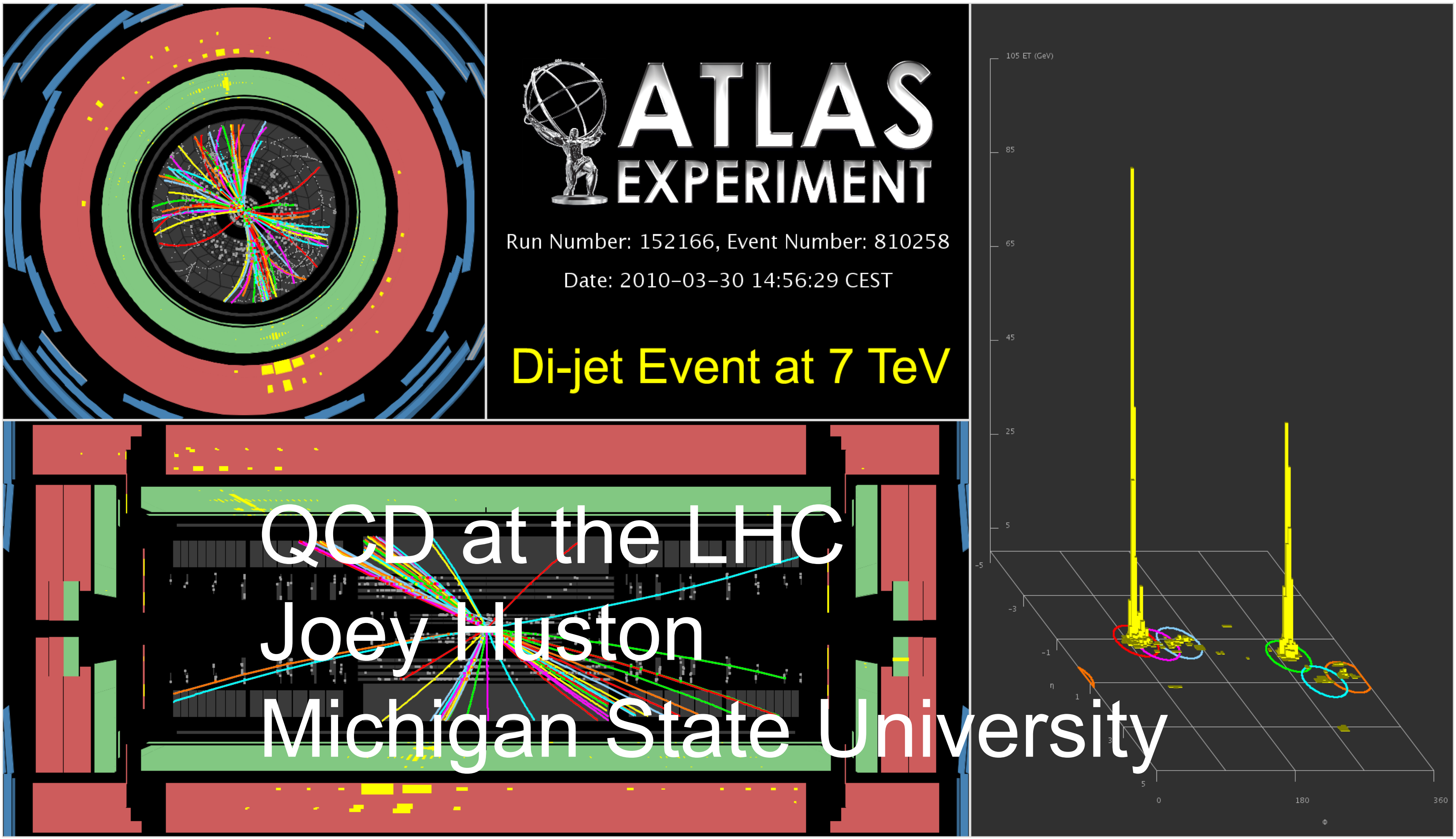

QCD at the LHC

Joey Huston
Michigan State University

Some references

INSTITUTE OF PHYSICS PUBLISHING REPORTS ON PROGRESS IN PHYSICS
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Hard interactions of quarks and gluons: a primer for LHC physics

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CHS

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over 1500 downloads so far

Abstract

In this paper, we will develop the perturbative framework for the calculation of hard-scattering processes. We will undertake to provide both a reasonably rigorous development of the formalism of hard-scattering of quarks and gluons as well as an intuitive understanding of the physics behind the scattering. We will emphasize the role of logarithmic corrections as well as power counting in α_S in order to understand the behaviour of hard-scattering processes. We will include 'rules of thumb' as well as 'official recommendations', and where possible will seek to dispel some myths. We will also discuss the impact of soft processes on the measurements of hard-scattering processes. Experiences that have been gained at the Fermilab Tevatron will be recounted and, where appropriate, extrapolated to the LHC.

(Some figures in this article are in colour only in the electronic version)

goal is to provide a reasonably global picture of LHC calculations (with rules of thumb)

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Review

Jets in hadron–hadron collisions

S.D. Ellis[a,*], J. Huston[b], K. Hatakeyama[c], P. Loch[d], M. Tönnesmann[e]

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arXiv:07122447 Dec 14, 2007

Abstract

In this article, we review some of the complexities of jet algorithms and of the resultant comparisons of data to theory. We review the extensive experience with jet measurements at the Tevatron, the extrapolation of this acquired wisdom to the LHC and the differences between the Tevatron and LHC environments. We also describe a framework (SpartyJet) for the convenient comparison of results using different jet algorithms.

Keywords: Jet; Jet algorithm; LHC; Tevatron; Perturbative QCD; SpartyJet

Contents

More references

Towards Jetography

Gavin P. Salam

LPTHE, UPMC Univ. Paris 6,
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Abstract

As the LHC prepares to start taking data, this review is intended to provide a QCD theorist's understanding and views on jet finding at hadron colliders, including recent developments. My hope is that it will serve both as a primer for the newcomer to jets and as a quick reference for those with some experience of the subject. It is devoted to the questions of how one defines jets, how jets relate to partons, and to the emerging subject of how best to use jets at the LHC.

explain it in 60 seconds

Jets are sprays of particles that fly out from certain high-energy collisions—for instance, from violent collisions of protons and antiprotons at Fermilab's Tevatron accelerator, or in the similar proton-proton collisions that will take place at CERN's Large Hadron Collider.

These collisions create very energetic quarks and gluons; as they travel away from the collision point, they emit more gluons, which can split into even more gluons. This results in a relatively narrow cascade, or jet, of particles.

In the last stage of jet creation, quarks and gluons combine to form particles such as protons, pions, and kaons. By measuring these end products, physicists can determine the properties of a jet, and thus the details of the collision that produced it. Scientists expect to see jets in the signatures of almost every interesting collision at the Large Hadron Collider.

The most violent collisions will produce jets with the highest momentum, and these can be used to probe the smallest distances within the colliding protons, less than one-billionth of a billionth of a meter. Physicists hope they can use these most energetic jets to look inside the quarks that make up protons.

Joey Huston, Michigan State University

*"When you're a jet,
you're a jet all the way,
from your first gluon split
to your last K decay..."*

Symmetry
A joint Fermilab/SLAC publication
PO Box 500
MS 206
Batavia Illinois 60510
USA

symmetry

Five lectures

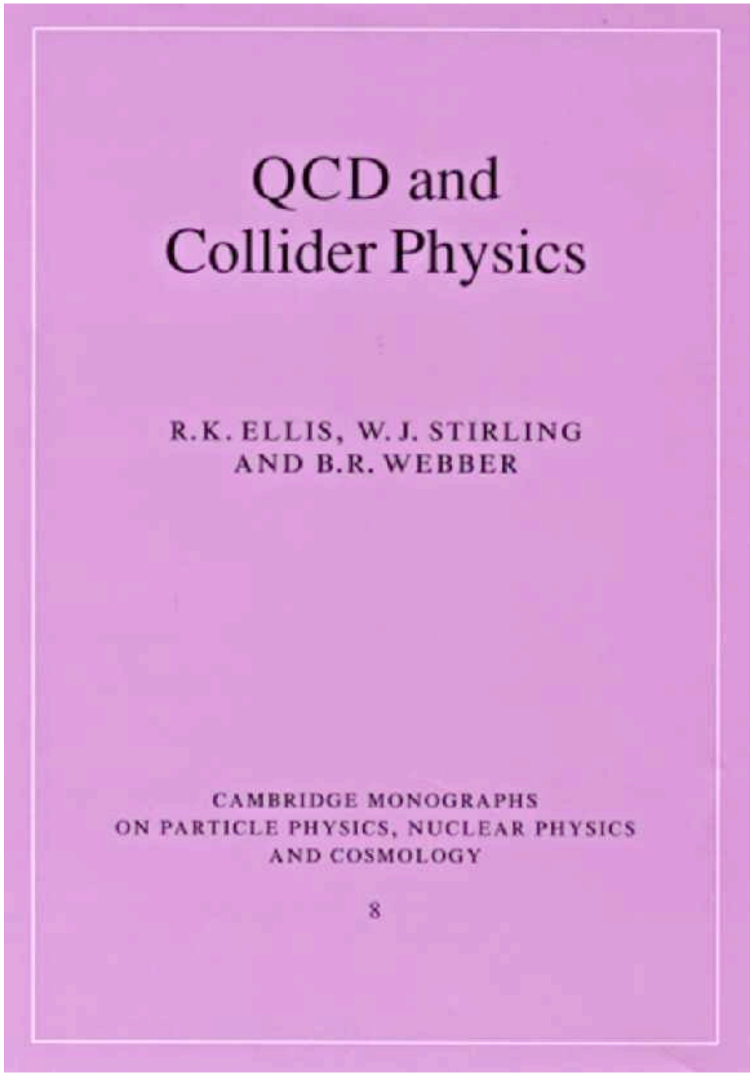
1. Introduction to pQCD-May 10
2. Higher order calculations-May 17
3. Parton distributions-May 24
4. Jet algorithms-June 1
5. On to the LHC-June 7

Caveat

- I'm not a theorist

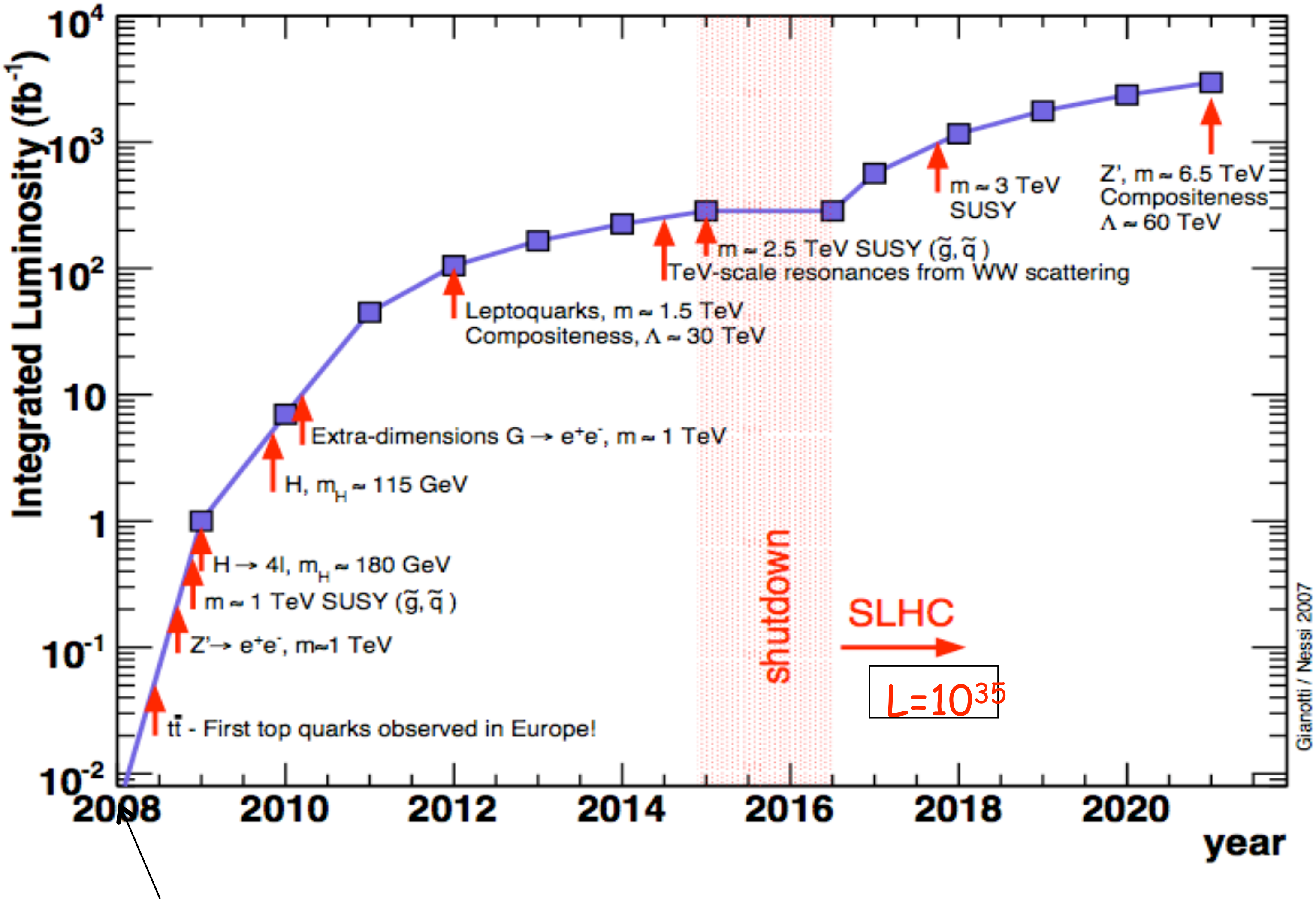
I'm an experimentalist; note the hard-hat

- I don't even play one on TV
- So my lectures are not going to be in as much technical detail as a theorist would
  - because I probably wouldn't get the details right
  - and I like the intuitive "rules-of-thumb" approach better
- But there are references that do go into such detail, as for example the book to the right
  - often termed as the "pink book"



We'll look back on early LHC trouble in 15 years and laugh

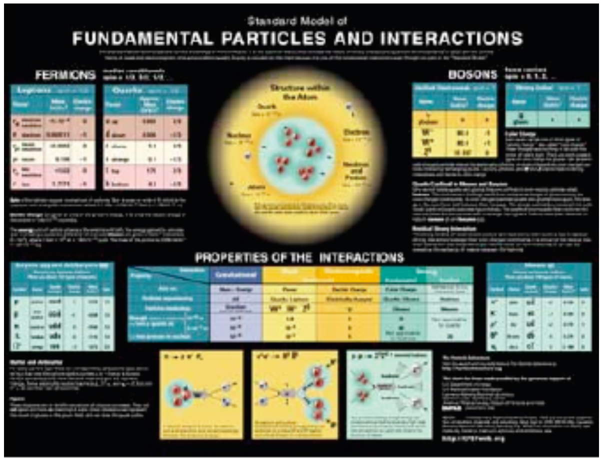
LHC vs time: a wild guess ...

