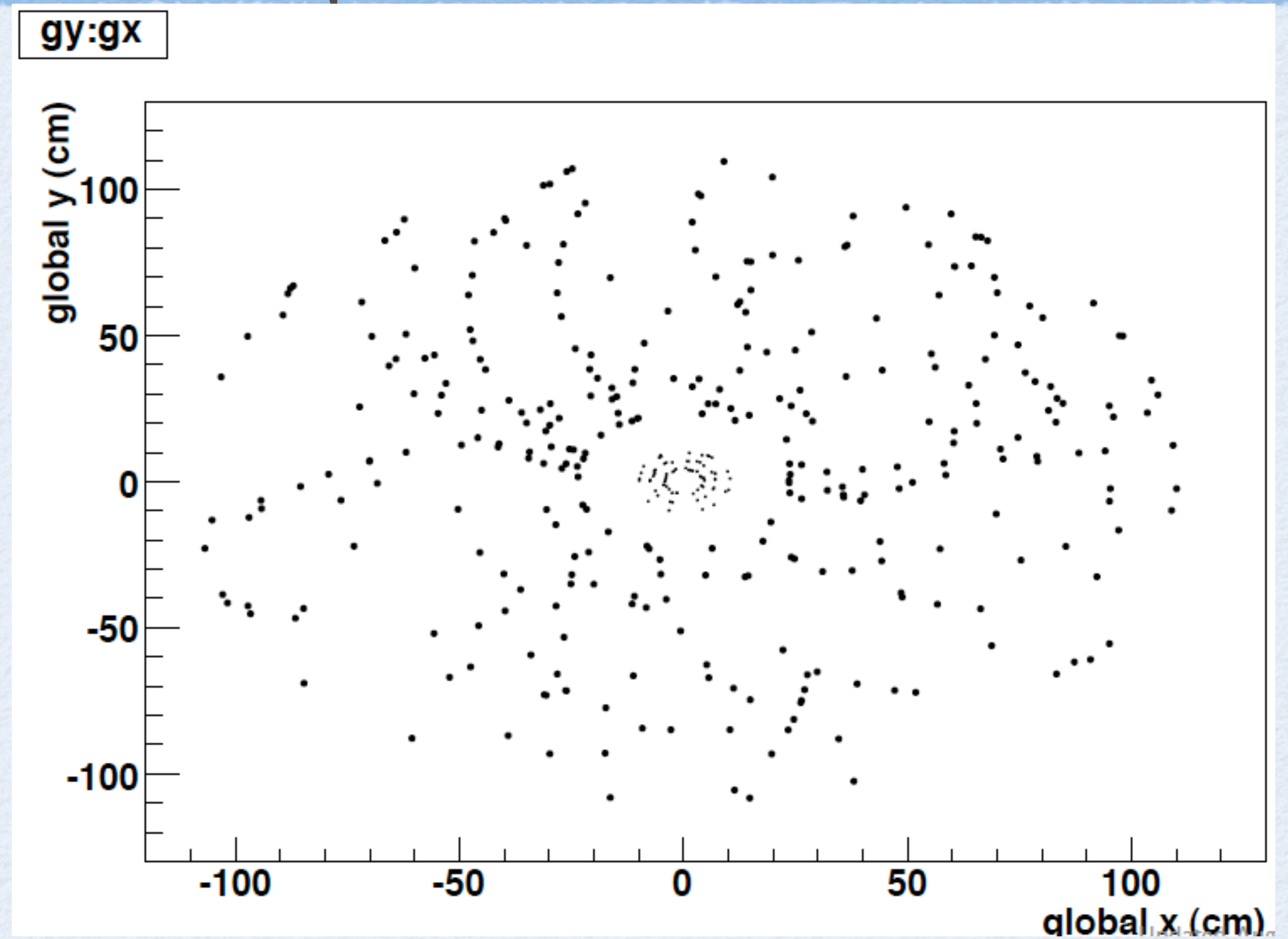
Why is the 2 strip cluster the most precise?

why does resolution get better for 2 strip?

