V_{us}

Paolo Franzini

Università di Roma, *La Sapienza*

Boston, 26 June 2004

1963

VOLUME 10, NUMBER 12—PHYSICAL REVIEW LETTERS—15 JUNE 1963

UNITARY SYMMETRY AND LEPTONIC DECAYS

Nicola Cabibbo
CERN, Geneva, Switzerland
(Received 29 April 1963)

. . . we should call it the Cabibbo angle. . . Tini Veltman

We still do not know its value to better than 1%!!!

$$V_{us} = 0.9999\ldots \times \sin\theta_C, \text{ CUSB, 1983}$$

this is tex

OUTLINE

1. History
2. Where are we today
3. Errors
4. What is needed
5. KLOE
6. Conclusions

History

1947: Rochester and Butler: $K^0 \to \pi^+\pi^-$, $K^+ \to \pi^+\pi^0$

1953: Gell-Mann (Nisijima) strangeness

1963: Cabibbo: $s - d$ mixing. $\sin\theta_c = 0.26$ $(K_{\mu 2}/\pi_{\mu 2})$

1964: Gell-Mann, Zweig: 3 quarks

1970: GIM: 4 quarks, 2×2 matrix

1973: Kobayashi and Maskawa, 3 quark-family mixing

CKM quark mixing

In the Standard Model the weak current is given by

$$J_\mu^+ = (\bar{u}\ \bar{c}\ \bar{t})\gamma_\mu(1-\gamma_5)\mathbf{V}_{\mathrm{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

with

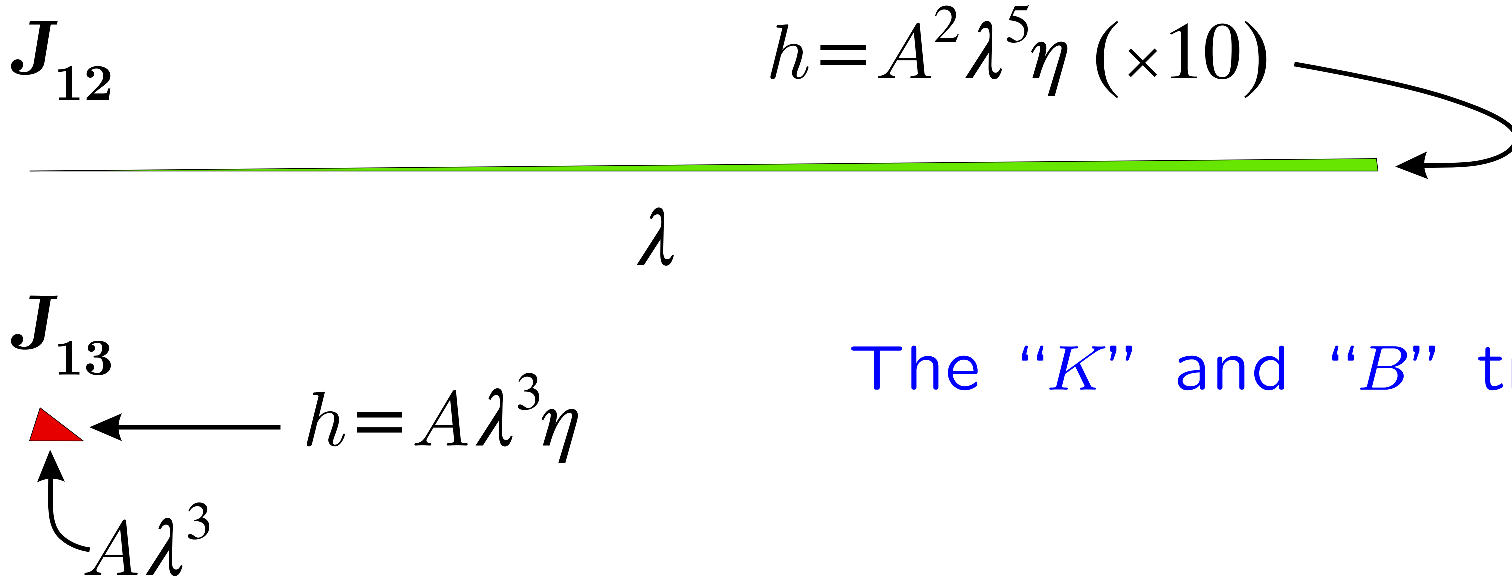
$$\mathbf{V}^\dagger\mathbf{V} = 1$$

which we would like to verify. Only four real #'s describe the weak interactions.

Verify, for instance, by proving the closing of triangles. . .

"Unitarity" triangle(s)

There are many triangles, they must all have the same area, also called the cost of $\not{C}\not{P}$, which is very poorly known



The "K" and "B" triangles

$\Delta S = \Delta Q = 1$ transitions, e.g. K meson decays, measure $|V_{us}|^2 = \lambda^2 = \sin^2\theta$.

$K_L \to \pi^0 \nu\bar{\nu}$ measures $(\Im(V_{td}V_{ts}))^2 = (A^2\lambda^5\eta)^2$

CKM and K mesons

The CP violation parameters add more information to the above. Usually they are shown as correlations in the "$\eta - \rho$" plane.

In fact, $K^0 - \overline{K}^0$ and $B^0 - \overline{B^0}$ mixing, together with V_{bu} had led to predictions for the angle β beautifully verified at the B-factories.

1 is better than 0

Instead of the "0's" in $\mathbf{V}^{\dagger}\mathbf{V}$ we can look at the "1's" The first row must satisfy

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

From $|V_{us}|$=0.2196±26 ($K_{\ell 3}$), $|V_{ud}|$=0.9734±0.0008 ($0^+ \to 0^+$ β-decays) and $|V_{ub}|^2$ of $\mathcal{O}(10^{-5})$ one finds

$$1 - |V_{ud}|^2 - |V_{us}|^2 (-|V_{ub}|^2) = 0.0042 \pm 0.0019$$

a $\sim$2.2σ deviation from 0.

Was 1.8σ in '96, 2.3σ in '02. Small changes in V_{ud} and its error.

By far the most stringent check of unitarity

It is beginning to change now.

$|V_{us}|$ from $K_{e3;\ (\mu 3)}$

Being 0→0 transitions, these decays are protected by the Ademollo-Gatto theorem, *i.e.* $SU(3)$ breaking effects are absent to lowest order in $m(s) - m(u,d)$.

From

$$J_\alpha(UD) = \bar{\mathbf{U}}\,\mathbf{V}_{\rm CKM}\,L_\alpha\,\mathbf{D} = \ldots V_{ud}\,\bar{u}L_\alpha d + V_{us}\,\bar{s}L_\alpha d \ldots$$

$$\langle\,\pi\,|J^H_\alpha|\,K\,\rangle = (p_K + p_\pi)_\alpha \times f_+(t)$$

$$f_+(t) = f_+(0) \times \left(1 + \lambda\frac{t}{\pi^2} + \ldots\right)$$

$$f_+(0) < 1$$

The decay width is:

$$\Gamma(K_{\ell 3}) = |V_{us}|^2 \times G_{\mathrm{F}}^2 \times F_1(\text{masses, slopes}) \times F_2(\text{corrections})^{\dagger}.$$

"from experiment" "calculations"

Apart from F_2

$$\frac{\delta|V_{us}|}{|V_{us}|} = \frac{1}{2}\frac{\delta\Gamma}{\Gamma}.$$

At present, however,

$$\frac{1}{2}\frac{\delta F_2}{F_2} > \frac{1}{2}\frac{\delta\Gamma}{\Gamma}.$$

mostly from $f_+(0)$. But there is hope.

†There are two "F_2" functions for $K_{\mu 3}$ decays

There are 6 "F_2" functions for hyperon decays

Master Formula

$$d\Gamma = \frac{G_F^2\, M_K^5}{768\,\pi^3}\,|V_{us}|^2\, S_{EW}\, |f_+^{K^0}(0)|^2\, C_K^2\, I_K^\ell\, [1+\delta^K_{SU(2)}+\delta^K_{em}]$$

S_{EW}: universal SD em correction at $\mu = m_\rho$, =1.0232

C_K: comb'n of clebsch's, =1 for $K_{S,\,L}$, $1/\sqrt{2}$ for $K^\pm$

$f_+^{K^0}(0) \le 1$: $SU(3)$ breaking correction

I_K^ℓ: Phase space integrals, depend on $\lambda_{+,0}$, etc

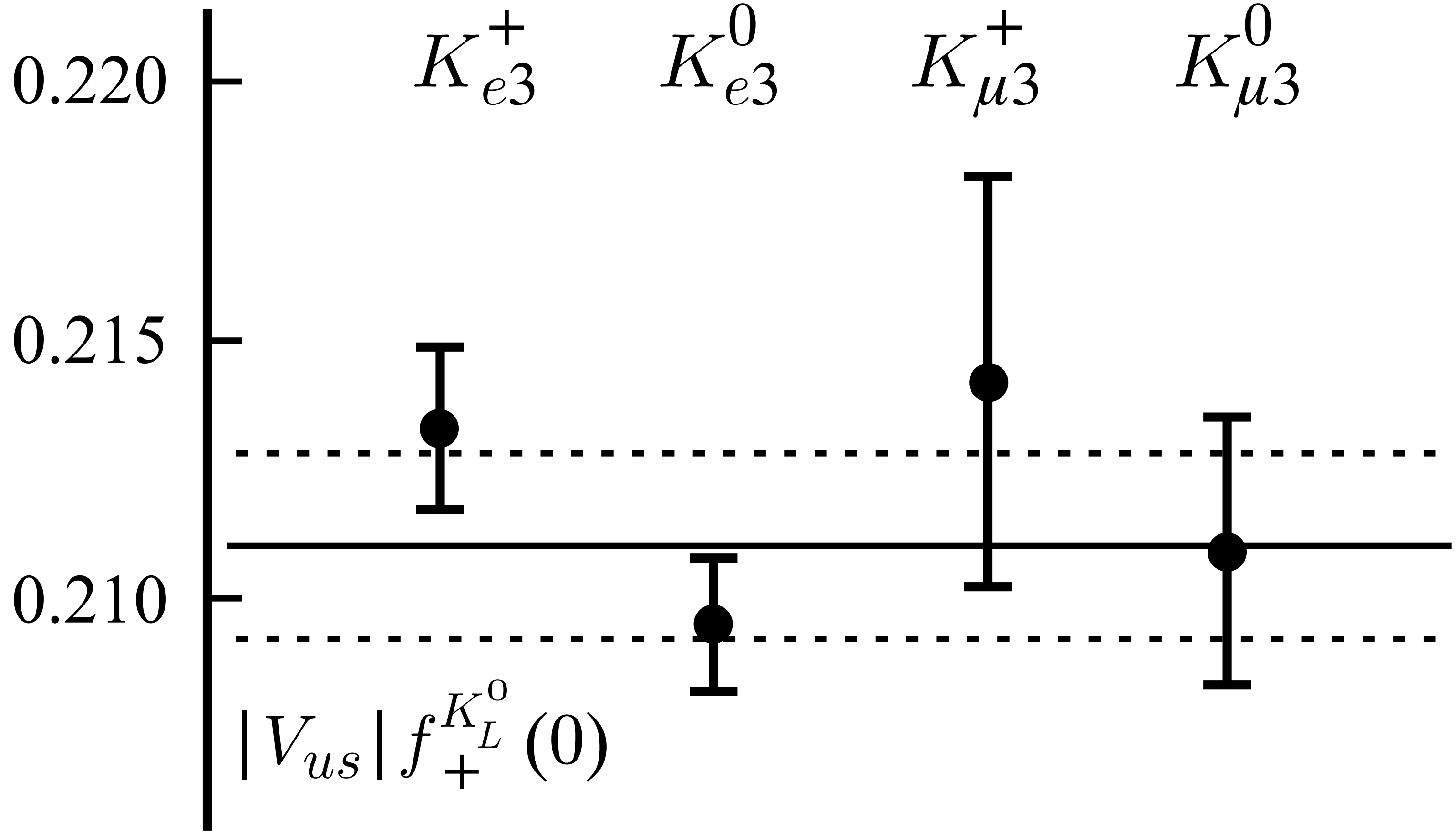
$\delta^K_{SU(2)}$: $SU(2)$ breaking $(m(u) \neq m(d))$ corrections

δ^K_{em}: long distance em correction

$f_+^{K^0}(0) = 0.961 \pm 0.08$, L&R, 1984, is the only widely accepted estimate still. (New lattice result, Becirevic et al., $f_+^{K^0}(0) = 0.960 \pm 0.005 \pm 0.007$. Also good match to $f(t)$ shape.)

