Lattice QCD and Flavor Physics

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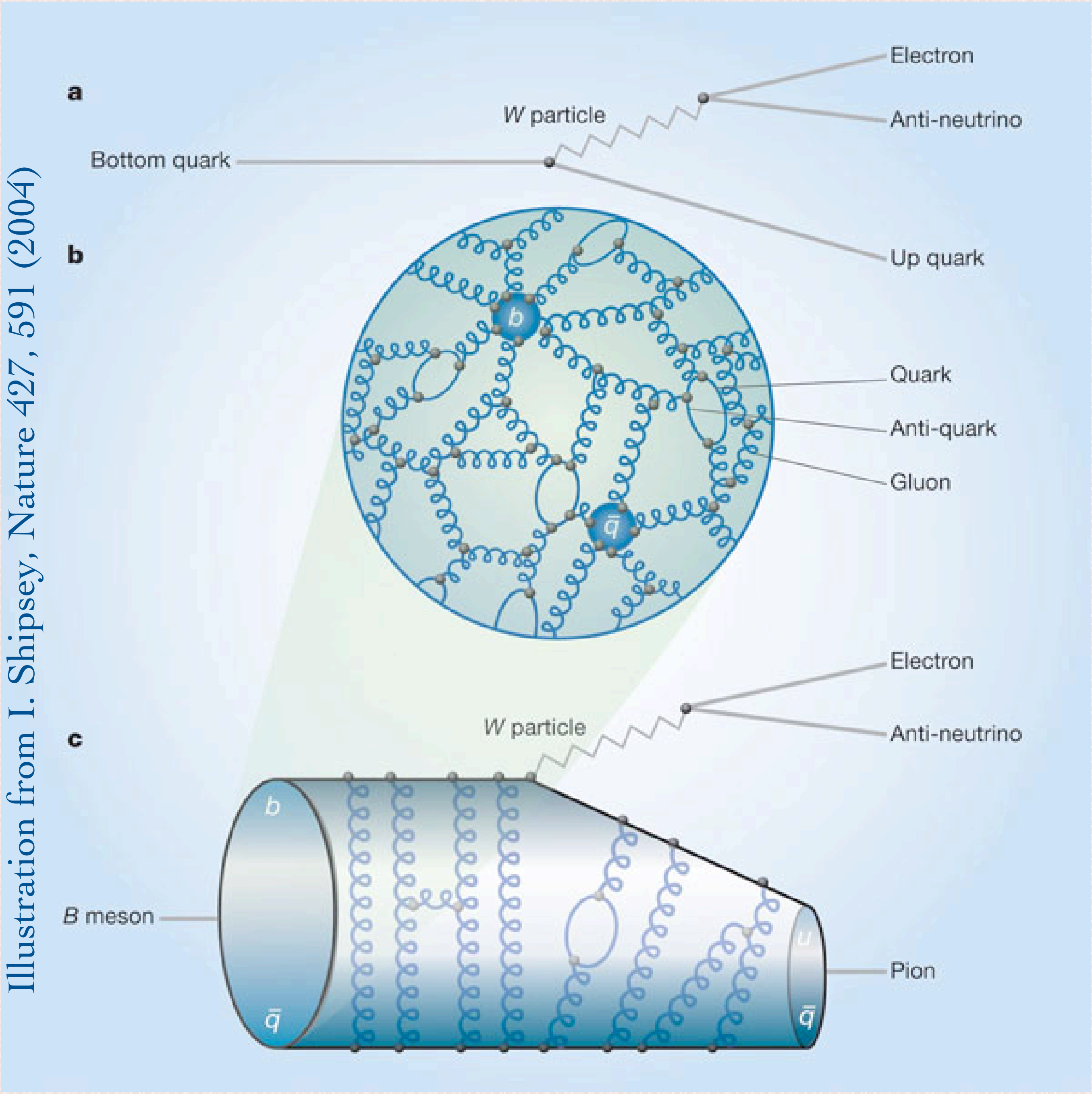
Physics in Collision 2004, Boston Univ.

Outline

- (Motivation: CKM matrix elements, Heavy-light & -onium spectra)
- A taste of lattice Monte Carlo calculations
- *Hot topic*: unquenching with “improved staggered” quarks
- Recent results

Wherefore LQCD for Flavor Physics?

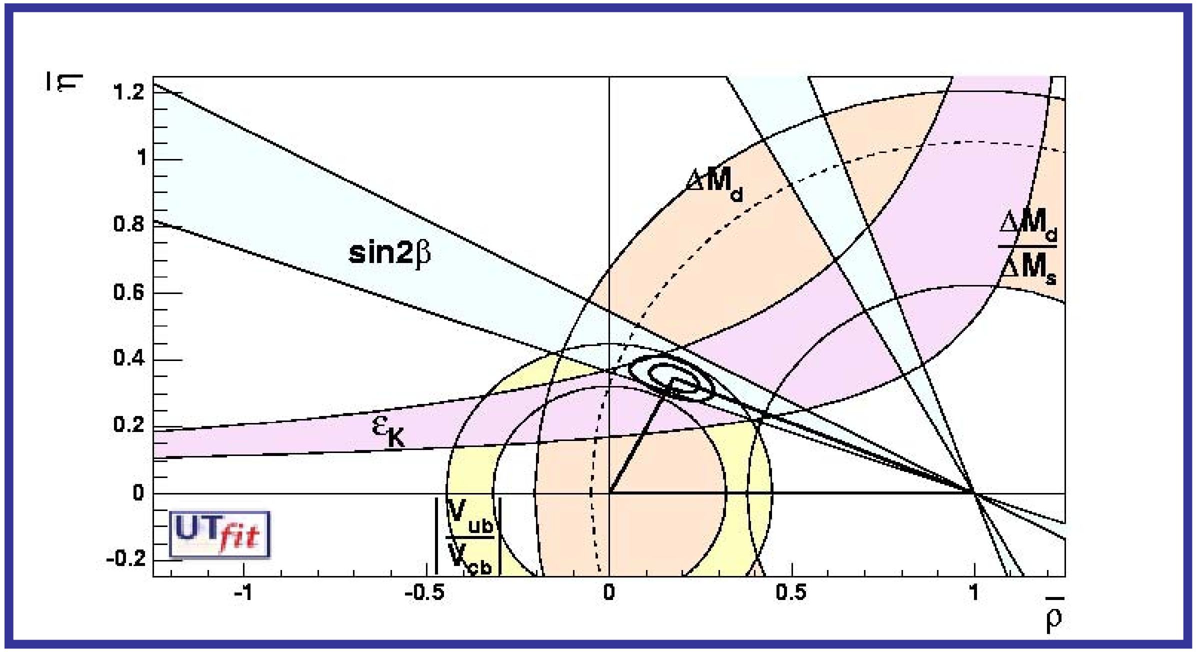


a. Weak decay of *b* to *u*

b. A *B*, not just a *b*

c. The whole decay

THE UNITARITY TRIANGLE ANALYSIS



5 CONSTRAINTS
2 PARAMETERS

Hadronic Matrix Elements from LATTICE QCD

$(b \to u)/(b \to c)$	$\bar{\rho}^2 + \bar{\eta}^2$	$f_+, F(1), \ldots$
ε_K	$\bar{\eta}[(1-\bar{\rho}) + P]$	B_K
Δm_d	$(1-\bar{\rho})^2 + \bar{\eta}^2$	$f_{B_d}^2 B_{B_d}$
$\Delta m_d / \Delta m_s$	$(1-\bar{\rho})^2 + \bar{\eta}^2$	ξ
$A(J/\psi\, K_S)$	$\sin 2\beta(\bar{\rho}, \bar{\eta})$	—

slide from V. Lubicz, Lattice 2004

Statistical FT & Quantum FT

$$Z = \int [d\psi][d\overline{\psi}][dU] \; e^{iS_M}$$ **define imaginary time coordinate** $$\rightarrow Z = \int [d\psi][d\overline{\psi}][dU] e^{-S_e}$$

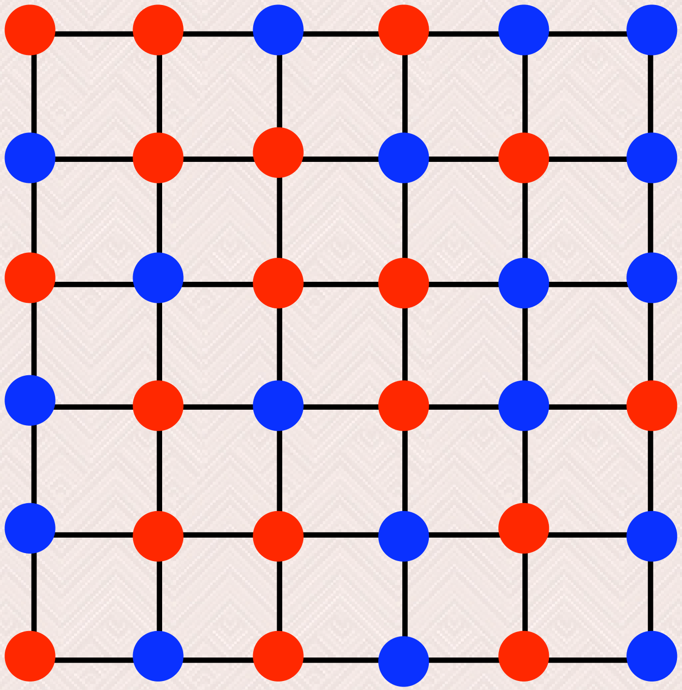
$$\langle J(y)J(x)\rangle = Z^{-1} \int [d\psi][d\overline{\psi}][dU] \; J(y)J(x) \; \exp\left(- \sum_x \overline{\psi}(\gamma^\mu D_\mu + m)\psi - \mathcal{L}_g \right)$$

Get rid of bizarre Grassman variables

$$Z = \int [dU] \; \det Q[U] \; e^{-S_g}$$

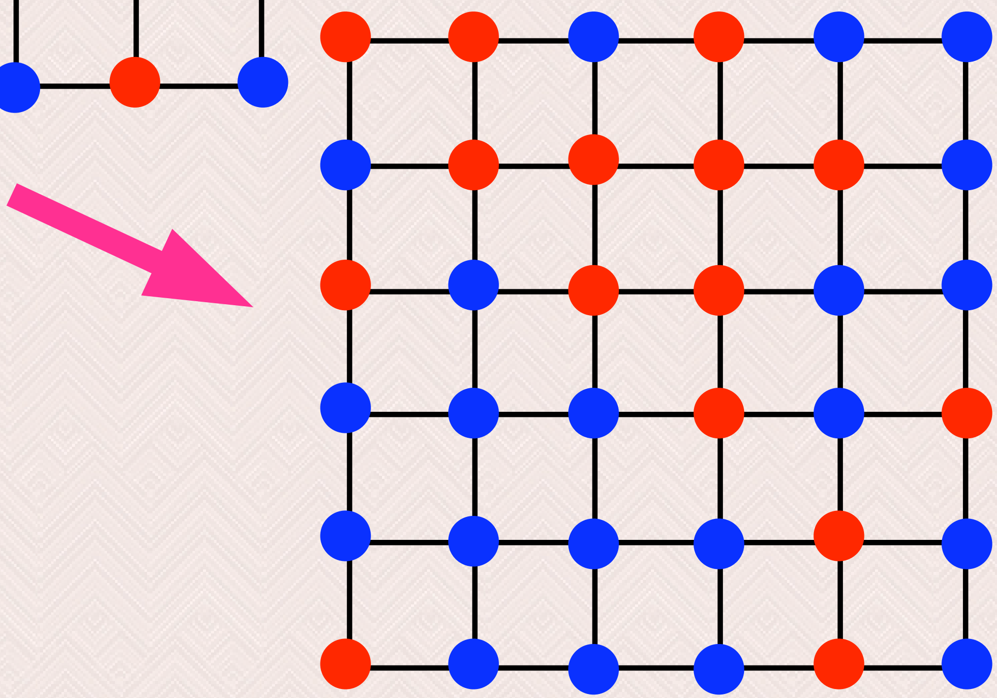
$$\langle J(y)J(x)\rangle = \int [dU] \; J(y)J(x) \; \det Q[U] \; e^{-S_g}$$