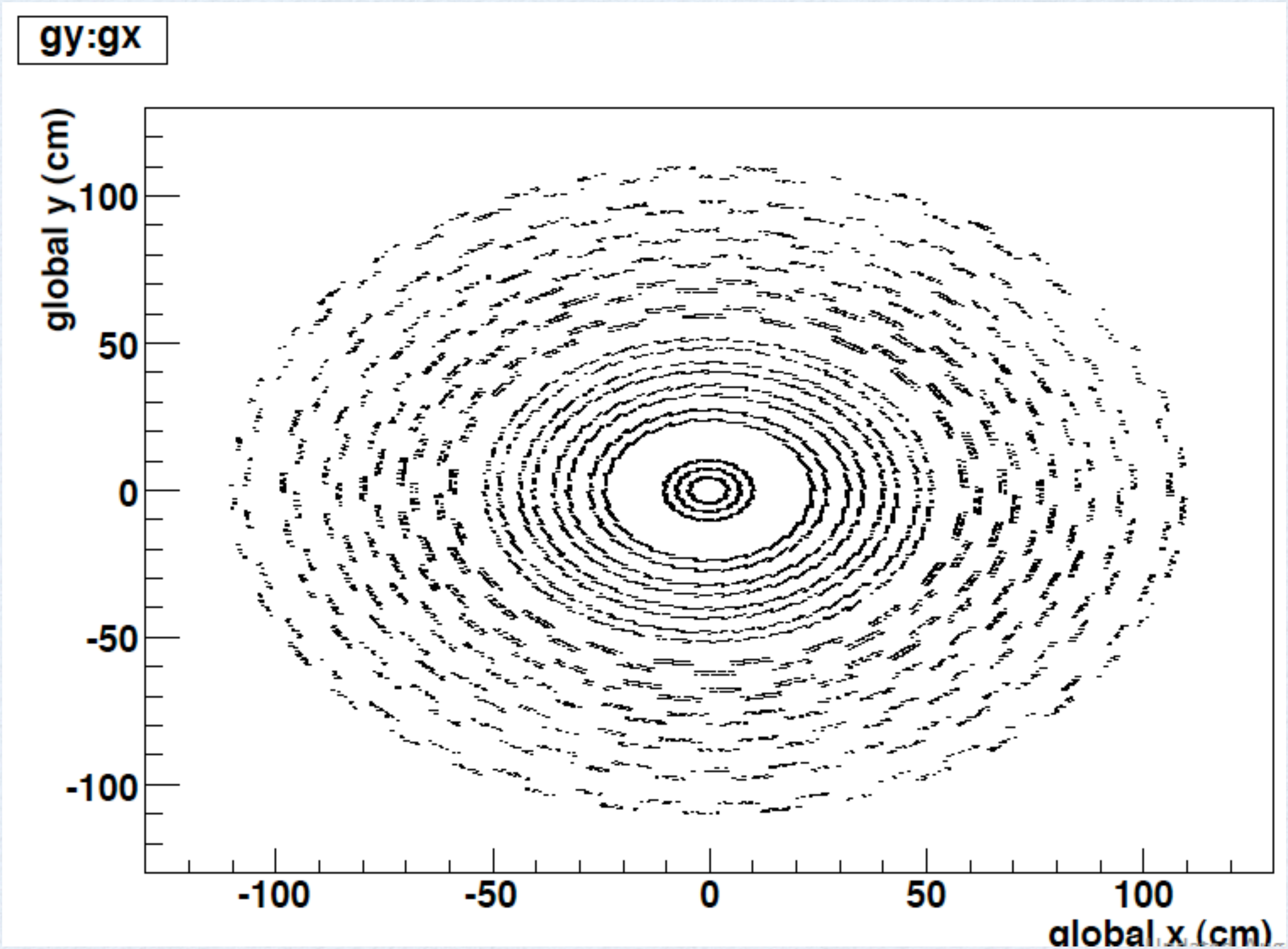


- Charge Sharing helps locate the position
- But too wide a cluster means less precision

Can you find the 50 GeV track



HOW ABOUT NOW?



Pattern Recognition

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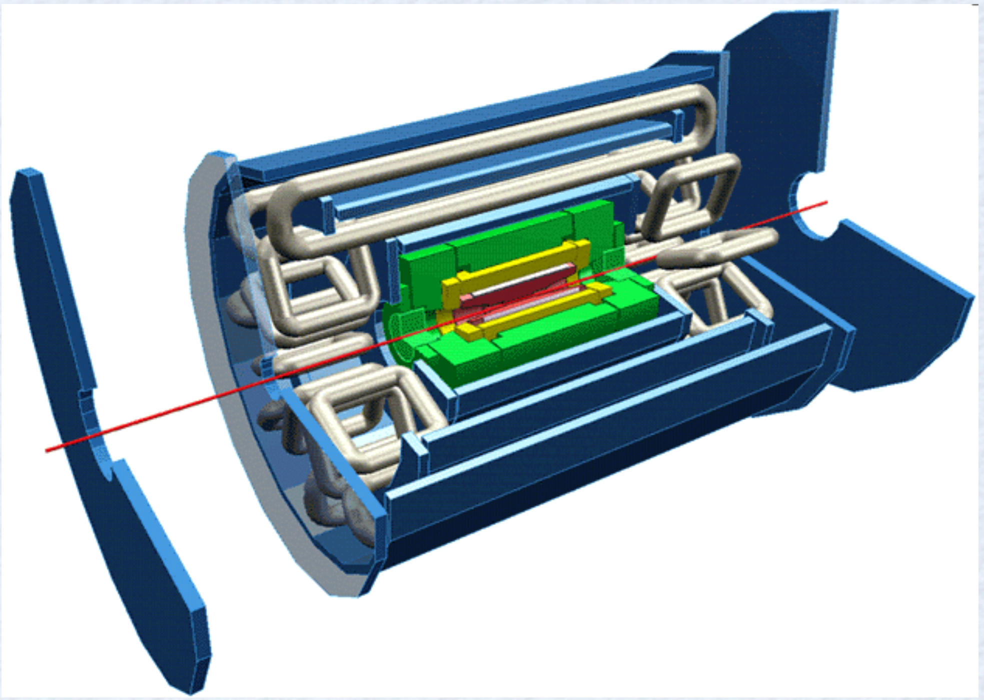
    - Continue that process until you have found some criterion for a ‘good track’
    - Once you select the hits on the track fit to those points

ATLAS MUONS

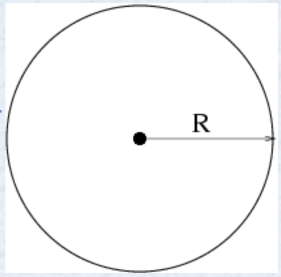
- Since muons penetrate material need to have a special system outside of the calorimeter
    - Large Volume Detector
    - Would like an independent momentum measurement -> B field (Toroidal)
- Over most of the detector we use drift tubes



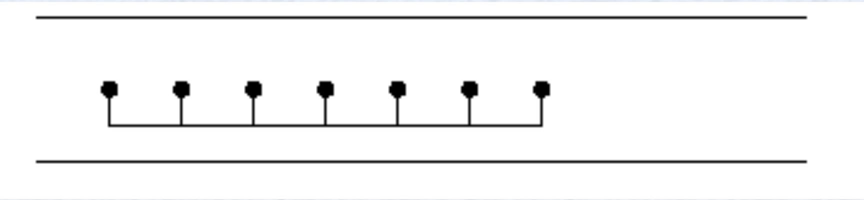
Basic Technology

- Volume of gas which a charged particle can ionize
- Voltage difference between wire and walls
- Charge drifts towards wire and signal is read out