Up to 2002

From PDG one then gets



CKM1, G. I. conv., 2002

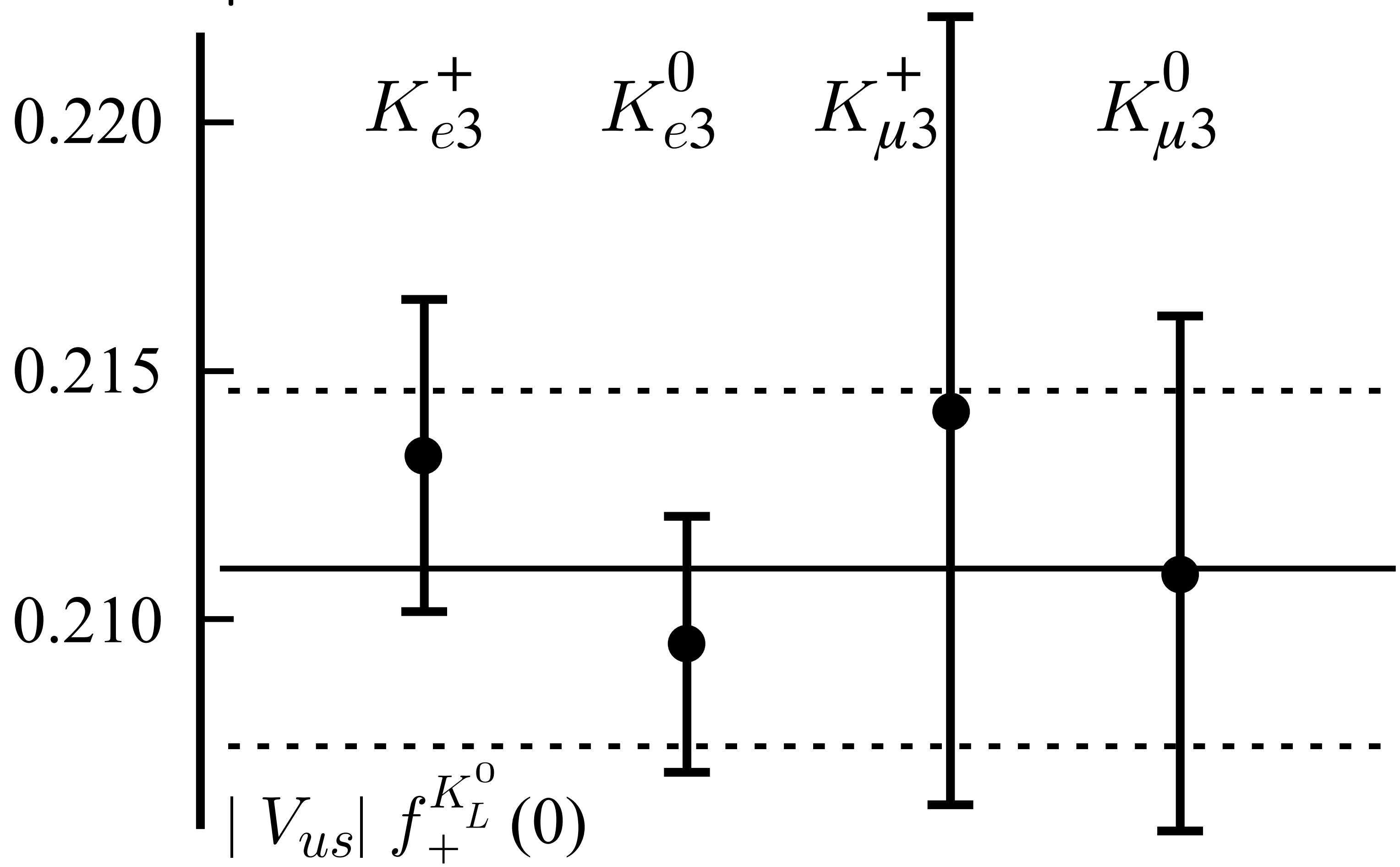
to be compared to $\sqrt{1-V_{ud}^2}\times f_+^{K^0}(0) = 0.2202 \pm 0.0037$

however

1. Γs are not measured. Values and $\delta\Gamma/2\Gamma$=0.5-0.6% - from PDG fit. Taking the PDG average for K_{e3}, the central value is quite different, higher. The error is also much larger.
2. For K_L, BR(e3)/BR(μ3) disagrees with value from slopes, λ_+, λ_0, and '01 KEK measurement by 4.4%
3. $\tau(K^\pm)$ not too reliable; dτ/τ=0.2 or 0.8%?
4. $\tau(K_L)$ is poorly known, dτ/τ=0.8%

It is *possible* that

a more realistic picture is



and the unitarity violation is swallowed by realistic errors.

this is tex

From PDG

$$\frac{\delta V_{us}}{V_{us}} = \frac{1}{2}\frac{\delta\Gamma}{\Gamma} \oplus \frac{1}{20}\frac{\delta\lambda_+}{\lambda_+} \oplus \frac{\delta f_+(0)}{f_+(0)} \qquad (K_{e3})$$

but, too often

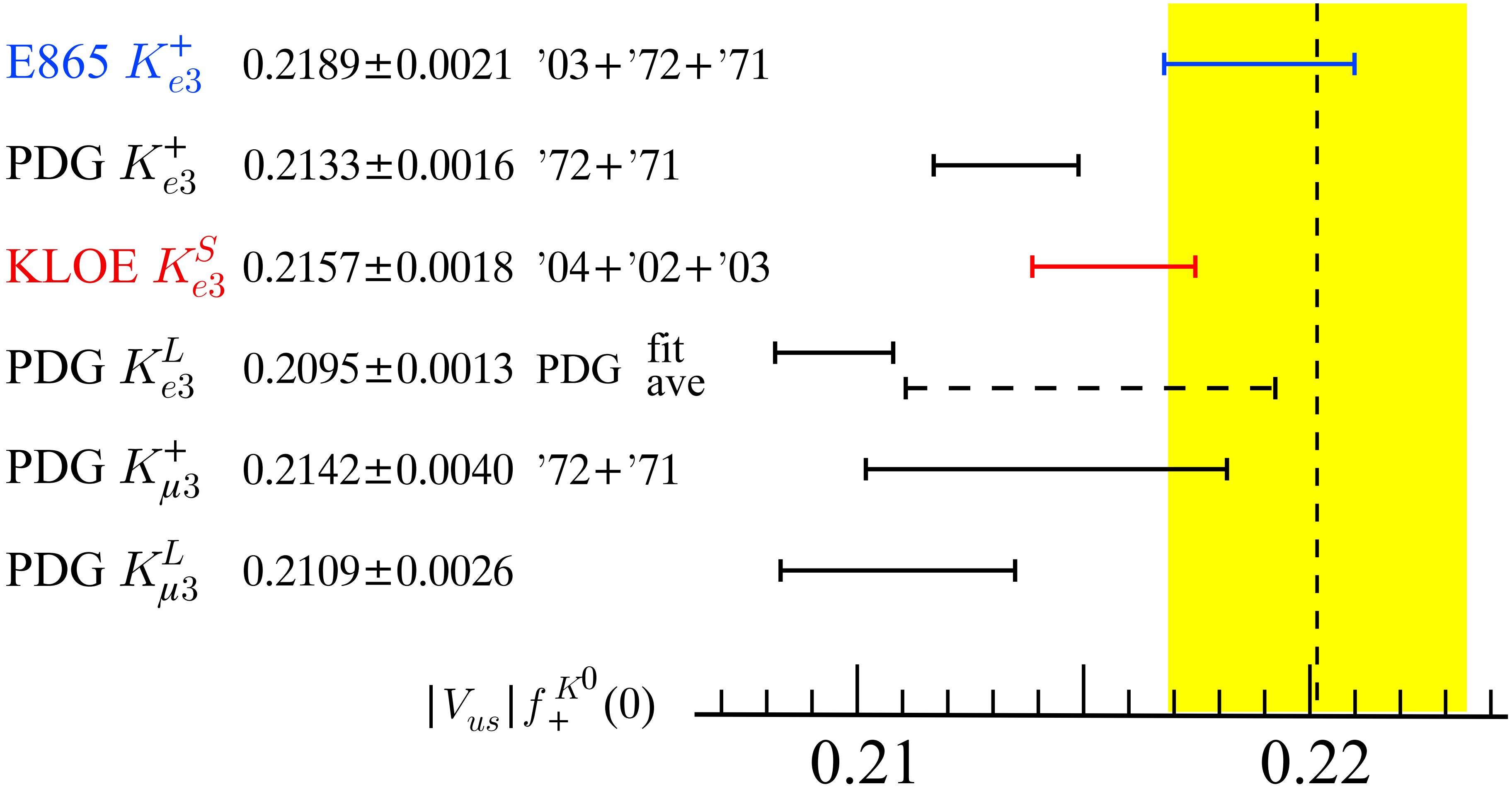
$$\frac{\delta\Gamma}{\Gamma} = \frac{\delta \mathrm{BR}}{\mathrm{BR}}\Big]_{\mathrm{meas.}} \oplus \frac{\delta \mathrm{BR}}{\mathrm{BR}}\Big]_{\mathrm{ref}} \oplus \frac{\delta\tau}{\tau}$$

Must

1. Avoid/improve $[\delta\mathrm{BR}/\mathrm{BR}]_{\mathrm{ref}}$
2. Improve on τ
3. Measure λ_+ (also λ_0, and λ', $\lambda''\ldots$)

There are important question about the slope and curvature parameters, which I will however ignore.

2003 – 2004

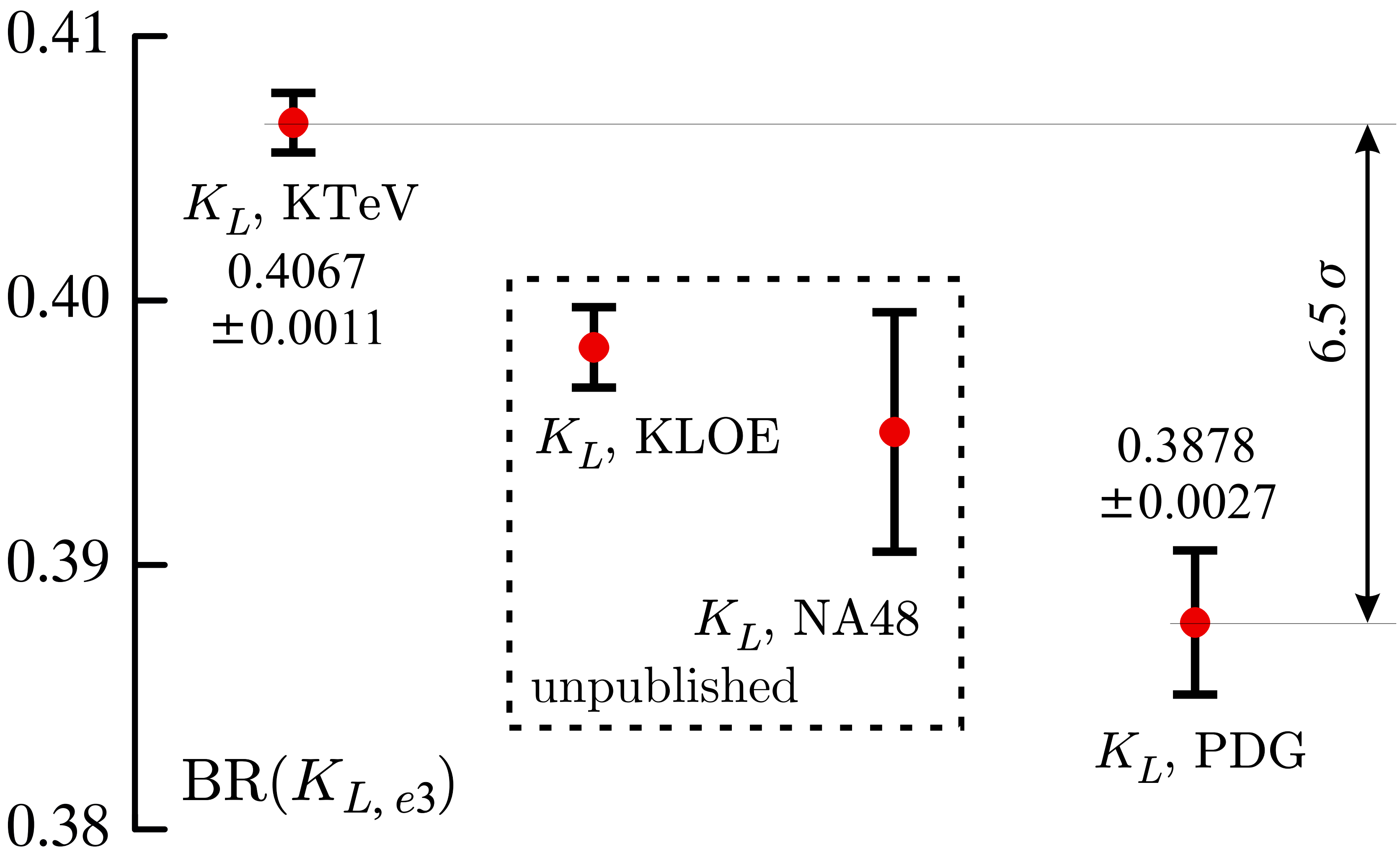


1. No problem with unitarity, with new results.
2. E865 depends on tainted PDG fit results.
3. KLOE, K_S, uses all new BR measurements.