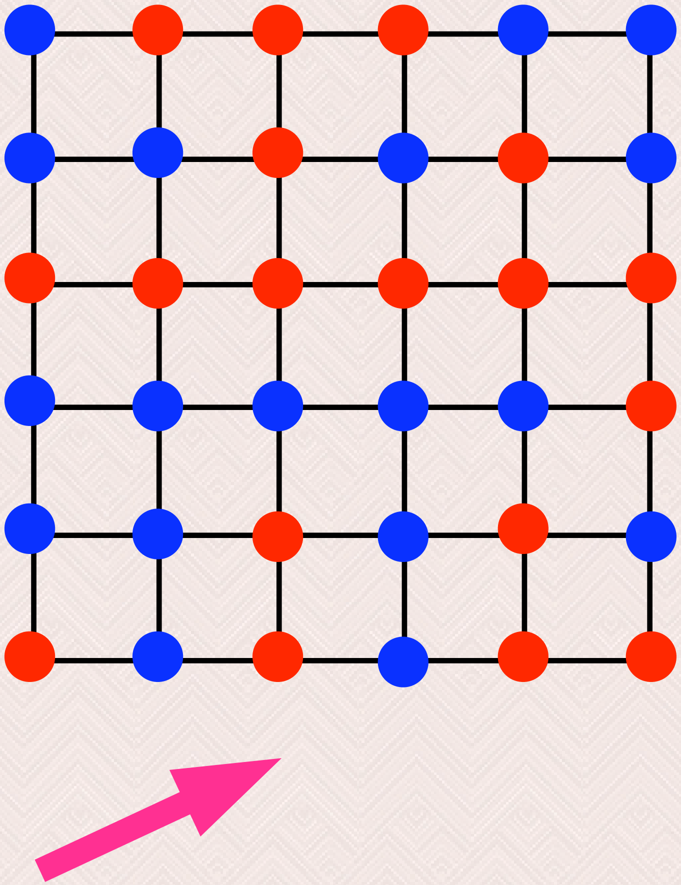
Monte Carlo - Ising Model



Important
Samples
Dominate

$$Z = \mathrm{Tr}\, e^{-\beta H}$$



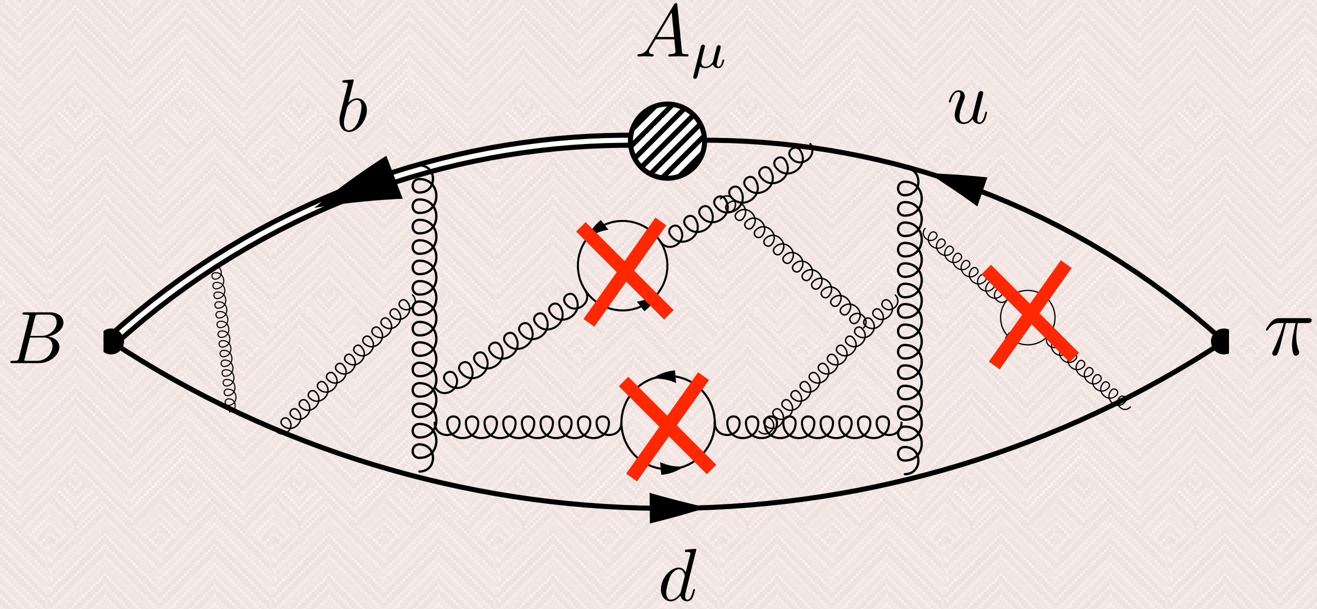


$$\langle O \rangle = Z^{-1} \mathrm{Tr}\, O e^{-\beta H}$$

$$= \frac{1}{N} \sum_n O_n$$

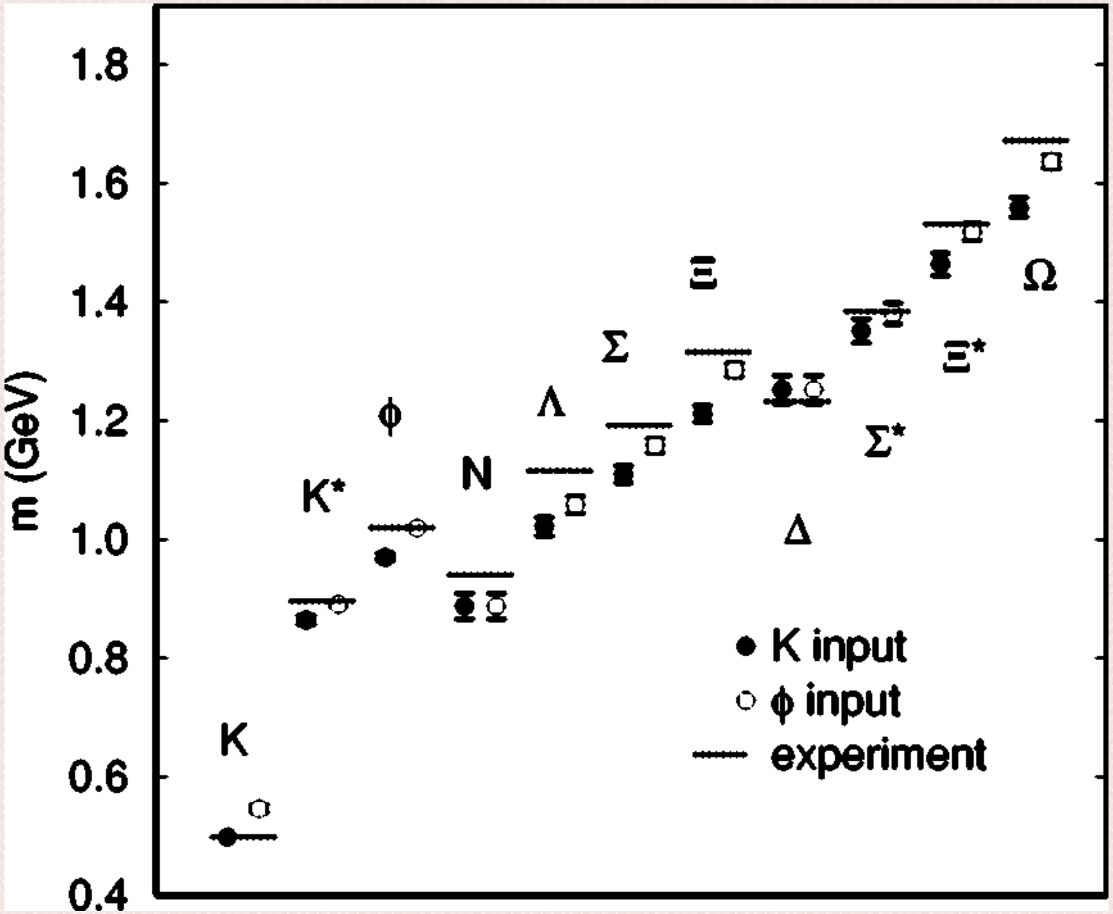
Correlation function for B to pi

in the Quenched or Valence Approximation



$$\langle J_\pi(y) A_\mu(0) J_B(x) \rangle \;=\; \int [dU]\, J_\pi(y) A_\mu(0) J_B(x)\; \det Q[U]\, e^{-S_{\rm glue}}$$

Quenched Light Hadron Spectrum



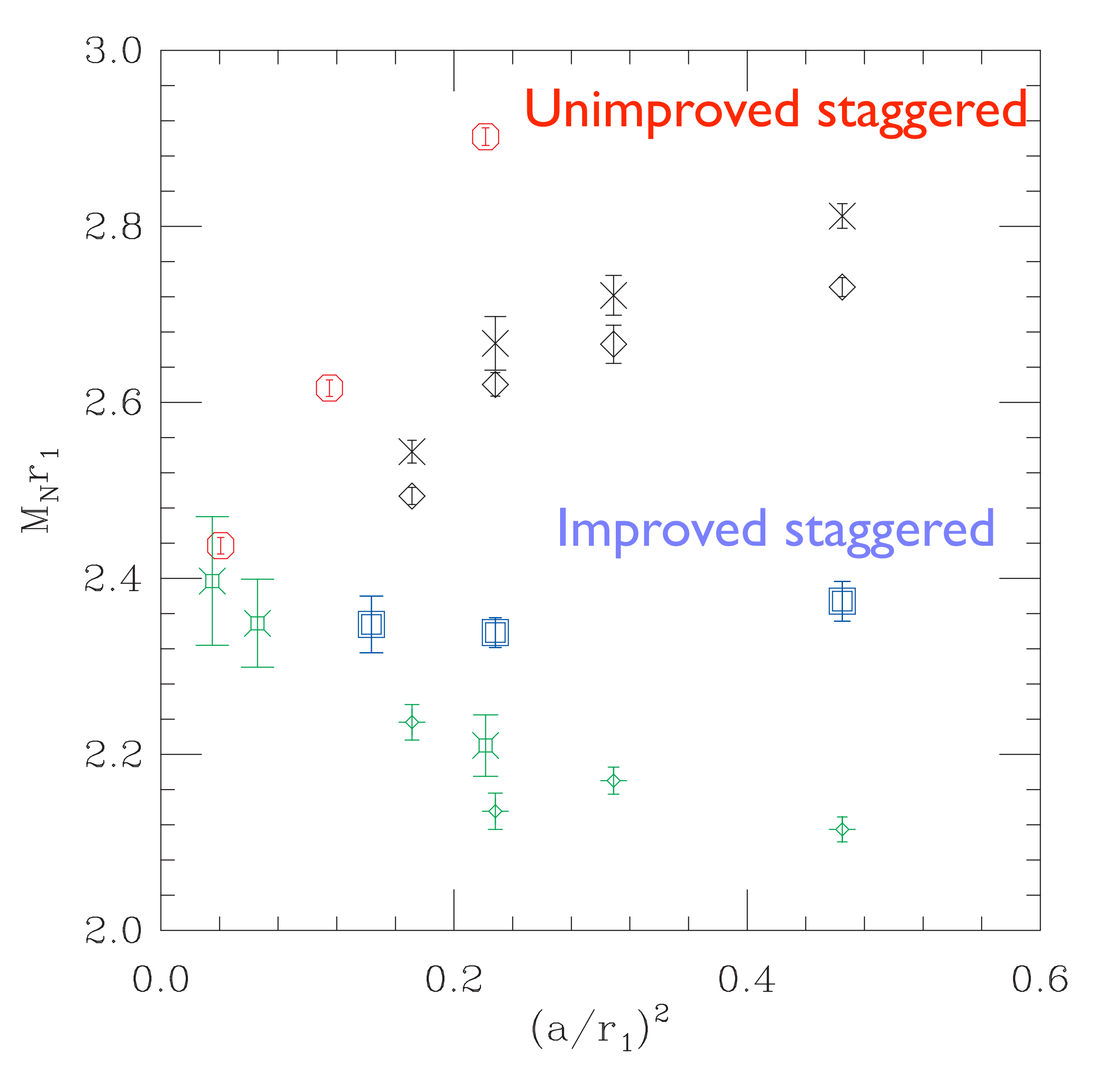
- Triumph of Force -- explored all systematic effects within quenched approximation
- Disagreement with experiment at 10% level
- Ambiguity in setting lattice spacing and strange quark mass