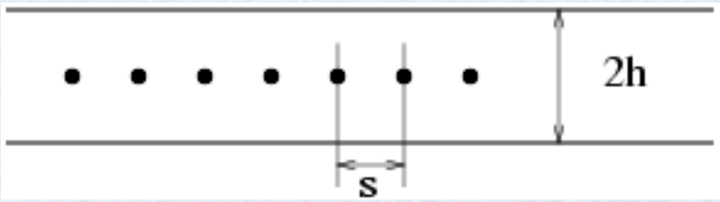
Tube Geiger- Müller, 1928



Multi Wire Geometry, in H. Friedmann 1949

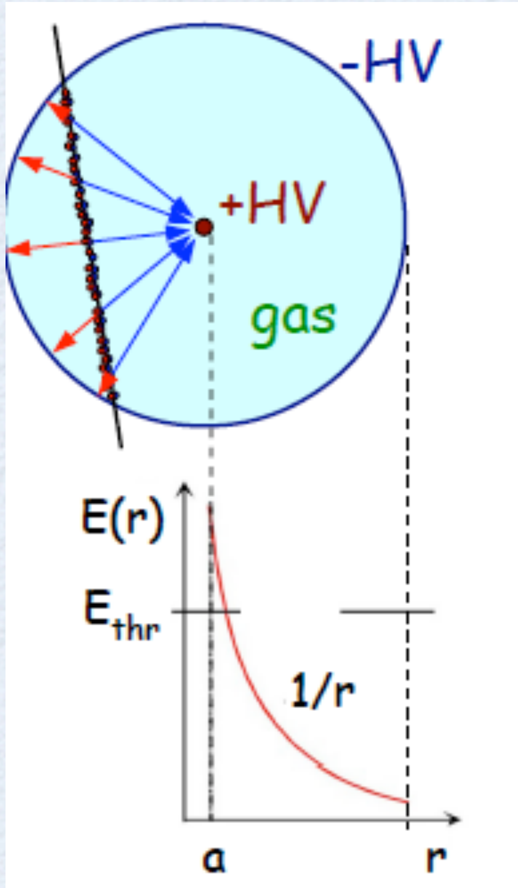


G. Charpak 1968



Basic Operation

- few mm to few cm diameter
- thin wire run down the center under tension
- apply voltage to the wire (few kV)
- ionizing particle creates 'primary' ionization
    - ions drift toward wall
    - electrons drift toward wall
- Strong field near wire creates 'avalanche' effect as primary ionization causes secondary ionization



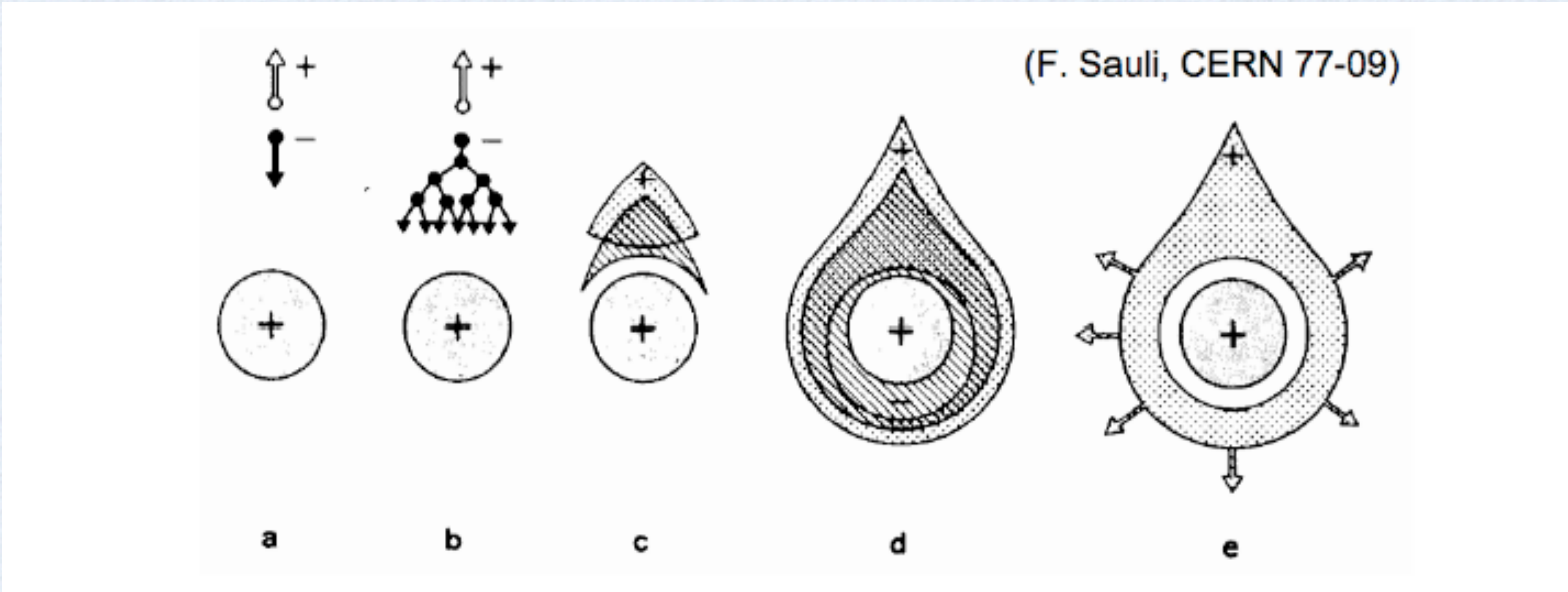
Signal Formation

- Recall from basic E&M
- Moving charges cause change of potential energy and a voltage pulse on the wire