this is tex

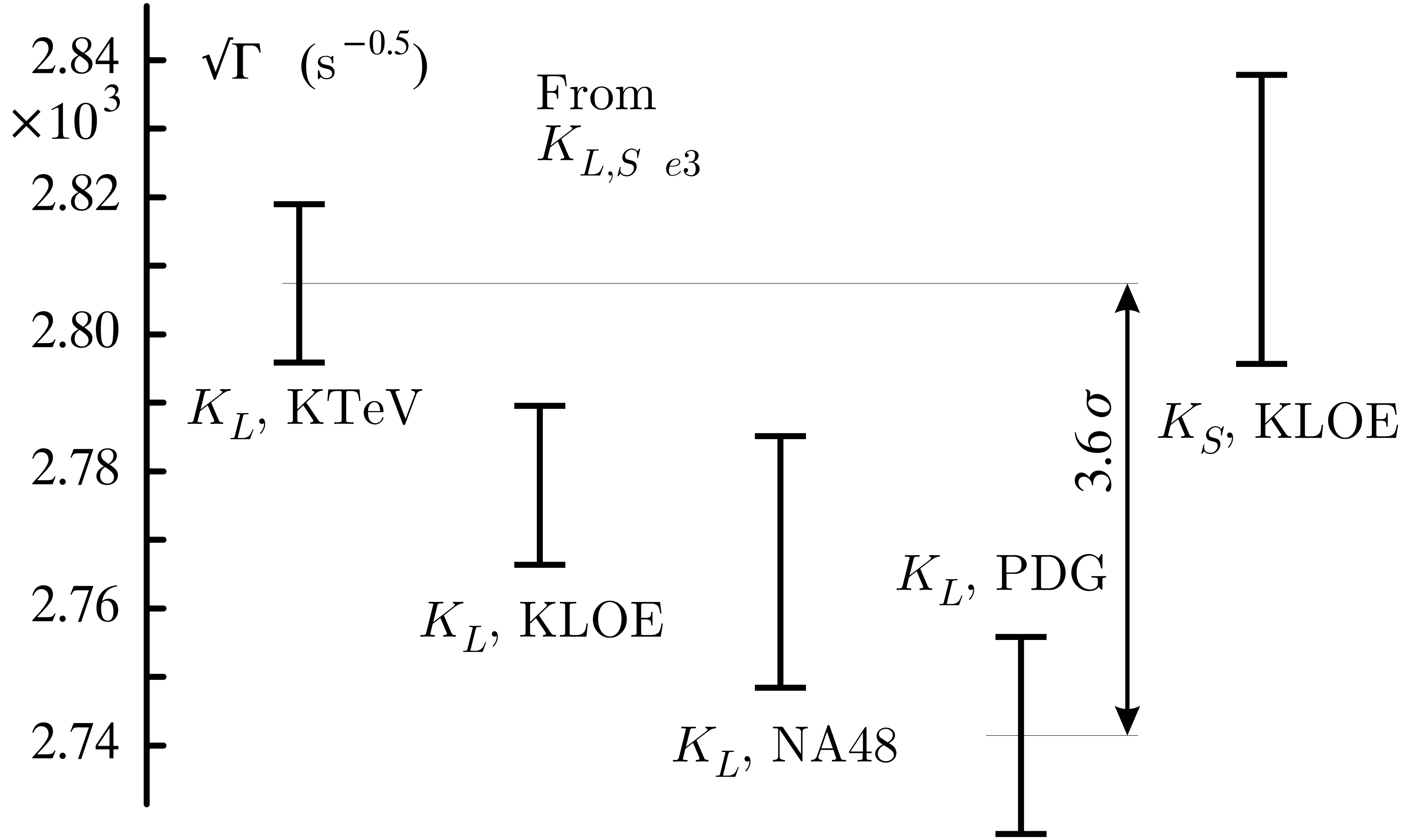
Very recent results

KTeV has just submitted their new results for publication. There is a huge discrepancy with PDG fit. Other results are on the way.

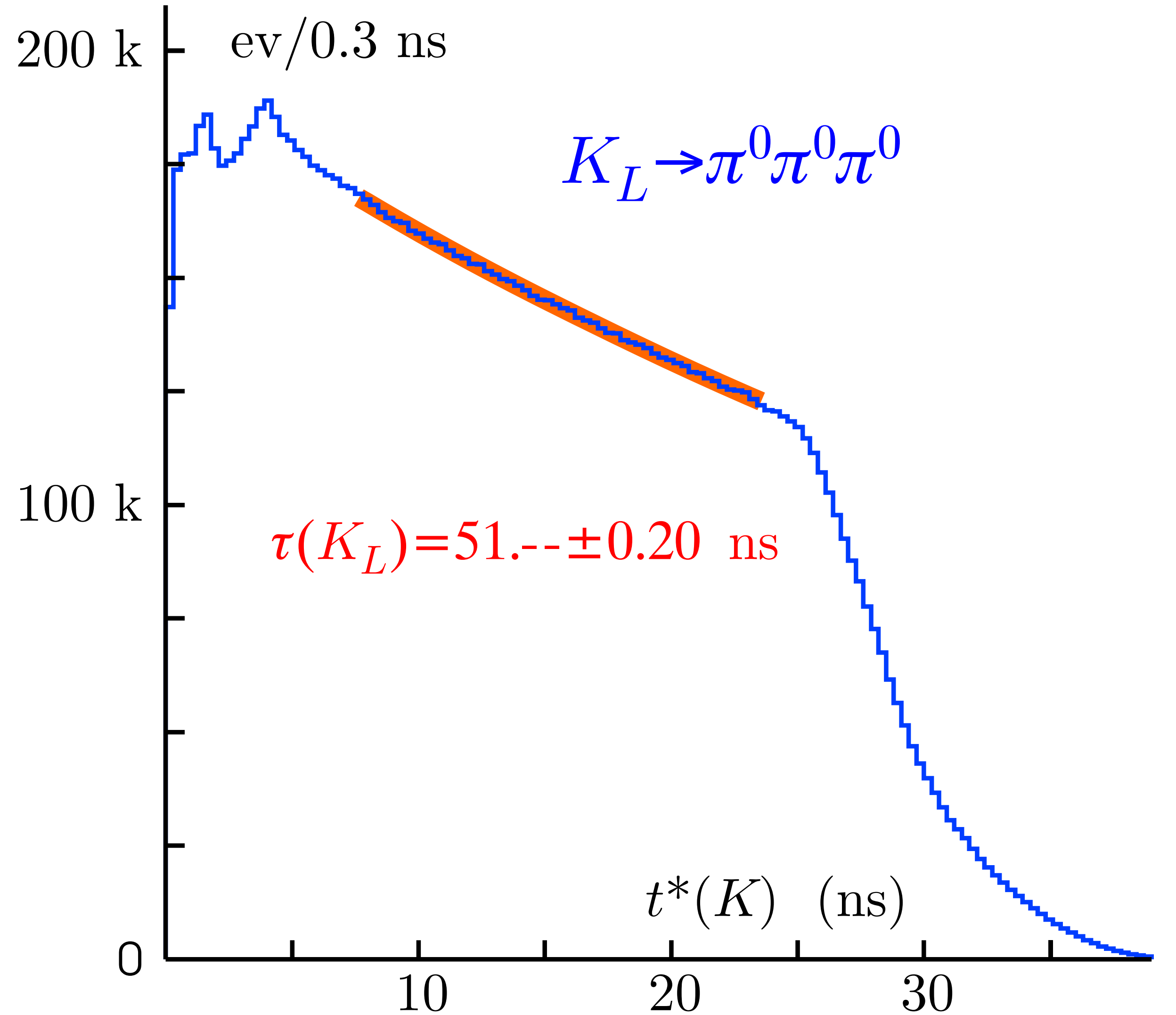


With K_L liftime

The much improved accuracy is however spoiled by the poor knowledge of $\tau(K_L)$



Coming soon

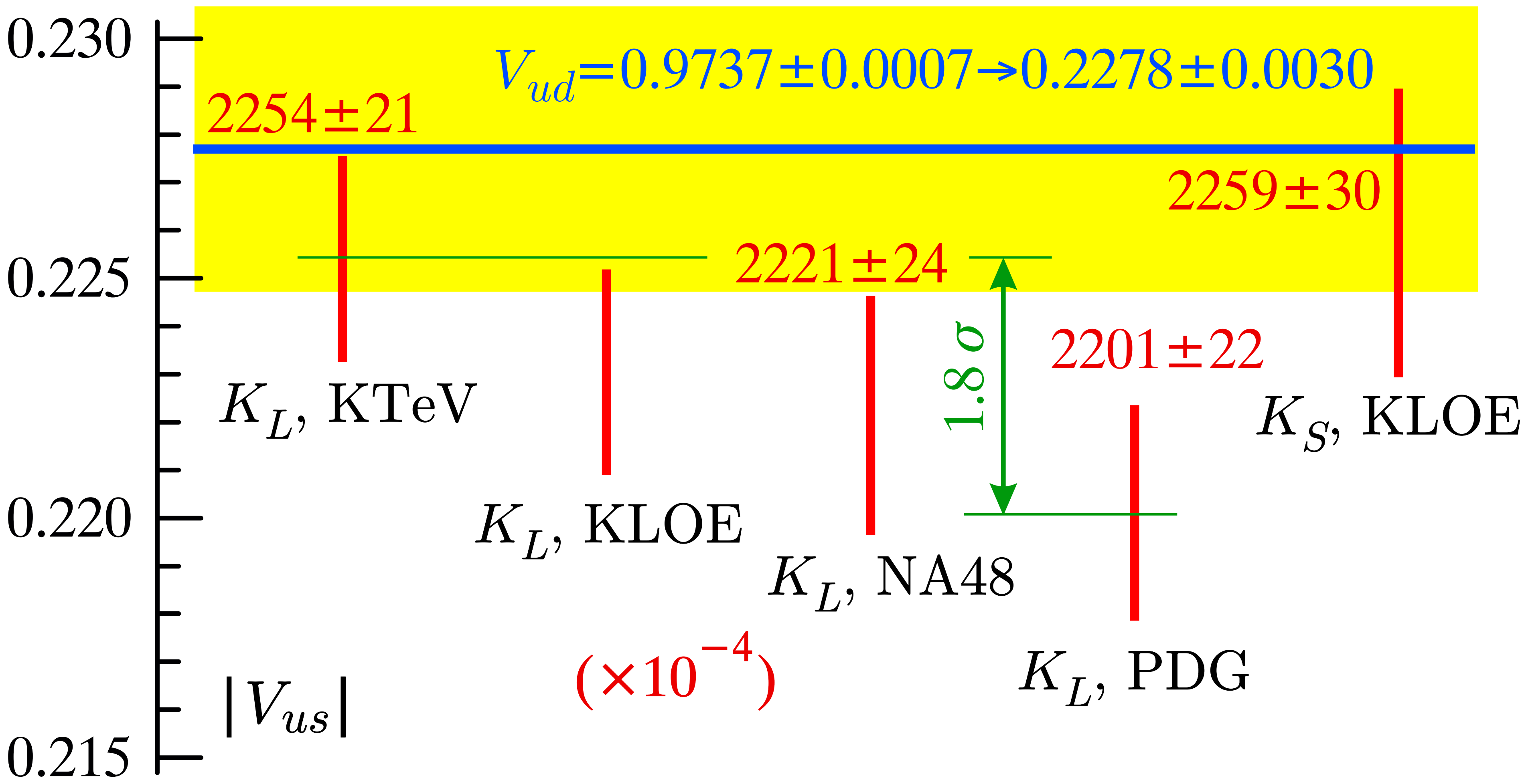


Exp	L
NA48/KTeV	0.01λ
KLOE	0.3λ

PDG (Vosburg, '72): $\tau(K_L) = 51.5 \pm 0.4$ ns

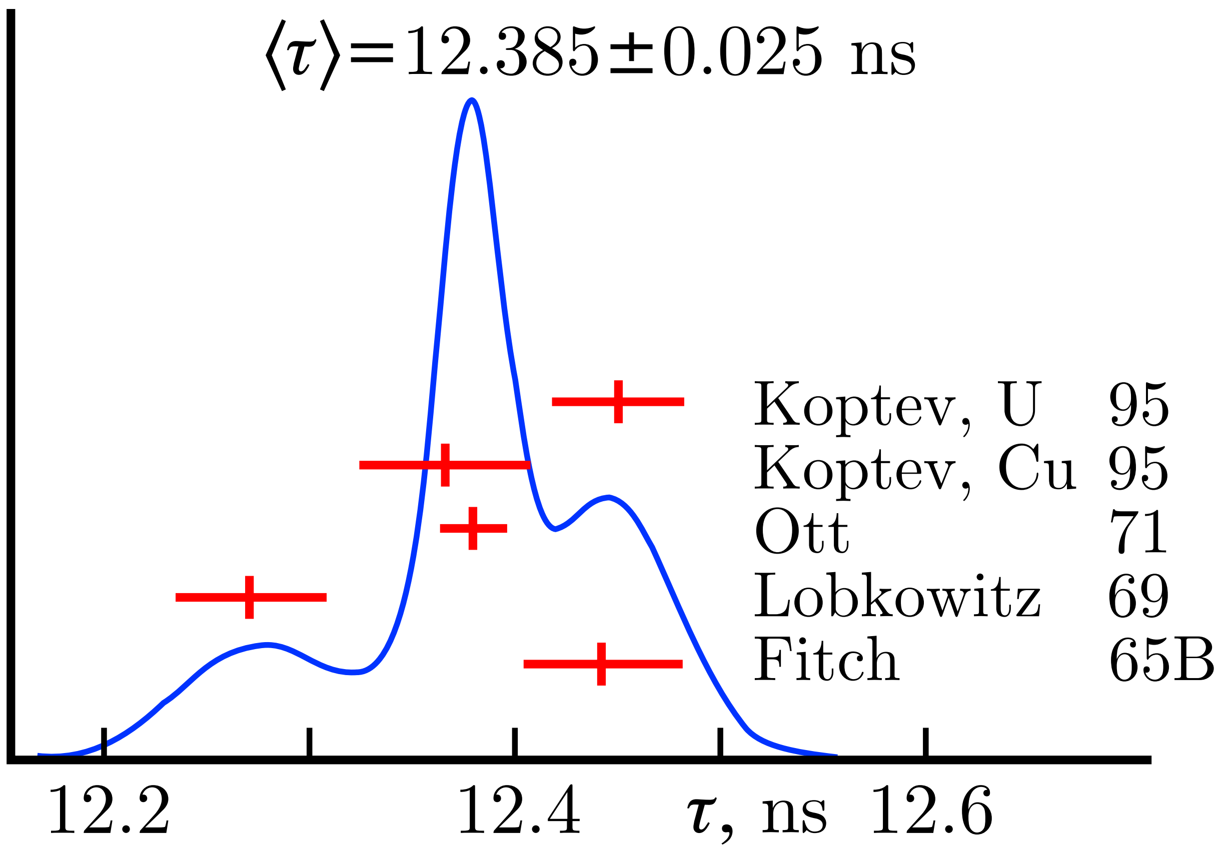
KLOE: $\tau(K_L) = 51.\text{--} \pm 0.20$ ns