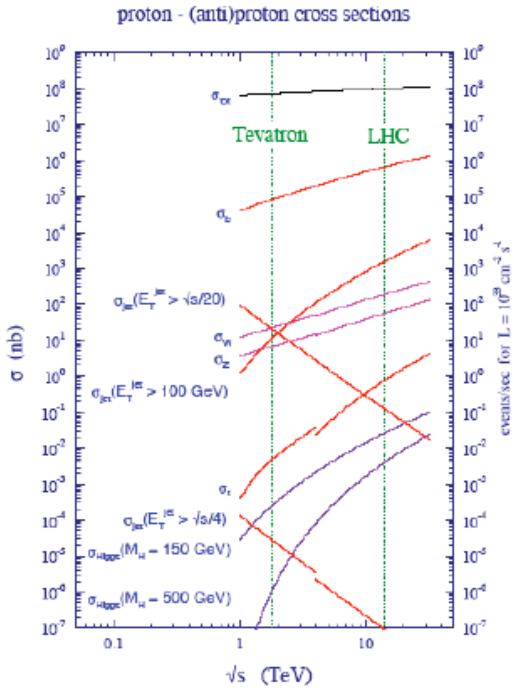


you are here (even though it's now 2010); first W's, first Z

Understanding cross sections at the LHC

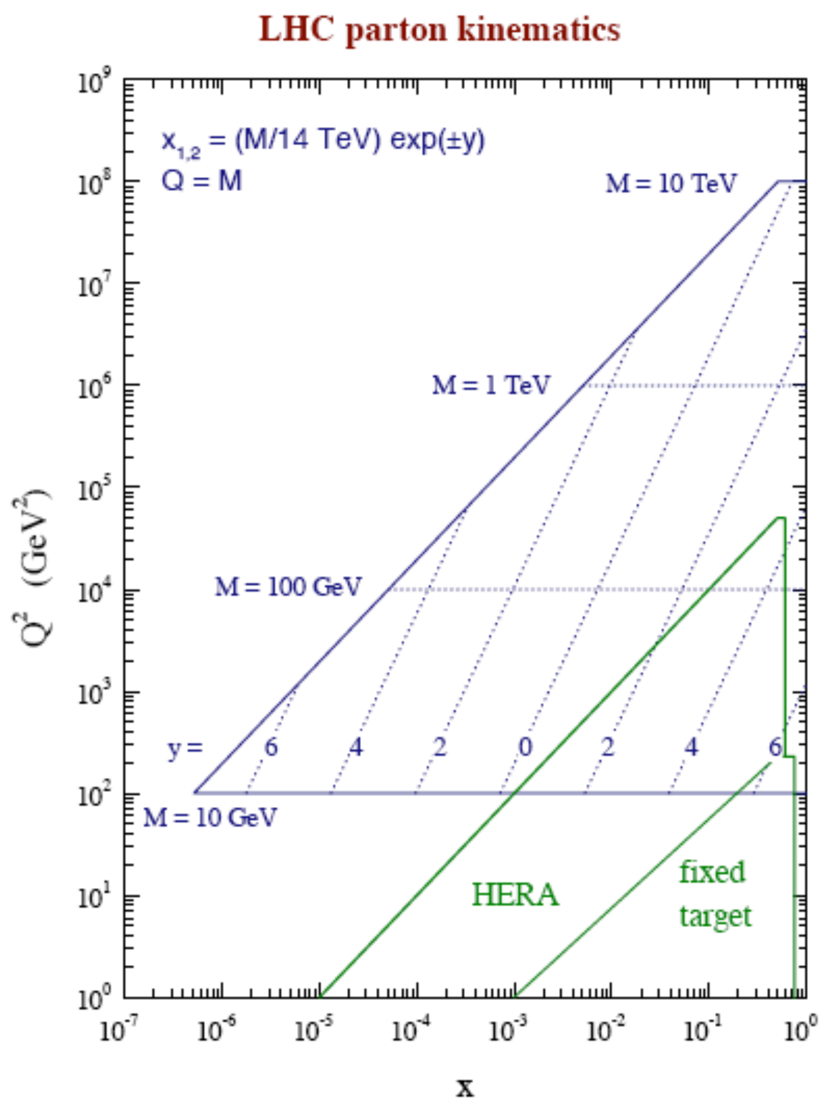
- But before we can laugh, we have to understand the Standard Model at the LHC
- We're all looking for BSM physics at the LHC
- Before we publish BSM discoveries from the early running of the LHC, we want to make sure that we measure/understand SM cross sections
  - detector and reconstruction algorithms operating properly
  - SM backgrounds to BSM physics correctly taken into account
  - and, in particular, that QCD at the LHC is properly understood





Cross sections at the LHC

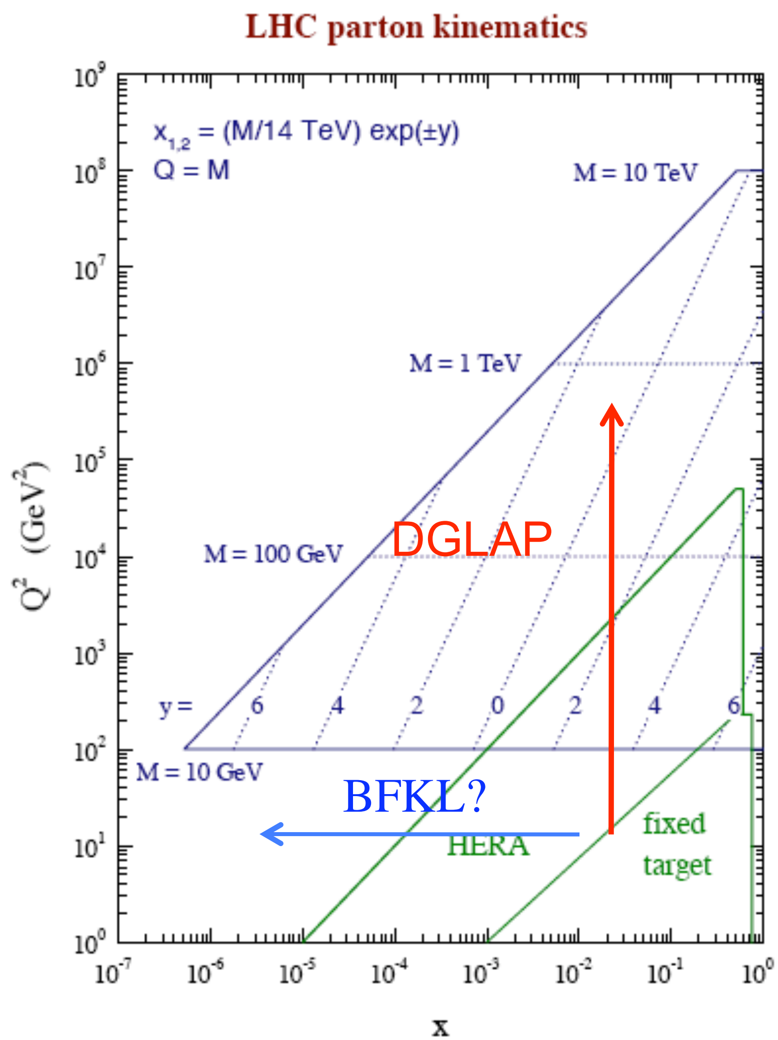
- Experience at the Tevatron is very useful, but scattering at the LHC is not necessarily just “rescaled” scattering at the Tevatron
- Small typical momentum fractions x for the quarks and gluons in many key searches
  - dominance of gluon and sea quark scattering
  - large phase space for gluon emission and thus for production of extra jets
  - intensive QCD backgrounds
  - or to summarize,…lots of Standard Model to wade through to find the BSM pony



Cross sections at the LHC

- Note that the data from HERA and fixed target cover only part of kinematic range accessible at the LHC
- We will access pdf's down to 10^{-6} (crucial for the underlying event) and Q^2 up to 100 TeV^2
- We can use the DGLAP equations to evolve to the relevant x and Q^2 range, but…
  - we're somewhat blind in extrapolating to lower x values than present in the HERA data, so uncertainty may be larger than currently estimated
  - we're assuming that DGLAP is all there is; at low x BFKL type of logarithms may become important (more later about DGLAP and BFKL)

$$\frac{d\sigma}{dM^2 dy} = \frac{\hat{\sigma}_0}{Ns}\left[\sum_k Q_k^2 \left(q_k(x_1, M^2)\bar{q}_k(x_2, M^2) + [1 \leftrightarrow 2]\right)\right]$$

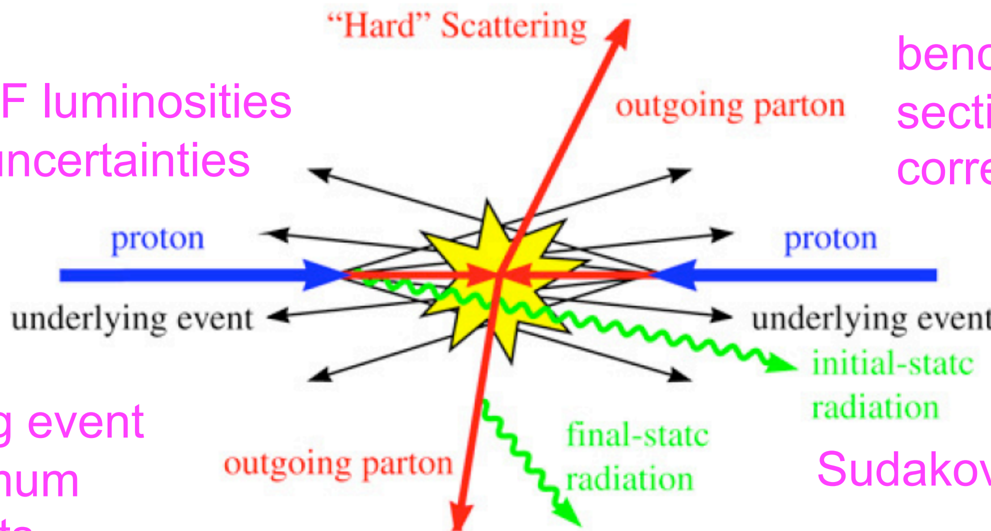


Understanding cross sections at the LHC

LO, NLO and NNLO calculations
K-factors

PDF's, PDF luminosities
and PDF uncertainties

benchmark cross
sections and pdf
correlations



underlying event
and minimum
bias events

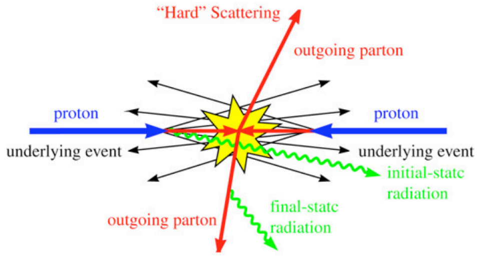
Sudakov form factors

jet algorithms and jet reconstruction

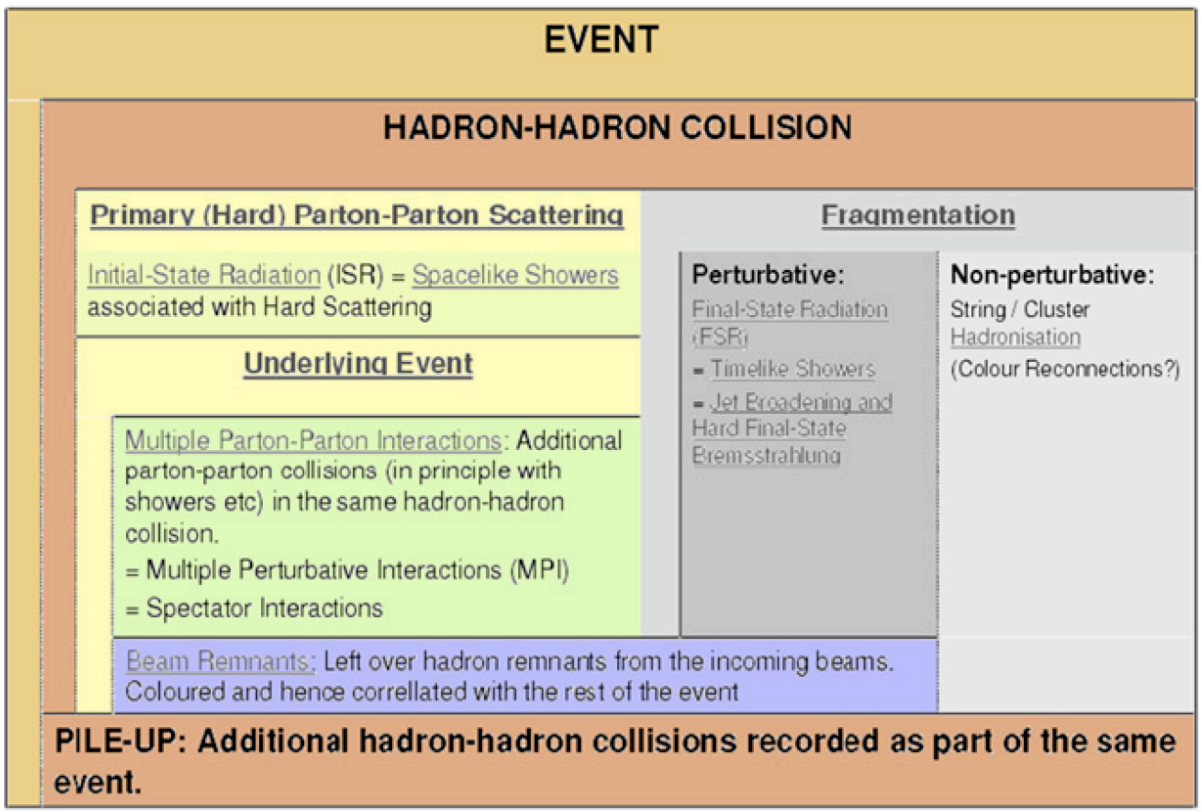
I'll try to touch on all of these topics in these lectures.

Most experimenters are/will still mostly use parton shower
Monte Carlo for all predictions/theoretical comparisons
at the LHC. I'll try to show that there's more than that.

Some definitions



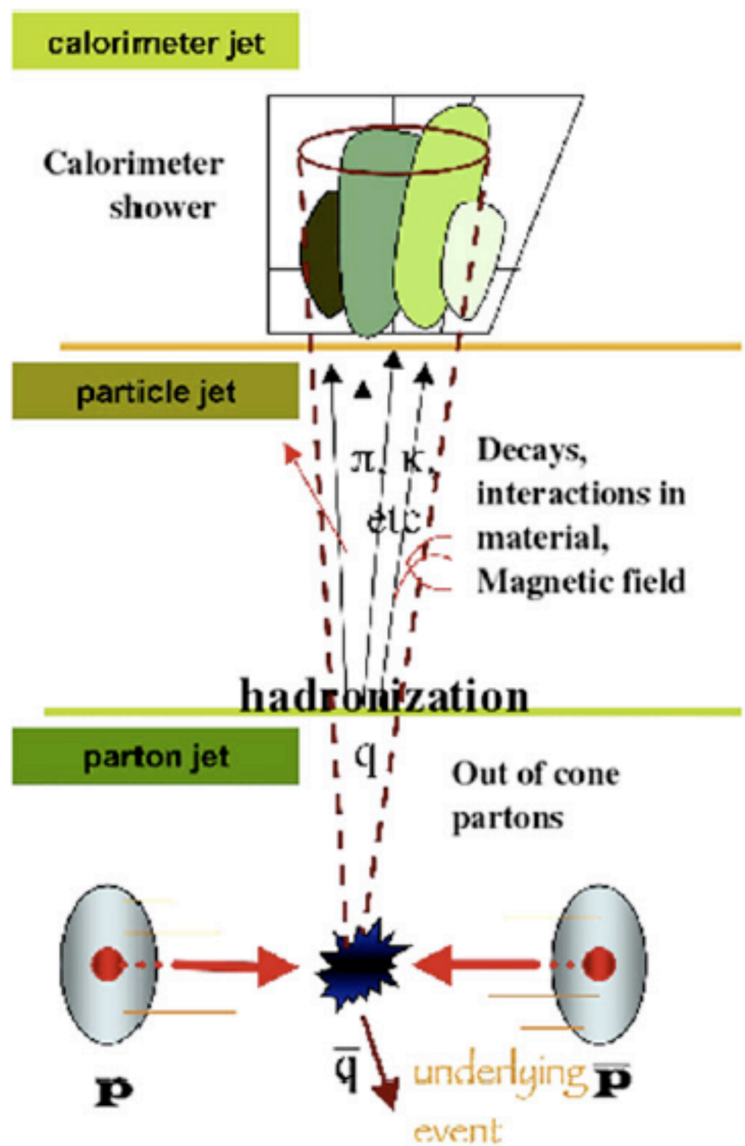
Dictionary of Hadron Collider Terminology



- The fundamental challenge to interpret experimentally observed final states is that pQCD is most easily applied to the short-distance degrees of freedom, i.e. to quarks and gluons, while the long-distance degrees of freedom seen in the detectors are color-singlet bound states
- The overall scattering process evolves from the incoming long-distance hadrons in the beams, to the short-distance scattering process, to the long-distance outgoing final states
- The separation of these steps is essential both conceptually and calculationally

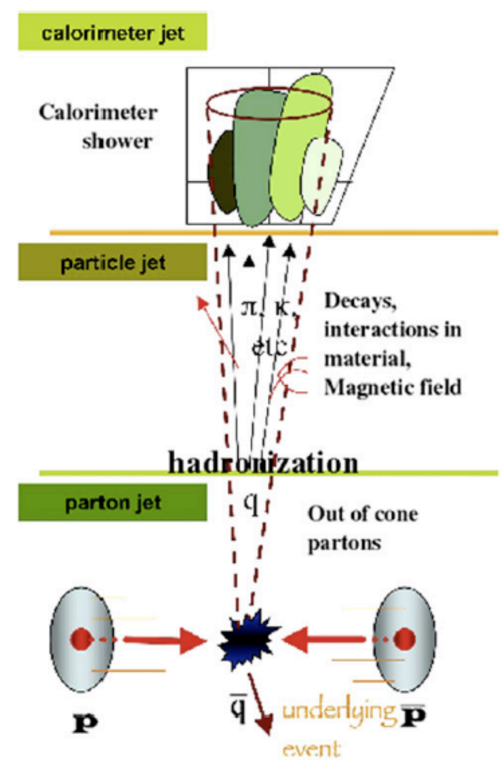
…and a word about jets

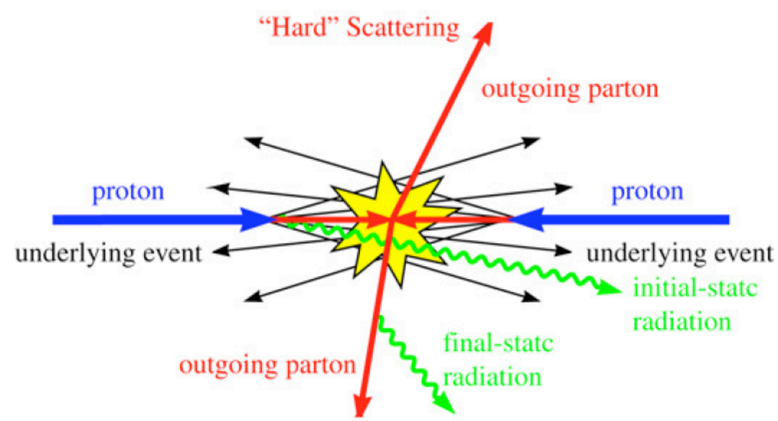
- Most of the interesting physics signatures at the Tevatron and LHC involve final states with jets of hadrons
- A jet is reconstructed from energy depositions in calorimeter cells and/or from charged particle track momenta, and ideally is corrected for detector response and resolution effects so that the resultant 4-vector corresponds to that of the sum of the original hadrons
- The jets can be further corrected , for hadronization effects, back to the parton(s) from which the jet originated,or the theory can be corrected to the hadron level
- The resultant measurements can be compared back to parton shower predictions, or to the short-distance partons described by fixed-order pertubative calculations



