PY541 Problem Set 6: Due Thursday, December 12, 2002 at 2:30 PM in mailbox of Nicolas Giovambattista, the grader, or at his office SCI240

1) Write down all diagrams corresponding to the sum of all 4-clusters. Also write an explicit expression as a sum of 9-dimensional integrals for this quantity (in terms of an arbitrary function $f(r)$). Write the corresponding cluster integral, b_4.

2) Show that the first order Joule-Thomson coefficient of a gas is given by the formula:

$$\left(\frac{\partial T}{\partial P}\right)_H = \frac{N}{C_P}\left(-T\frac{d(\lambda^3 b_2)}{dT} + b_2\lambda^3\right). \tag{1}$$

3) Consider a one-dimensional gas of N particles. (i.e. the particles are confined to move along a line only. The line has some long length, L.) Assume that the particles interact with the potential:

$$\begin{aligned} u(r) &= \infty, \quad r < r_0 \\ &= 0, \quad r > r_0. \end{aligned} \tag{2}$$

a) Calculate the (ordinary) partition function and the equation of state exactly.

b) Calculate b_2 and b_3 for this potential and from these the expansion of the equation of state:

$$P \approx nkT[1 - b_2 n\lambda + (4b_2^2 - 2b_3)n^2\lambda^2 + \ldots]. \tag{3}$$

Verify that this result is consistent with the expansion of your exact result in a) in powers of the density, n.

Hint: In 1 dimension we define:

$$\begin{aligned} b_2 &\equiv \frac{1}{2\lambda}\int_{-\infty}^{\infty} dx f(x) \\ b_3 - 2b_2^2 &\equiv \frac{1}{3!\lambda^2}\int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} dx_2 f(x_1)f(x_2)f(x_1 - x_2). \end{aligned} \tag{4}$$

(These are just the three-dimensional definitions with the number of powers of λ changed to make the integrals dimensionless. I have corrected here an error in my lecture notes in the second integral.)

4) Consider a gas (in 3 dimensions) with the interaction potential:

$$\begin{aligned} u(r) &= \infty, \quad r < r_0 \\ &= -u_0, \quad r_0 < r < r_1 \\ &= 0, \quad r > r_1. \end{aligned} \tag{5}$$

Calculate exactly the cluster integral, b_2 for this gas.