$$\Phi(r) = -\frac{V_0}{ln(\frac{b}{a})} ln\frac{r}{a}$$

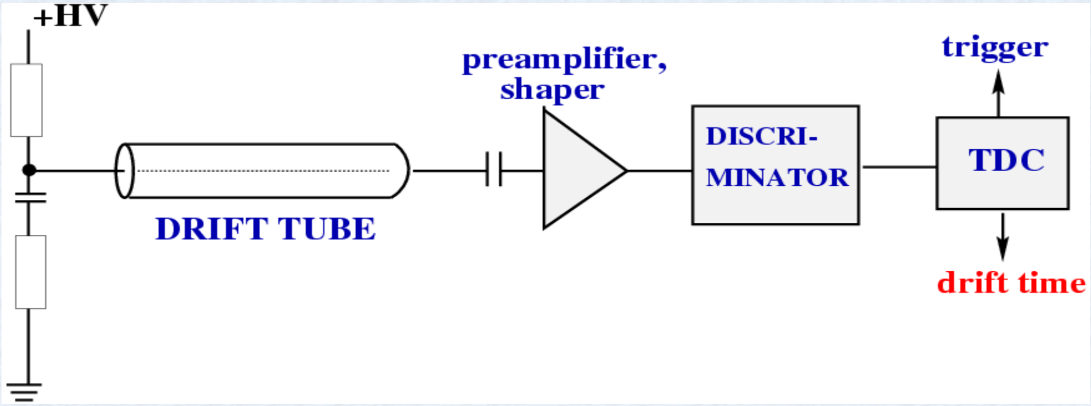
$$E(r) = \frac{V_0}{ln\frac{b}{a}}\frac{1}{r}$$