…another word about jets

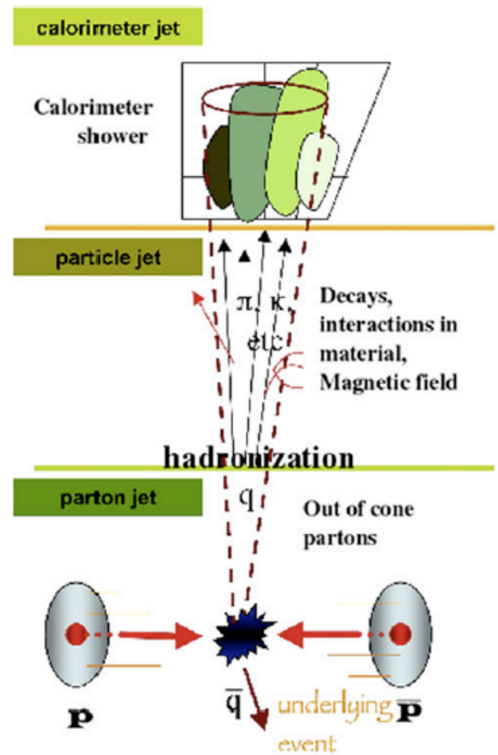
- We pick out from the incident beam particles, the short-distance partons that participate in the hard collision
- The partons selected can emit radiation prior to the short distance scattering leading to initial state radiation
- The remnants of the original hadrons, with one parton removed, will interact with each other, producing an underlying event
- Next comes the short-distance, large momentum transfer scattering process that may change the character of the scattering partons, and/or produce more partons
  - the cross section for this step is calculated to fixed order in pQCD

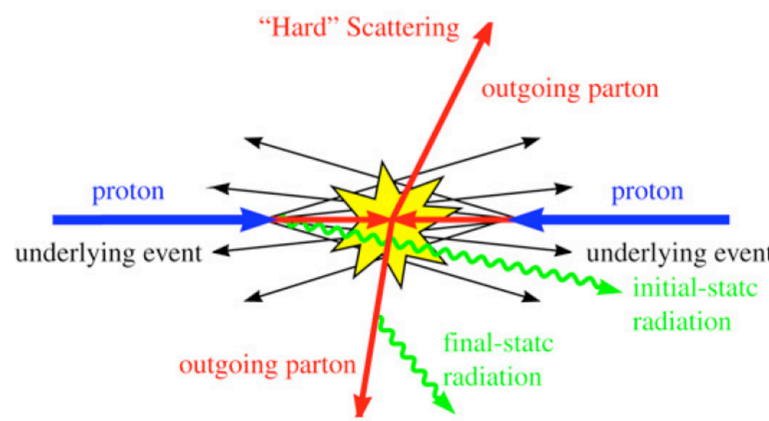




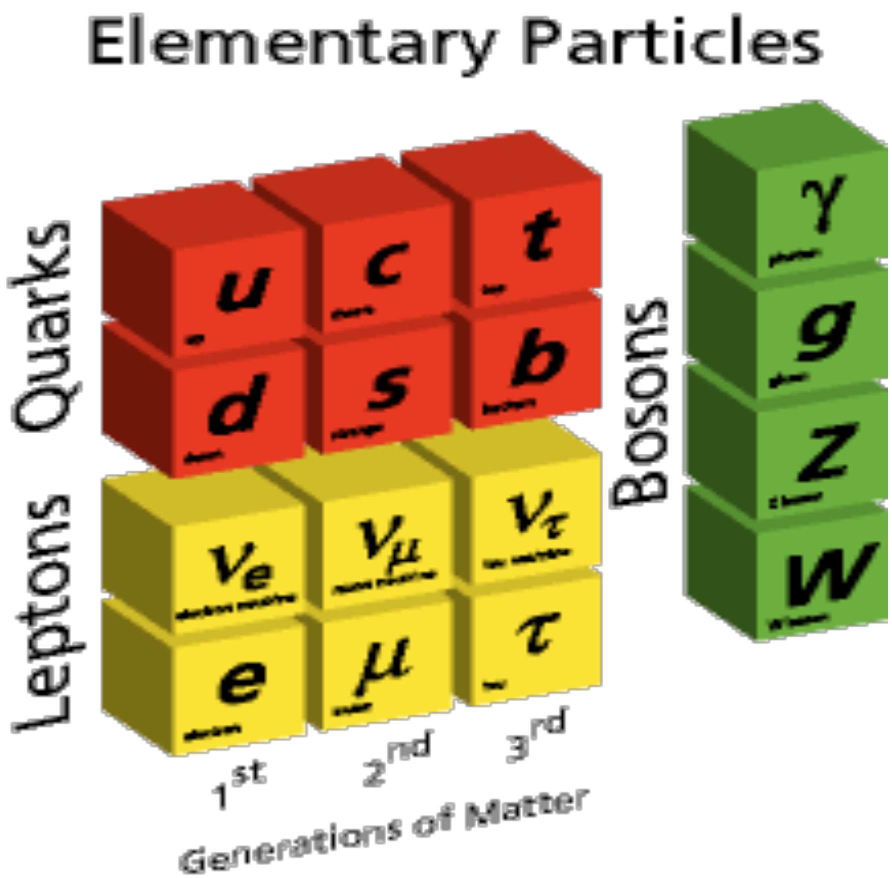
…still another word about jets

- Then comes another color radiation step, when many new gluons and quark pairs are added to the final state
- The final step in the evolution to the long distance states involves a nonperturbative hadronization process that organizes the colored degrees of freedom
- This non-perturbative hadronization step is accomplished in a model-dependent fashion





Back to the Standard Model



The Standard Model has been extremely successful, although admittedly incomplete.

In these lectures, we're most interested in QCD and thus the force carrier of the strong force (the gluon) and its interaction with quarks (and with itself).

Start with the QCD Lagrangian…

The (Classical) QCD Lagrangian

$$L_{QCD} = -\frac{1}{4} F_{\alpha\beta}^{B} F_{B}^{\alpha,\beta} + \sum_{f} \bar{q}_{f,a} \left(iD_{\mu}\gamma^{\mu} - m_{f} \right)_{ab} q_{f,b}$$

$f \rightarrow$ n_f flavors

describes the interactions of spin ½ quarks with mass m, and massless spin 1 gluons

$$F_{\alpha\beta}^{B} = \left[\partial_{\alpha} A_{\beta}^{B} - \partial_{\beta} A_{\alpha}^{B} - g f^{BCD} A_{\alpha}^{C} A_{\beta}^{D} \right]$$

field strength tensor derived from gluon field A; 3rd term is the non-Abelian term (QCD≠QED)

Acting on the triplet and octet fields, respectively, the covariant derivative is

$$\left(D_{\mu} \right)_{ab} = \partial_{\mu}\delta_{ab} + ig\left(t^{C} A_{\mu}^{C} \right)_{ab} \; ; \left(D_{\mu} \right)_{CD} = \partial_{\mu}\delta_{CD} + ig\left(T^{B} A_{\mu}^{B} \right)_{CD}$$

The matrices for the fundamental (t_{ab}^{B}) and adjoint (T_{CD}^{B}) representations carry the information about the Lie algebra

$$\left[t^{B}, t^{C} \right] = if^{BCD} t^{D} \; ; \left[T^{B}, T^{C} \right] = if^{BCD} T^{D} \; ; \quad \left(T^{B} \right)_{CD} = -if^{BCD} \; ;$$

(f^{BCD} is the structure constant of the group)

$$Tr\left[t^{B} t^{C} \right] = \frac{\delta^{BC}}{2} \equiv T_{R}\delta^{BC} \; ; \; t_{ab}^{B} t_{bc}^{B} = \frac{4}{3}\delta_{ac} \equiv C_{F}\delta_{ac} \; ;$$

$$Tr\left[T^{B} T^{C} \right] = 3\delta^{BC} \equiv C_{A}\delta^{BC}$$

$$C_{A} = N_{colors} = 3$$

$$C_{F} = \frac{N_{colors}^{2} - 1}{2N_{colors}} = \frac{4}{3}$$

for SU(3)

…thanks to Steve Ellis for the next few slides

Feynman Rules:

Propagators – (in a general gauge represented by the parameter λ; Feynman gauge is λ = 1; this form does not include axial gauges)

Quark 

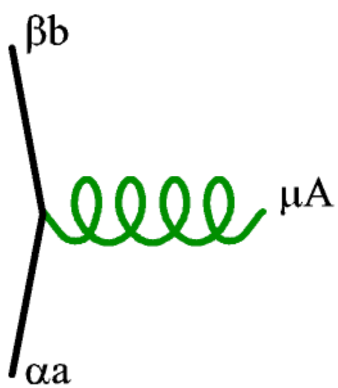
$$\frac{i\delta_{ab}}{(\gamma^{\mu}q_{\mu} - m)_{\alpha\beta}} = \frac{i(\gamma^{\mu}q_{\mu} + m)_{\alpha\beta}\,\delta_{ab}}{q^2 - m^2}$$

Gluon 

$$\frac{-i}{q^2}\delta_{AB}\left[g^{\mu\nu} - (1-\lambda)\frac{q^{\mu}q^{\nu}}{q^2}\right]$$

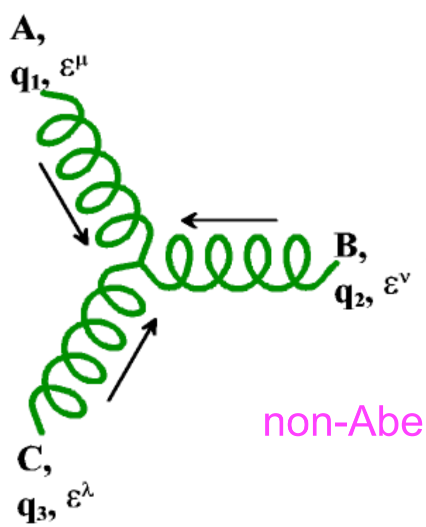
Vertices –

Quark – gluon



$$-ig\gamma^{\mu}{}_{\alpha\beta}\frac{t^{A}{}_{ab}}{2}$$

3 gluons

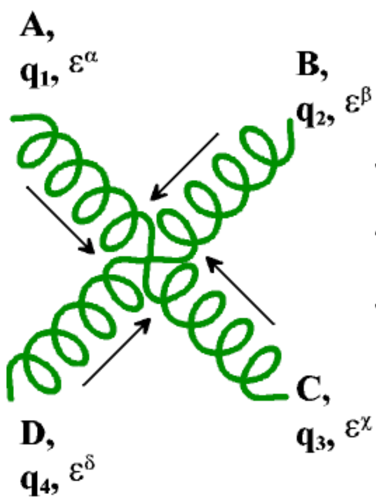


$$-g f^{ABC}[(q_1-q_2)^{\lambda}g^{\mu\nu} + (q_2-q_3)^{\mu}g^{\nu\lambda} + (q_3-q_1)^{\nu}g^{\mu\lambda}]$$

non-Abelian coupling; not present in QED

Feynman Rules II:

4 gluons



$$-ig^2 f^{EAD} f^{EBC}[g^{\alpha\beta} g^{\chi\delta} - g^{\alpha\chi} g^{\beta\delta}]$$

$$-ig^2 f^{EAC} f^{EBD}[g^{\alpha\beta} g^{\chi\delta} - g^{\alpha\delta} g^{\beta\chi}]$$

$$-ig^2 f^{EAB} f^{ECD}[g^{\alpha\chi} g^{\beta\delta} - g^{\alpha\delta} g^{\beta\chi}]$$

pQCD 101 - Use QCD Lagrangian to Correct the Parton Model

- Naïve QCD Feynman diagrams exhibit infinities at nearly every turn, as they must in a conformal theory with no "bare" dimensionful scales (ignore quark masses for now).***

First consider life in the Ultra-Violet – short distance/times or large momenta (the Renormalization Group at work):

- The UV singularities mean that the theory

➢ does *not* specify the strength of the coupling in terms of the "bare" coupling in the Lagrangian

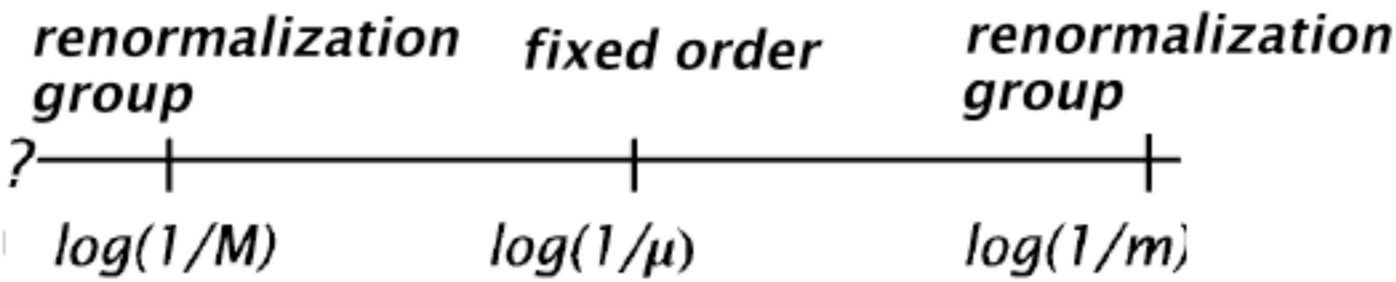
➢ does specify how the coupling varies with scale [$\alpha_s(\mu)$ measures the "charge inside" a sphere of radius $1/\mu$]

***** Typical of any renormalizable gauge field theory. This is one reason why String theorists want to study something else! We will not discuss the issue of choice of gauge. Typically axial gauges ($\hat{n} \cdot \vec{A} = 0$) yield diagrams that are more parton-model-like, so-called physical gauges.**

Consider a range of distance/time scales – $1/\mu$

- use the renormalization group below some (distance) scale 1/m (perhaps down to a GUT scale 1/M where theory changes?) to sum large logarithms ln[M/μ]

- use fixed order perturbation theory around the physical scale 1/μ ~ 1/Q (at hadronic scale 1/m things become non-perturbative, above the scale M the theory may change)



← Short distance   Long distance →

new physics??   perturbative   non-perturbative

Strong coupling constant α_s

An important component of all QCD cross sections

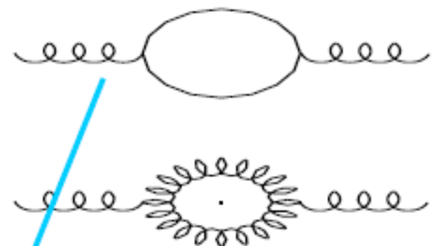
QED:

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 + \frac{\beta_0}{2\pi}\alpha(\mu^2)\log\frac{Q^2}{\mu^2}}$$

$$\beta_0 = \frac{2}{3}$$

Low resolution:
charge is screened
by ee-pairs
High resolution :
charge is big

QCD:



$$\beta_0 = \frac{2}{3} T_F N_F - \frac{11}{3} C_A = \frac{2 N_F - 11 N_C}{3}$$

generated by
$q\bar{q}$ fluctuations
(as in QED)

gluonicselfinteraction

N_F number of fermions
N_c number of colours
$T_F = \frac{1}{2}, C_A = N_c$ colour factors

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + b_0\, \alpha_s(\mu^2) \log\frac{Q^2}{\mu^2}}$$

positive term, since $b_0 < 1$

where $b_0 = \frac{-\beta_0}{4\pi}$

$\beta_0 < 0$ for $N_F \le 16$
$\rightarrow$ anti - screening
charge is spread - out
by gluons, i.e. at infinite
resolution charge is
very small

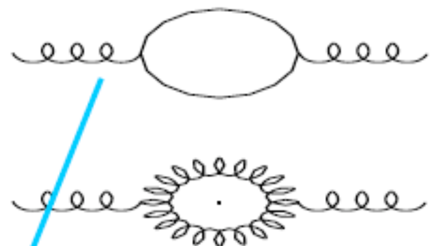
I borrowed the next few slides from someone in ATLAS, but lost track who.