V_{us} today



0.230
0.225
0.220
0.215
V_{ud}=0.9737±0.0007→0.2278±0.0030
2254±21
2259±30
2221±24
2201±22
1.8 σ
K_L, KTeV
K_L, KLOE
K_L, NA48
K_L, PDG
K_S, KLOE
(×10^{-4})
$|V_{us}|$

Must check $\tau(K^\pm)$



From PDG

Notes

Ott *et al.*, claim stat. err$< 1/\sqrt{N}$

Koptev is inconsistent with itself

Other ways

- Hyperon leptonic decays. Cabibbo et al. find $V_{us} = 0.2250 \pm 0.002$, without having to apply any $SU(3)$ breaking corrections, estimated to be .975-.987. This is not well understood.
- τ-decays should allow a good determination of the Wsu coupling. The result is $V_{us} = 0.2210 \pm 0.0026$ in poor agreement with unitarity.
- The ratio $\Gamma(\pi \to \mu\nu)/\Gamma(\pi \to \mu\nu)$, using recent calculations of f_K/f_π yields $|V_{us}/V_{ud}|^2$=0.0527±0.0015, in good agreement with unitarity and the value of V_{ud}.

$|V_{us}|$ from $\pi_{\mu 2}$ and $K_{\mu 2}$

$$\Gamma(\pi \to \mu\bar{\nu}_\mu(\gamma)) = \frac{G_\mu^2|V_{ud}|^2}{8\pi} f_\pi^2 m_\pi m_\mu^2 \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2 \left[1 + \frac{\alpha}{\pi} C_\pi\right]$$

$$\Gamma(K \to \mu\bar{\nu}_\mu(\gamma)) = \frac{G_\mu^2|V_{us}|^2}{8\pi} f_K^2 m_K m_\mu^2 \left(1 - \frac{m_\mu^2}{m_K^2}\right)^2 \left[1 + \frac{\alpha}{\pi} C_K\right]$$

$$\frac{\Gamma(K \to \mu\nu(\gamma))}{\Gamma(\pi \to \mu\nu(\gamma))} = \left|\frac{V_{us}}{V_{ud}}\right|^2 \times G_1(\text{masses, etc}) \times \frac{f_K}{f_\pi} \times G_2(\text{corrections})$$

$V_{us} = 0.2238 \pm 0.0003_{\text{exp!!??}} \pm 0.0004_{\text{rc}} \pm 0.0030_{\text{LQCD}}$ – Davies et al., (2004)

W. Marciano

$\phi \to K_S K_L$

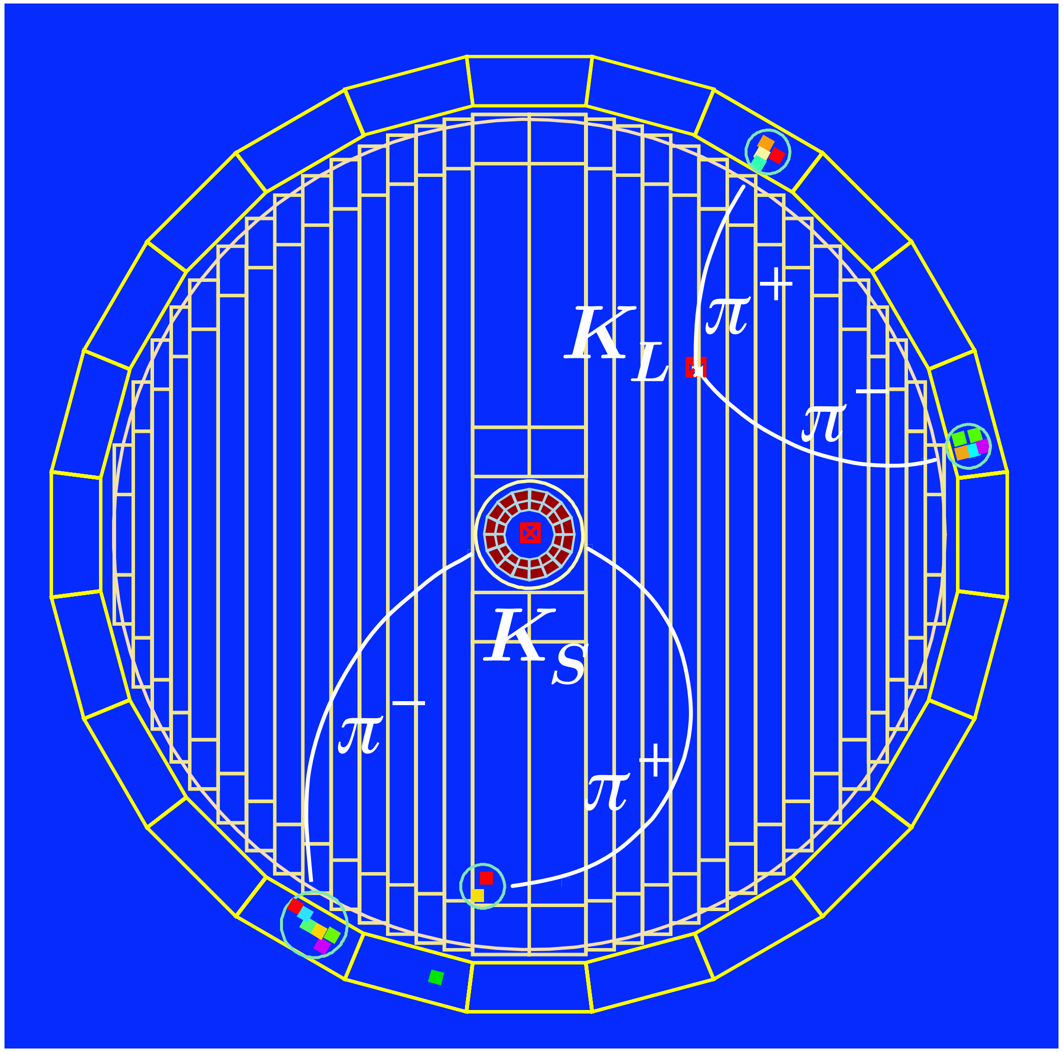
From $e^+e^- \to \phi \to K_S K_L$ one gets

1. Monochromatic
2. Pure
3. Tagged

beams of K_L, K_S and $K^{\pm}$ mesons.

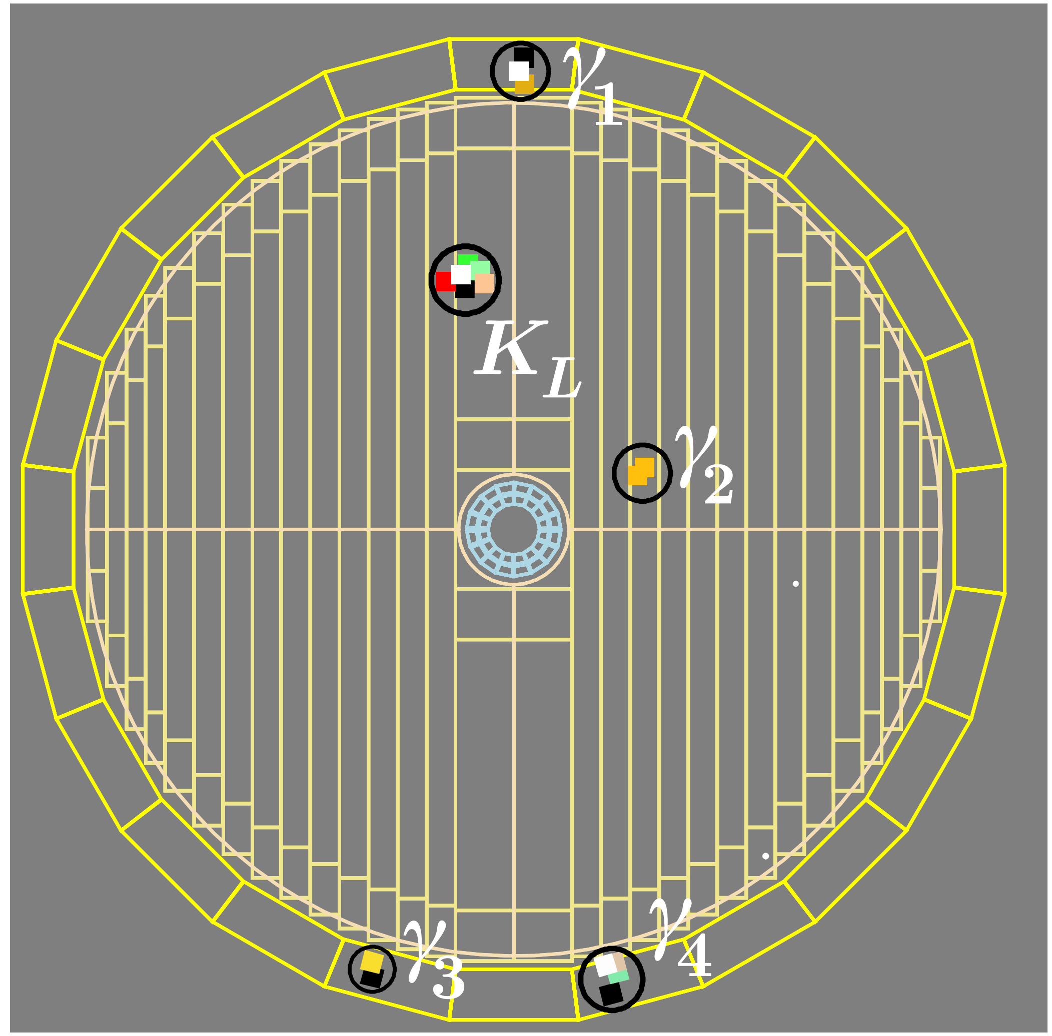
This offers unique possibility for measuring absolute branching ratios as well as lifetimes of all kaon species.