AVALANCHE



Electrons move quickly to wire,
ions more slowly to tube surface

Voltage pulse



$$dW = qE(r)dr = CV_0 dV$$

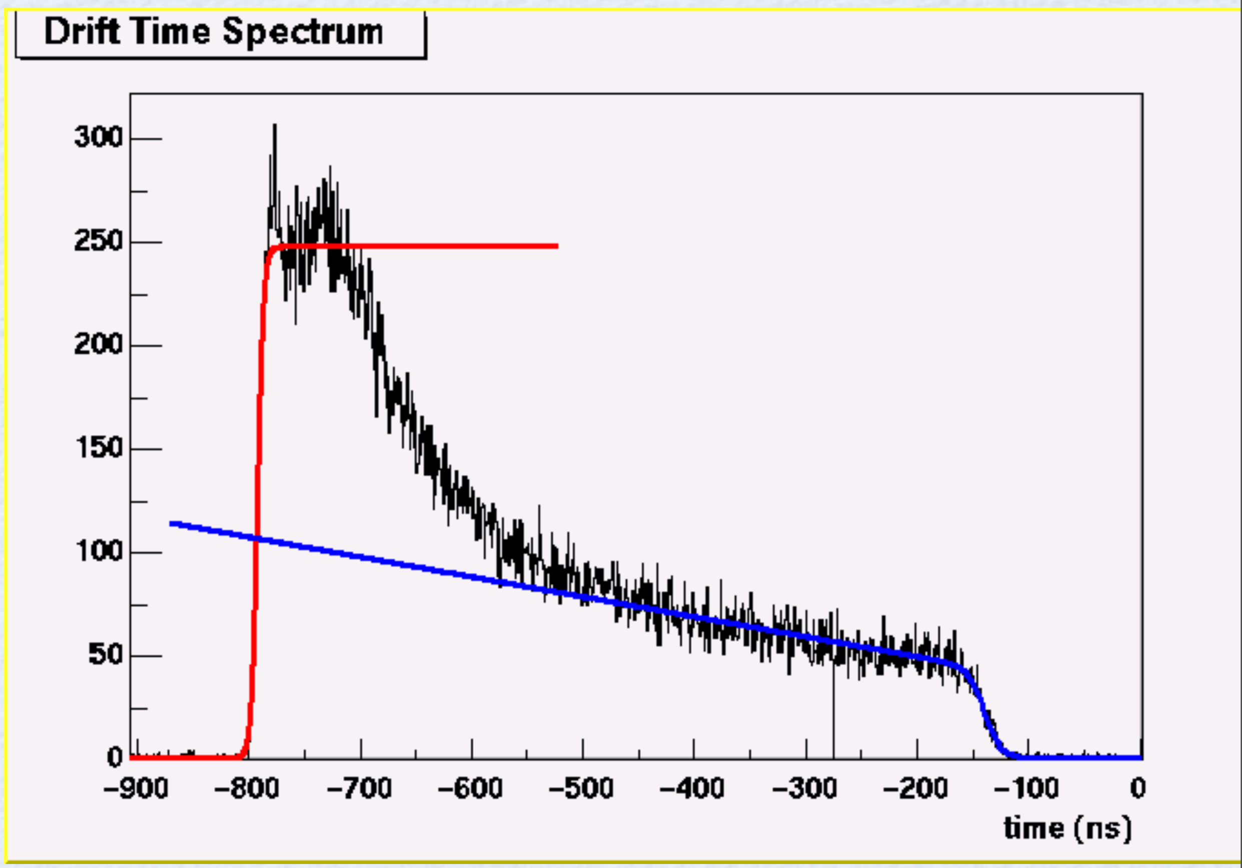
$$dV = \frac{q}{CV_0}E(r)dr$$

$$\Delta V = -\frac{q}{C}$$

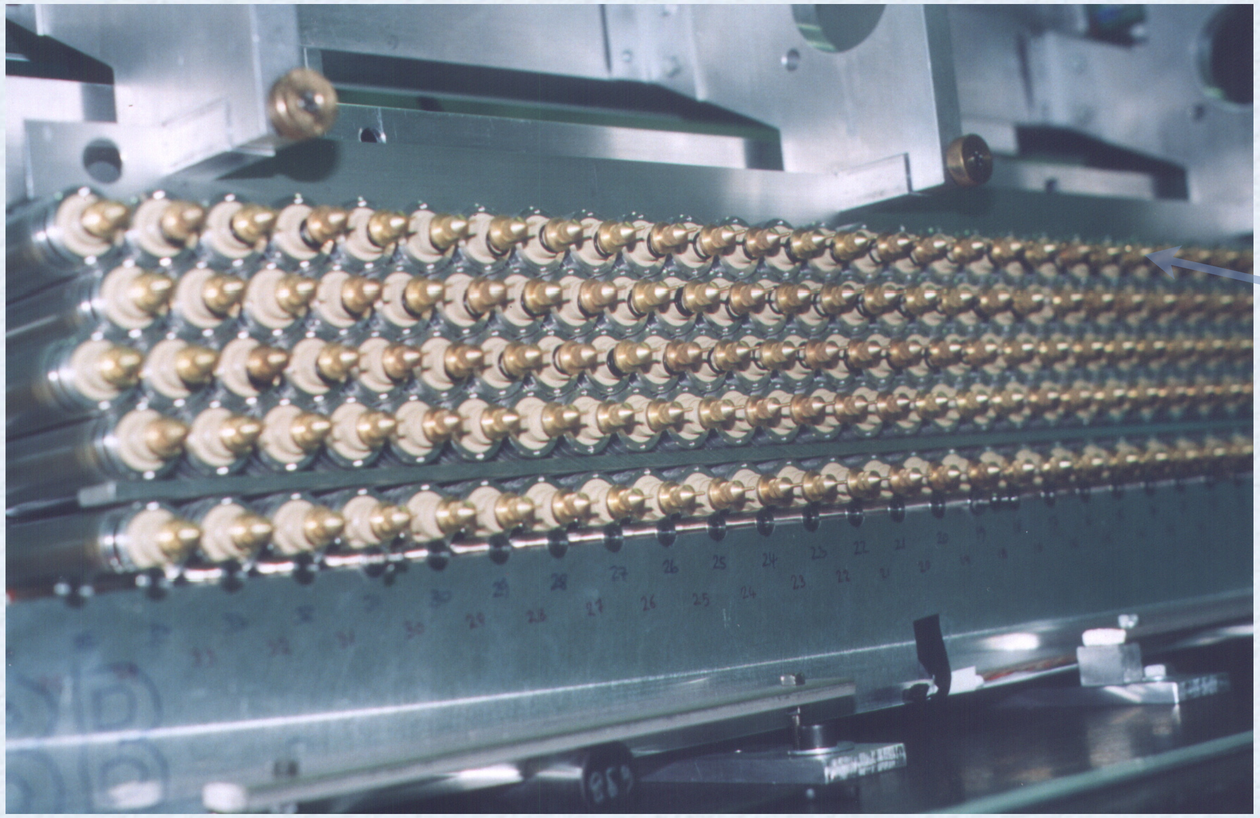
- Measure 2 basic quantities:
  - voltage drop and hence charge
  - time for leading edge of avalanche to reach the wire