Note there can be different conventions, i.e. β_o can be defined so it's positive, but then $b_o = +\beta_o/4\pi$

It's important that the β function is negative

An important component of all QCD cross sections

QCD:



$$\beta_0 = \frac{2}{3} T_F N_F - \frac{11}{3} C_A = \frac{2 N_F - 11 N_C}{3}$$

ated by
ctuations
QED)

gluonicselfinteraction

N_F number of fermions
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$\beta_0 < 0$ for $N_F \leq 16$
$\rightarrow$ anti - screening
charge is spread - out
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α_s and Λ

At 1-loop :

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 + b_0\, \alpha(\mu^2) \log \frac{Q^2}{\mu^2}} \quad \text{with} \quad b_0 = \frac{33 - 2N_F}{12\,\pi}$$

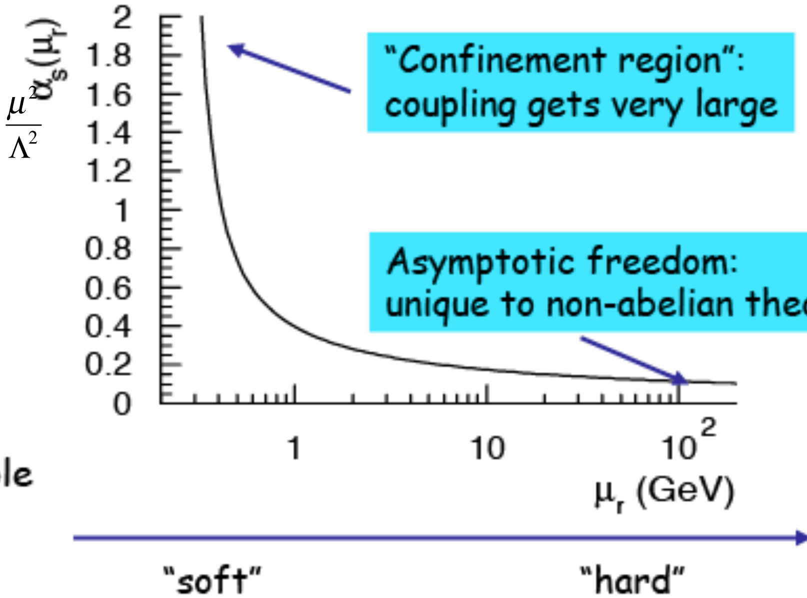
μ is arbitrary parameter (left-over from renormalisation)

Choose $\mu = \Lambda$: point where effective coupling becomes large

$$\Lambda^2 = \mu^2 \exp(-1/b_0\, \alpha_s(\mu^2)) \quad \text{or} \quad \alpha_s(\mu^2) = \frac{1}{b_0 \log \frac{\mu^2}{\Lambda^2}}$$

Therefore :

$$\alpha_s(Q^2) = \frac{1}{b_0 \log \frac{\mu^2}{\Lambda^2} + b_0 \log \frac{Q^2}{\mu^2}} = \frac{1}{b_0 \log \frac{Q^2}{\Lambda^2}}$$



"Confinement region": coupling gets very large

Asymptotic freedom: unique to non-abelian theories

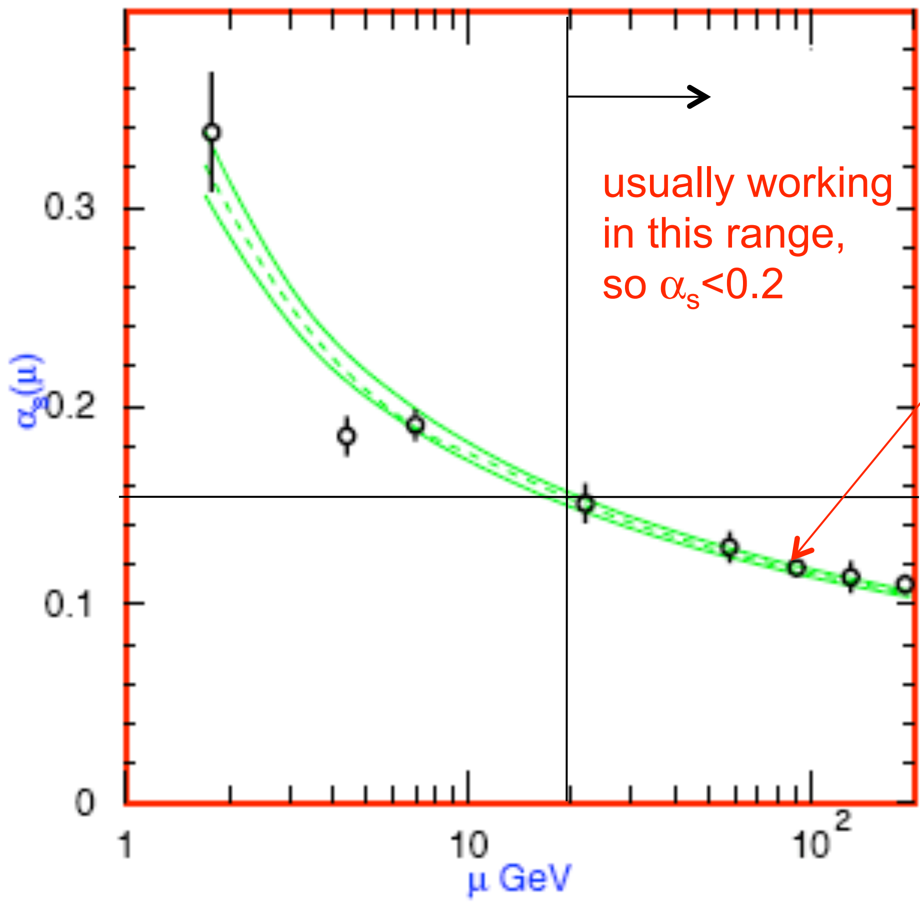
$Q^2 >> \Lambda^2$: $\alpha_s(Q^2)$ small $\rightarrow$ perturbative QCD applicable

$Q^2 \approx \Lambda^2$: quark and gluons form bound states

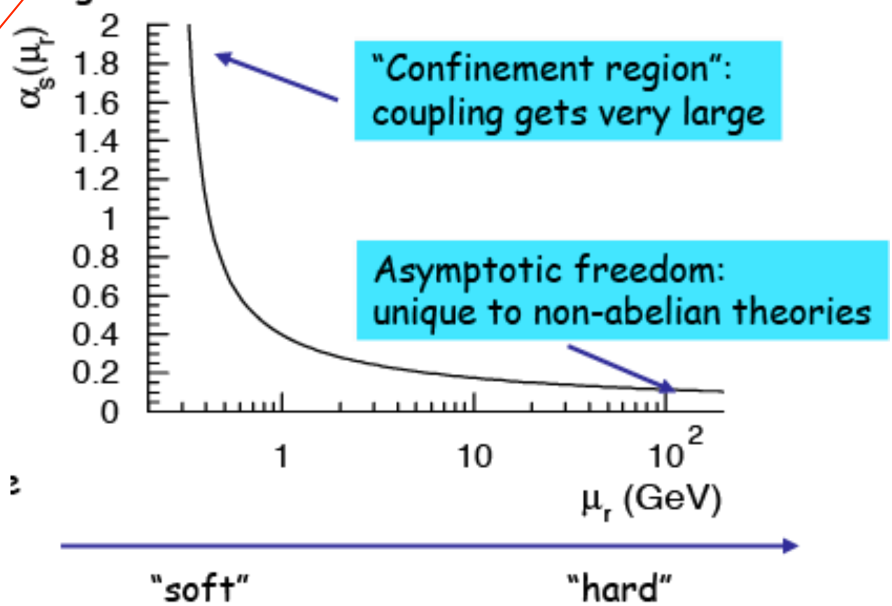
Λ is free parameter of theory,
has to be determined by experiment
$\rightarrow$ expected to be of order of hadron mass

QCD explains confinement of colour and allows calculations of hard hadronic processes via perturbative expansion of coupling !

see www-theory.lbl.gov/~ianh/alpha/alpha.html



@ scale of m_Z, world average for α_s is 0.118 (NLO) and 0.130 (LO); α_s(NNLO_~α_s(NLO) It's more common now to quote α_s at a scale of m_Z than to quote Λ

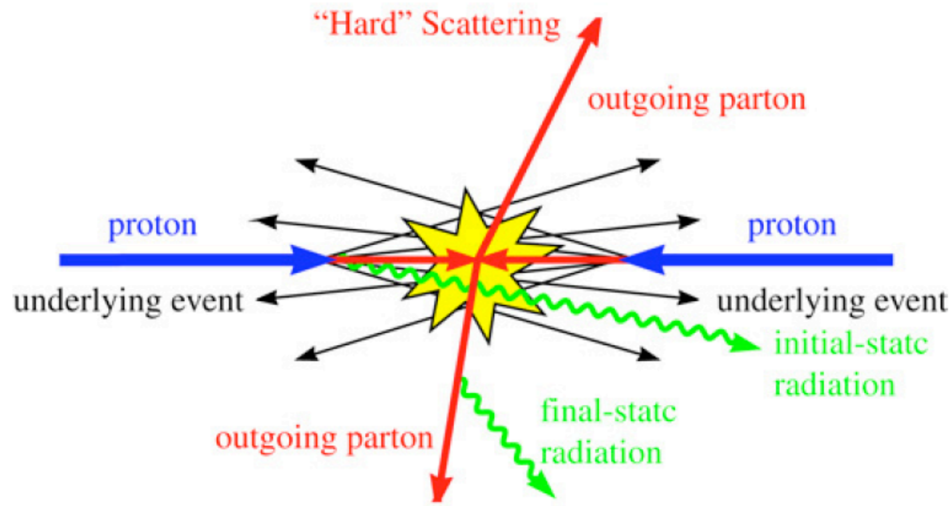


Λ is free parameter of theory, has to be determined by experiment
→ expected to be of order of hadron mass

QCD explains confinement of colour and allows calculations of hard hadronic processes via perturbative expansion of coupling !

Factorization

- Factorization is the key to perturbative QCD
  - the ability to separate the short-distance physics and the long-distance physics
- In the pp collisions at the LHC, the hard scattering cross sections are the result of collisions between a quark or gluon in one proton with a quark or gluon in the other proton
- The remnants of the two protons also undergo collisions, but of a softer nature, described by semi-perturbative or non-perturbative physics



The calculation of hard scattering processes at the LHC requires:
(1)knowledge of the distributions of the quarks and gluons inside the proton, i.e. what fraction of the momentum of the parent proton do they have
->parton distribution functions (pdf's)
(2) knowledge of the hard scattering cross sections of the quarks and gluons, at LO, NLO, or NNLO in the strong coupling constant α_s

Parton distributions

- The momentum of the proton is distributed among the quarks and gluons that comprise it
  - about 40% of the momentum is with gluons, the rest with the quarks
- We'll get back to pdf's for more detail later, but for now notice that the gluon distribution dominates at small momentum fractions (x), while the (valence) quarks dominate at high x

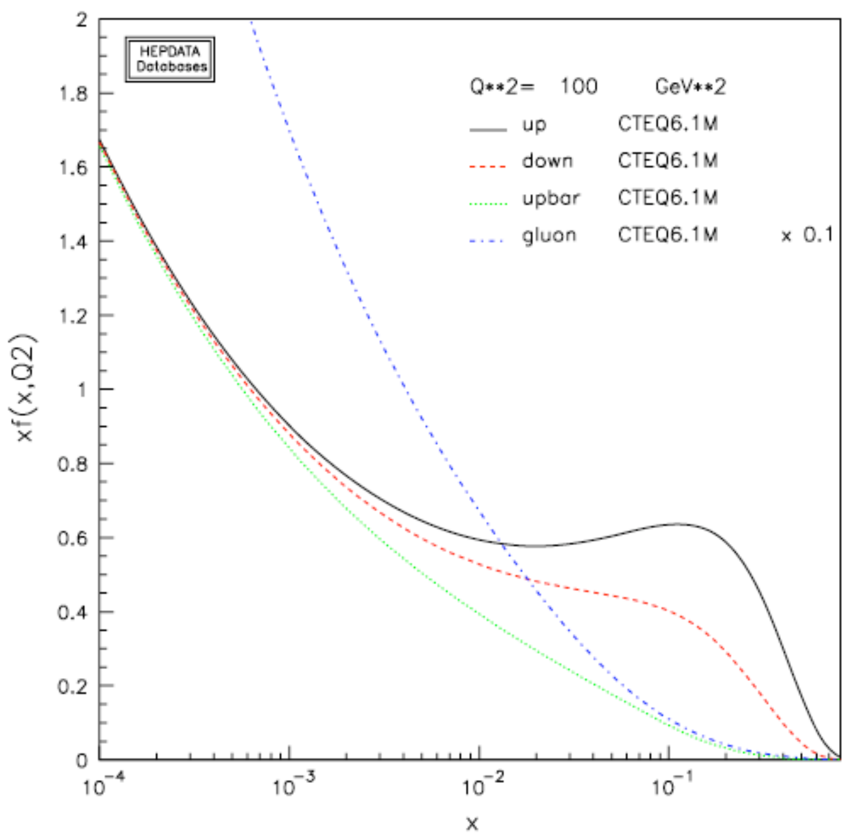
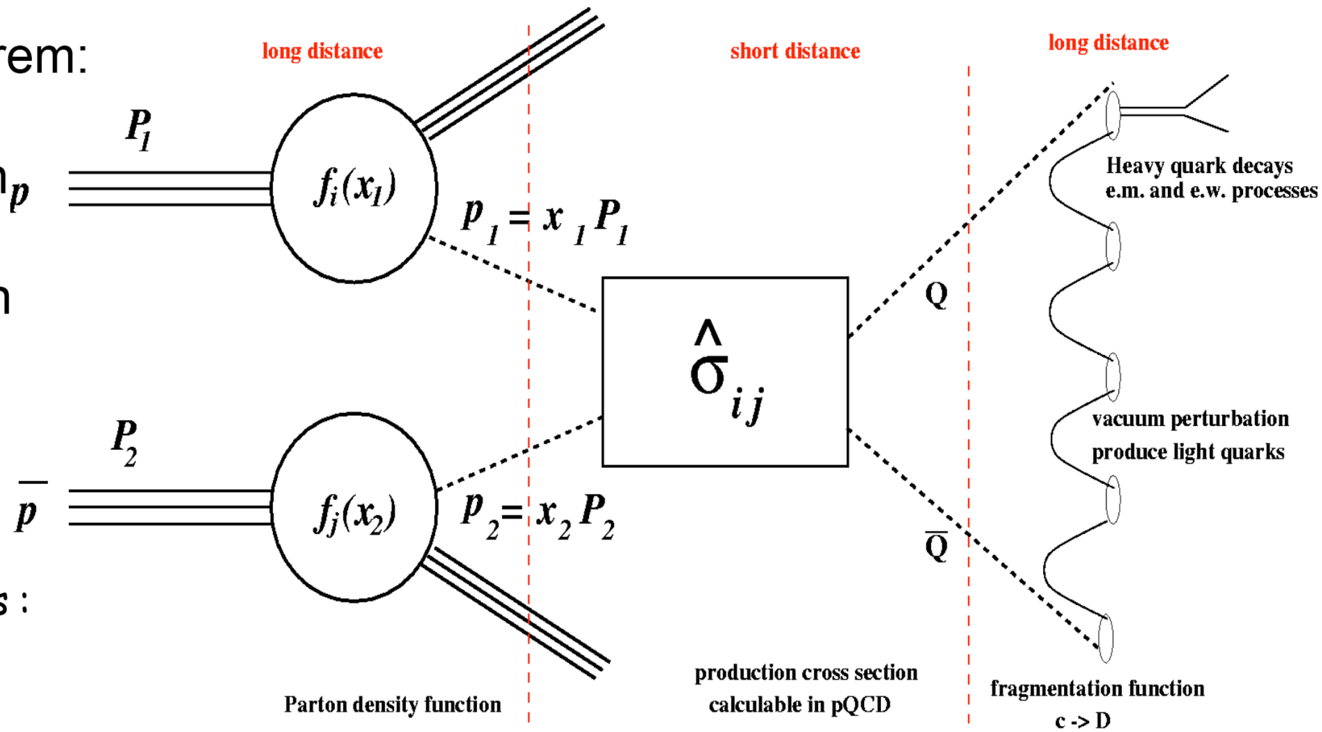


Figure 27. The CTEQ6.1 parton distribution functions evaluated at a Q of 10 GeV.