S. Aoki, *et al.*,(CP-PACS) PRD **67** (2003)

A Smattering of Staggering

- Naive fermion discretization has 15 extra states (“doublers”)
- $$G(p)a \; = \; \frac{1}{i\sum_\mu \gamma_\mu \sin(p_\mu a)}$$
- Staggered quarks are cheap to simulate because they turn the doubling problem into an asset -- spin diagonalization
- Remaining doubler degrees-of-freedom (4) interpreted as “tastes” (artificial flavors)
- “Fourth-root trick” used to get right number of sea quarks
  No proof showing this is theoretically sound or unsound

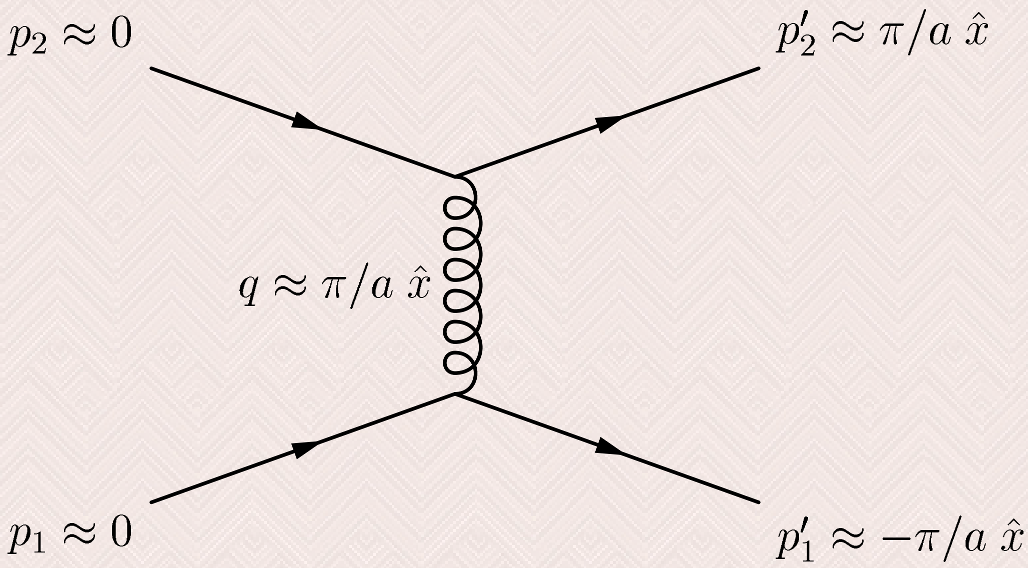
Nucleon mass vs. lattice spacing



- Quenched
- Funny units (r_1 = 0.13 fm)
- Heavy masses

MILC Collaboration, hep-lat/9912018

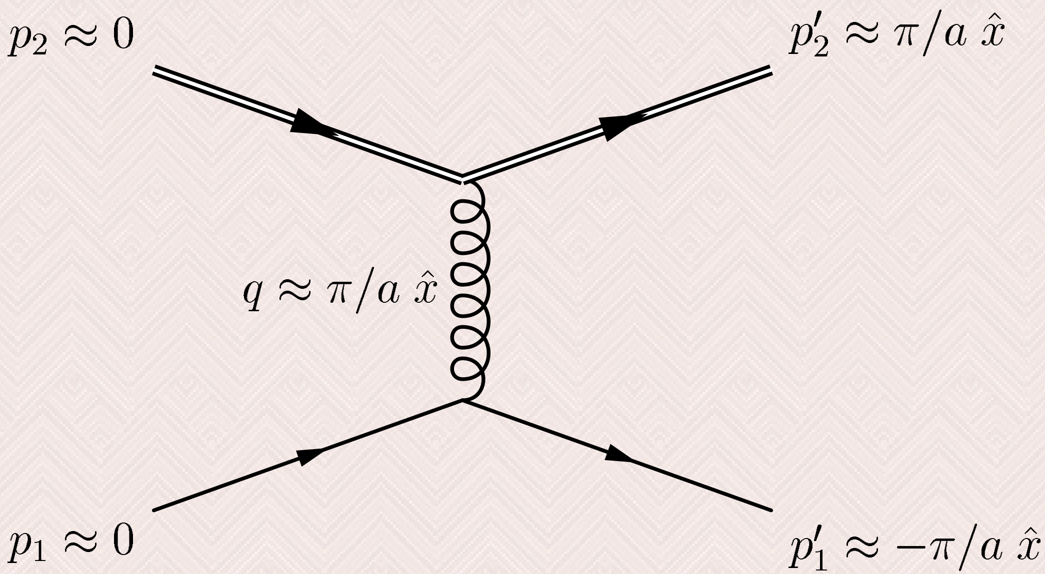
Improving staggered fermions



- Exchange of hard "lattice-y" gluons break "taste" and generate large scaling violations
- Large momentum modes are the offenders: can suppress them using perturbation theory
- "Fat links"

$$V_\mu(x) = \left[\prod_{\nu\neq\mu} \left(1 + \frac{a^2 \nabla_\nu^{(2)}}{4} \right) \right] \Bigg|_{\mathrm{sym'd}} U_\mu(x)$$

Heavy-staggered mesons



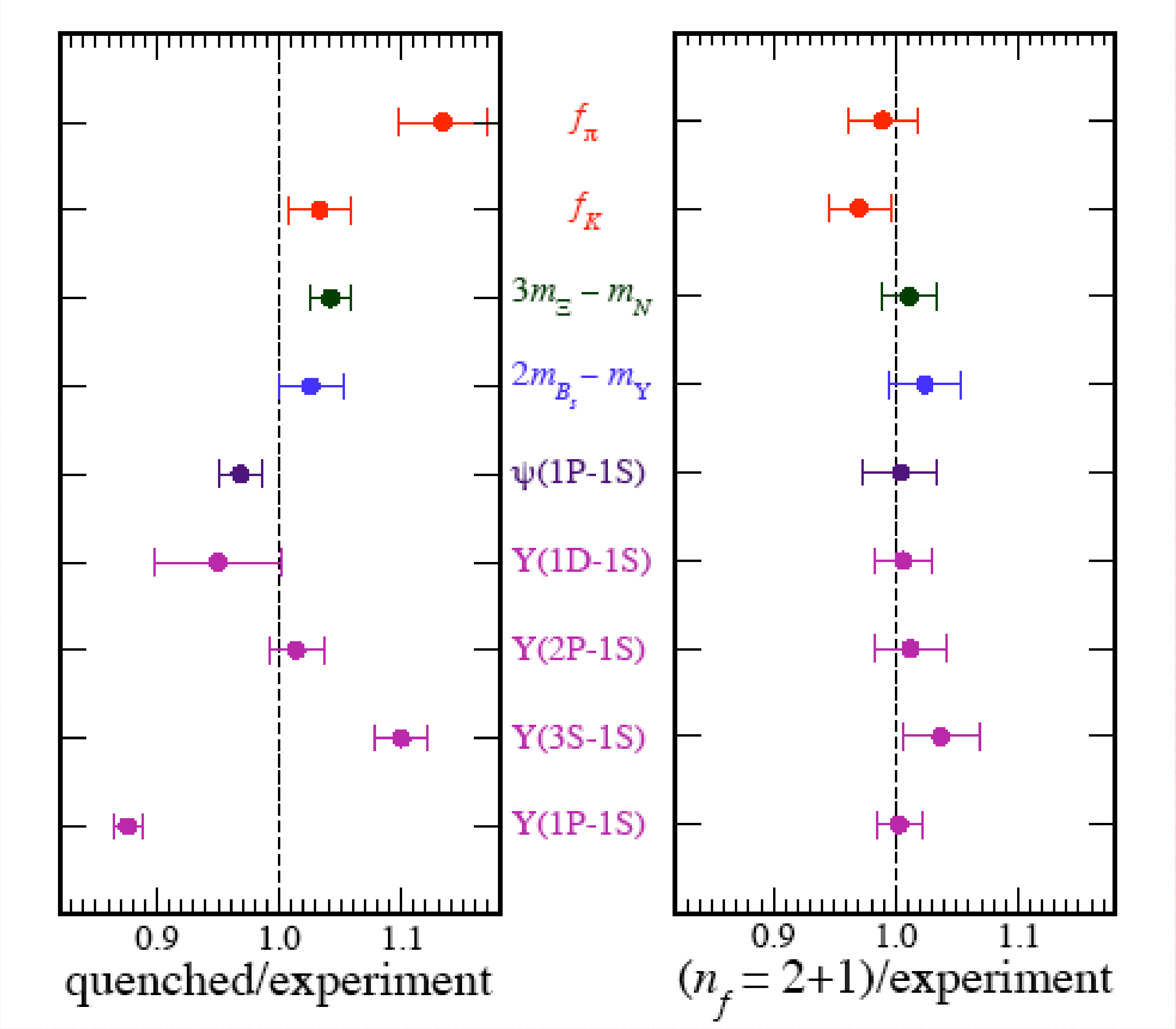
$$\left\langle \overline{\Psi}(x)\gamma_5\psi(x)\ \overline{\psi}(0)\gamma_5\Psi(0) \right\rangle$$

$$G_\psi(x,y) = g_\chi(x,y)\ \prod_\mu \gamma_\mu^{x_\mu}\ \gamma_{4-\mu}^{y_{4-\mu}}$$

- NRQCD/FNAL heavy quarks are not doubled
- Taste-breaking is negligible
- Compute corr'n fns using naive light fermions

M.W. *et al.*, PRD **67**, 054505 (2003)

Quenched vs. Light Improved Staggered

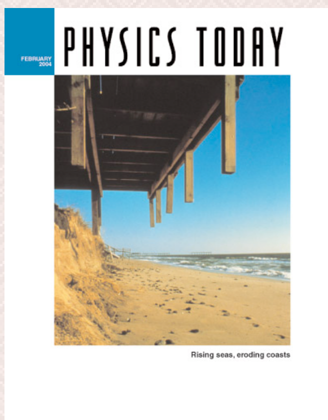


C. Davies, *et al.* PRL **92**, 022001 (2004)

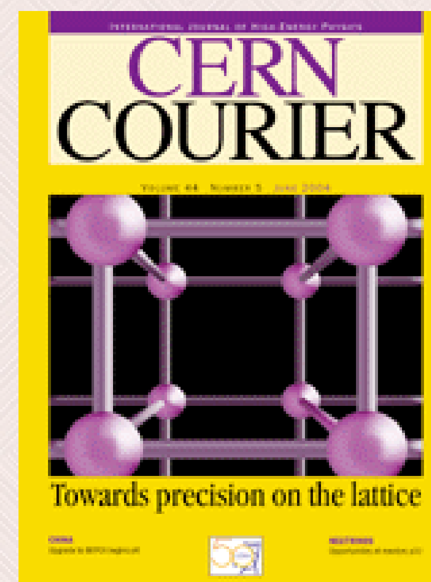
Getting some good press...

A. Cho, "Calculating the Incalcuable," *Science*, **300**, p. 1076 (16 May 2003)

I. Shipsey, "Lattice Window on the Strong Force," *Nature*, **427**, p. 591 (12 February 2004)



C. DeTar & S. Gottlieb, "Lattice QCD Comes Of Age," *Physics Today*, (February 2004)



C. Davies, "Joining Up the Dots with the Strong Force," *Cern Courier*, **44** (June 2004)

The Grail of Purity

- Theoretically sound algorithm
- Good chiral properties - Ginsparg-Wilson-Luescher symmetry
- Simulate on large volumes, small lattice spacings, physical sea quark masses (or close enough for chi-PT)

Sea quarks and states of Sin

- Quenched

  Theoretically wrong. 10-20% disagreement with experiment.