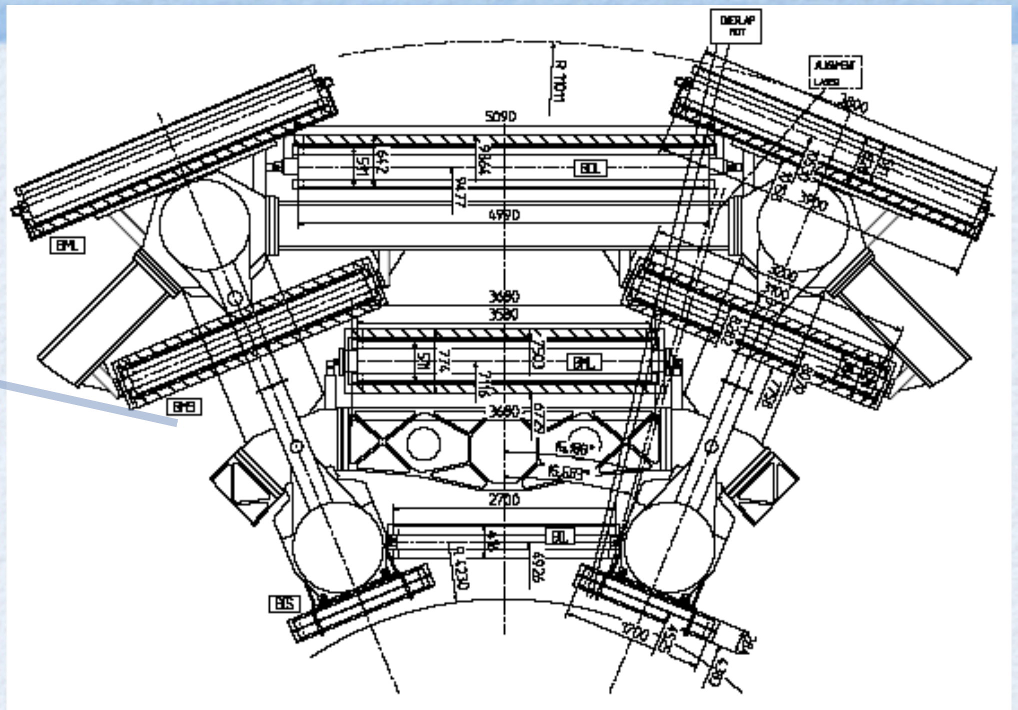
Time Distribution

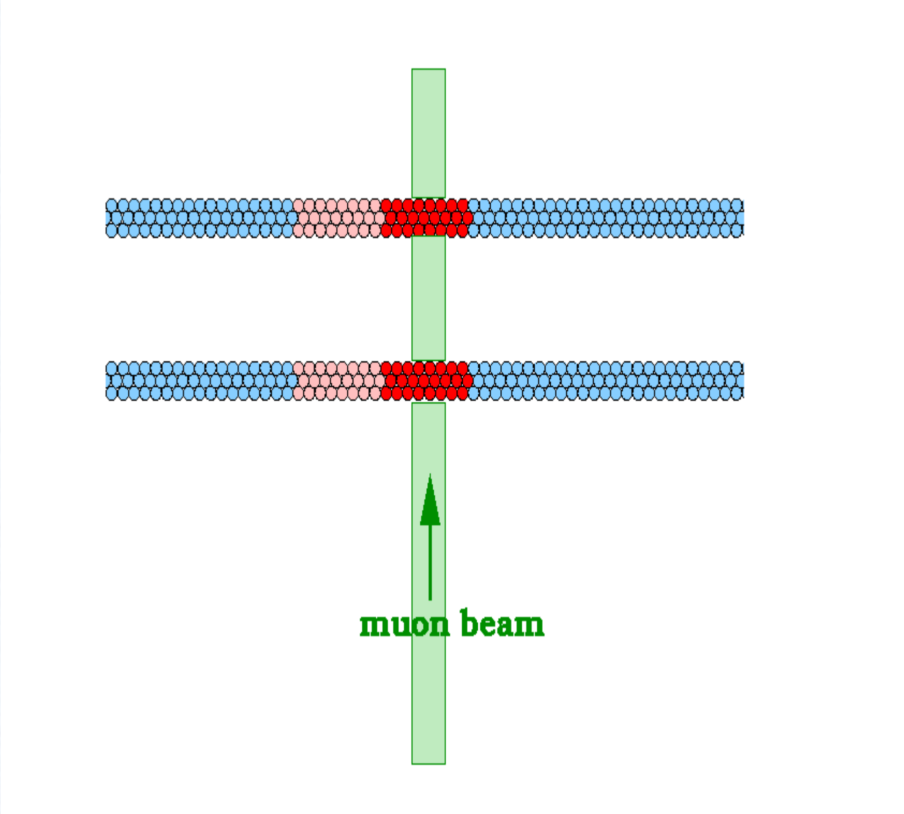
- What we measure is the drift time.
- By knowing the relationship between the time measured of the leading pulse and how far from the wire the ions we create a so called r(t) function which tells us how far we are away from the wire



ARRANGEMENT



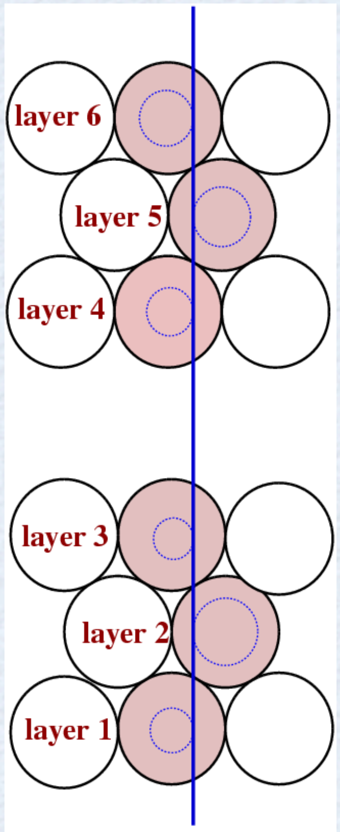




Arrange tubes in chambers of layers to make measurements at different points along trajectory

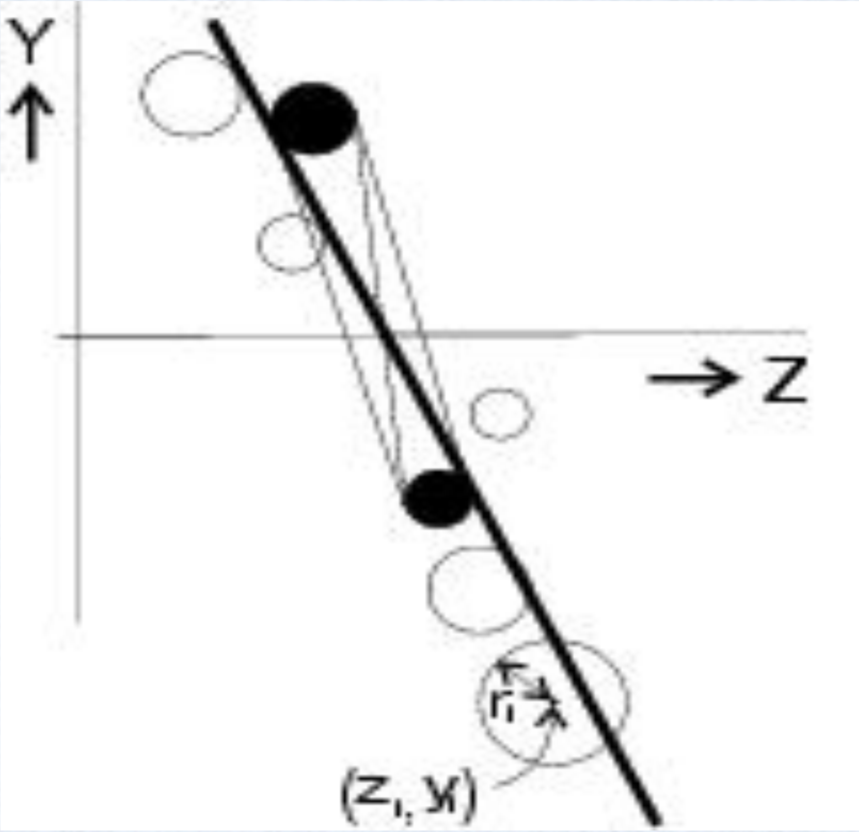
Reconstructing the Path

- For each tube that the muon passes through we get a time measurement from tube which we convert into a distance from the wire
  - Note - we don't actually know where along that circle the charge came from - called "Drift Circle"
  - This is resolved by pattern recognition!
- In this case the trajectory is fairly obvious



