

Energy Loss

Peter Fisher

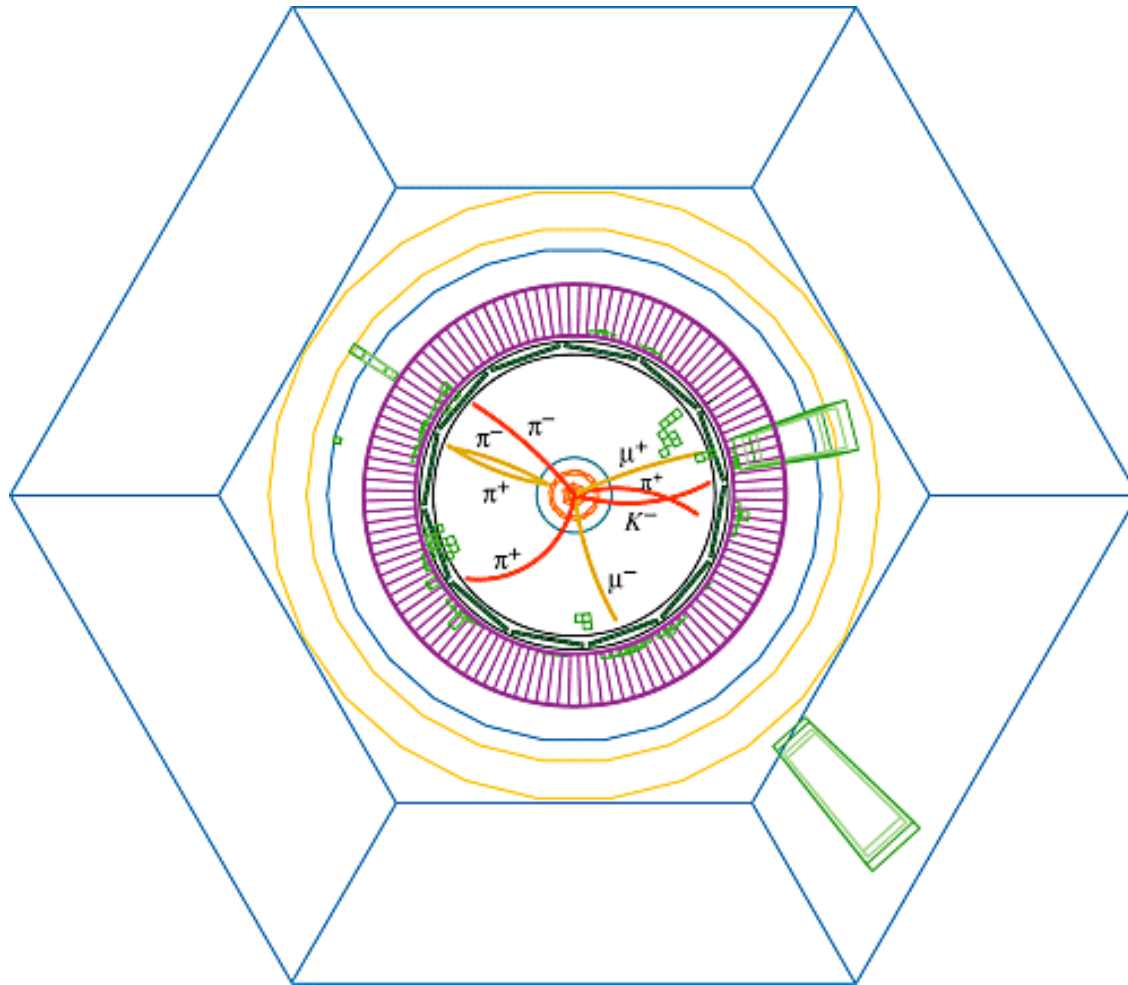
MIT

August 13, 2009

Outline

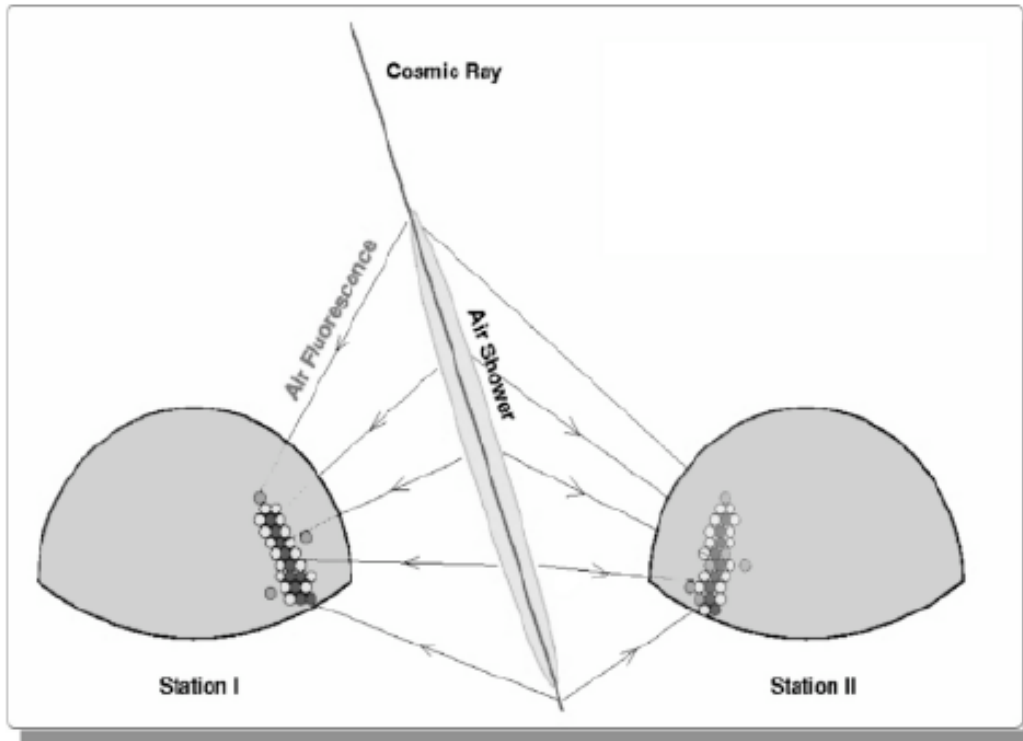
- Why study energy loss?
- Cases
 - Ionization, TPCs
 - Cerenkov, very high energy neutrinos
 - Transition, TRT, AMS/TRD
 - Nuclear Recoil, CDMS, CRESST
- Key numbers
- References

Why study energy loss?