The first events



$K_S \to \pi^+\pi^-$, $K_L \to \pi^+\pi^-$

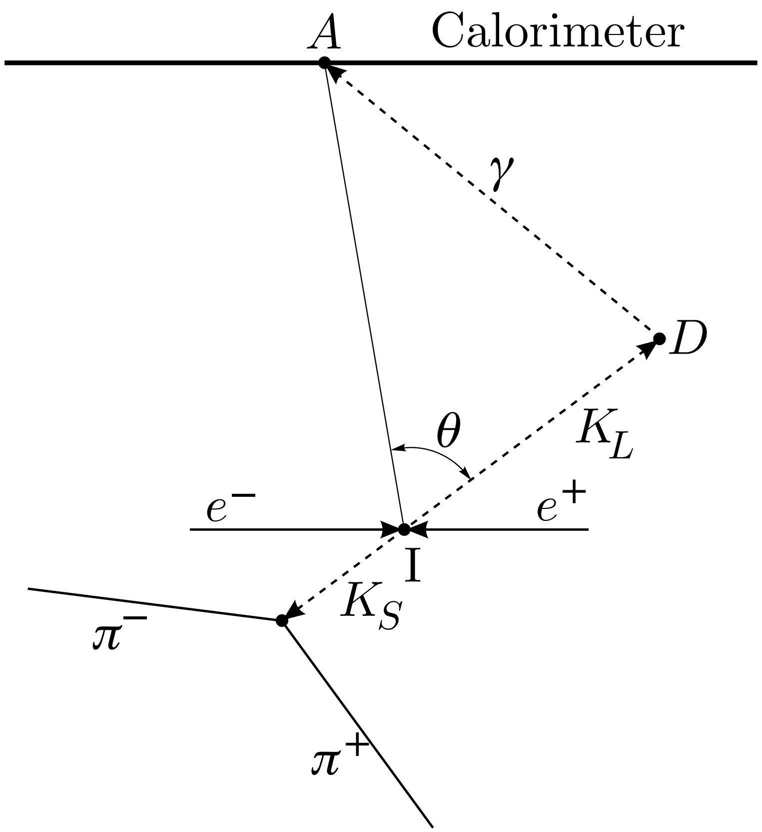
$\not{C}\not{P}$



$K_S \to \pi^0\pi^0$, K_L interacts in ECal

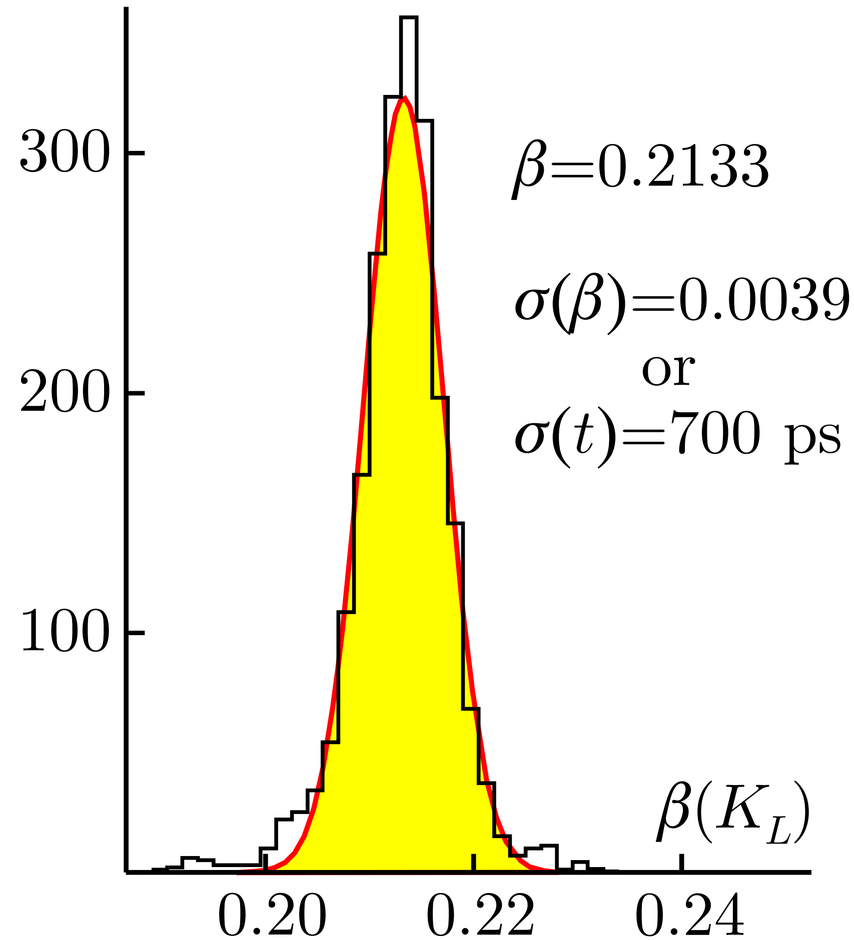
CP-conserving

KLOE performance



For tagged K_L measure path from $t(I \to A)$ and K_S direction

Tag K_S from "K_L-*crash*"

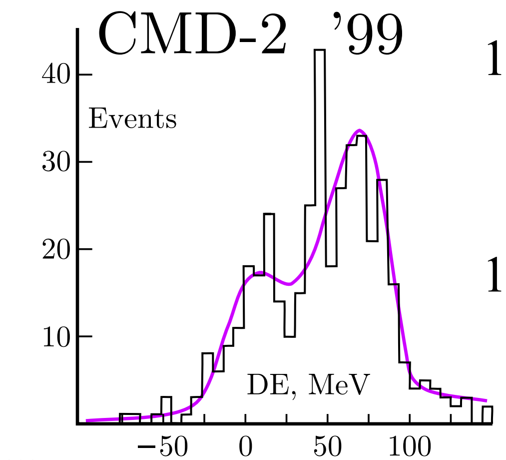


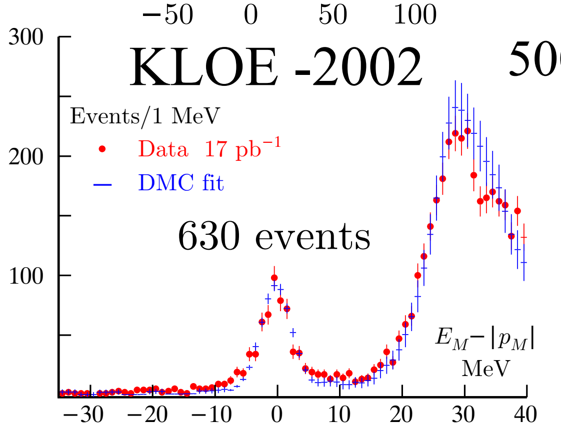
δW(DAΦNE)=1 MeV gives $\delta\beta$=0.004

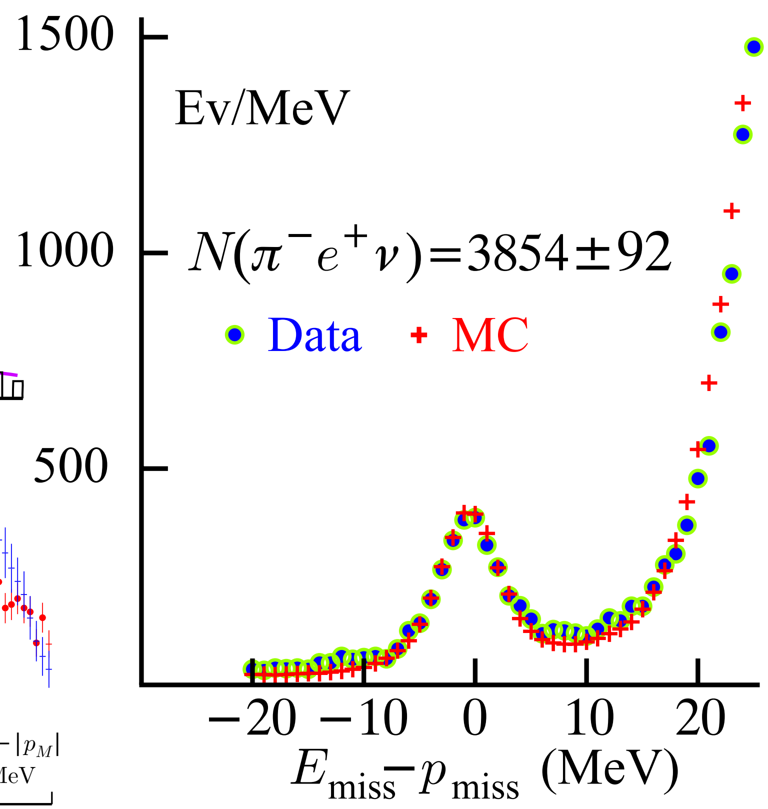
Pure K_S beam

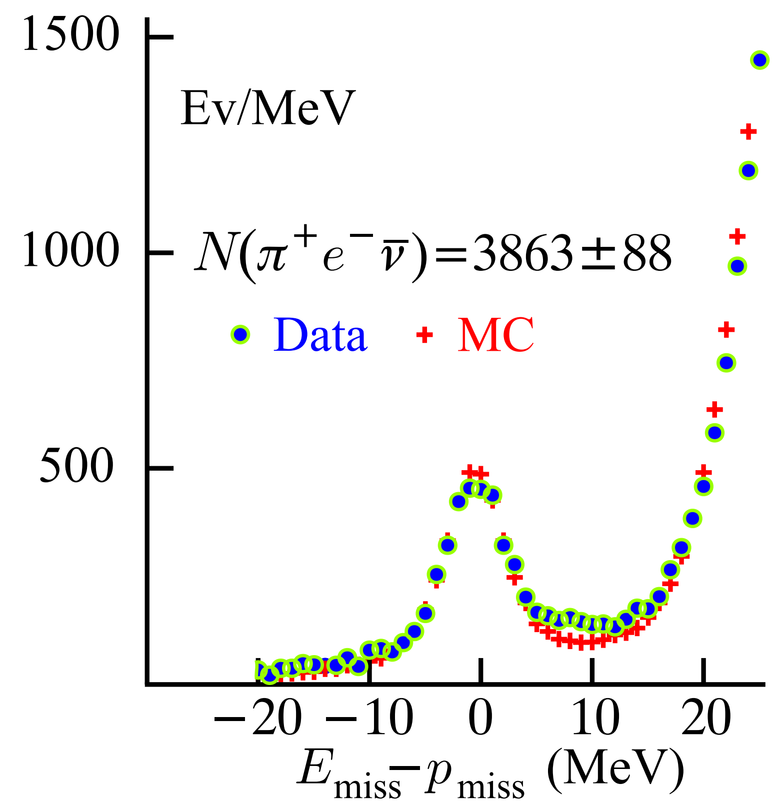
$K_S \to \pi e \nu$

For tagged K_S, $\pi^+ e^- \bar{\nu}$ and $\pi^- e^+ \nu$ are identified by TOF(+), TOF(−) and kinematics closure.



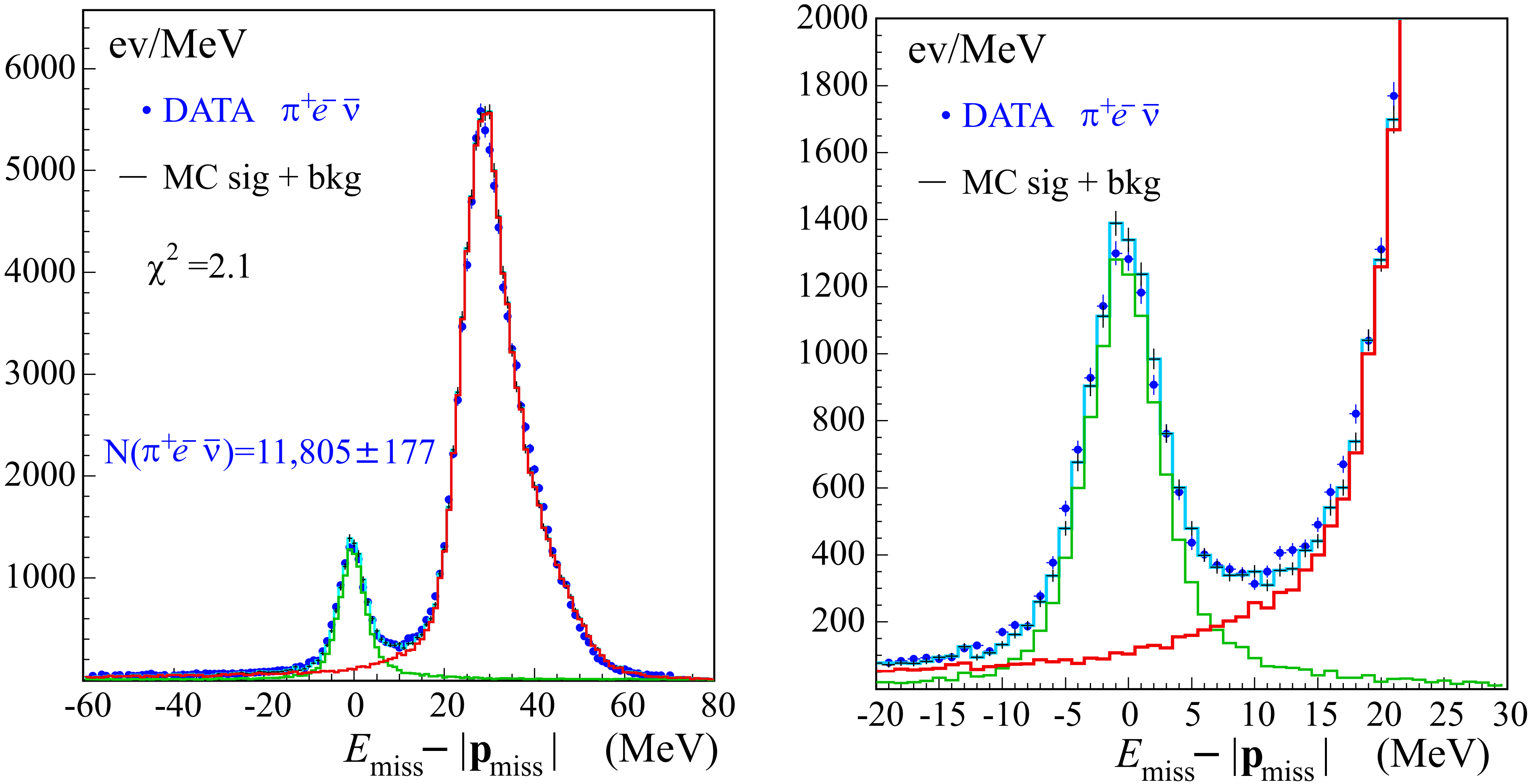






$K_S \to \pi e \nu(\gamma)$

IR finite radiative corrections are necessary for 1. agreement with shape, 2. correct event counting and 3. determination of BR($\pi e \nu[\gamma]$).