Segment Fitting

- Over a small distance can approximate the trajectory as a straight line
- For each pair of hits:
  - Take outer most hits in a chamber and draw 4 tangential lines
  - Draw tangent lines to circle and see how far 'inside hits' are from line
  - Fit hits to a line
- Take segment with most hits and best fit



HOW DO WE FIT?

$$\chi^2 = \sum_i^N \frac{(y_i - \lambda_i)^2}{\sigma_i^2}$$

- Recall our definition of a chi2
- If we have a model of our data we want to minimize that chi2

Straight Line Fit

- here our model is that the particle goes in a straight line so that if we know the x position we can predict the y position with two parameters
  - a, b

$$y = a + bx$$

DEFINITIONS

- For the linear case this just becomes:

$$\chi^2(a,b) = \sum_i^N \left(\frac{y_i - a - bx_i}{\sigma_i}\right)^2$$

- How do we minimize it ?

DEFINITIONS

- For the linear case this just becomes:
- How do we minimize it ?
  - Take derivative with respect to the parameters a and b so that we can find the values a and b which minimize the chi2
  - We are assuming we know the data y,x and the uncertainty on each point

$$\chi^2(a,b) = \sum_i^N \left(\frac{y_i - a - bx_i}{\sigma_i}\right)^2$$

$$0 = \frac{\delta\chi^2}{\delta a} = -2\sum_i^N \frac{y_i - a - bx_i}{\sigma_i^2}$$

$$0 = \frac{\delta\chi^2}{\delta b} = -2\sum_i^N \frac{x_i(y_i - a - bx_i)}{\sigma_i^2}$$

Solving it

- First some definitions:
- Such that our minimization becomes:

$$S = \sum_i^N \frac{1}{\sigma_i^2}$$

$$S_x = \sum_i^N \frac{x_i}{\sigma_i^2}$$

$$S_y = \sum_i^N \frac{y_i}{\sigma_i^2}$$

$$S_{xx} = \sum_i^N \frac{x_i^2}{\sigma_i^2}$$

$$S_{xy} = \sum_i^N \frac{x_i y_i}{\sigma_i^2}$$

$$0 = \frac{\delta\chi^2}{\delta a} = -2\sum_i^N \frac{y_i - a - bx_i}{\sigma_i^2}$$

$$0 = \frac{\delta\chi^2}{\delta b} = -2\sum_i^N \frac{x_i(y_i - a - bx_i)}{\sigma_i^2}$$

$$aS + bS_x = S_y$$

$$aS_x + bS_{xx} = S_{xy}$$

Solving for a,b

Defining $\Delta = SS_{xx} - S_x^2$

$$a = \frac{S_{xx}S_y - S_xS_{xy}}{\Delta}$$

$$b = \frac{SS_{xy} - S_xS_y}{\Delta}$$

Homework: Show that this is true!

Not quite done...

- It is not good enough to have an estimate of the parameters a and b.

Not quite done...

- It is not good enough to have an estimate of the parameters a and b.
- We must also have an estimate of the uncertainty on a and b and the 'goodness of fit'

Propagation of Errors

- Recall that for any function f(a,b) we can expand in a Taylor series and write:

$$f \approx f^0 + \frac{\delta f}{\delta a} a + \frac{\delta f}{\delta b} b + ...$$

- And hence

$$\sigma_f^2 = |\frac{\delta f}{\delta a}|^2 \sigma_a^2 + |\frac{\delta f}{\delta b}|^2 \sigma_b^2 + 2 \frac{\delta f}{\delta a} \frac{\delta f}{\delta b} cov_{ab}$$

PROPAGATION

$$\sigma_f^2 = |\frac{\delta f}{\delta a}|^2\sigma_a^2 + |\frac{\delta f}{\delta b}|^2\sigma_b^2 + 2\frac{\delta f}{\delta a}\frac{\delta f}{\delta b}cov_{ab}$$

- Depends on functional dependence of the function
  - eg f has is much more sensitive to a if

$$f(x;a) = a^5 x$$

  - Then if

$$f(x;a) = a + x$$

For the case at hand..