Factorization theorem

Factorisation Theorem:
PDF is universal
Once extracted can calculate any cross-section within same theoretical scheme

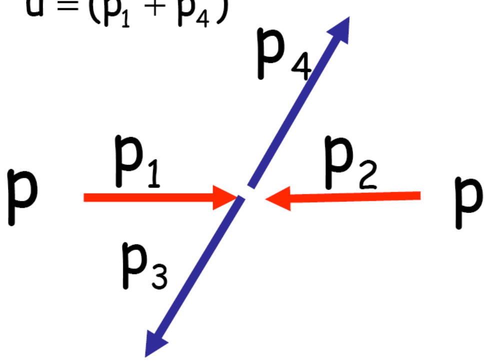


Mandelstamm variables :

$$\hat{s} = (p_1 + p_1)^2$$

$$\hat{t} = (p_1 + p_3)^2$$

$$\hat{u} = (p_1 + p_4)^2$$



$$\sigma = \sum_{ij} \int dx_1\, dx_2\, d\hat{t}\; f_i(x_1,\mu^2)\, f_j(x_2,\mu^2)\, d\hat{\sigma}_{ij}/d\hat{t}$$

$$\text{e.g.: qq-scattering in LO:}\quad d\hat{\sigma}_{ij}/d\hat{t} = \frac{\pi}{\hat{s}^2}\frac{4}{9}\alpha_s^2\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2} \propto \frac{1}{p_T^2}$$

→ Diverges for low P_T → 0

Factorization theorem

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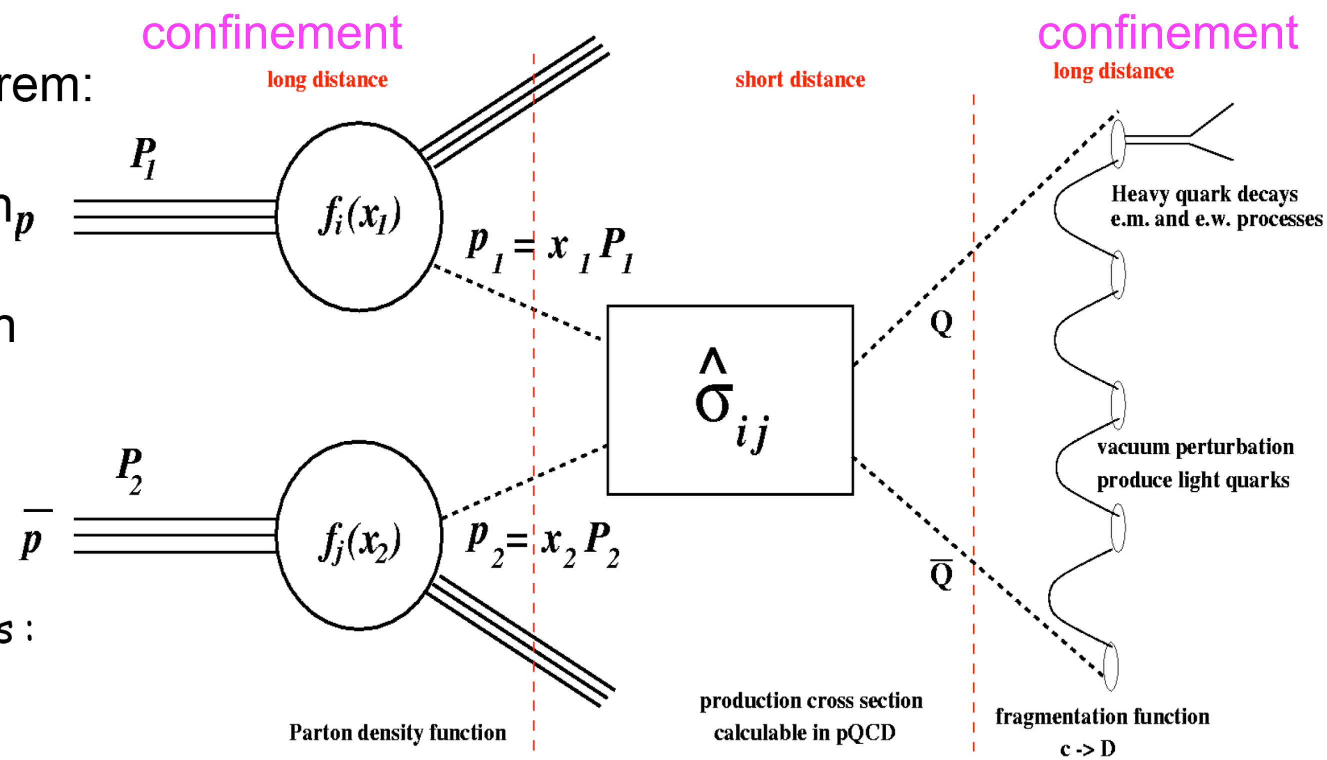

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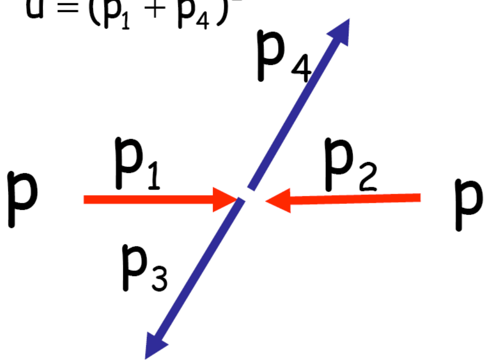

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$$\text{e.g. : qq - scattering in LO :}\quad d\hat{\sigma}_{ij}/d\hat{t} = \frac{\pi}{\hat{s}^2}\frac{4}{9}\alpha_s^2 \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} \propto \frac{1}{p_T^2}$$

→ Diverges for low P_T→0

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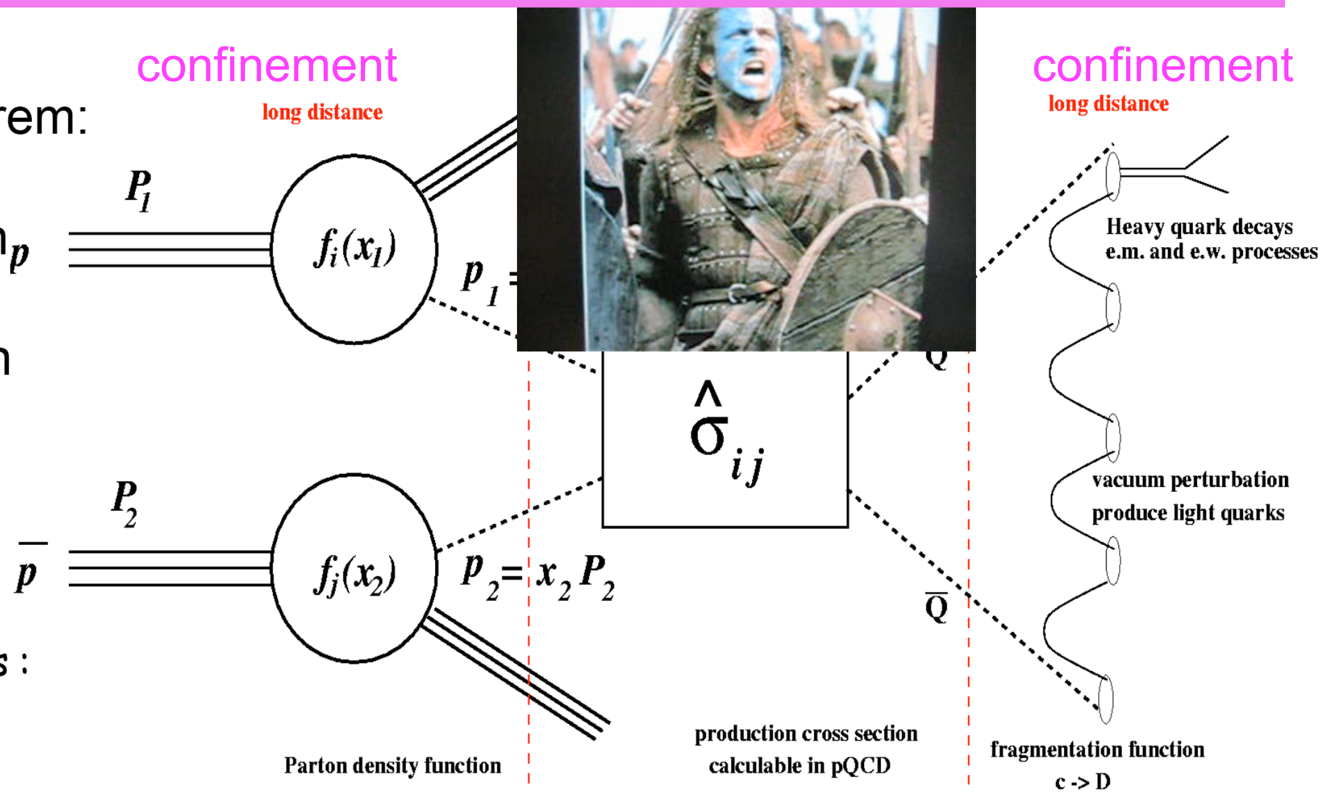

Mandelstamm variables :

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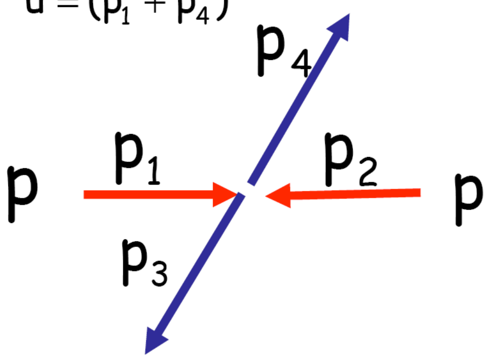


$$\sigma = \sum_{ij} \int dx_1\, dx_2 d\hat{t}\; f_i(x_1,\mu^2)\, f_j(x_2,\mu^2)\, d\hat{\sigma}_{ij}/d\hat{t}$$

$$\text{e.g.: qq-scattering in LO:}\quad d\hat{\sigma}_{ij}/d\hat{t} = \frac{\pi}{\hat{s}^2}\frac{4}{9}\alpha_s^2 \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} \propto \frac{1}{p_T^2}$$

→ Diverges for low $P_T \rightarrow 0$

Go back to some SM basics: Drell Yan

- Consider Drell-Yan production
  - write cross section as

$$\sigma_{AB} = \int \mathrm{d}x_a \mathrm{d}x_b \; f_{a/A}(x_a) f_{b/B}(x_b) \; \hat{\sigma}_{ab\to X}$$

  - where $X=l^+l^-$
- Potential problems appeared to arise from when perturbative corrections from real and virtual gluon emissions were calculated
  - but these logarithms were the same as those in structure function calculations and thus can be absorbed, via DGLAP equations in definition of parton distributions, giving rise to logarithmic violations of scaling
  - can now write the cross section as

where x_a is the momentum fraction of parton a in hadron A, and x_b the momentum fraction of parton b in hadron B, and Q is a scale that measures the hardness of the interaction

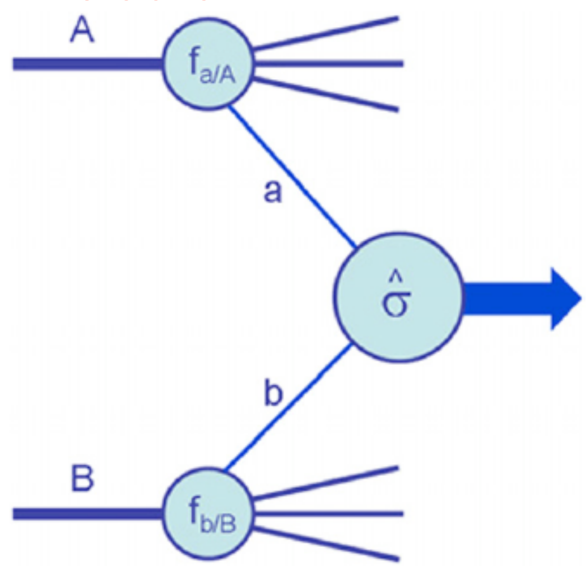


Figure 1. Diagrammatic structure of a generic hard-scattering process.

$$\sigma_{AB} = \int \mathrm{d}x_a \mathrm{d}x_b \; f_{a/A}(x_a, Q^2) f_{b/B}(x_b, Q^2) \; \hat{\sigma}_{ab\to X}$$

…but

- Key point is that all logarithms appearing in Drell-Yan corrections can be factored into renormalized (universal) parton distributions
  - factorization
- But finite corrections left behind after the logarithms are not universal and have to be calculated separately for each process, giving rise to order α_s^n perturbative corrections
- So now we can write the cross section as

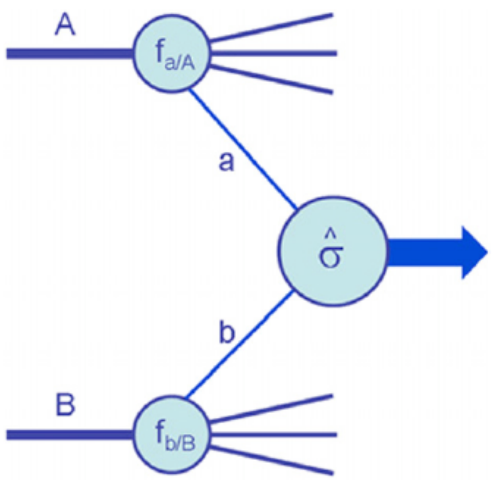


Figure 1. Diagrammatic structure of a generic hard-scattering process.

also depends on μ_R and μ_F, so as to cancel scale dependence in PDF's and α_s, to this order

$$\sigma_{AB} = \int dx_a dx_b \; f_{a/A}(x_a, \mu_F^2) \; f_{b/B}(x_b, \mu_F^2) \; \times \; [\, \hat{\sigma}_0 + \alpha_S(\mu_R^2)\, \hat{\sigma}_1 + \cdots]_{ab \to X} .$$

- where μ_F is the factorization scale (separates long and short-distance physics) and μ_R is the renormalization scale for α_s
- choose $\mu_R = \mu_F \sim Q$

An all-orders cross section has no dependence on μ_F and μ_R; a residual dependence remains (to order α_s^{n+1}) for a finite order (α_s^n) calculation

DGLAP equations

- Parton distributions used in hard-scattering calculations are solutions of DGLAP equations (or in Italy the AP equations)
  - the DGLAP equations determine the Q^2 dependence of the pdf's

DGLAP equations sum leading powers of $[\alpha_s \log\mu^2]^n$ generated by multiple gluon emission in a region of phase space where the gluons are strongly ordered in transverse momentum ($\log \mu >> \log (1/x)$)

For regions in which this ordering is not present (e.g. low x at the LHC), a different type of resummation (BFKL) may be needed

$$\frac{\partial q_i(x,\mu^2)}{\partial \log \mu^2} = \frac{\alpha_S}{2\pi} \int_x^1 \frac{dz}{z} \left\{ P_{q_i q_j}(z,\alpha_S) q_j(\frac{x}{z},\mu^2) + P_{q_i g}(z,\alpha_S)\; g(\frac{x}{z},\mu^2) \right\},$$

$$\frac{\partial g(x,\mu^2)}{\partial \log \mu^2} = \frac{\alpha_S}{2\pi} \int_x^1 \frac{dz}{z} \left\{ P_{g q_j}(z,\alpha_S) q_j(\frac{x}{z},\mu^2) + P_{gg}(z,\alpha_S) g(\frac{x}{z},\mu^2) \right\},$$

  - the splitting functions have the perturbative expansions

$$P_{ab}(x,\alpha_S) = P_{ab}^{(0)}(x) + \frac{\alpha_S}{2\pi} P_{ab}^{(1)}(x) + \cdots .$$

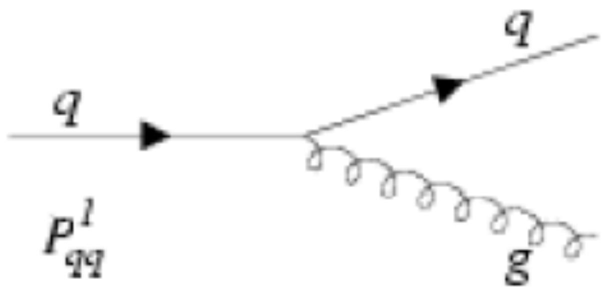
Thus, a full NLO calculation will contain both $\hat{\sigma}_1$ (previous slide) and $P_{ab}^{(1)}$

Altarelli-Parisi splitting functions

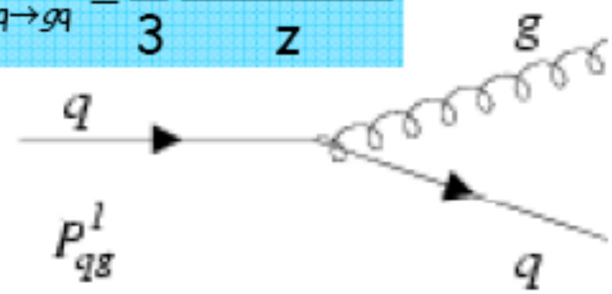
Note that the emitted gluon likes to be soft

Altarelli-Parisi splitting functions:

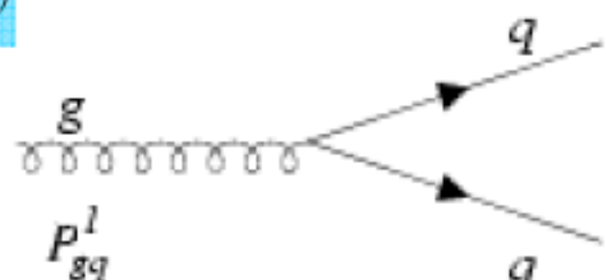
$$P_{q\to qg} = \frac{4}{3}\left(\frac{1+z^2}{1-z}\right)$$



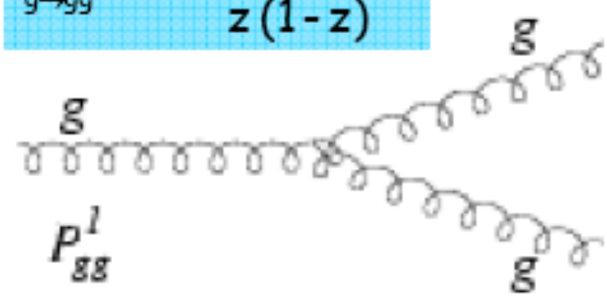
$$P_{q\to gq} = \frac{4}{3}\frac{1+(1-z)^2}{z}$$



$$P_{g\to q\bar{q}} = \frac{n_f^2}{2}\left(z^2+(1-z)^2\right)$$



$$P_{g\to gg} = 3\frac{(1-z(1-z))^2}{z(1-z)}$$



We'll also encounter the A-P splitting functions later, when we discuss parton showering and Sudakov form factors

Here the emitted gluon can be soft or hard

Kinematics

- Double differential cross section for production of a Drell-Yan pair of mass M and rapidity y is given by

$$\frac{d\sigma}{dM^2 dy} = \frac{\hat{\sigma}_0}{Ns}\left[\sum_k Q_k^2 \left(q_k(x_1, M^2)\bar{q}_k(x_2, M^2) + [1 \leftrightarrow 2]\right)\right]$$

