PY541 Practice Problems on Quantum Gases

1) Start with the expression derived in class for the grand potential of a Bose or Fermi gas expressed as an integral over energy, obtained by a change of variables from the momentum. By integrating by parts, derive the exact formula for a Bose or Fermi gas:

$$-\Omega \equiv PV = (2/3)E. \tag{1}$$

2) Consider a dilute Bose or Fermi gas such that $e^{\mu\beta} << 1$. Taylor expand Ω to second order in $e^{\mu\beta} << 1$. Keeping only the first term, solve for $N(\mu, T, V)$ and from this obtain the equation of state: $P(V, T)$ and the energy $E(T, V)$. Show that these agree with the results from the Maxwell-Boltzmann approximation that we made earlier. Now keep the second order term in the expansion and derive the leading correction to the ideal gas law, $P = NkT/V$. You should find a correction to $P(V, T, N)$ which is second order in $(N/V)^2$. Comment on whether the statistical interaction corresponds to an effective attraction or repulsion for bosons or fermions.
NB- You can find this all worked out in Landau and Lifshitz pages 163-165.
3) Calculate the equation of state $P(T, V, N)$ for a Bose gas below the transition temperature. You should find a result which is independent of V. Show that this implies infinite compressibility.
NB- This one is in Pathria and Landau & Lifshitz.