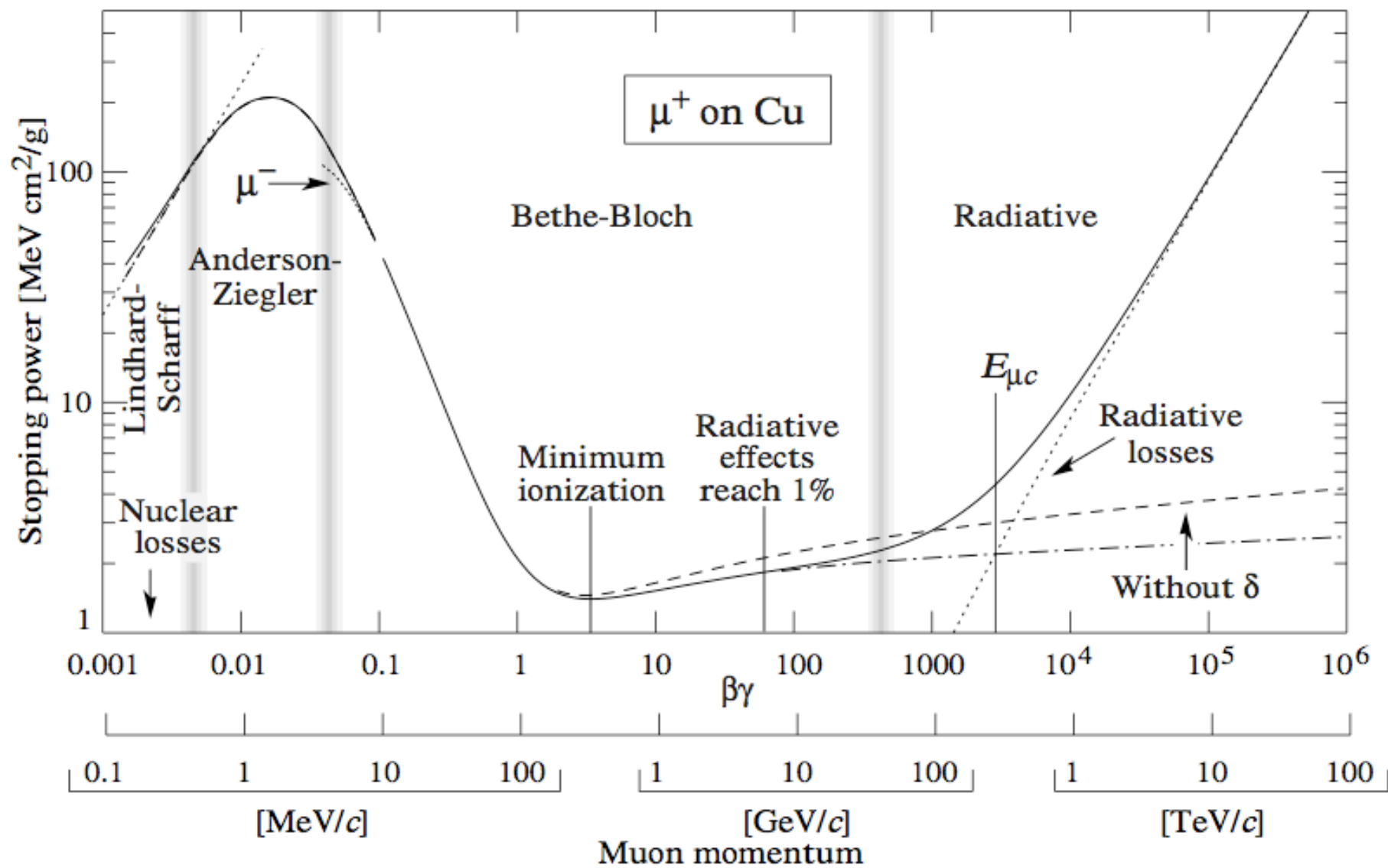


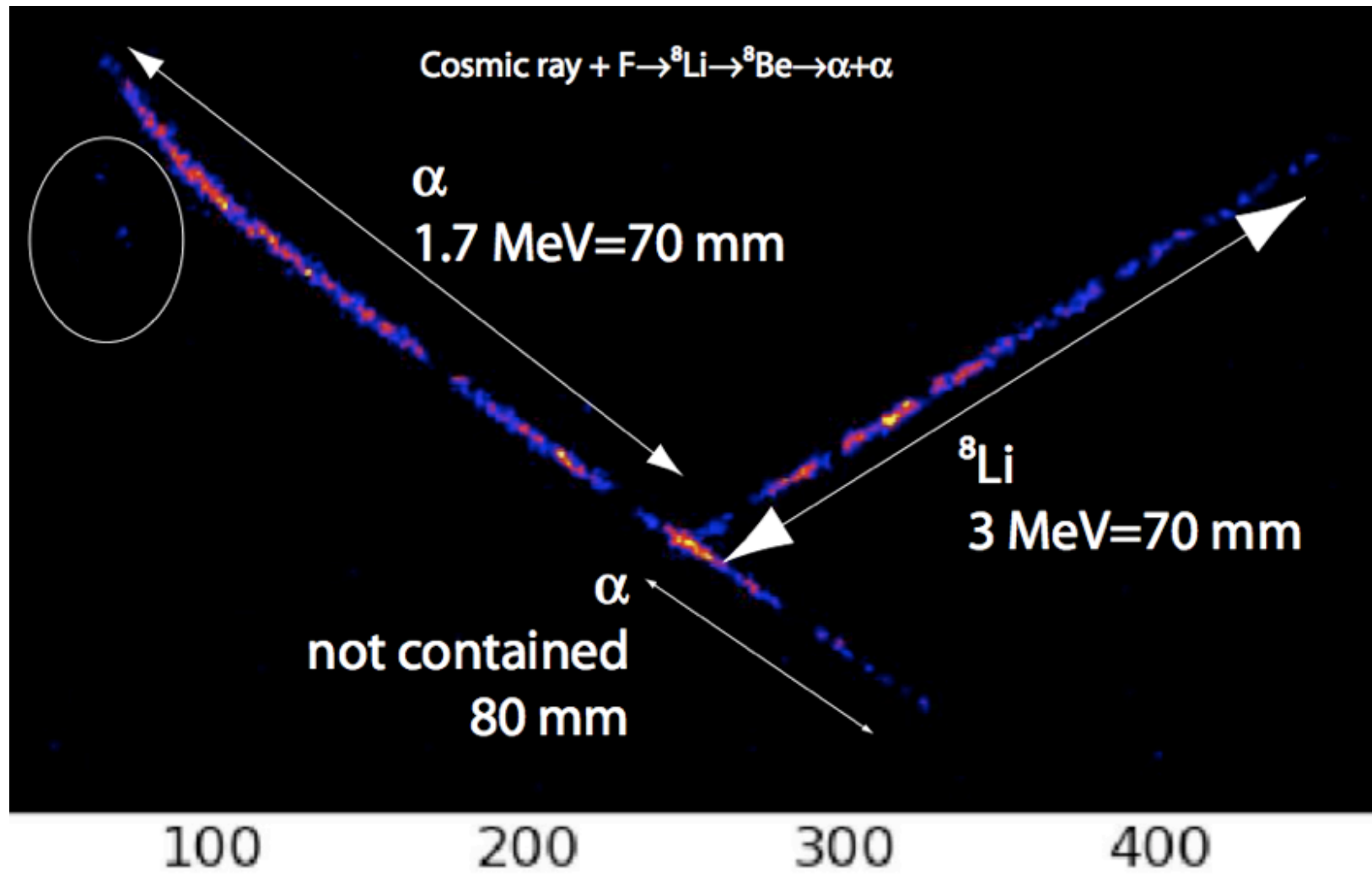
Even the very best detectors don't tell you what the particles are!

