## **PY541** Practice Problems

1) Determine the leading divergence of the isothermal compressibility of an ideal Bose gas as  $T \to T_c$ , the critical temperature. Show that it has the form:

$$K_T \to \frac{A}{(T - T_c)^p},$$
(1)

for some exponent p and some constant factor, A. Find the values of p and A.

2) Calculate the non-linear paramagnetic magnetization of an ideal Fermi gas of electrons (with energy  $E = p^2/2m$ ). Show that the leading correction to linear behavior at zero temperature is of the form:

$$M/V = \chi B + AB^3,\tag{2}$$

and find the constant A. (Warning: you must consider the shift of the Fermi energy due to the field.)

3) Consider an ideal gas of particles with energy-momentum relation:

$$E = Ap^4, (3)$$

for some constant A, in three dimensions.

a) If the particles were bosons, would there be a Bose condensation transition?

b) Assuming that the particles are fermions, calculate the pressure at T = 0 in terms of the density.