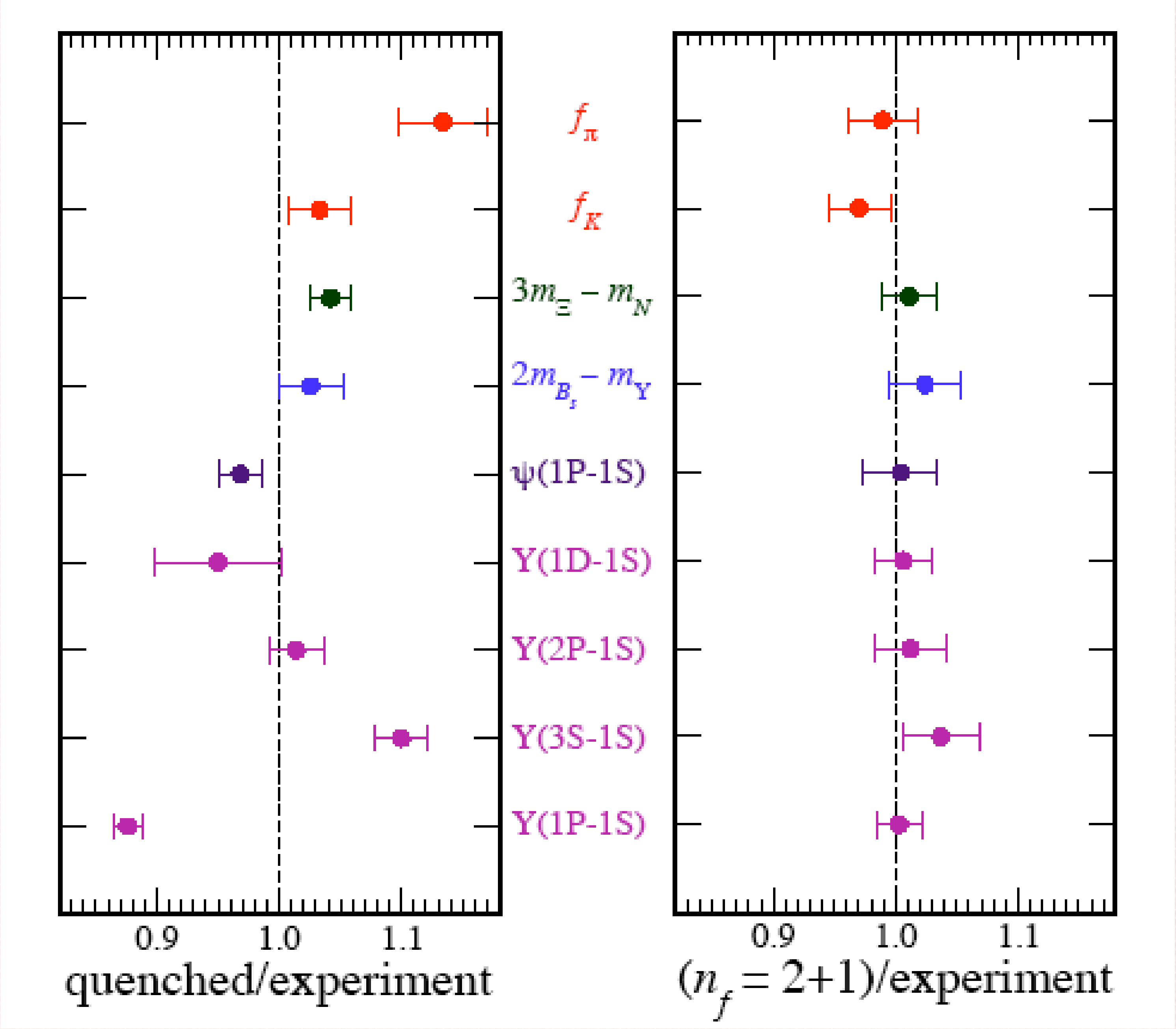
- Lighter staggered

  Theoretically uncertain. Agreement with experiment within quoted uncertainties. Permits simulation inside chiral regime

- Heavier Wilson, twisted-mass, domain wall, overlap, fixed point

  Theoretically sound. More costly, so heavier mass required. Extrapolation to physical sea quark masses: inside chiral regime???

Quenched vs. Light Improved Staggered



C. Davies, *et al.* PRL **92**, 022001 (2004)

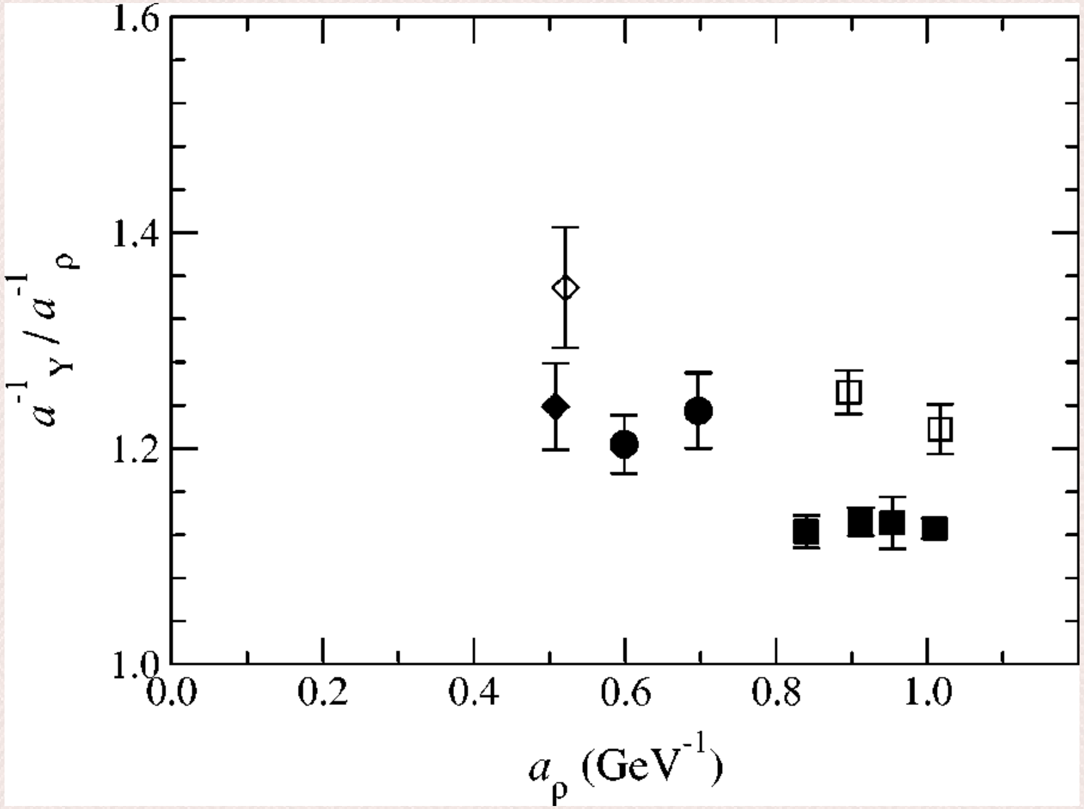
f_{B_s}

f_{B_s} with NRQCD

Ref	n_f	Configs	result (MeV)	
1	2	CP-PACS $m_{ud}^{\mathrm{sea}} \geq m_s/2$	$242 \pm 9 \pm 34$	
2	2	JLQCD $m_{ud}^{\mathrm{sea}} \geq m_s/2$	$215 \pm 9 \pm 13$	
3	2+1	MILC $m_{ud}^{\mathrm{sea}} \geq m_s/4$	$260 \pm 7 \pm 28$	

1. A. Ali Khan, *et al.* PRD 64 (2001)
2. S. Aoki, *et al.* PRL 91 (2003)
3. M.W., *et al.* PRL 92 (2004)

Lattice spacing ambiguity



(open points = quenched)

Solid points = 2 flavor

- CP-PACS - 2 flavors of clover (tadpole coeff)
- Upsilon 1P-1S splitting vs. rho mass to set lattice spacing
- JLQCD simulation sees scale agreement using m_ρ, f_K, r_0

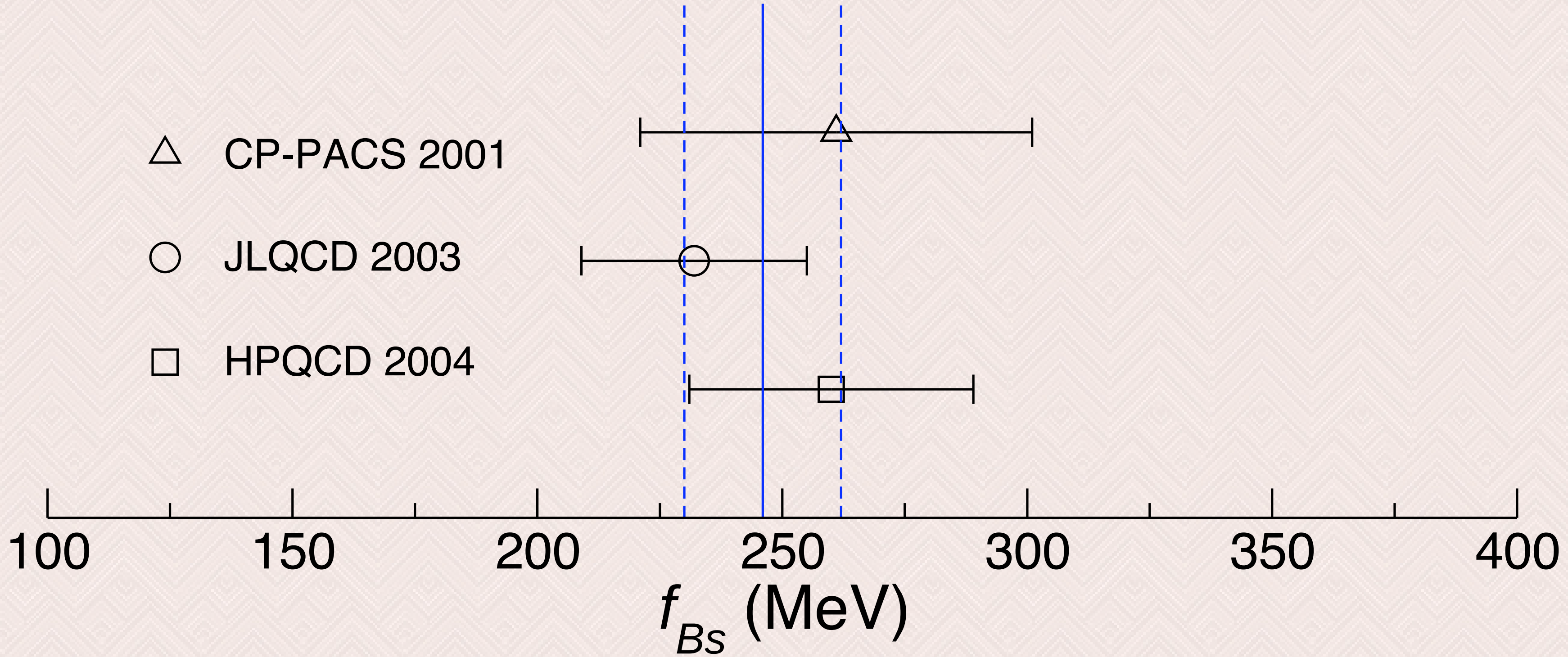
A. Ali Khan, et al. PRD 64 (2001)

f_{B_s} with NRQCD

Ref	n_f	Configs	result (MeV)	scale ambiguity
1	2	CP-PACS $m_{ud}^{\mathrm{sea}} \geq m_s/2$	$242 \pm 9 \pm 34$	$+38$ -0
2	2	JLQCD $m_{ud}^{\mathrm{sea}} \geq m_s/2$	$215 \pm 9 \pm 13$	$+34$ -0
3	2+1	MILC $m_{ud}^{\mathrm{sea}} \geq m_s/4$	$260 \pm 7 \pm 28$	$< 4\%$

1. CP-PACS: A. Ali Khan, *et al*. PRD 64 (2001)
2. JLQCD: S. Aoki, *et al.* PRL 91 (2003)
3. HPQCD: M.W., *et al*. PRL 92 (2004)

