Exponential Decay

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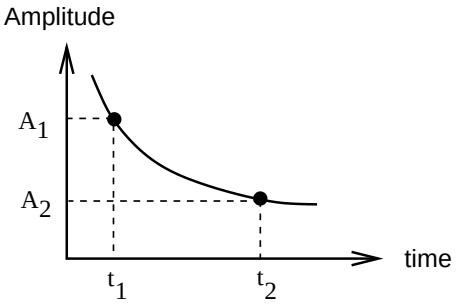
For most physical systems the amplitude of an oscillation decays exponentially. Thus the sound from an impulsively excited instrument (plucked string, drum, etc.) will also exhibit exponential decay. This note tells you how to take two points on an exponentially decaying waveform a find the characteristic decay time.

In its most simple form, this exponential decay is given by

$$A = A_0 e^{-t/\tau}$$

where τ is the "mean life" or characteristic decay time for the sound. If you make measurements of the amplitudes at two times, t_1 and t_2, you get amplitudes A_1 and A_2, as shown in the picture below.



Now

$$A_1 = A_0 e^{-t_1/\tau} \quad \text{and} \quad A_2 = A_0 e^{-t_2/\tau}$$

so we can divide and get

$$\frac{A_1}{A_2} = \frac{e^{-t_1/\tau}}{e^{-t_2/\tau}}$$

or

$$\frac{A_1}{A_2} = e^{-(t_1 - t_2)/\tau}$$

Taking the natural log of each side we get

$$\ln \frac{A_1}{A_2} = \frac{-(t_1 - t_2)}{\tau}$$

which we solve for τ and get

$$\tau = \frac{(t_2 - t_1)}{\ln \frac{A_1}{A_2}}$$