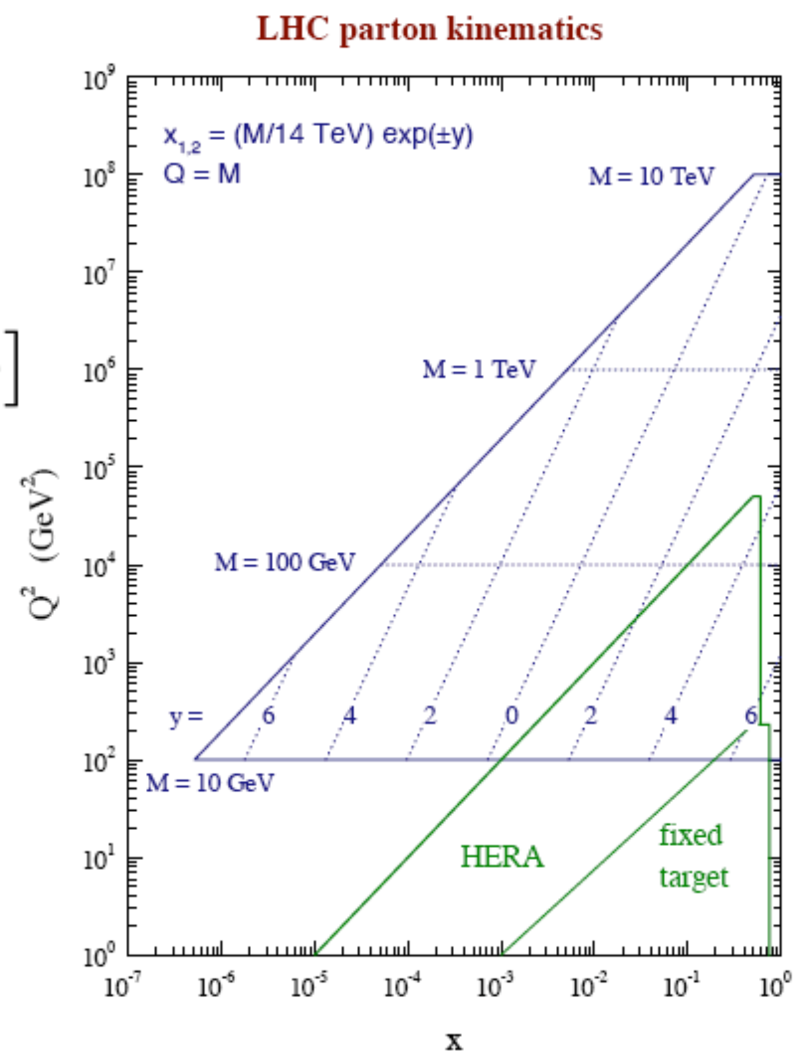
  - where

$$\hat{\sigma}_o = \frac{4\pi\alpha^2}{3M^2}$$

  - and

$$x_1 = \frac{M}{\sqrt{s}}\, e^{y}\,, \qquad x_2 = \frac{M}{\sqrt{s}}\, e^{-y}.$$

- Thus, different values of M and y probe different values of x and Q^2



W/Z production

- Cross sections for on-shell W' Z production (in narrow width limit) given by

$$\hat{\sigma}^{q\bar{q}'\to W} = \frac{\pi}{3}\sqrt{2}G_F M_W^2 |V_{qq'}|^2 \delta(\hat{s} - M_W^2),$$
$$\hat{\sigma}^{q\bar{q}\to Z} = \frac{\pi}{3}\sqrt{2}G_F M_Z^2 (v_q^2 + a_q^2)\delta(\hat{s} - M_Z^2),$$

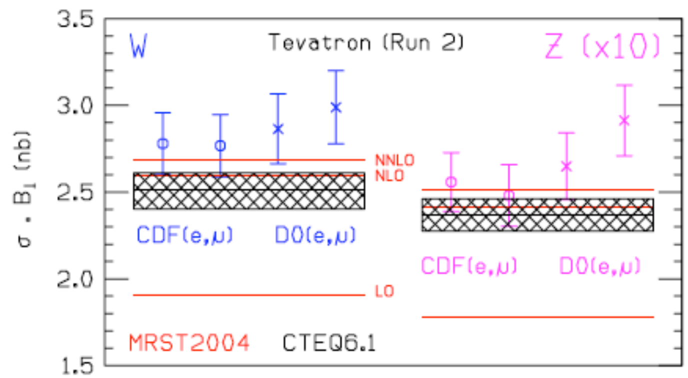


Figure 4. Predictions for the W and Z total cross sections at the Tevatron and LHC, using MRST2004 [10] and CTEQ6.1 pdfs [11], compared with recent data from CDF and D0. The MRST predictions are shown at LO, NLO and NNLO. The CTEQ6.1 NLO predictions and the accompanying pdf error bands are also shown.

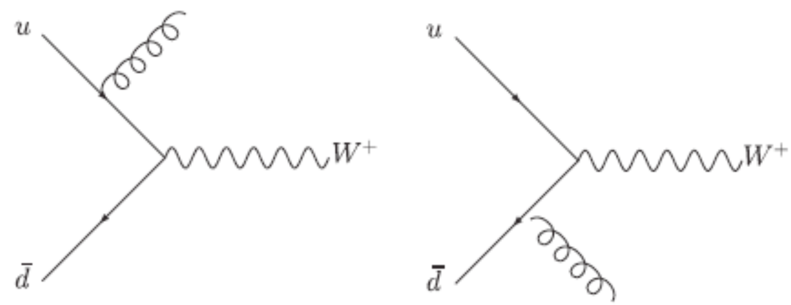
- Where $V_{qq'}$ is appropriate CKM matrix element and v_q and a_q are the vector and axial coupling of the Z to quarks
- Note that at LO, there is no α_s dependence; EW vertex only
- NLO contribution to the cross section is proportional to α_s; NNLO to α_s^2…

LO->NLO is a large correction at the Tevatron
NLO->NNLO is a fairly small (+) correction

W/Z cross sections have small experimental systematic errors with theory errors (pdf's/higher orders) also under reasonable(?) control

W/Z p_T distributions

- Most W/Z produced at low p_T, but can be produced at non-zero p_T due to diagrams such as shown on the right; note the presence of the QCD vertex, where the gluon couples (so one order higher)



$$\sum |\mathcal{M}^{q\bar{q}' \to Wg}|^2 = \pi \alpha_S \sqrt{2} G_F M_W^2 |V_{qq'}|^2 \frac{8}{9} \frac{\hat{t}^2 + \hat{u}^2 + 2M_W^2 \hat{s}}{\hat{t}\hat{u}},$$

$$\sum |\mathcal{M}^{gq \to Wq'}|^2 = \pi \alpha_S \sqrt{2} G_F M_W^2 |V_{qq'}|^2 \frac{1}{3} \frac{\hat{s}^2 + \hat{u}^2 + 2\hat{t} M_W^2}{-\hat{s}\hat{u}},$$

- Sum is over colors and spins in initial state, averaged over same in final state
- Transverse momentum distribution is obtained by convoluting these matrix elements with pdf's in usual way

Note that 2->2 matrix elements are singular when final state partons are soft or collinear with initial state partons (soft and collinear->double logarithms)

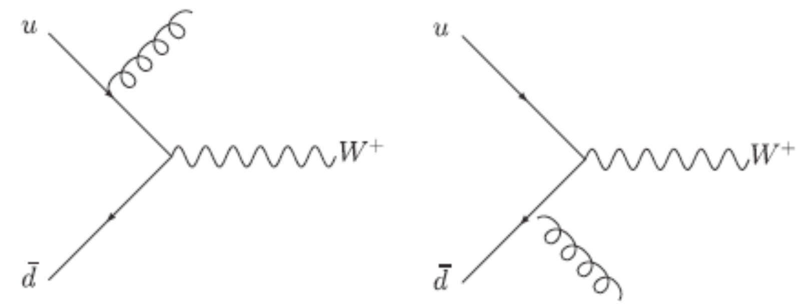
Related to poles at $\hat{t}=0$ and $\hat{u}=0$

But singularities from real and virtual emissions cancel when all contributions are included, so NLO is finite

W/Z p_T distributions

- Back to the 2->2 subprocess

$$|\mathcal{M}^{u\bar{d}\to W+g}|^2 \sim \left(\frac{\hat{t}^2 + \hat{u}^2 + 2Q^2\,\hat{s}}{\hat{t}\,\hat{u}}\right)$$



  - where Q^2 is the virtuality of the W boson

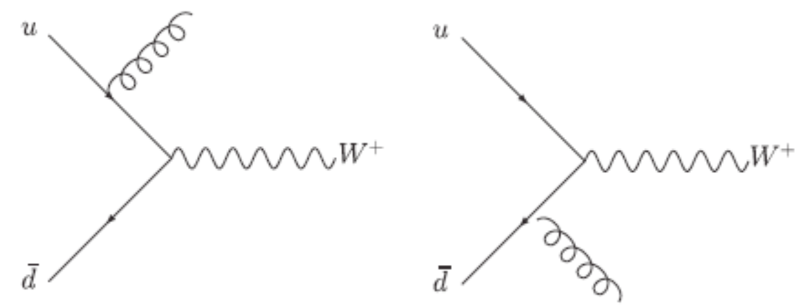
- Convolute with pdf's

$$\sigma = \int \mathrm{d}x_1 \mathrm{d}x_2\, f_u(x_1, Q^2)\, f_{\bar{d}}(x_2, Q^2)\, \frac{|\mathcal{M}|^2}{32\pi^2 \hat{s}}\, \frac{\mathrm{d}^3 p_W}{E_W}\, \frac{\mathrm{d}^3 p_g}{E_g}\, \delta(p_u + p_{\bar{\mathrm{d}}} - p_g - p_W)$$

W/Z p_T distributions

- Transform into differential cross section

$$\frac{d\sigma}{dQ^2 dy\, dp_T^2} \sim \frac{1}{s}\int dy_g\, f_u(x_1, Q^2) f_{\bar{d}}(x_2, Q^2) \frac{|\mathcal{M}|^2}{\hat{s}}$$



  - where we have one integral left over, the gluon rapidity
- Note that $p_T^2 = \hat{t}\hat{u}/\hat{s}^2$
  - thus, leading divergence can be written as $1/p_T^2$ (Brems)
- In this limit, behavior of cross section becomes

$$\frac{d\sigma}{dQ^2 dy\, dp_T^2} \sim \frac{2}{s}\frac{1}{p_T^2}\int dy_g\, f_u(x_1, Q^2) f_{\bar{d}}(x_2, Q^2) + (\text{sub-leading in } p_T^2)$$

- As p_T of W becomes small, limits of y_g integration are given by +/- $\log(s^{1/2}/p_T)$
- The result then is

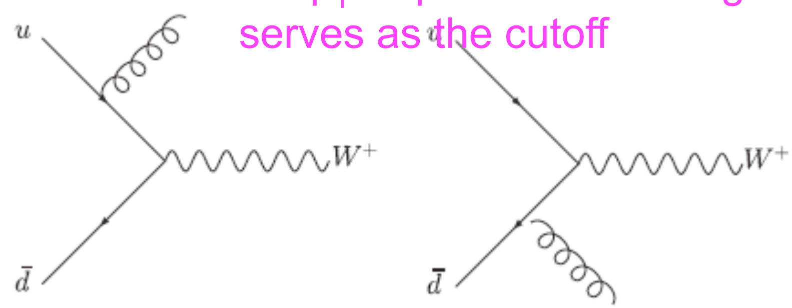
$$\frac{d\sigma}{dQ^2 dy\, dp_T^2} \sim \frac{\log(s/p_T^2)}{p_T^2}$$

…diverges unless we apply a p_T^{min} cut; so we end up with a distribution that depends not only on α_s but on α_s times a logarithm: universal theme

Rapidity distributions

- Now look at rapidity distributions for jet for two different choices of p_T^{min}
- Top diagrams imply that gluon is radiated off initial state parton at an early time (ISR)
- With collinear pole, this would imply that these gluons would be emitted primarily at forward rapidities
- But the distributions look central
- The reason is that we are binning in p_T and not in energy, and the most effective place to convert from E to p_T is at central rapidities
- Suppose I re-draw the Feynman diagrams as shown to the right
  - is there a difference from what is shown at the top of the page?

the p_T requirement of the gluon serves as the cutoff



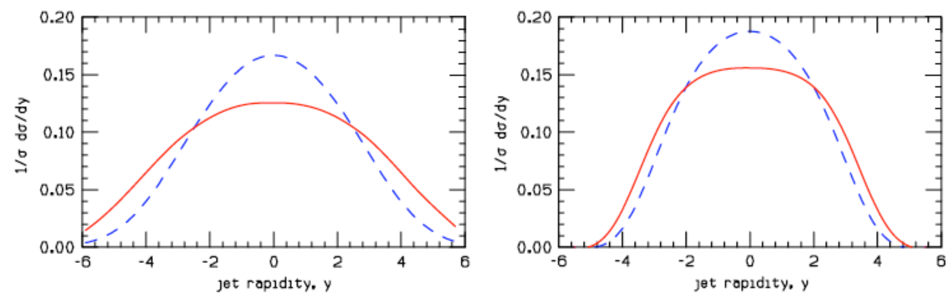
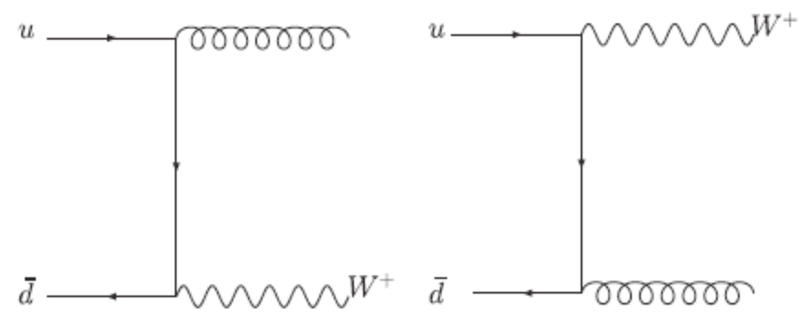
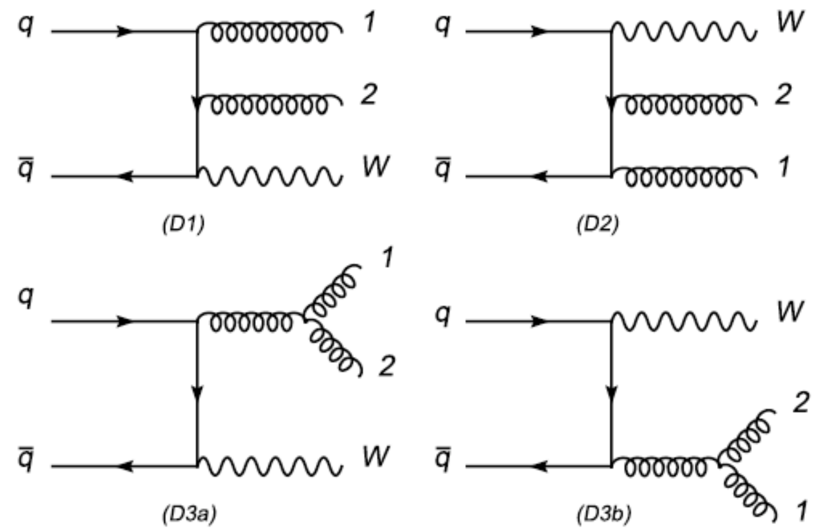


Figure 9. The rapidity distribution of the final-state parton found in a lowest-order calculation of the $W+1$ jet cross section at the LHC. The parton is required to have a p_T larger than 2 GeV (left) or 50 GeV(right). Contributions from $q\bar{q}$ annihilation (solid red line) and the qg process (dashed blue line) are shown separately.