Weighted Average

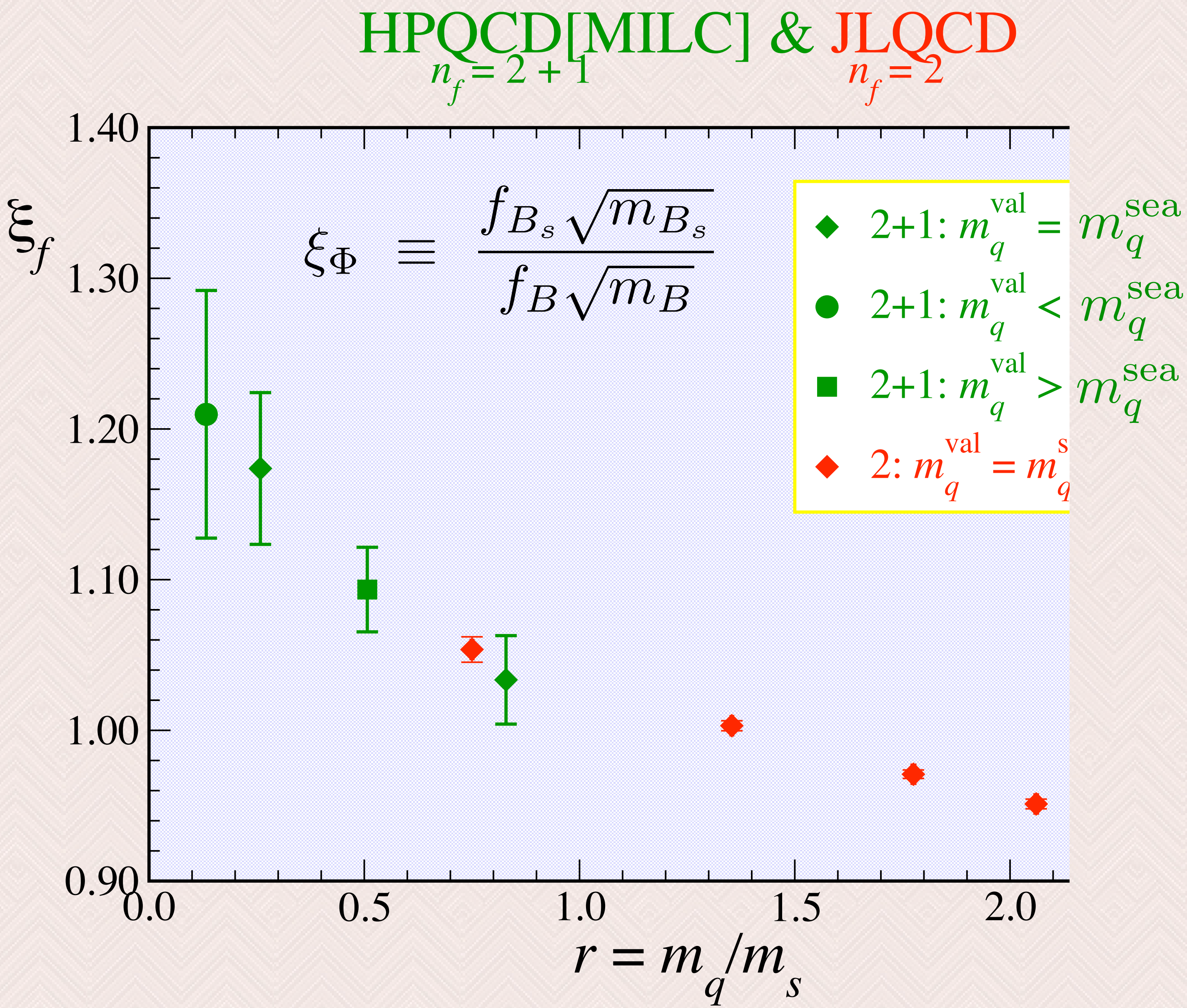


$$f_{B_s} = 246 \pm 16 \text{ MeV}$$

M.W., Lattice 2004

$$f_B$$

From A. Kronfeld, Lattice 2003

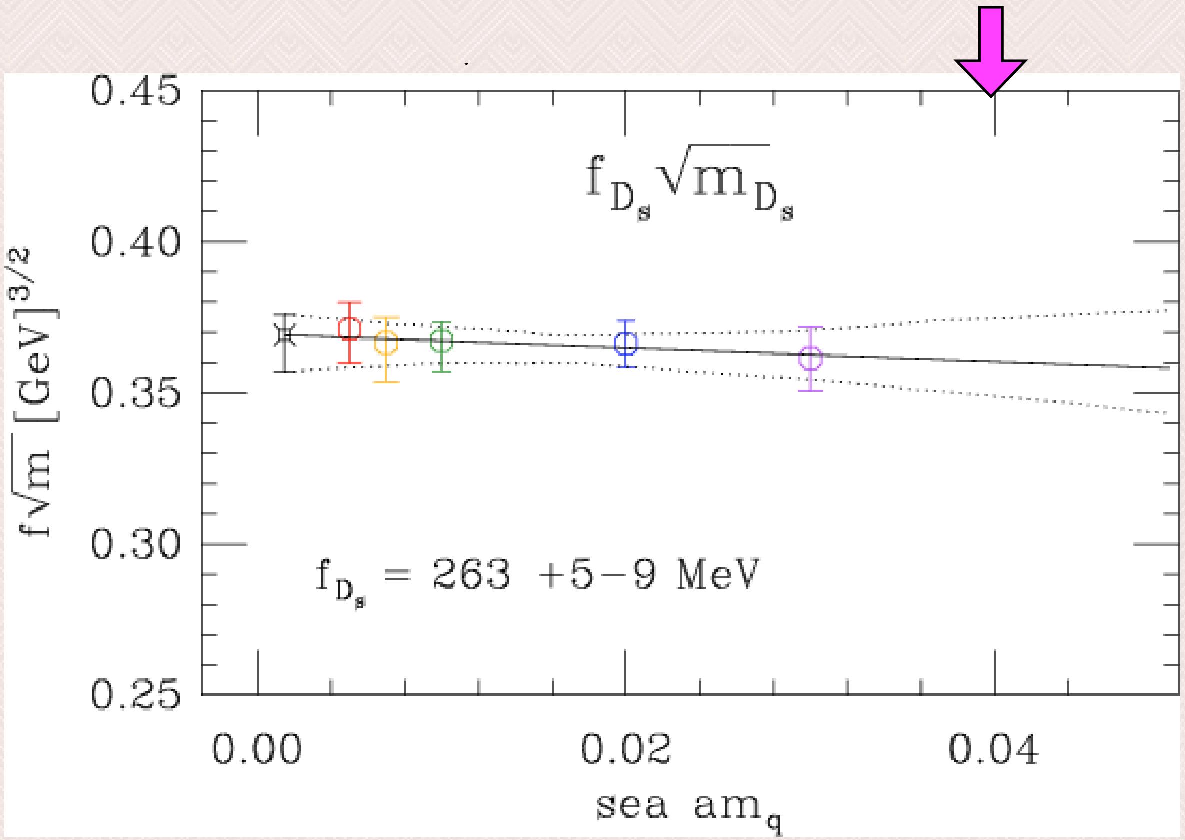


$$f_{D_s}$$

f_{D_s} by FNAL/MILC **PRELIMINARY**

Linear extrapolation in sea quark

physical m_s



$$f_{D_s} = 263 \begin{array}{c} +5 \\ -9 \end{array} \pm 24 \text{ MeV}$$

- n_f= 2+1 impr. staggered

  $m_{ud}^{sea} \geq m_s/8$

- coarser MILC lattices

  $a \approx 0.12$ fm

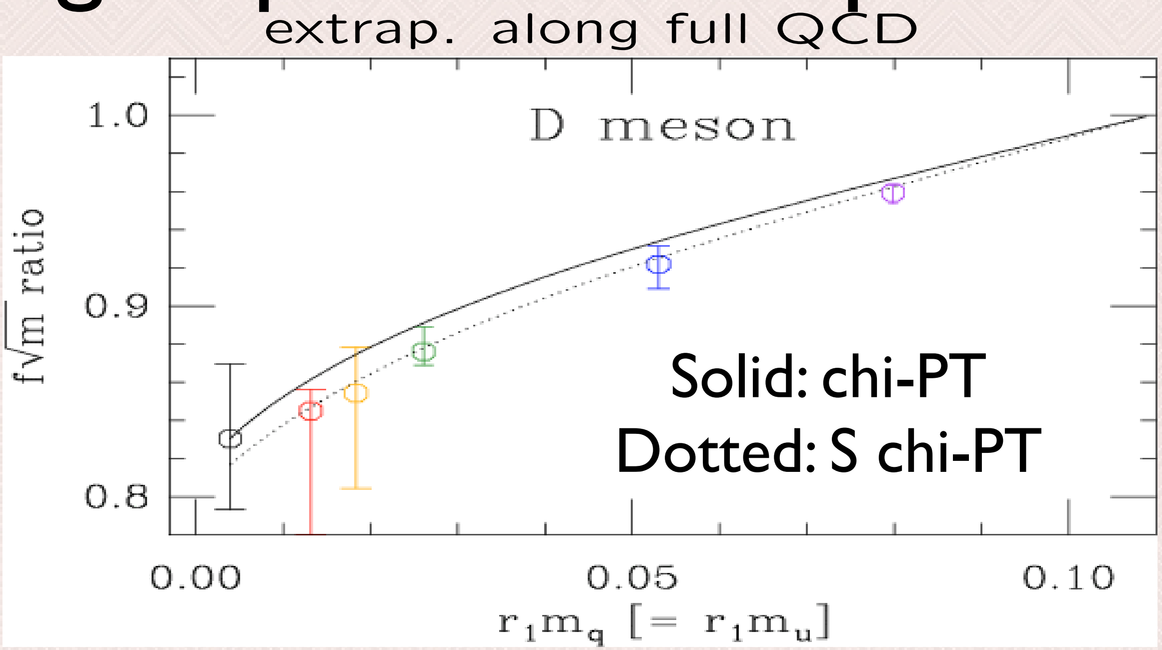
- Volume $\approx (2.4 \text{ fm})^3$

- Mesons: FNAL-heavy impr. staggered light (AsqTad)

J. Simone, Lattice 2004

$$\int D$$

Light quark mass dependence



$$\xi_\Phi = \frac{f_{D_s}\sqrt{m_{D_s}}}{f_D\sqrt{m_D}} = 1.20 \pm 0.06_{\rm stat} \pm 0.06_{\rm sys}$$

$$f_{D_s} = 263 \begin{matrix} +5 \\ -9 \end{matrix} \pm 24 \text{ MeV}$$

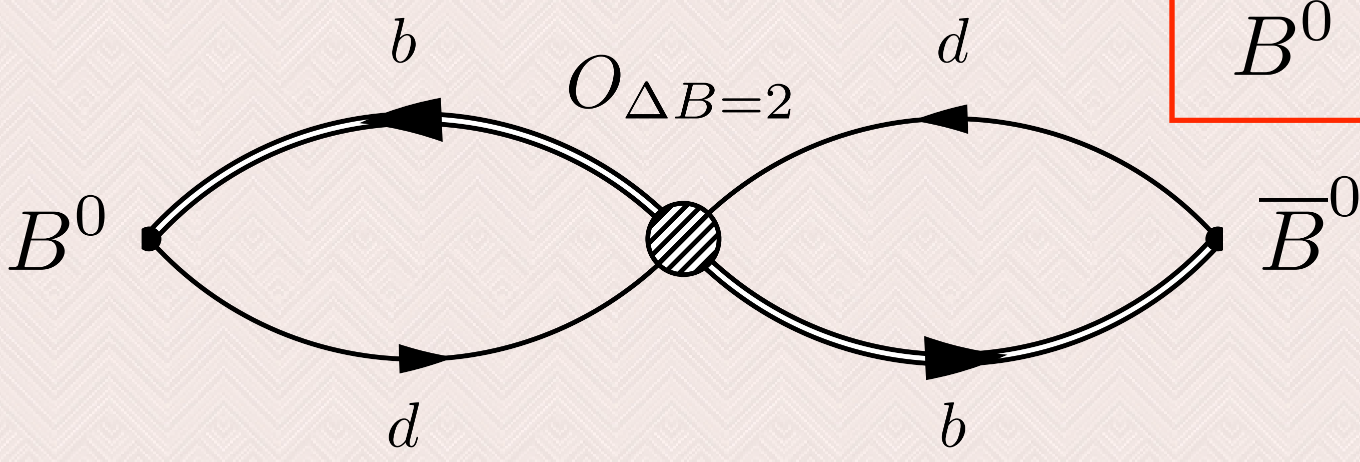
$$f_D = 224 \begin{matrix} +10 \\ -14 \end{matrix} \pm 22 \text{ MeV}$$

Preliminary

J. Simone, Lattice 2004

$$V_{td}$$

$B^0 - \overline{B}^0$ mixing



$$\langle \overline{B}^0_s | (\bar{b}s)_{V-A} (\bar{b}s)_{V-A} | B^0_s \rangle = \frac{8}{3} f^2_{B_s} m^2_{B_s} B_{B_s}$$

$$\langle \overline{B}^0_s | (\bar{b}s)_{S-P} (\bar{b}s)_{S-P} | B^0_s \rangle = -\frac{5}{3} \left(\frac{f_{B_s} m^2_{B_s}}{m_b + m_s} \right)^2 B_{S_s}$$