Charge asymmetry in $K_S \rightarrow \pi e \nu$

$\mathrm{BR}(\pi^- e^+ \nu) = (3.54 \pm 0.05 \pm 0.05) \times 10^{-4}$

$\mathrm{BR}(\pi^+ e^- \bar{\nu}) = (3.54 \pm 0.05 \pm 0.04) \times 10^{-4}$

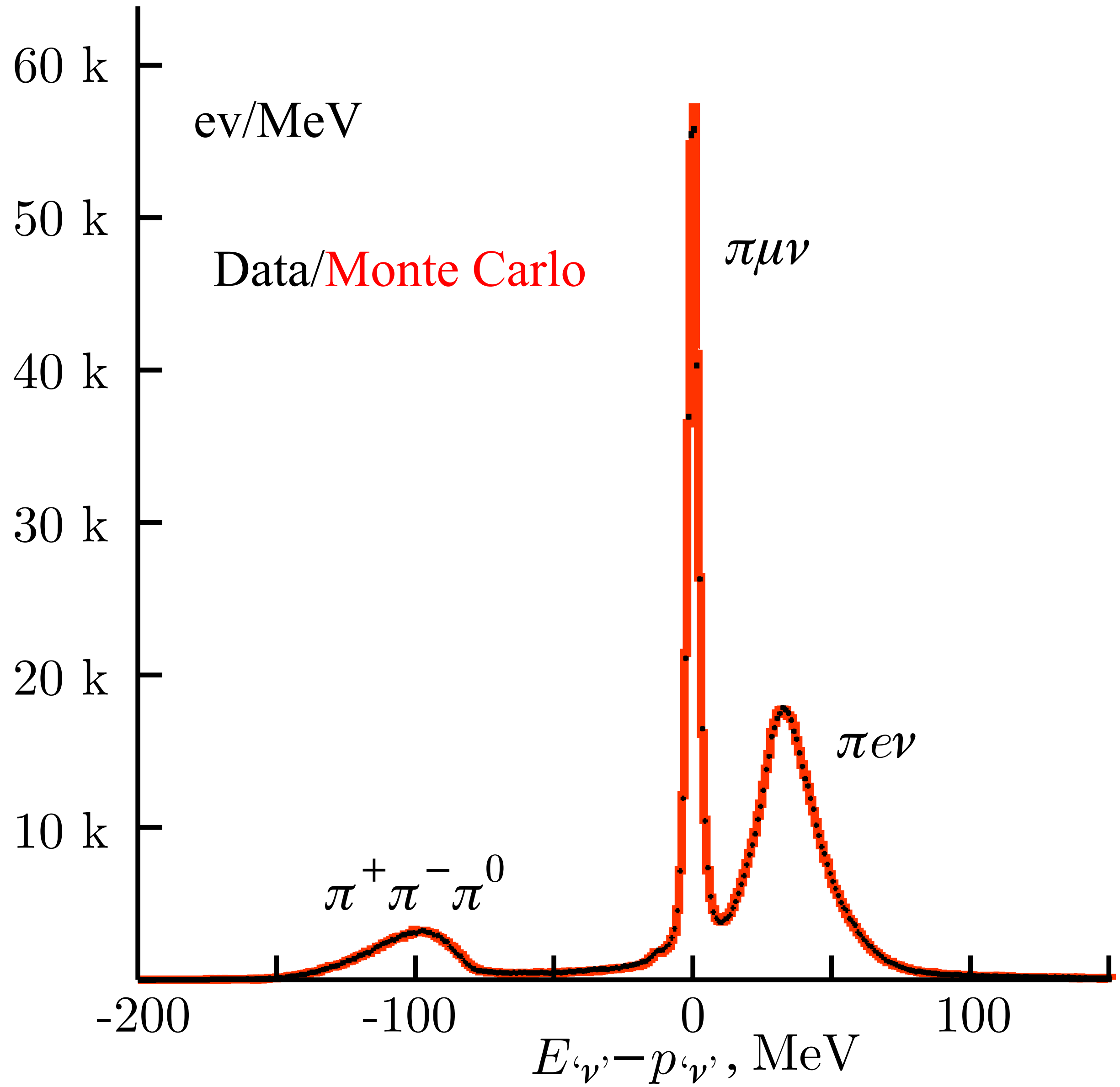
$\mathrm{BR}(\pi e \nu) = (7.09 \pm 0.07 \pm 0.08) \times 10^{-4}$

$$A = \frac{N^+ - N^-}{N^+ + N^-} = 2(\Re\epsilon \pm \Re\delta + \Re y \pm \Re x_-)$$

$A_S = (-2 \pm 9 \pm 6) \times 10^{-3}$ KLOE, 1st meas.

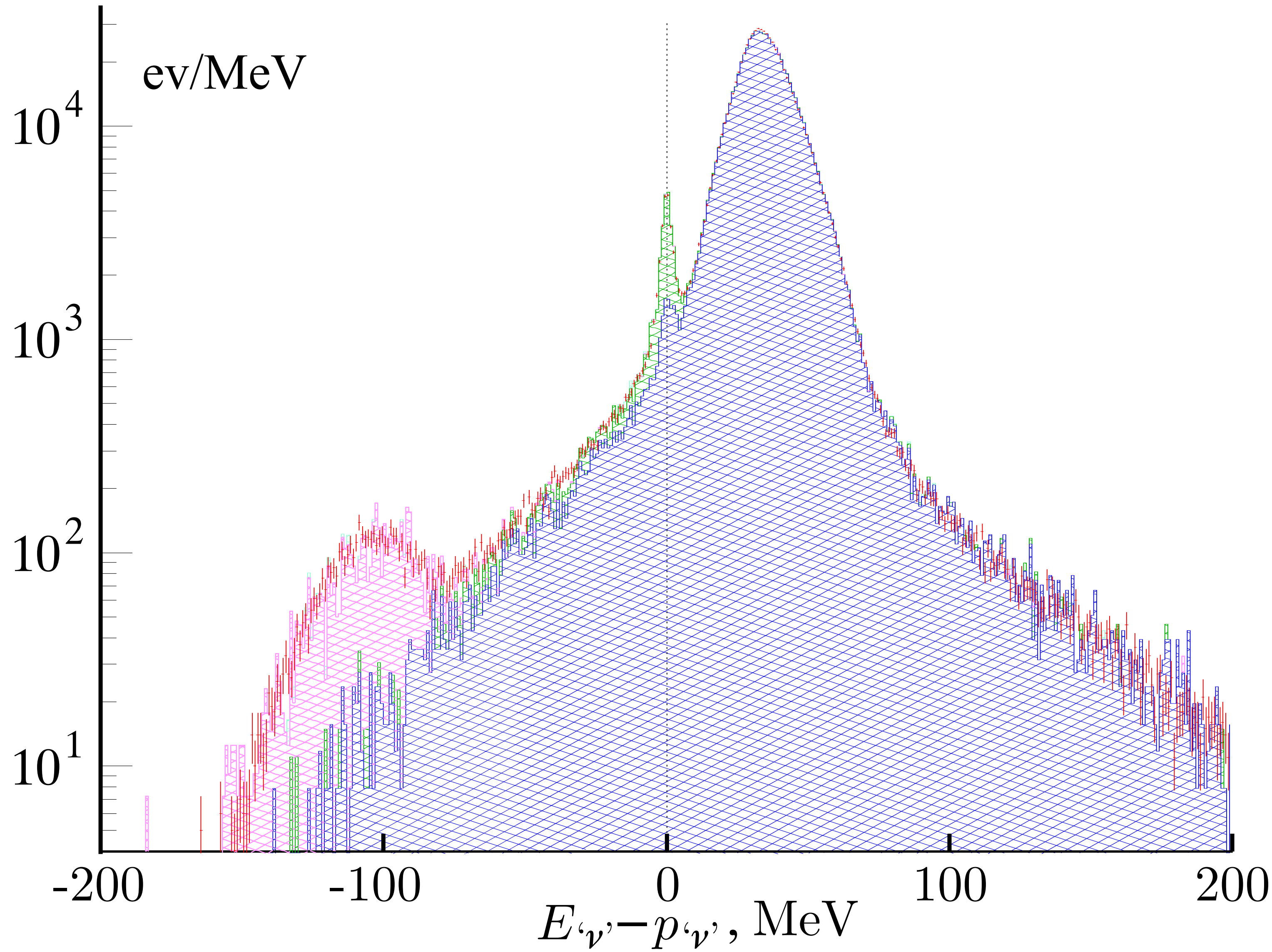
$A_L = (3.322 \pm 0.058 \pm 0.058) \times 10^{-3}$ KTeV