Now on to W + 2 jets

- For sake of simplicity, consider Wgg
- Let p_1 be soft
- Then can write



$$\mathcal{M}^{q\bar{q}\to Wgg} = t^A t^B (D_2 + D_3) + t^B t^A (D_1 - D_3).$$

- where t^A and t^B are color labels of p_1 and p_2

so the kinematic structures obtained from the Feynman diagrams are collected in the function D_1,D_2 and D_3, which are called color-ordered amplitudes

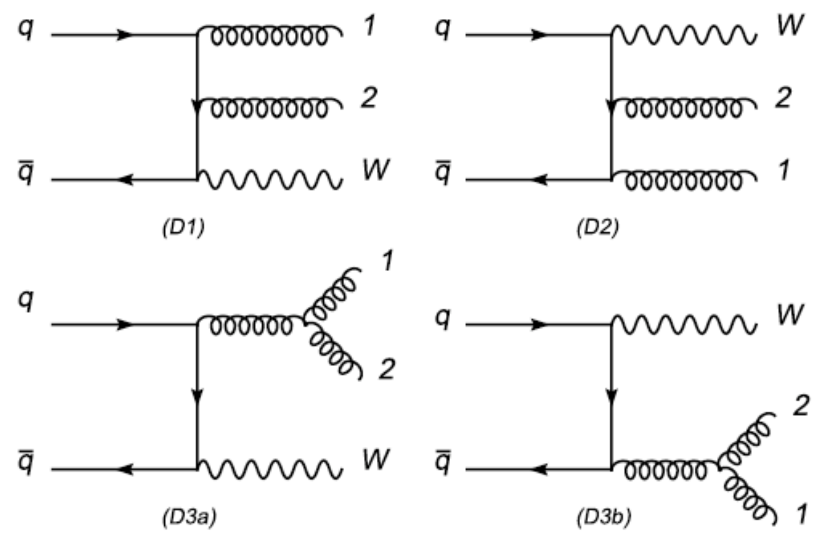
- Square the matrix amplitude to get

using $tr(t^A t^B t^B t^A)=NC_F^2$ and $tr(t^A t^B t^A t^B)=-C_F/2$

$$|\mathcal{M}^{q\bar{q}\to Wgg}|^2 = NC_F^2\left[|D_2 + D_3|^2 + |D_1 - D_3|^2\right] - C_F \,\mathrm{Re}\left[(D_2 + D_3)(D_1 - D_3)^\star\right]$$
$$= \frac{C_F N^2}{2}\left[|D_2 + D_3|^2 + |D_1 - D_3|^2 - \frac{1}{N^2}|D_1 + D_2|^2\right].$$

W + 2 jets

- Since p_1 is soft, can write D's (color-ordered amplitudes) as product of an eikonal term and the matrix elements containing only 1 gluon



$$D_2 + D_3 \longrightarrow \epsilon_\mu \left(\frac{q^\mu}{p_1.q} - \frac{p_2^\mu}{p_1.p_2} \right) \mathcal{M}_{q\bar{q}\to Wg},$$

$$D_1 - D_3 \longrightarrow \epsilon_\mu \left(\frac{p_2^\mu}{p_1.p_2} - \frac{\bar{q}^\mu}{p_1.\bar{q}} \right) \mathcal{M}_{q\bar{q}\to Wg},$$

  - where ε_μ is the polarization vector for gluon p_1

- Summing over gluon polarizations, we get

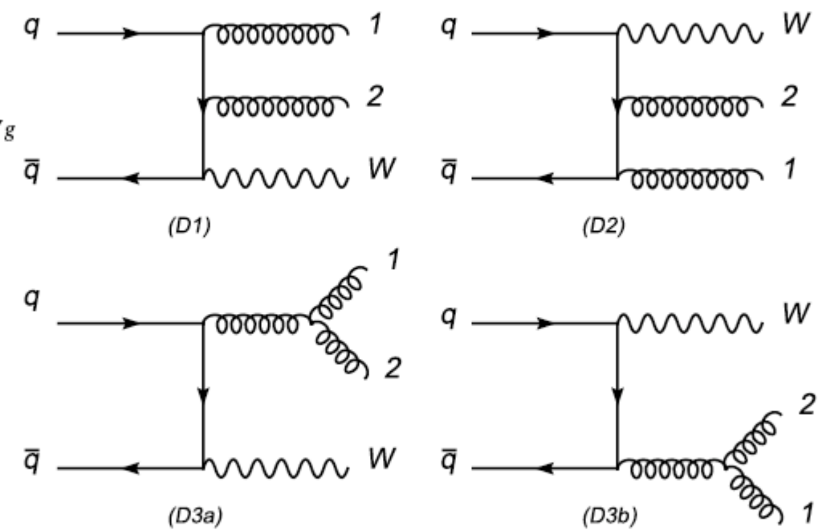
$$|\mathcal{M}^{q\bar{q}\to Wgg}|^2 \xrightarrow{\text{soft}} \frac{C_F N^2}{2} \left[[q\ p_2] + [p_2\ \bar{q}] - \frac{1}{N^2}[q\ \bar{q}] \right] \mathcal{M}^{q\bar{q}\to Wg}$$

  - where

$$\frac{a.b}{p_1.a\ p_1.b} \equiv [a\ b].$$

Color flow

$$|\mathcal{M}^{q\bar{q}\to Wgg}|^2 \xrightarrow{\text{soft}} \frac{C_F N^2}{2}\left[[q\ p_2]+[p_2\ \bar{q}]-\frac{1}{N^2}[q\ \bar{q}]\right]\mathcal{M}^{q\bar{q}\to Wg}$$



- The leading term (in number of colors) contains singularities along two lines of color flow-one connecting gluon p_2 to the quark and the other connecting it to the anti-quark
  - sub-leading term has singularities along the line connecting the quark and anti-quark
- It is these lines of color that indicate preferred direction for emission of additional gluons
  - needed by programs like Pythia/Herwig for example
  - sub-leading terms don't correspond to any unique color flow

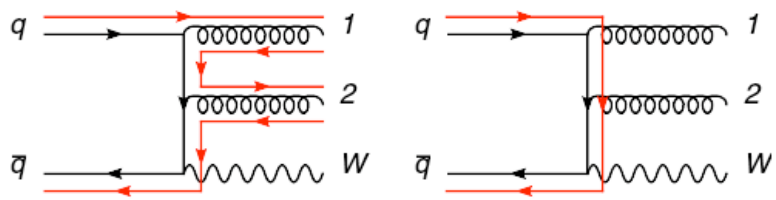


Figure 12. Two examples of colour flow in a $W + 2$ jet event, shown in red. In the left-hand diagram, a leading colour flow is shown. The right-hand diagram depicts the sub-leading colour flow resulting from interference.

Eikonal factors

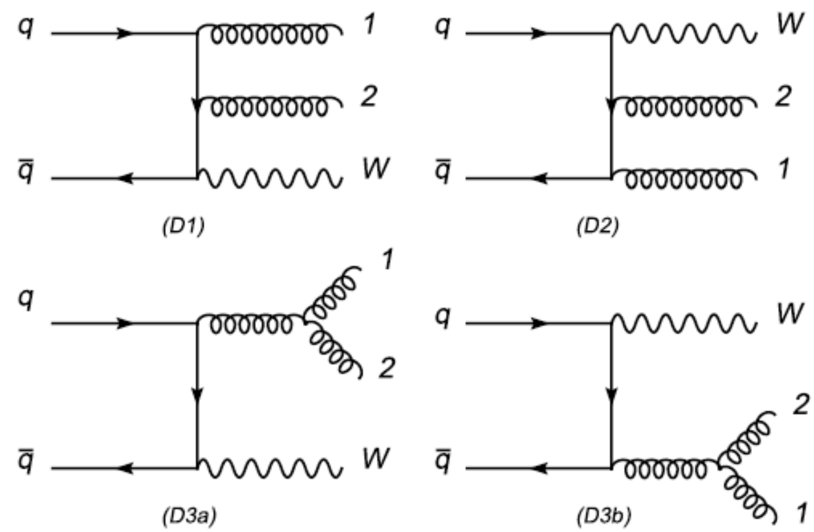
- Re-write

$$\frac{a.b}{p_1.a\ p_1.b} \equiv [a\ b],$$

- As

$$[a\ b]\,\mathrm{d}PS_{\mathrm{gluon}} = \frac{1}{E^2}\frac{1}{1-\cos\theta_a}E\mathrm{d}E\ \mathrm{d}\cos\theta_a$$

- It is clear that the cross section diverges either as $\cos\theta_a$->1 (gluon is collinear to parton a) or as E->0
  - similar for parton b
- Each divergence is logarithmic and regulating the divergence by providing a fixed cutoff (in angle or energy) will produce a single logarithm from collinear configurations and another from soft ones
  - so again the double logs



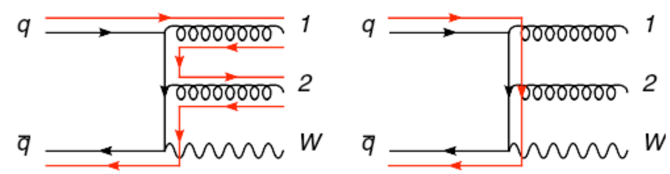


Figure 12. Two examples of colour flow in a $W+2$ jet event, shown in red. In the left-hand diagram, a leading colour flow is shown. The right-hand diagram depicts the sub-leading colour flow resulting from interference.

Logarithms

- You can keep applying this argument at higher orders of perturbation theory
- Each gluon that is added yields an additional power of α_s, and via the eikonal factorization outlined, can produce an additional two logarithms
- So can write the W + jets cross section as

$$d\sigma = \sigma_0(W + 1\text{ jet})\left[1 + \alpha_S(c_{12}L^2 + c_{11}L + c_{10}) + \alpha_S{}^2(c_{24}L^4 + c_{23}L^3 + c_{22}L^2 + c_{21}L + c_{20}) + \cdots\right]$$

  - where L represents the logarithm controlling the divergence, either soft or collinear
  - note that α_s and L appear together as $\alpha_s L$

- Size of L depends on criteria used to define the jets (min E_T, cone size)
- Coefficients c_{ij} depend on color factors
- Thus, addition of each gluon results in additional factor of α_s times logarithms
- In many (typically exclusive) cases, the logs can be large, leading to an enhanced probability for gluon emission to occur
- For most inclusive cases, logs are small and α_s counting may be valid estimator for production of additional jets

for W + jets

Re-shuffling

$$d\sigma = \sigma_0(W + 1\text{ jet})\left[1 + \alpha_S(c_{12}L^2 + c_{11}L + c_{10}) + \alpha_S{}^2(c_{24}L^4 + c_{23}L^3 + c_{22}L^2 + c_{21}L + c_{20}) + \cdots\right]$$

each gluon added has an additional factor of α_s and two additional logs (soft and collinear)

c_{ij} depend on color factors

- re-write the term in brackets as

$$\left[\cdots\right] = 1 + \alpha_S L^2 c_{12} + (\alpha_S L^2)^2 c_{24} + \alpha_S L c_{11}(1 + \alpha_S L^2 c_{23}/c_{11} + \cdots) + \cdots$$
$$= \exp\left[c_{12}\alpha_S L^2 + c_{11}\alpha_S L\right],$$

- Where the infinite series has been resummed into an exponential form
  - first term in expansion is called leading logarithm term, 2nd next-to-leading logarithm, etc
- Now can write out each contribution as a contribution in powers of α_s and logarithms

$$\sigma_W = \sigma_{W+0j} + \sigma_{W+1j} + \sigma_{W+2j} + \sigma_{W+3j} + \cdots$$

$$\sigma_{W+0j} = a_0 + \alpha_S(a_{12}L^2 + a_{11}L + a_{10}) + \alpha_S{}^2(a_{24}L^4 + a_{23}L^3 + a_{22}L^2 + a_{21}L + a_{20}) + \cdots$$

$$\sigma_{W+1j} = \alpha_S(b_{12}L^2 + b_{11}L + b_{10}) + \alpha_S{}^2(b_{24}L^4 + b_{23}L^3 + b_{22}L^2 + b_{21}L + b_{20}) + \cdots$$

$$\sigma_{W+2j} = \cdots.$$

as jet definitions change, size of the logs shuffle the contributions from one jet cross section to another, keeping the sum over all contributions the same; for example, as R decreases, L increases, contributions shift towards higher jet multiplicities

Re-shuffling

- Configuration shown to the right can be reconstructed as an event containing up to 2 jets (0,1,2), depending on jet definition and momenta of the partons.
- For a large value of R_{cone}, this is one jet; for a smaller value, it may be two jets
- The matrix elements for this process contain terms proportional to $\alpha_s \log(p_{T3}/p_{T4})$ and as $\log(1/\Delta R_{34})$, so min values for transverse momentum and separation must be imposed

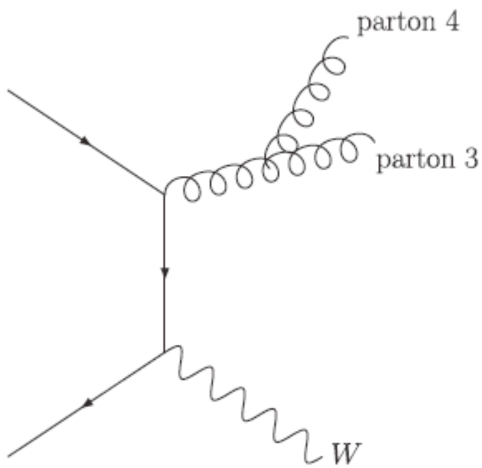


Figure 13. A final-state configuration containing a W and 2 partons. After the jet definition has been applied, either zero, one or two jets may be reconstructed.

$$\sigma_{W+0j} = a_0 + \alpha_S(a_{12}L^2 + a_{11}L + a_{10}) + \alpha_S{}^2(a_{24}L^4 + a_{23}L^3 + a_{22}L^2 + a_{21}L + a_{20}) + \cdots$$

$$\sigma_{W+1j} = \alpha_S(b_{12}L^2 + b_{11}L + b_{10}) + \alpha_S{}^2(b_{24}L^4 + b_{23}L^3 + b_{22}L^2 + b_{21}L + b_{20}) + \cdots$$

$$\sigma_{W+2j} = \cdots.$$

At given fixed order in perturbation theory, results are independent of jet parameters and we can say

$$\sigma_W^{LO} = a_0,$$

$$\sigma_W^{NLO} = \alpha_S\,(a_{10} + b_{10})$$

NLO calculations

- NLO calculation requires consideration of all diagrams that have an extra factor of α_s
  - real radiation, as we have just discussed
  - virtual diagrams (with loops)
- For virtual diagram, have to integrate over loop momentum
  - but result contains IR singularities (soft and collinear), just as found for tree-level diagrams

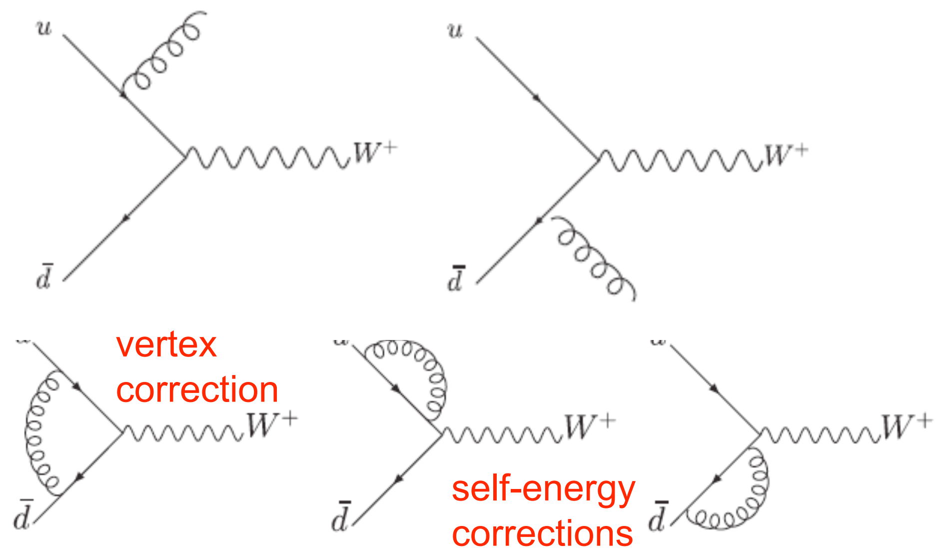


Figure 14. Virtual diagrams included in the next-to-leading order corrections to the Drell–Yan production of a W at hadron colliders.