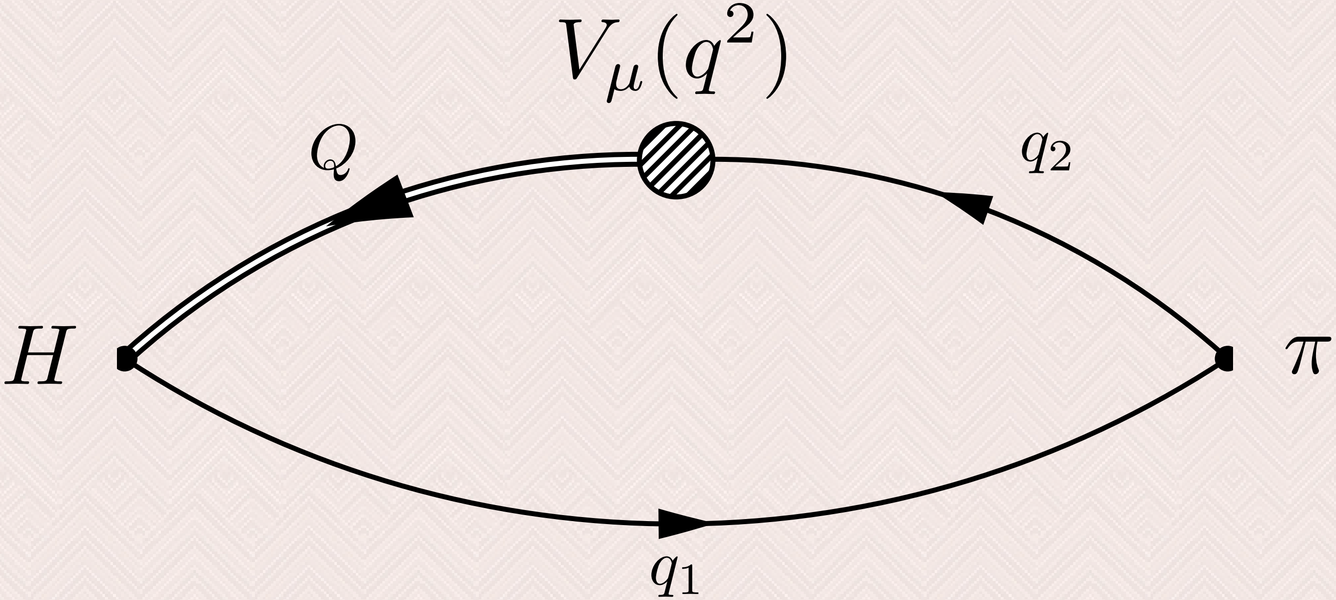
- JLQCD, PRL 91 (2003); N. Yamada, Lattice 2001:
- $$B_{S_s}(m_b) = 0.86(3)(7)$$

$$B_{B_d}(m_b) = 0.836(27)(^{+\ 0}_{-27})(56),$$
$$B_{B_s}(m_b) = 0.850(22)(^{+18}_{-\ 0})(57)(^{+5}_{-0}),$$
$$\frac{B_{B_s}}{B_{B_d}} = 1.017(16)(^{+53}_{-\ 0})(17)(^{+6}_{-0}).$$

- A. Gray (HPQCD), Lattice 2004 - Calculation underway
- Chiral symmetry reduces mixings NRQCD+KS or Tsukuba+DWF (N. Yamada, Lattice 2004)

$$V_{cd}$$

Semileptonic 3 point function

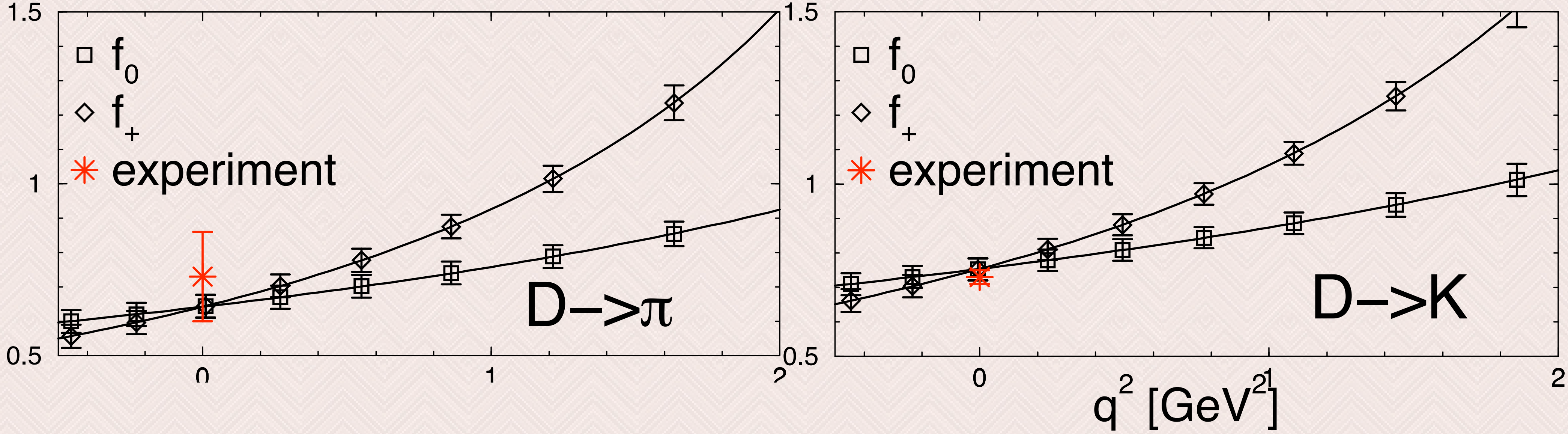


$$\langle \pi | V_\mu | H \rangle \;\equiv\; f_+(q^2) \left(p_\pi + p_H - \frac{m_H^2 - m_\pi^2}{q^2} q \right)_\mu$$

$$+\; f_0(q^2) \;\frac{m_H^2 - m_\pi^2}{q^2}\, q_\mu$$

$$\equiv\; \sqrt{2 m_H} \left(f_{||}(E_\pi)\, v_\mu \;+\; f_\perp(E_\pi)\, p_{\perp,\mu} \right)$$

q^2 dependence



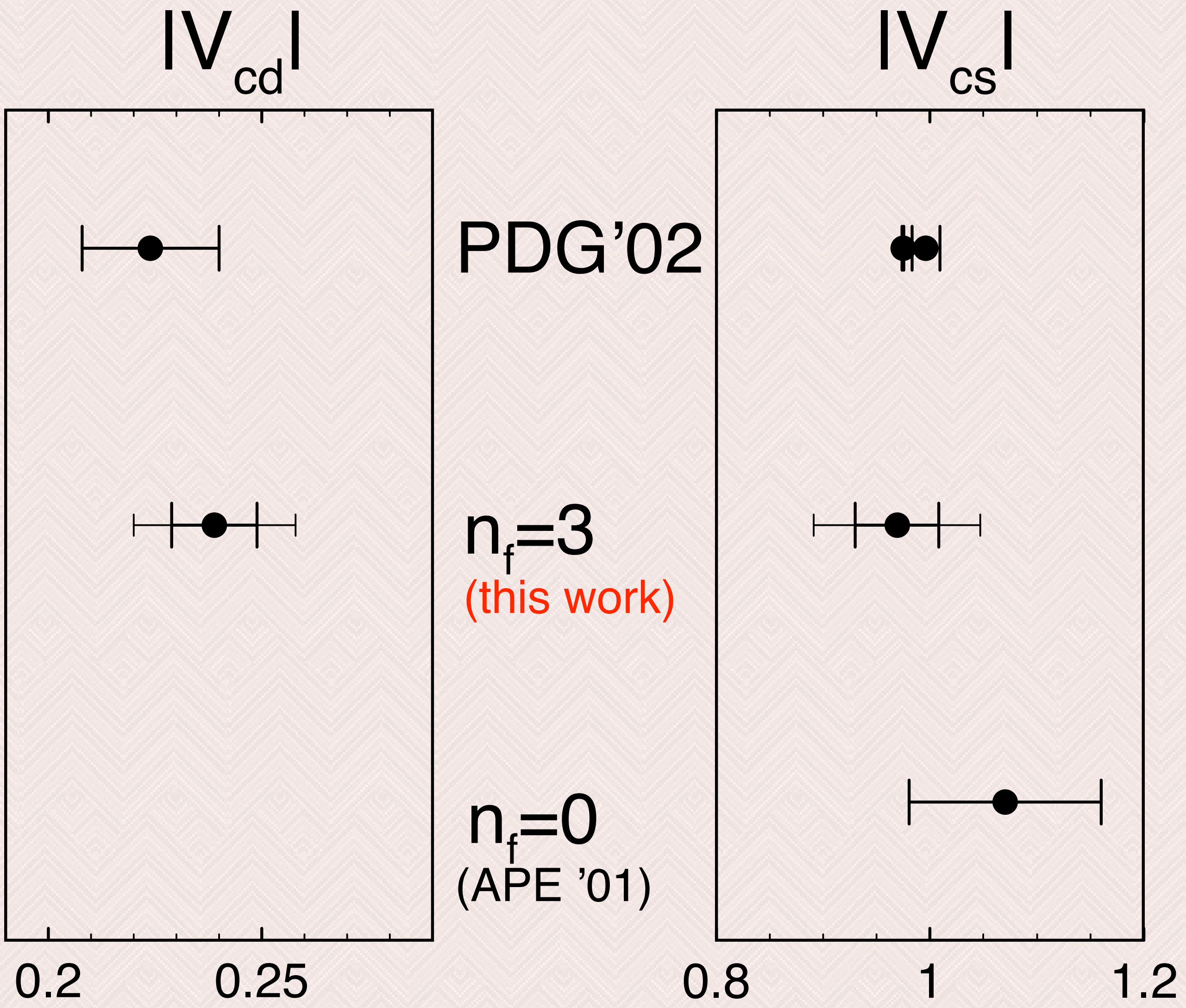
- Fit to Becirevic-Kaidalov ansatz
- Nearly final results:

$$f_+^{D\to\pi}(0) \;=\; 0.64(3)(5), \quad f_+^{D\to K} \;=\; 0.73(3)(6)$$

- Largest uncertainty due to heavy quark discretization: 7%

M. Okamoto, Lattice 2004

Combining f.f. with experiment



$$|V_{cd}| = 0.239(10)(19)(20), \quad |V_{cs}| = 0.969(39)(78)(24)$$

M. Okamoto, Lattice 2004

$$V_{cb}$$

$B \to Dl\nu$ decay

$$\langle D|V^\mu|B\rangle = \sqrt{m_B m_D} \times [h_+(w)(v_B + v_D)^\mu + h_-(w)(v_B - v_D)^\mu],$$

where $w = v_B \cdot v_D$.

$$\frac{d\Gamma(B \to Dl\nu)}{dw} \propto |\mathcal{F}_{B\to D}(w)|^2 |\mathbf{V_{cb}}|^2$$

$$\mathcal{F}_{B\to D}(w) = h_+(w) - \frac{m_B - m_D}{m_B + m_D} h_-(w).$$

We focus on the zero recoil limit ($w = 1$).

Ratio method (S. Hashimoto *et.al.* '99)

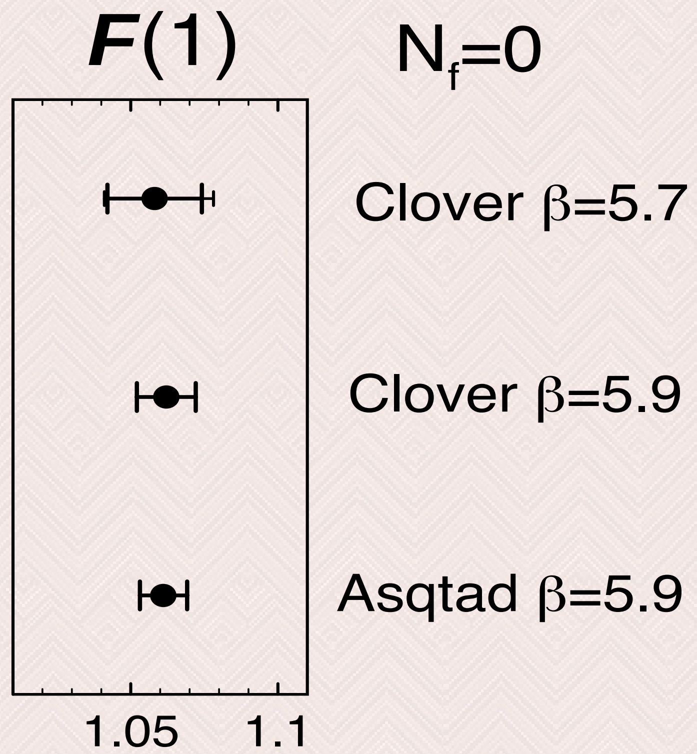
$$\frac{C^{DV_0B}(t)C^{BV_0D}(t)}{C^{DV_0D}(t)C^{BV_0B}(t)} \to \frac{\langle D|V_0|B\rangle\langle B|V_0|D\rangle}{\langle D|V_0|D\rangle\langle B|V_0|B\rangle} = |h_+^{B\to D}(1)|^2$$

$\mathcal{F}(1) = 1$ in $B = D$ limit.