$K_L \to \pi \ell \nu(\gamma)$ with KLOE



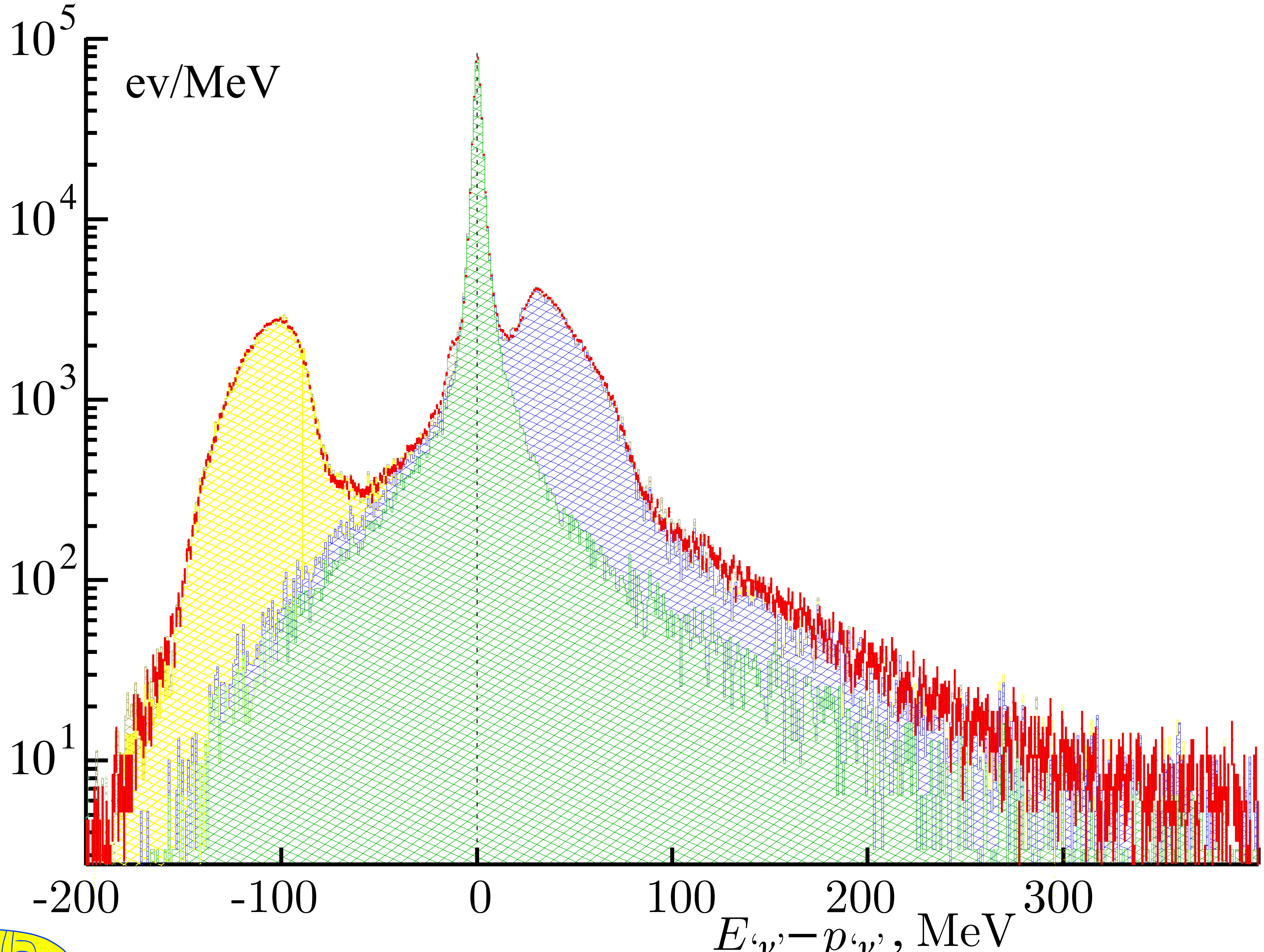
this is tex

$K_L \to \pi \ell \nu(\gamma)$, MC comparison

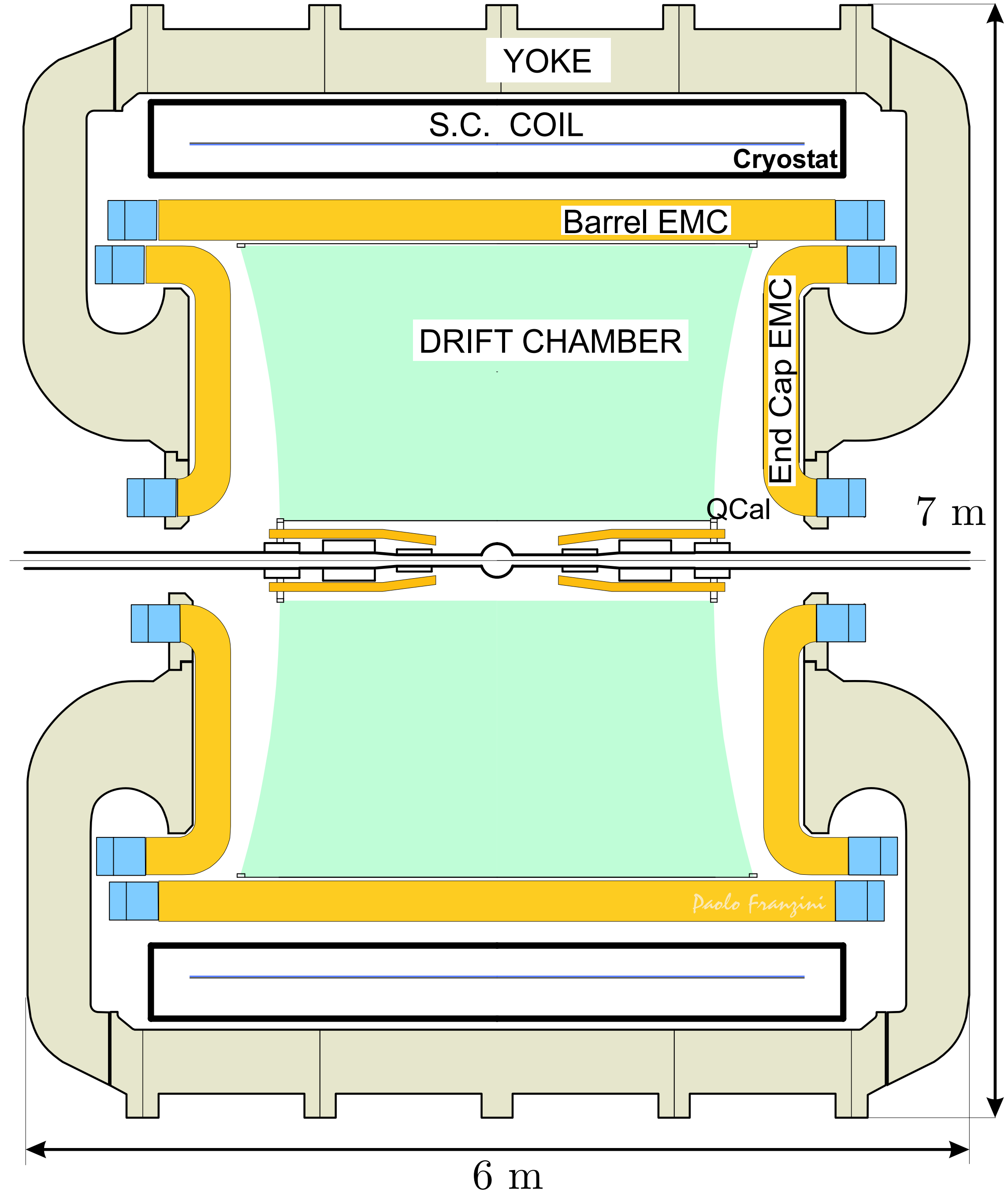


this
is
tex

$K_L \to \pi\ell\nu(\gamma)$, MC comparison



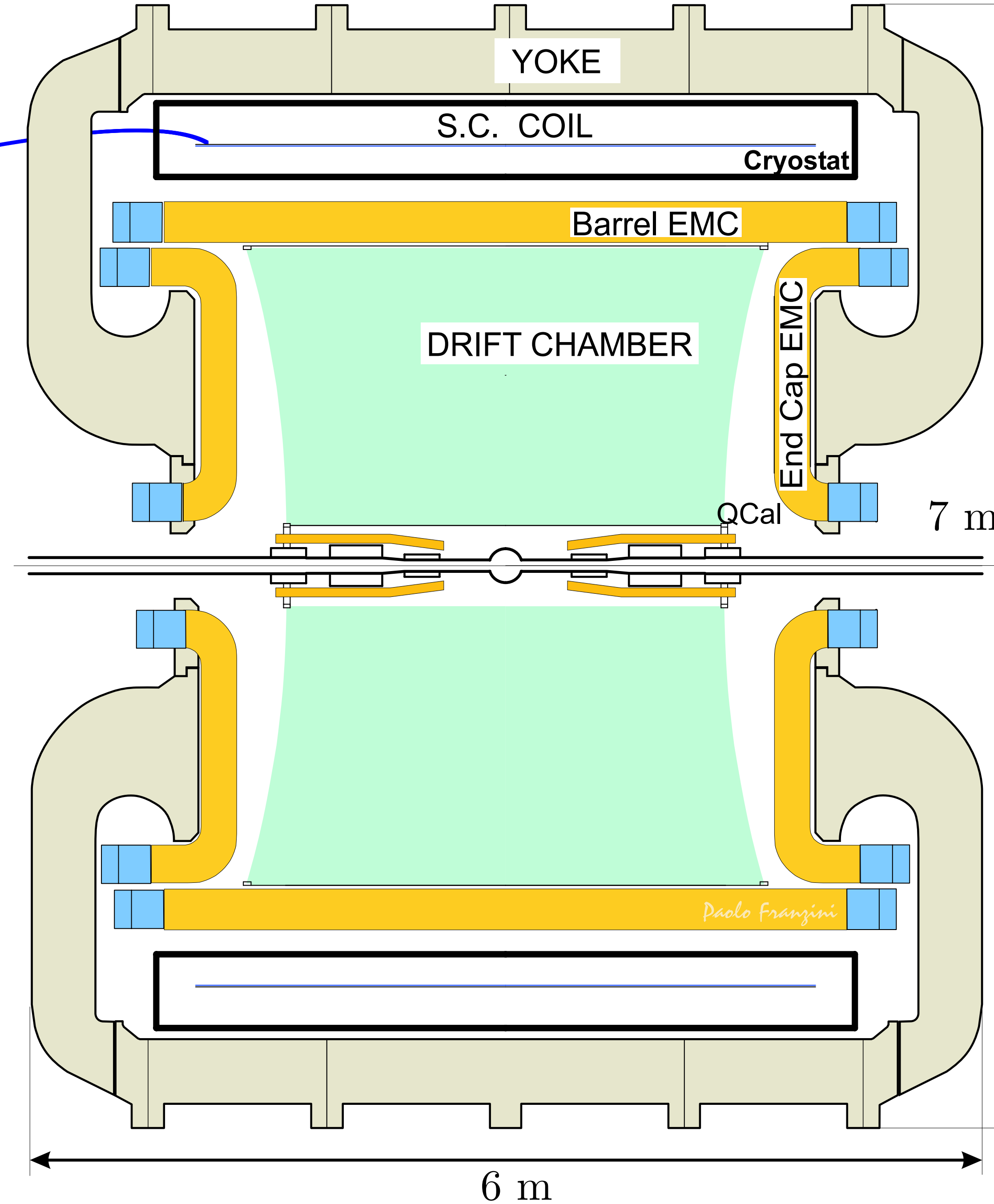
KLOE



this
is
tex

Magnet

SC Coil, B=0.6 T



this is tex

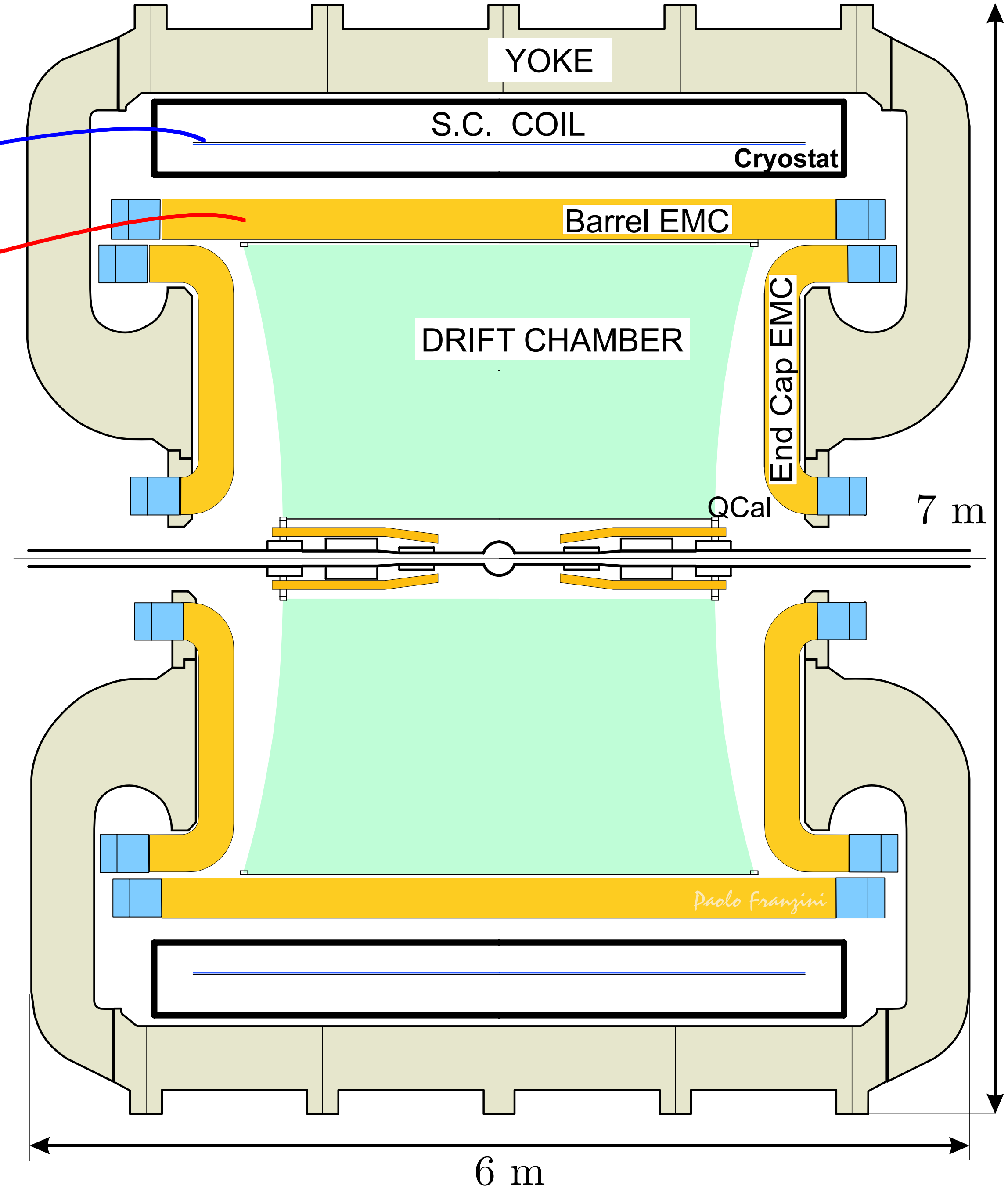
Magnet

SC Coil, B=0.6 T

EM Calor.

Pb–scint fiber

4880 pm



this
is
tex

Magnet

SC Coil, B=0.6 T

EM Calor.