$O(\alpha_s)$ virtual corrections in NLO cross section arise from interference between tree level and one-loop virtual amplitudes

Advantages of NLO

- Less sensitivity to unphysical input scales, i.e. renormalization and factorization scales
- First level of prediction where normalization can be taken seriously
- More physics
  - parton merging gives structure in jets
  - initial state radiation
  - more species of incoming partons
- Suppose I have a cross section σ calculated to NLO ($O(\alpha_s^n)$)
- Any remaining scale dependence is of one order higher ($O(\alpha_s^{n+1})$)
  - in fact, we know the scale dependent part of the $O(\alpha_s^{n+1})$ cross section before we perform the complete calculation, since the scale-dependent terms are explicit at the previous order

$$
\begin{aligned}
\frac{d\sigma}{dE_T} &= \alpha_s(\mu_R)^2 A \\
&+ \alpha_s(\mu_R)^3 \left(B + 2b_0 L A\right) \\
&+ \alpha_s(\mu_R)^4 \left(C + 3b_0 L B + (3b_0^2 L^2 + 2b_1 L) A\right)
\end{aligned}
$$

Inclusive jet prod at NNLO

with $L = \log(\mu_R/E_T)$ and b_i the known beta function coefficients.

we know A and B, not C

LO has monotonic scale dependence

non-monotonic at NLO

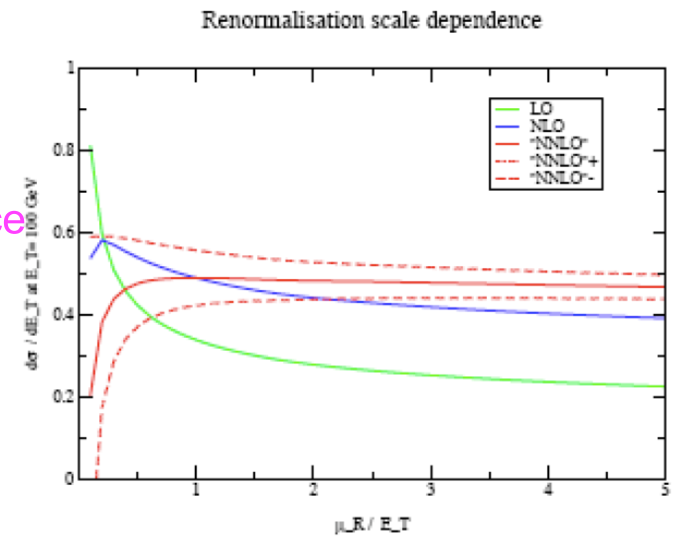
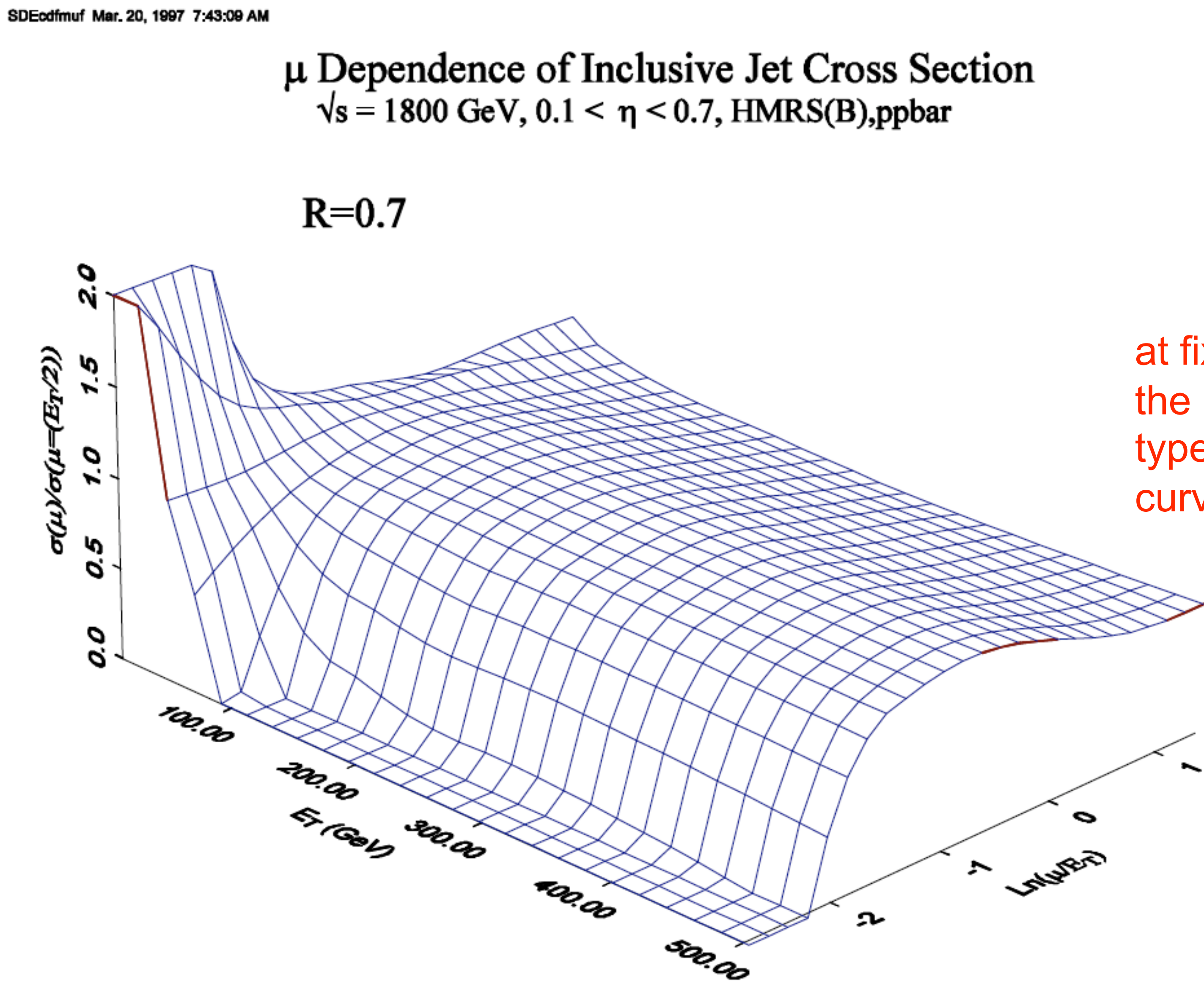


Figure 11: Single jet inclusive distribution at $E_T = 100$ GeV and $0.1 < |\eta| < 0.7$ at $\sqrt{s} = 1800$

The NNLO coefficient C is unknown. The curves show the guesses $C = 0$ (solid) and $C = \pm B^2/A$ (dashed).

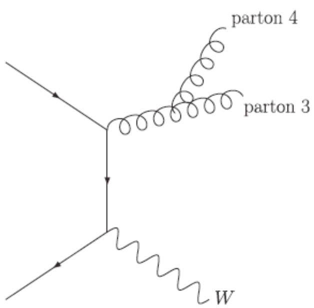
Predictions tend to be more reliable at higher E_T



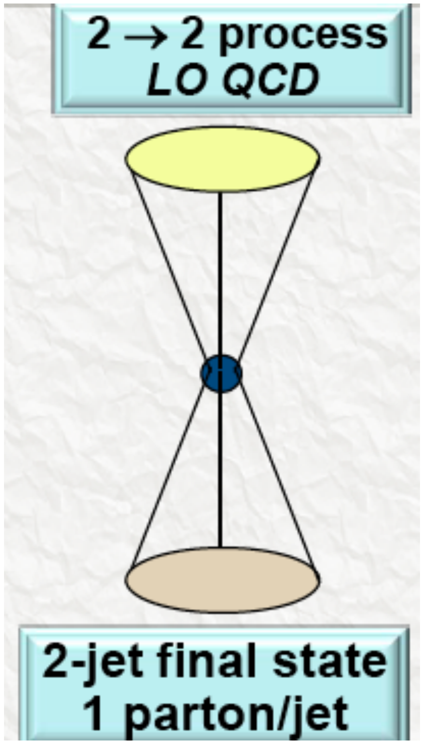
at fixed E_T, note the parabolic type shape for the curve

Jet algorithms at LO

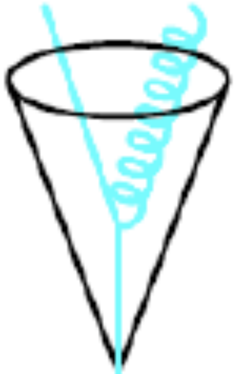
- At (fixed) LO, 1 parton = 1 jet
  - why not more than 1? I have to put a ΔR cut on the separation between two partons; otherwise, there's a collinear divergence. LO parton shower programs effectively put in such a cutoff
  - remember



$$\log\left(\frac{1}{\Delta R_{34}}\right)$$



- But at NLO, I have to deal with more than 1 parton in a jet, and so now I have to talk about how to cluster those partons
  - i.e. jet algorithms



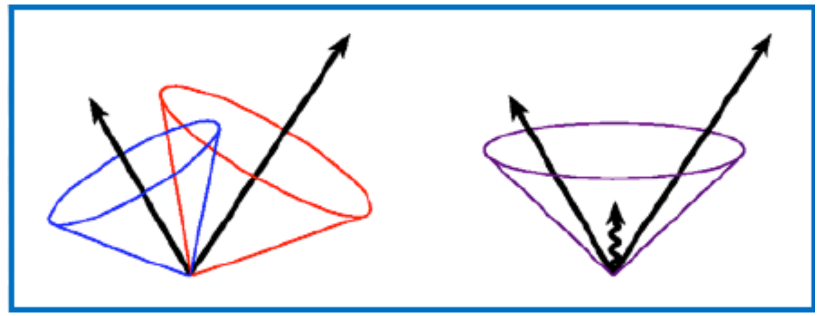
Jet algorithms at NLO

- At (fixed) LO, 1 parton = 1 jet
  - why not more than 1? I have to put a ΔR cut on the separation between two partons; otherwise, there's a collinear divergence. LO parton shower programs effectively put in such a cutoff
- At NLO, there can be two partons in a jet, life becomes more interesting and we have to start talking about jet algorithms to define jets
  - the addition of the real and virtual terms at NLO cancels the divergence.
- A jet algorithm is based on some measure of localization of the expected collinear spray of particles
- Start with an inclusive list of particles/partons/calorimeter towers/topoclusters
- End with lists of same for each jet
- …and a list of particles… not in any jet; for example, remnants of the initial hadrons
- Two broad classes of jet algorithms
  - cluster according to proximity in space: cone algorithms
  - cluster according to proximity in momenta: k_T algorithms

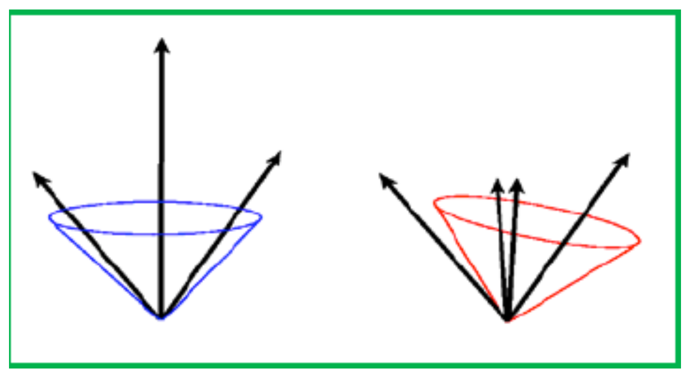
What do I want out of a jet algorithm?

- It should be fully specified, including defining in detail any pre-clustering, merging and splitting issues
- It should be simple to implement in an experimental analysis, and should be independent of the structure of the detector
- It should be boost-invariant
- It should be simple to implement in a theoretical calculation
  - it should be defined at any order in perturbation theory
  - it should yield a finite cross section at any order in perturbation theory
  - it should yield a cross section that is relatively insensitive to hadronization effects

- It should be IR safe, i.e. adding a soft gluon should not change the results of the jet clustering

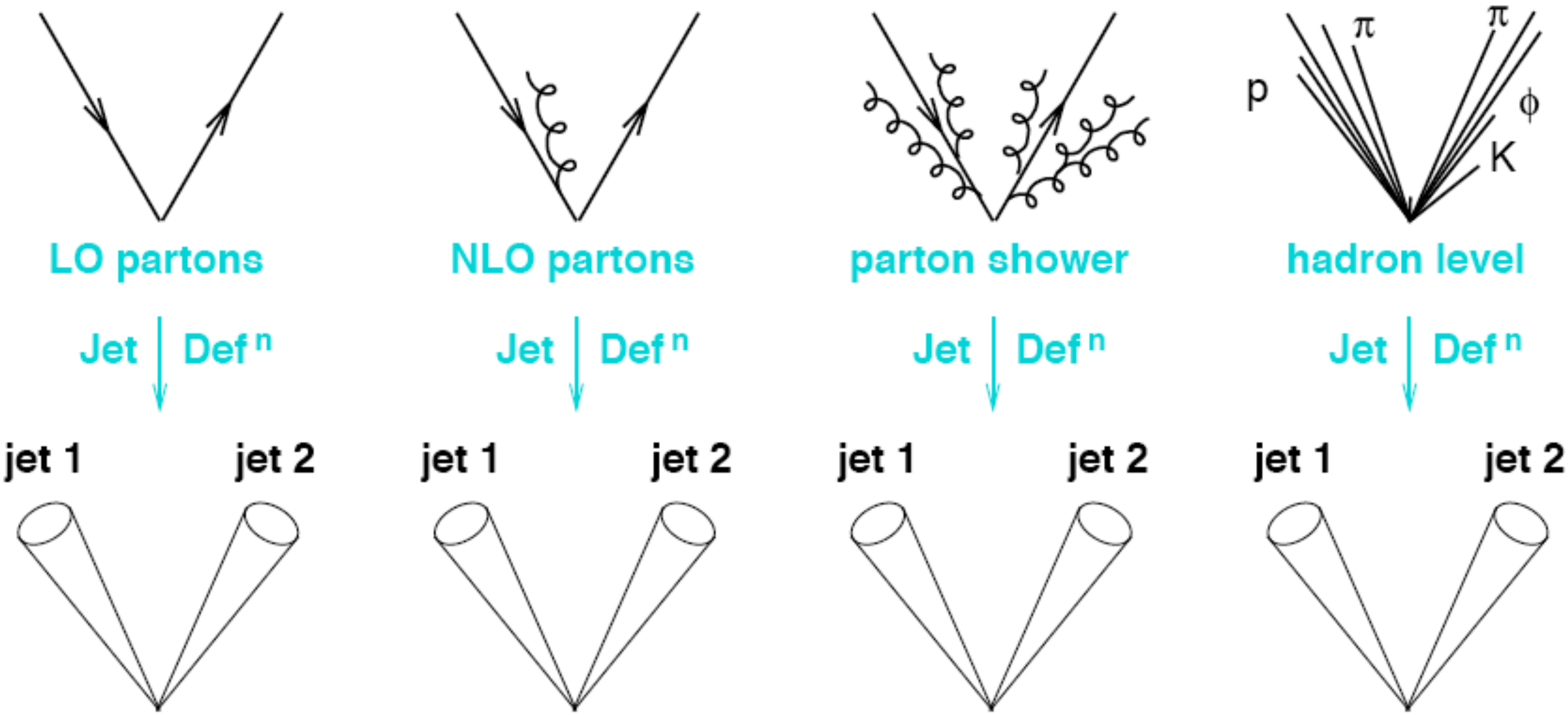


- It should be collinear safe, i.e. splitting one parton into two collinear partons should not change the results of the jet clustering



Jet algorithms

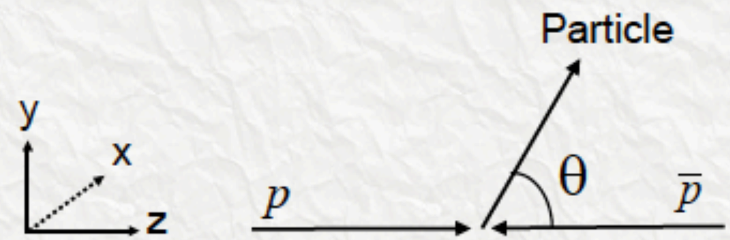
- The algorithm should behave in a similar manner (as much as possible) at the parton, particle and detector levels



Projection to jets should be resilient to QCD effects

Some kinematic definitions

Rapidity (y) and Pseudo-rapidity (η)



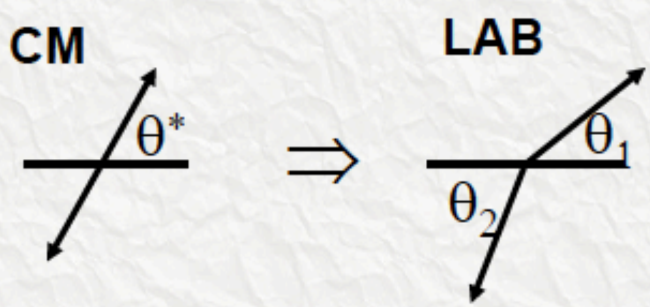
$$y \equiv \frac{1}{2}\ln\frac{E+p_z}{E-p_z} = \frac{1}{2}\ln\frac{1+\beta\cos\theta}{1-\beta\cos\theta}$$

$$\beta\cos\theta = \tanh y \quad \text{where } \beta = p/E$$

In the limit $\beta \to 1$ (or $m << p_T$) then

$$\eta \equiv y\big|_{m=0} = \frac{1}{2}\ln\frac{1+\cos\theta}{1-\cos\theta} = -\ln\tan\frac{\theta}{2}$$

LAB System ≠ parton-parton CM system



Δη and p_T are invariant under longitudinal boosts

Some kinematic definitions

To satisfy listed requirements for jet algorithms, use p_T, y and ϕ to characterize jets

Transverse Energy/Momentum

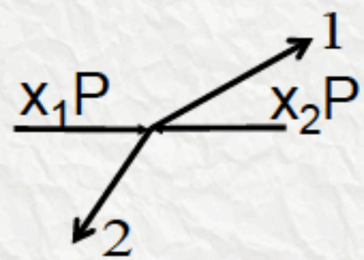
$$E_T^2 \equiv p_x^2 + p_y^2 + m^2 = p_T^2 + m^2 = E^2 - p_z^2$$

$$p_z = E \tanh y$$
$$E = E_T \cosh y$$
$$p_z = E_T \sinh y$$

$$p_T \equiv p \sin\theta \xrightarrow[m \to 0]{} E_T$$

Invariant Mass

$$M_{12}^2 \equiv (p_1^\mu + p_2^\mu)(p_{1\mu} + p_{2\mu})$$
$$= m_1^2 + m_2^2 + 2(E_1 E_2 - \boldsymbol{p}_1 \cdot \boldsymbol{p}_2)$$
$$\xrightarrow[m_1, m_2 \to 0]{} 2E_{T1}E_{T2}(\cosh\Delta\eta - \cos\Delta\phi)$$



Partonic Momentum Fractions

$$x_1 = \left(e^{\eta_1} + e^{\eta_2}\right) E_T / \sqrt{s}$$
$$x_2 = \left(e^{-\eta_1} + e^{-\eta_2}\right) E_T / \sqrt{s}$$

$$\text{Parton CM (energy)}^2 \to \hat{s} = x_a x_b s$$

$$x_T \equiv 2E_T / \sqrt{s} = x_{1,2}(\eta_{1,2} = 0)$$

$$0 < x_1, x_2 < 1$$
$$x_T^2 < x_1 x_2 < 1$$