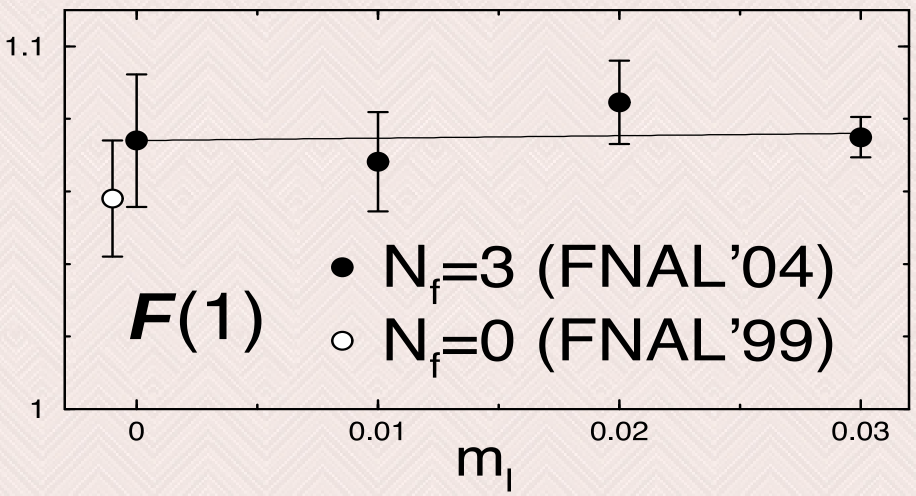
slide from M. Okamoto, Lattice 2004

$B \to D$ results (preliminary)

lattice spacing effect



dynamical quark effect



$$\mathcal{F}^{n_f=3}_{B\to D}(1) = 1.074(18)(15)$$

Using Belle's branching ratio

$$|V_{cb}| \times 10^2 = 3.83(07)(06)(64)$$

Unitarity check: $(|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2)^{1/2} = 1.00(4)(8)(2)$

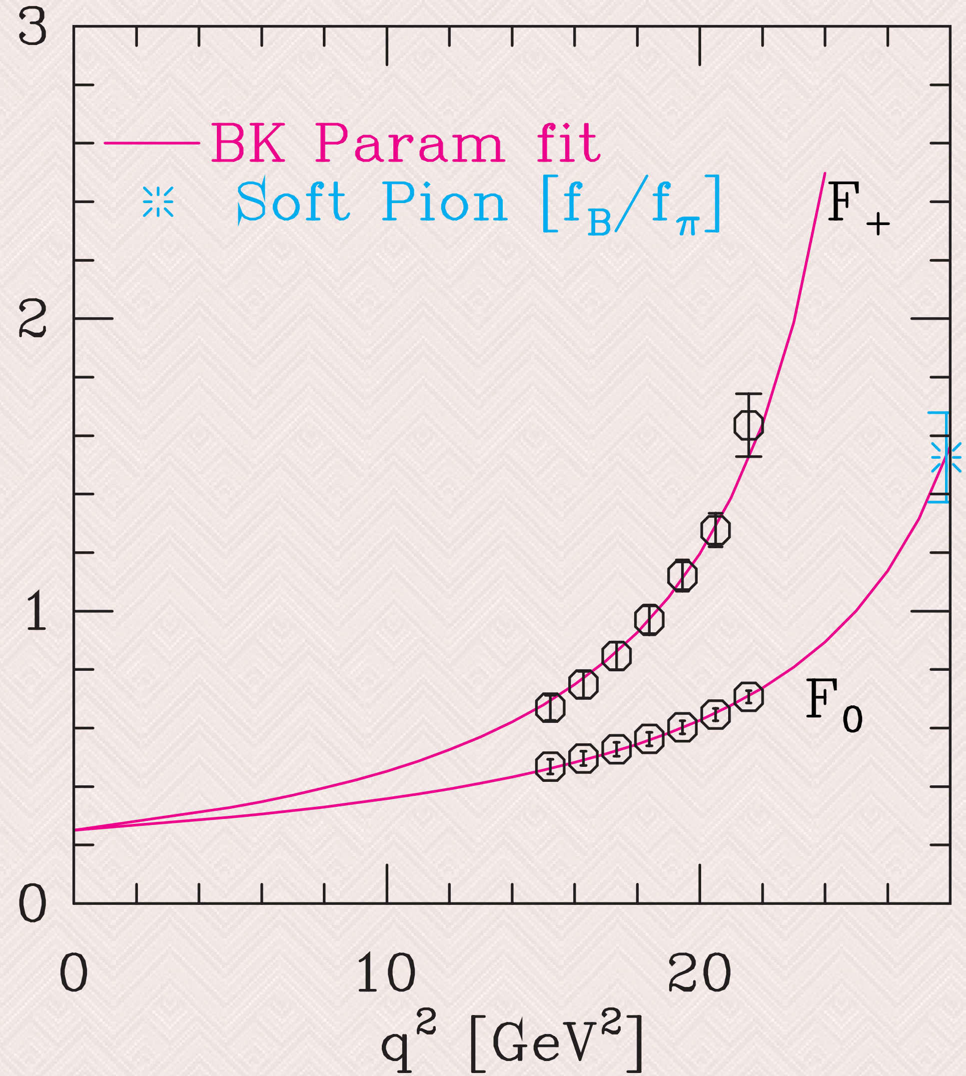
slide from M. Okamoto, Lattice 2004

$$V_{ub}$$

q^2 dependence

- Fit to Becirevic-Kaidalov ansatz (B* pole plus effective pole)
- Result:

$$f_0(0) \ = \ f_+(0) \ = \ 0.25(2)$$



J. Shigemitsu, E. Gulez, Lattice 2004

Estimating $|V_{ub}|$

CLEO (hep-ex/0304019) has published the branching fractions,

$$\mathcal{B}(B^0 \to \pi^-, l^+\nu) =$$
$$(1.33 \pm 0.18 \pm 0.11 \pm 0.01 \pm 0.07) \times 10^{-4}$$

for full range $0 \leq q^2 \leq q^2_{max}$ and

$$\mathcal{B}(q^2 \geq 16GeV^2) =$$
$$(0.25 \pm 0.09 \pm 0.04 \pm 0.01 \pm 0.03) \times 10^{-4}$$

Using lattice determination of $f_+(q^2)$ one can integrate

$$\frac{1}{|V_{ub}|^2}\frac{d\Gamma}{dq^2} = \frac{G_F^2}{24\pi^3} p_\pi^3 |f_+(q^2)|^2$$

to get $\frac{\Gamma}{|V_{ub}|^2}$ $\Longrightarrow$ $\boxed{|V_{ub}|}$

Estimating $|V_{ub}|$ (cont'd)

Our preliminary results are (E.Gulez)

$$\frac{\Gamma}{|V_{ub}|^2} = \begin{cases} 5.80(93)\ ps^{-1} & 0 \leq q^2 \leq q^2_{max} \\ 1.31(16)\ ps^{-1} & 16GeV^2 \leq q^2 \end{cases}$$

hence,

$$|V_{ub}| = \begin{cases} 3.86(35)(62) \times 10^{-3} & 0 \leq q^2 \leq q^2_{max} \\ 3.52(70)(42) \times 10^{-3} & 16GeV^2 \leq q^2 \end{cases}$$

The errors (exper.)(lattice) are still tentative.

A recent review (Ali 2003) quotes an average of CLEO, BELLE and BABAR inclusive results

$$|V_{ub}|_{(inclusive)} = 4.32(57) \times 10^{-3}$$

slides from J. Shigemitsu, Lattice 2004

$$V_{us}$$

$|V_{us}|$ from leptonic decays and lattice

W. Marciano, hep-ph/0402299

- Experimental rates for $\pi \to \mu \overline{\nu}_\mu(\gamma) \quad K \to \mu \overline{\nu}_\mu(\gamma)$

- yield

$$\frac{|V_{us}|^2 f_K^2}{|V_{ud}|^2 f_\pi^2} = 0.07602 \pm 0.00023_{\rm expt} \pm 0.00027_{\rm rad}$$

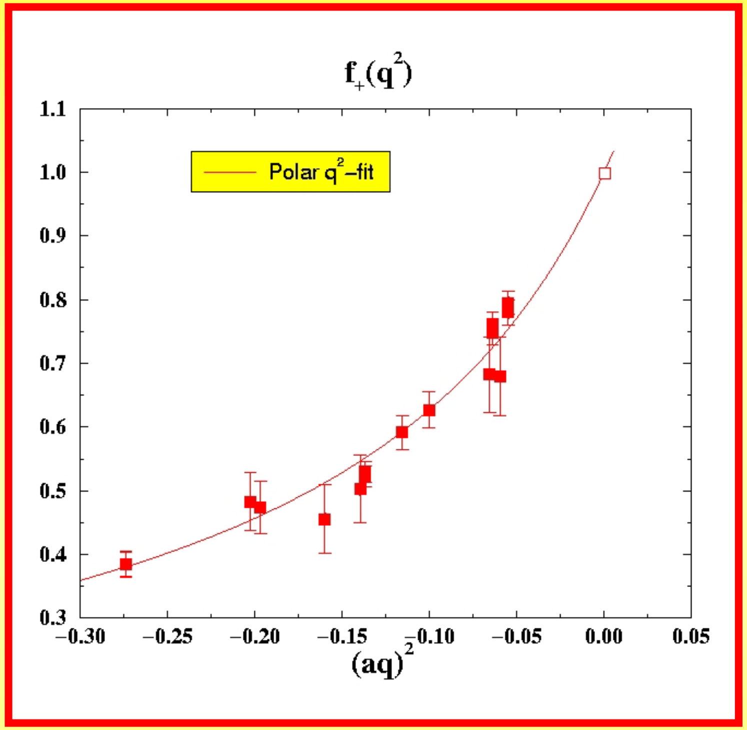
- $f_K/f_\pi = 1.210 \pm 0.004_{\rm stat} \pm 0.013_{\chi,a}$ C. Bernard, Lattice 2004

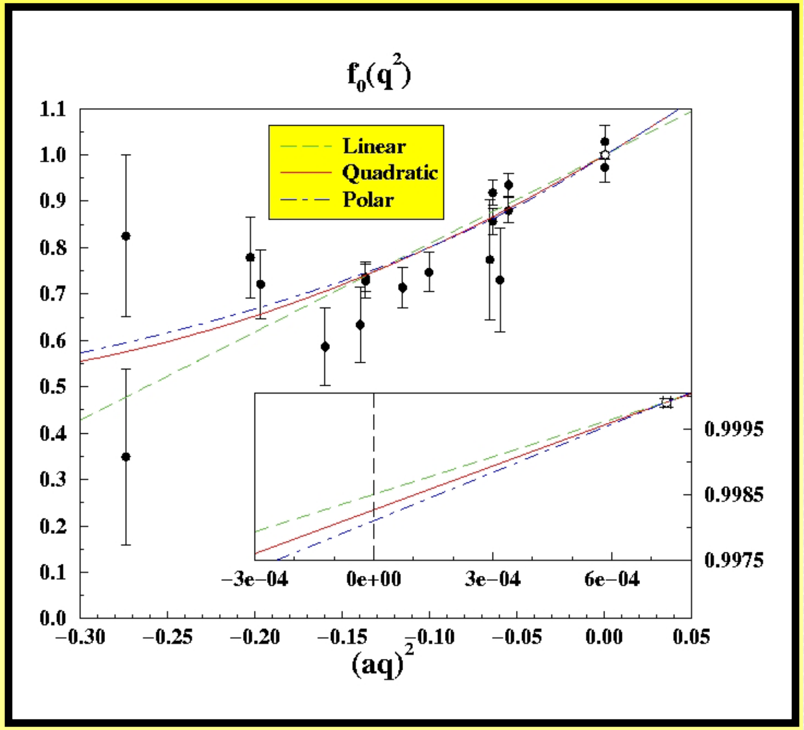
$$\Rightarrow \quad |V_{us}| = 0.2219 \pm 0.0026_{\rm lattice}$$