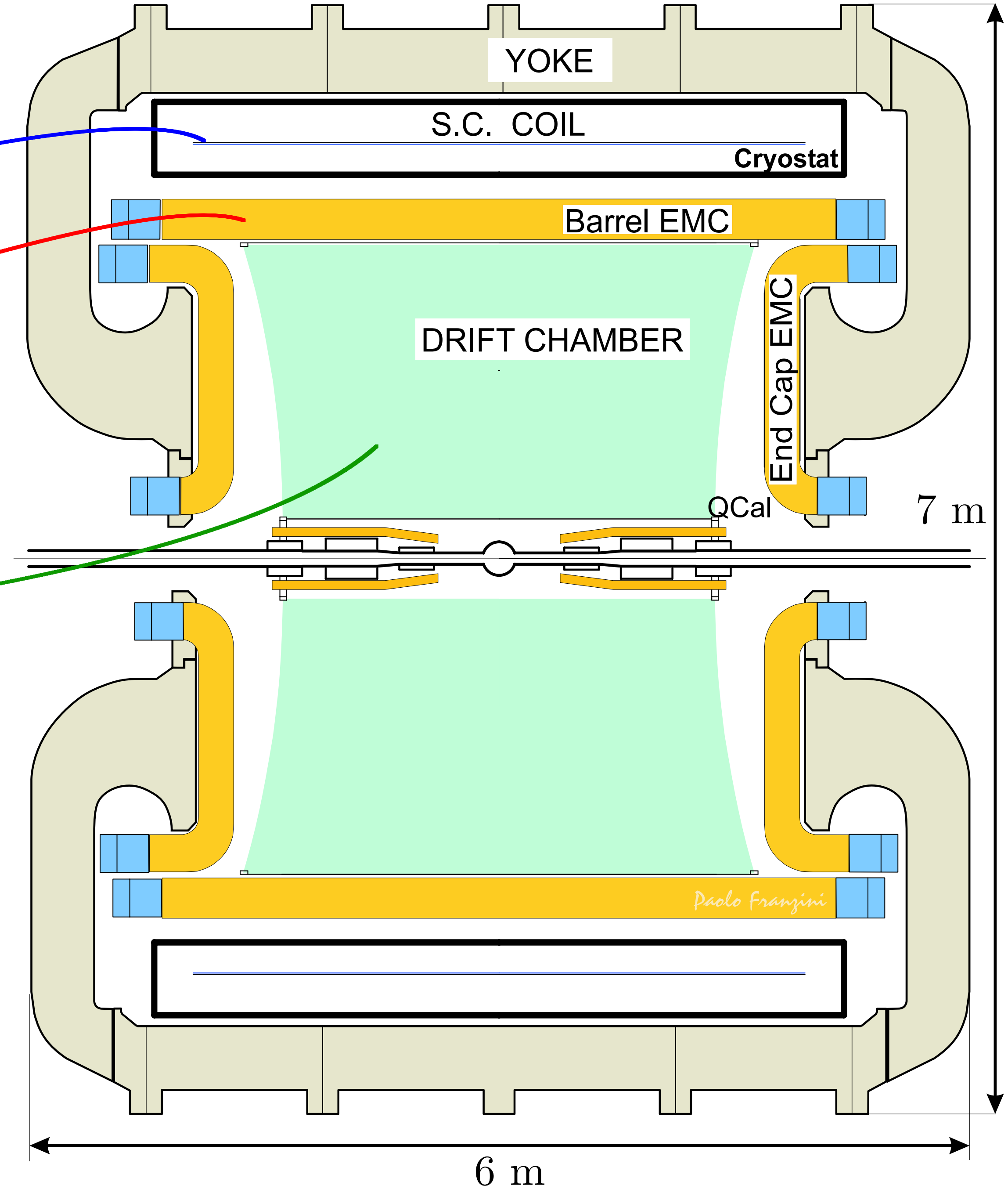
Pb–scint fiber

4880 pm

Drift Ch.

12582 sense wires

52140 tot wires



Magnet

SC Coil, B=0.6 T

EM Calor.

Pb–scint fiber

4880 pm

Drift Ch.

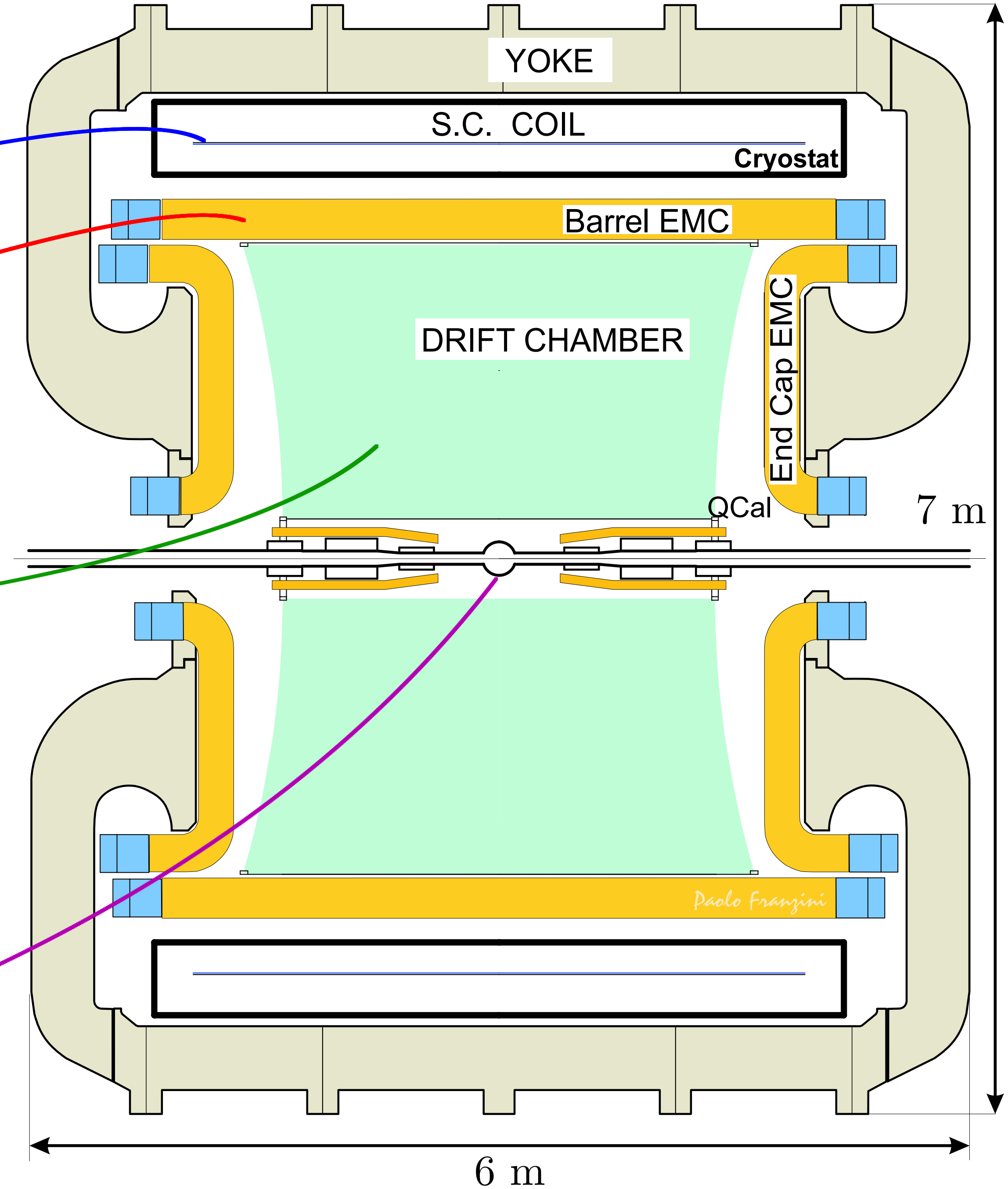
12582 sense wires

52140 tot wires

dE/dx

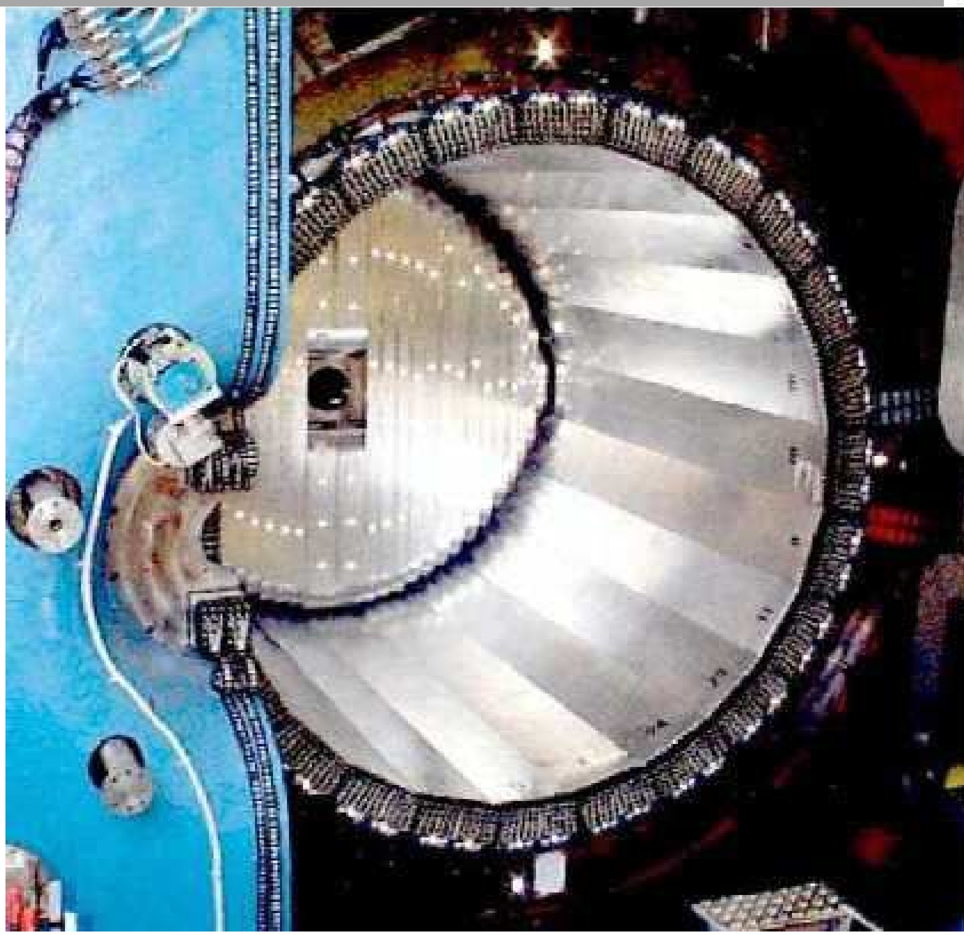
Al-Be beam pipe

r=10 cm, 0.5 mm thick



Drift chamber

$$\sigma_E/E = 5.7\%/\sqrt{E\ (\mathrm{GeV})}$$
$$\sigma_t = 54/\sqrt{E\ (\mathrm{GeV})}\ \mathrm{ps}$$



$$\sigma(p_\perp)/p_\perp = 0.4\%$$
$$\sigma_{x,y} = 150\ \mu\mathrm{m};\ \sigma_z = 2\ \mathrm{mm}$$

EM Calorimeter

To get to $\delta|V_{us}|/|V_{us}|$ of $\mathcal{O}(0.1\%)$

Must measure

1. $\Gamma(K_L,\ K^{\pm})_{e,\mu 3}$ to $\leq 0.2\%$
2. $\tau(K^{\pm},\ K_L)$ to $\leq 0.2\%$
3. $\lambda_+(K_L,\ K^{\pm})$ to $\leq 4\%$
4. $\lambda_0(K_L,\ K^{\pm})$ to $\leq 4\%$

KTeV, NA48, KLOE

Must compute

I-spin corrections to $<0.1\%$

SU(3) corr., $f(0)$ to $<0.1\%$

f_K/f_π to $<0.1\%$

LATTICE?

χPT?

Lattice results appears today particularly promising. $f_+(0)$ has been computed on the lattice by the Rome group with the same result as L&R, 2 unquenched calculations under way. f_K/f_π.

THEN

1. Comparison of K_L and $K^{\pm}$ verifies I-spin corrections
2. $|V_{us}|$ with $|V_{ud}|$ verifies $SU(3)$ corrections.

And, when you can believe everything, you can check unitarity.

The rewards are large, whether it is chiral perturbations or lattice calculations, one can check for the first time calculations of hadronic effects. It is quite time that we learned how to do them.

A gross violation of CKM unitarity would certainly be a big surprise. There are quantities, such as the K_S-K_L mass difference and the branching ratio for $K_L \to \mu^+ \mu^-$, which make such surprises quite unlikely.

I would be very careful before accepting the present, almost gone, discrepancy 1-$|V_{ud}|^2 - |V_{ud}|^2$=.99??$\pm$0.00??. Better measurements are becoming available and we need more experience in computing corrections both for β-decay and strange decays.

"Possible" problems with the quark mixing scheme, as well as with the SM, have gone away thus far.

THE END

What form factor?

$$f(t) = 1 + b \times t/m(\pi^2) + d \times t^2/m(\pi^4) \quad \text{or} = M^2/(M^2 - t)$$

$$\int\!\!\int \rho \, \mathrm{d}x \, \mathrm{d}z =$$

$$0.563371 + 1.9467\, b + 2.6990\, b^2 + 5.37985\, d + 18.5425\, bd + 36.4182\, d^2 =$$

$$0.563371 + 0.055481 + 0.002185 + 0.004035 + 0.000396 + 0.000020$$

$$8.87\% \qquad 0.35\% \qquad 0.65\% \qquad 0.06\% \qquad 0.003\%.$$

Fits with slope, slope + curvature and pole all give different phase space integrals.

Correlations

$$\begin{pmatrix} \overline{(\delta\lambda)^2} & \overline{\delta\lambda\delta\lambda'} \\ \overline{\delta\lambda'\delta\lambda} & \overline{(\delta\lambda)^2} \end{pmatrix} = \frac{1}{N}\begin{pmatrix} 0.475 & -0.121 \\ -0.121 & 0.035 \end{pmatrix} = \frac{1}{N}\begin{pmatrix} 0.689^2 & -0.121 \\ -0.121 & 0.187^2 \end{pmatrix}$$

Compared to $\delta\lambda = 0.24/\sqrt{N}$ and $\delta\lambda' = 0.065/\sqrt{N}$ for no correlations, both the λ and λ' errors are approximately tripled and the correlation is ~100% in the PDG notation.