So

$$\sigma_f^2 = \sum_i^N \sigma_i^2 (\frac{\delta f}{\delta y_i})^2$$

with

$$\frac{\delta a}{\delta y_i} = \frac{S_{xx} - S_x x_i}{\sigma_i^2 \Delta}$$

$$\frac{\delta b}{\delta y_i} = \frac{S x_i - S_x}{\sigma_i^2 \Delta}$$

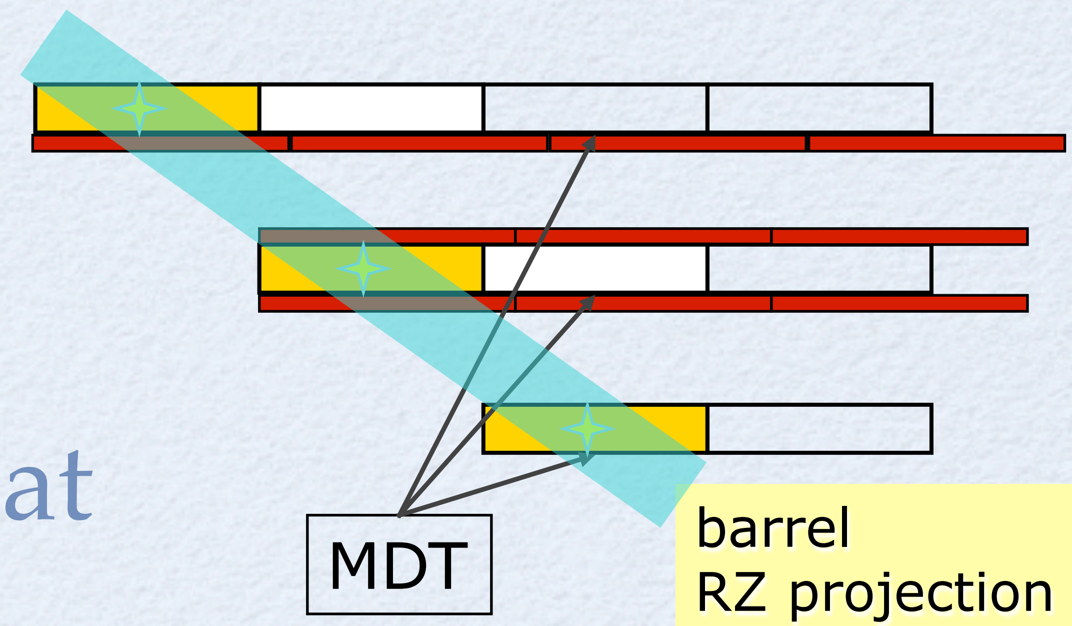
by summing:

$$\sigma_a^2 = \frac{S_{xx}}{\Delta}$$

$$\sigma_b^2 = \frac{S}{\Delta}$$

Match segments

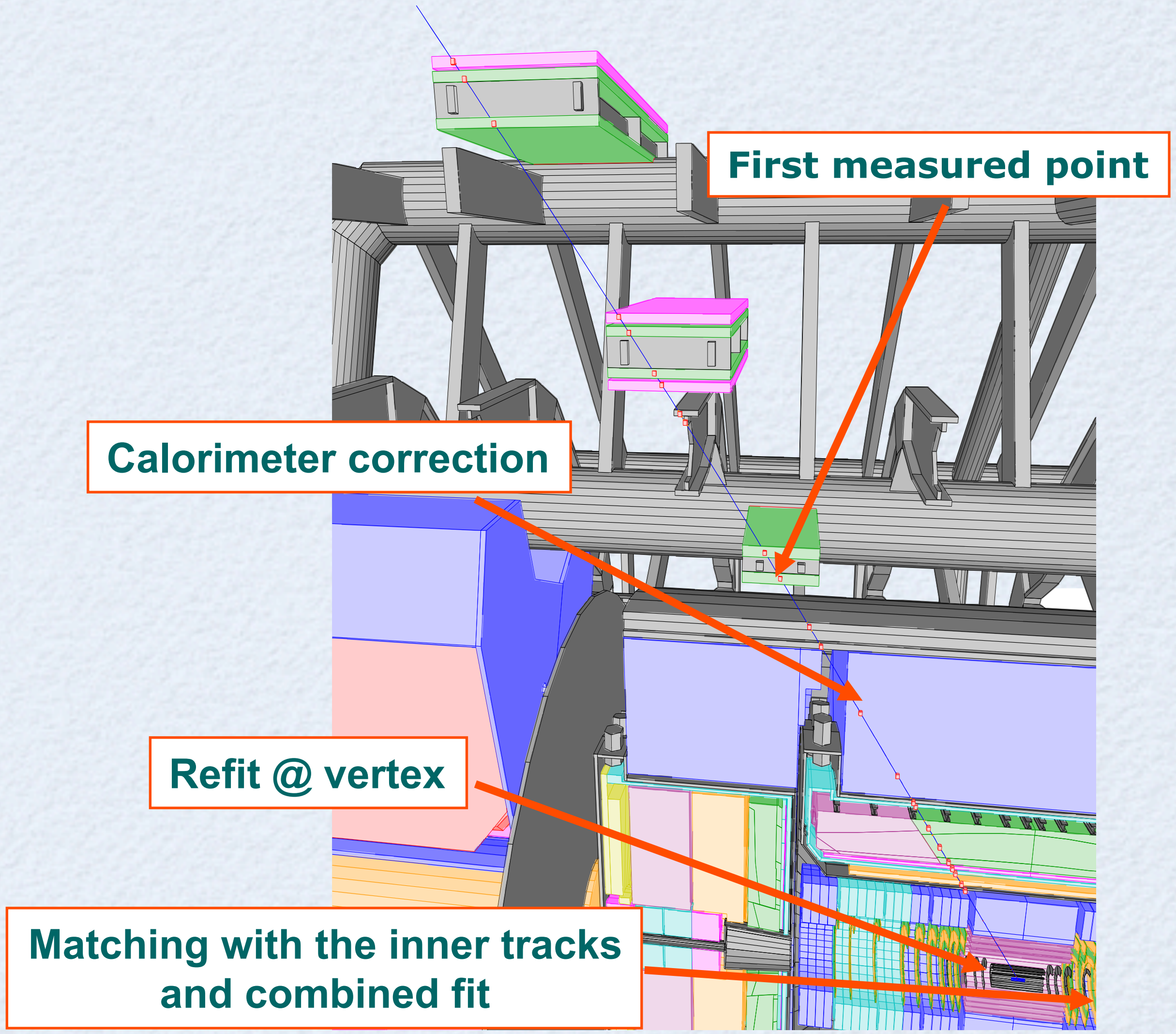
- Match segments from different layers of the spectrometer
- Look for pairs or triplets that point to each other
- Try all possibilities - and fit resulting hits to a track



Fit back to ID

- Extrapolate track back to production point
- Match with track from inner detector
- Fit combined track

Package *Muid*



Next time

- More fun with likelihoods
- practice with some calculations