- f_K/f_π from lattice competitive to experiment + $|V_{us}|$

- Complementary to semileptonic decay analysis

K to pi form factors vs. q^2





Comparison of polar fits:

LQCD:	$\lambda_+ = (25 \pm 2)\,10^{-3}$	$\lambda_0 = (12 \pm 2)\,10^{-3}$
KTeV:	$\lambda_+ = (24.11 \pm 0.36)\,10^{-3}$	$\lambda_0 = (13.62 \pm 0.73)\,10^{-3}$

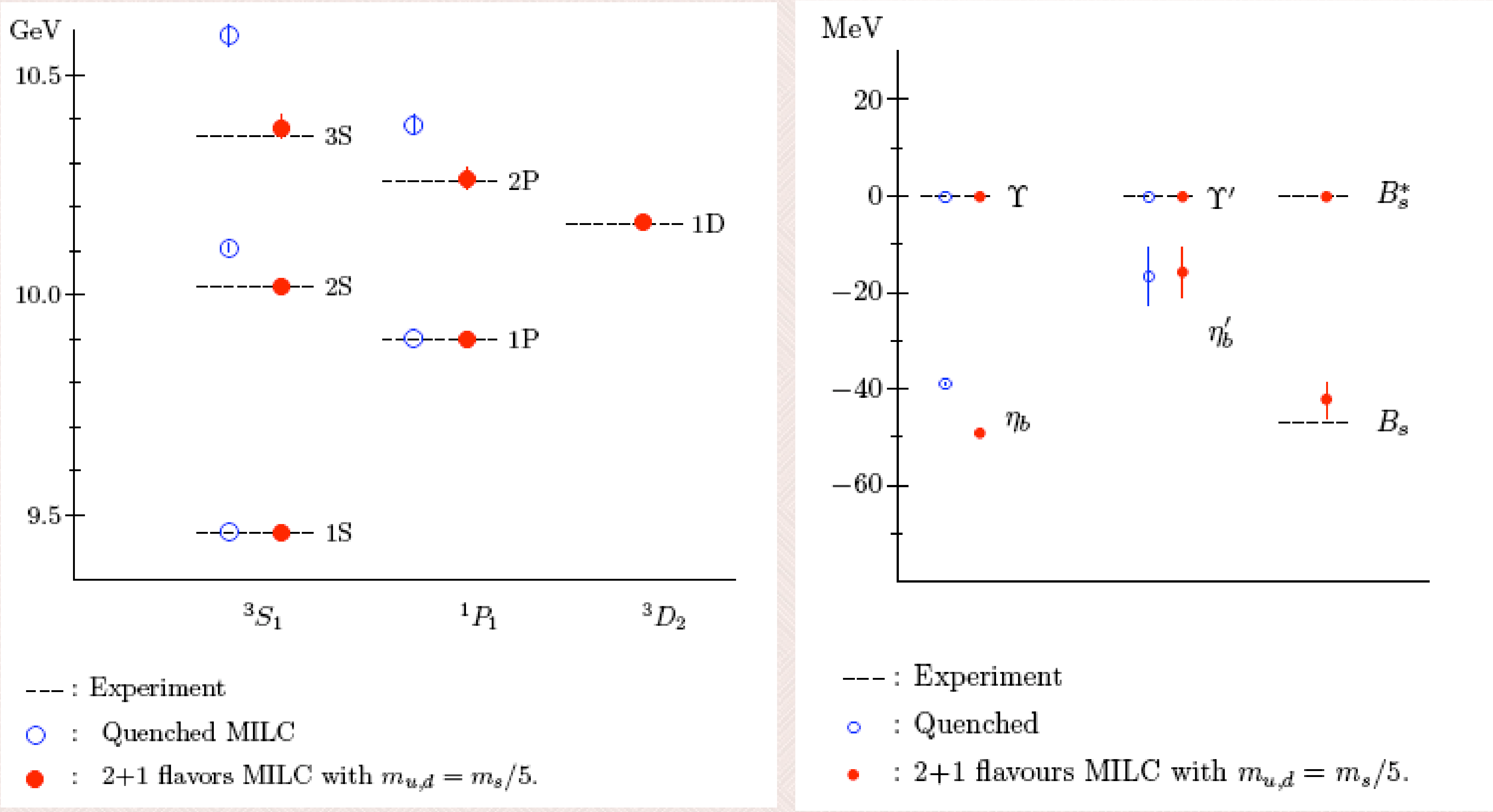
slide from V. Lubicz, Lattice 2004

$$f_+^{K^0\pi^-}(0) = 0.960 \pm 0.005_{\rm stat} \pm 0.007_{\rm sys} \pm 0.???_{\rm quench}$$

D. Becirevic, *et al.*, hep-ph/0403217

$$\bar{b}b$$

Bottomonium Spectrum

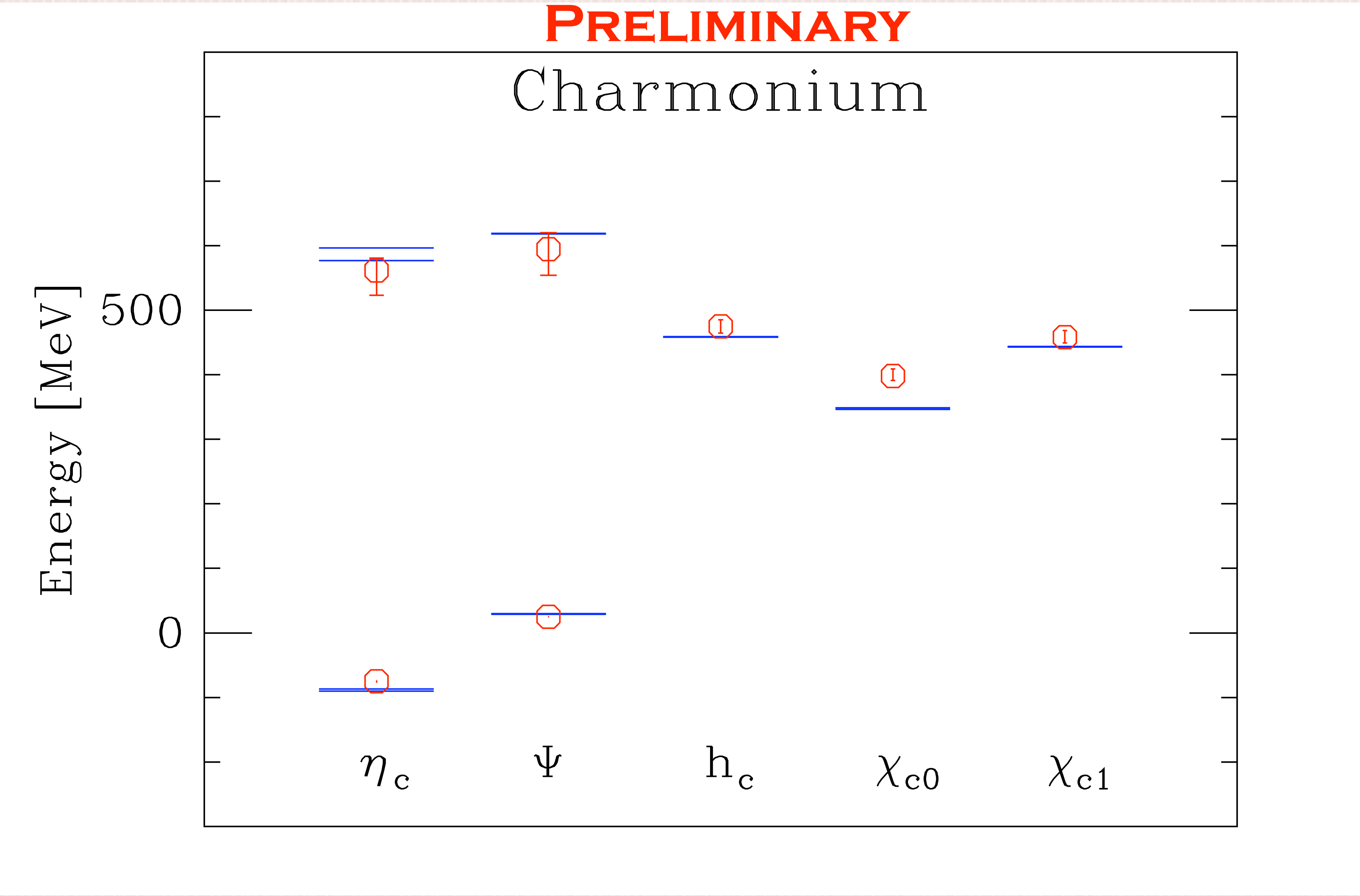


Spin-averaged splittings

Hyperfine splittings

PRELIMINARY

$$\bar{c}c$$



- Zero is spin average of 1S states
- Results in physical sea quark mass limit (mild dependence)

J. Simone, *et al.* (FNAL)

$$B_c$$

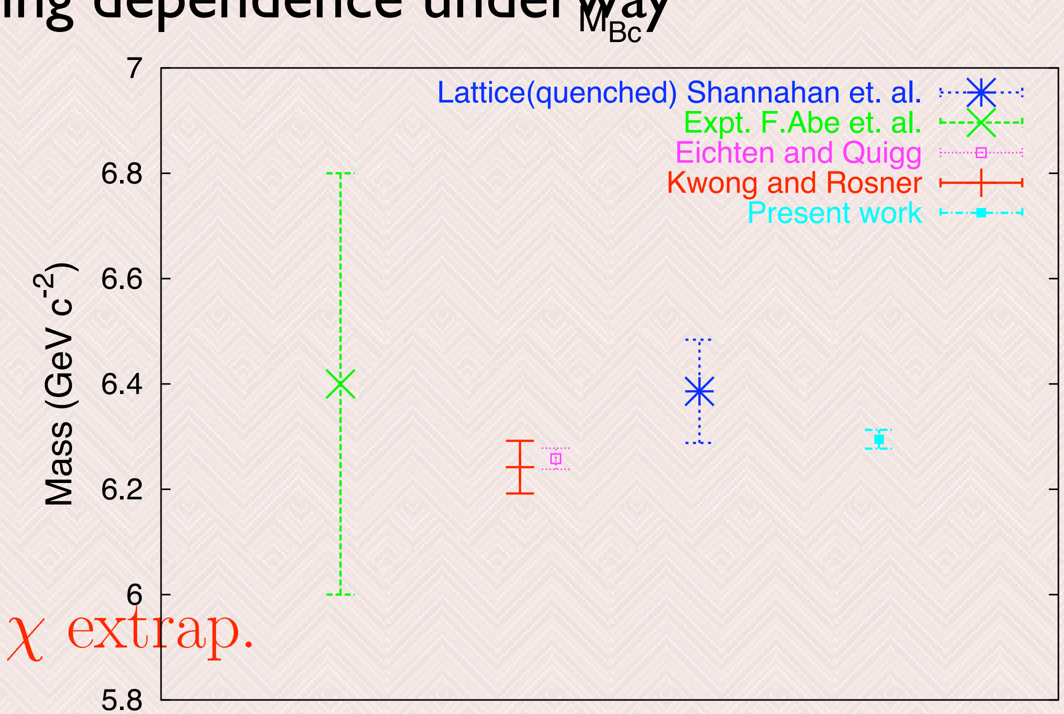
B_c meson mass

PRELIMINARY

- Quarkonium baseline $m_{B_c} - \frac{1}{2}(m_\psi + m_\Upsilon)$
- Heavy-light meson baseline $m_{B_c} - (m_{D_s} + m_{B_s})$
- Further study of lattice spacing dependence underway

$$M_{B_c} = 6.295(3)(10)(10)(c)(3)$$

Stat; Syst; M_b; M_c; χ extrap.



I. Allison, *et al.*, (Glasgow/FNAL), Lattice 2004

Summary

- Lattice QCD is needed to account for hadronic contributions to flavor-changing interactions
- Improved staggered sea quarks allow unquenched simulations with light masses $\approx m_s/8$
- Preliminary results/work in progress
  - Heavy-light decay constants
  - Neutral B mixing
  - Semileptonic form factors for heavy-light mesons
  - Neutral K mixing
- Calculations which are predictions for CLEO-c will bolster those which are not experimentally accessible directly

Q&A