Why (cont.)?



Understanding energy loss mechanisms can lead to new experiments - Fly's Eye (later HiRES) relies only on fluorescence and clear air to detect the very highest energy particles.





Nomenclature

- N - number density
- Z - number protons
- $A=N+Z$
- m_N - nucleon mass
- m - electron mass
- ρ - mass density, radial coordinate
- T - deposited energy
- E - incident energy
- dx - medium thickness
- ε - dielectric constant
- θ - scattering angle in lab
- $\alpha=e^2/hc/2\pi=1/137$
- ω - angular frequency

Single particle kinematics

$$\beta = \frac{v}{c}$$

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}} \Rightarrow m = \frac{p}{\gamma\beta c}$$

$$E = \gamma mc^2$$

$$p = \gamma\beta mc$$

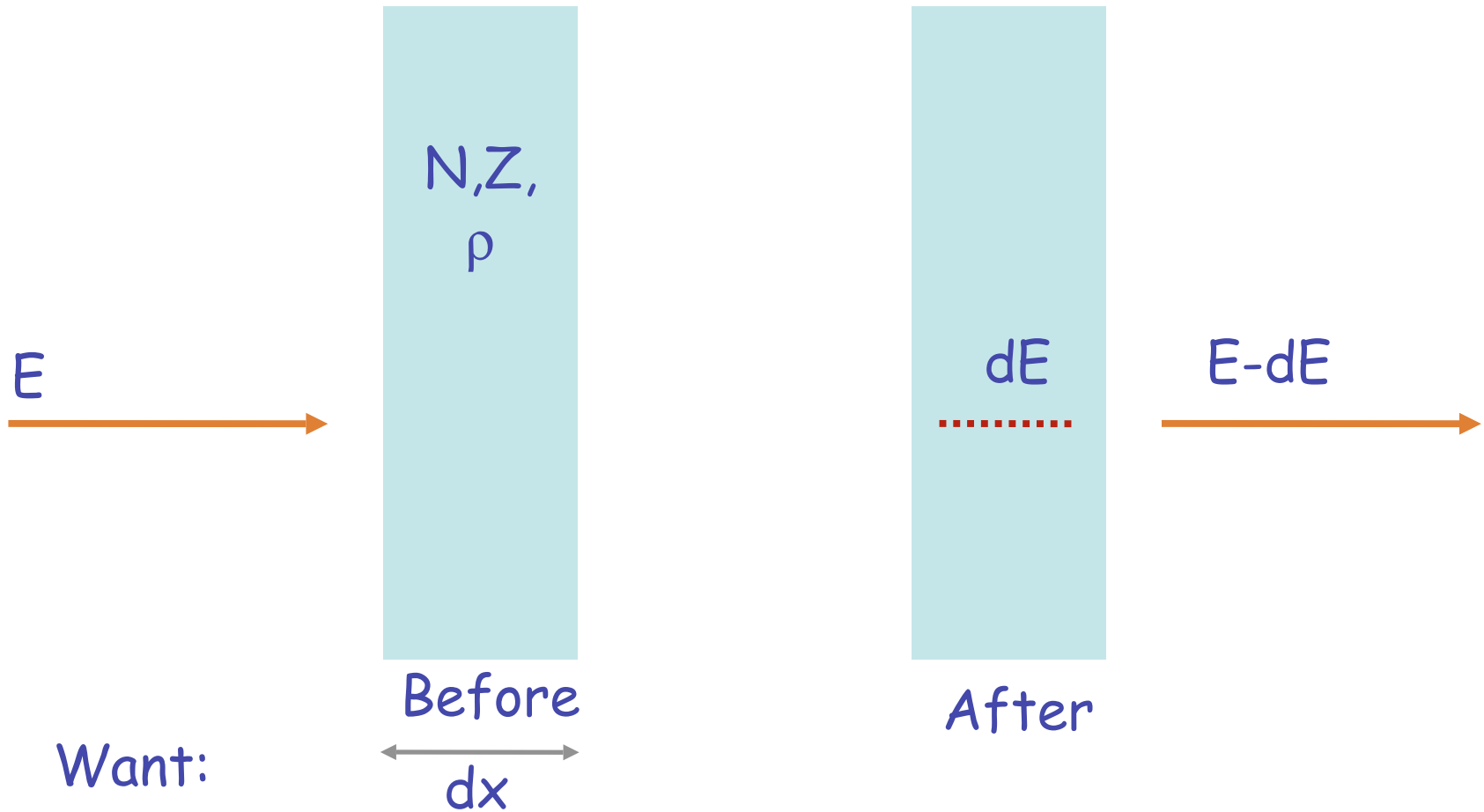
• Identifying a particle is (generally) the same as measuring its mass

• Need to measure two quantities to find m .

In a magnetic field:

$$p_{\perp} = \frac{0.3 \text{ GeV}}{\text{T} \cdot \text{m}} B \rho$$

General Problem

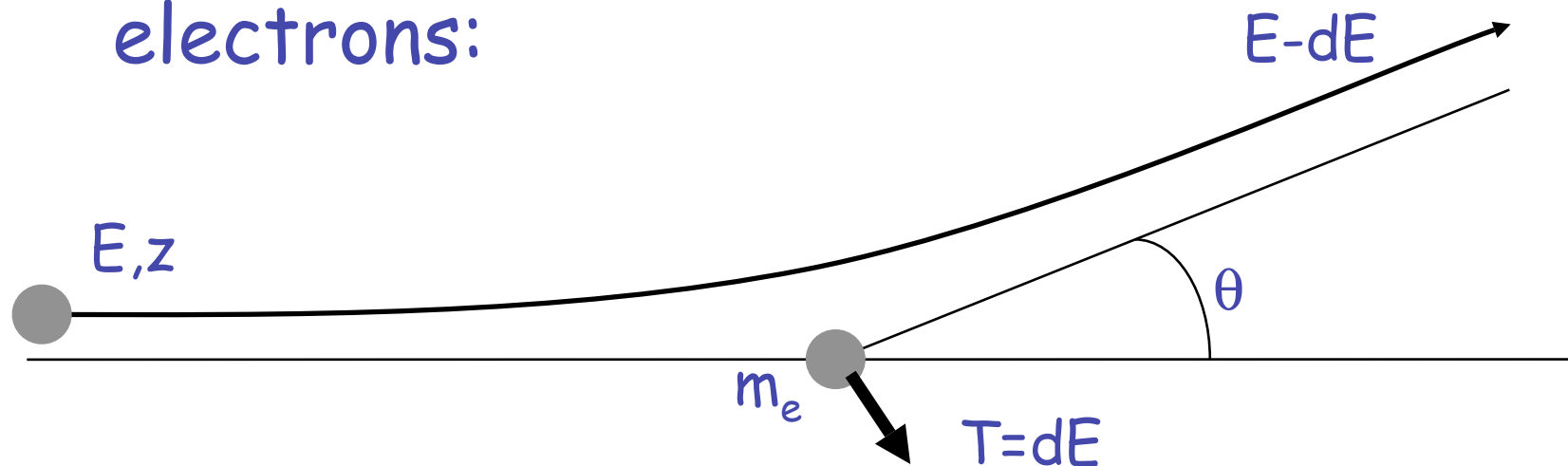


Want:

- dE/dx - particle
- Effect on medium - detection

1a. Ionization loss - adding single collisions

Simplest approach - treat as a series of
Coulomb collisions with atomic
electrons:



Rutherford:

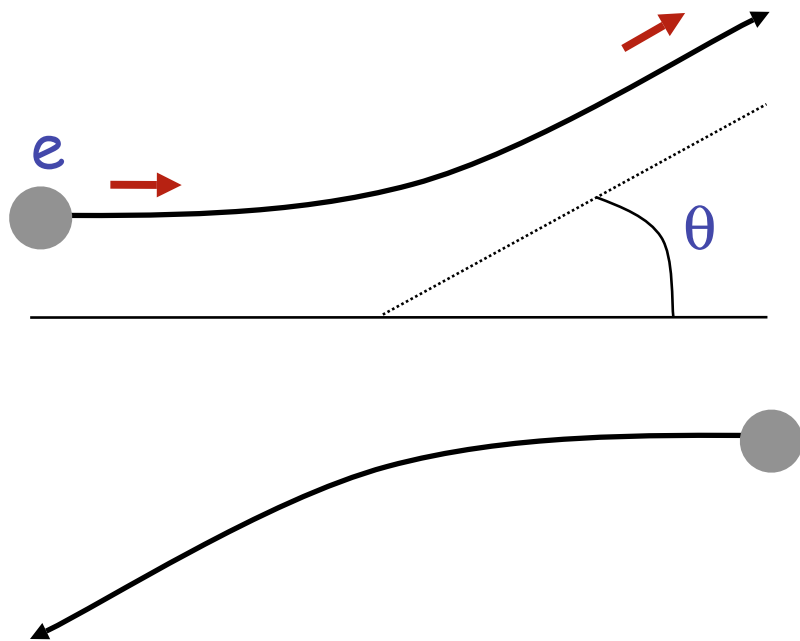
$$\frac{d\sigma}{d\Omega} = \frac{z^2 e^4}{4 p v \sin^4 \theta / 2}$$

Kinematics

- Some trig: $\sin^2 \frac{\theta}{2} = \frac{1}{2}(1 - \cos \theta)$
- Momentum transfer: $Q^2 = -(p - p')^2 = 4p^2 \sin^2 \frac{\theta}{2} = 2mT$
- Recoil kinetic energy: $T = \frac{Q^2}{2m}$
- Solid angle: $d\Omega = 2\pi d(\cos \theta) = \frac{\pi}{2p^2} dQ^2$

$$\frac{d\sigma}{dQ^2} = 4\pi \frac{z^2 e^4}{\beta c Q^4} \Rightarrow \frac{d\sigma}{dT} = \frac{2\pi z^2 e^4}{mc^2 \beta T^2}$$

A quantum correction for spin



- Spin of electron rotated through angle θ during collision

- Average over initial, sum over final states gives probability factor

$$|A|^2 = (1 - \beta^2 \sin^2 \theta / 2)$$

$$\Rightarrow \frac{d\sigma}{dT} = \frac{2\pi z^2 e^4}{mc^2 \beta^2 T^2} \left(1 - \beta^2 \frac{T}{T_{\max}} \right) \text{ and } T_{\max} = 2\gamma^2 \beta^2 mc^2$$

Sum over many scatters...

- Sum over N scattering sites particle passes
- Minimum energy $\epsilon \gg E_{\text{binding}}$

$$\frac{dE}{dx} = \int_{\epsilon}^{T_{\text{max}}} \left(N \frac{d\sigma}{dT} \right) T dT = 2\pi NZ \frac{e^4}{mc^2 \beta^2} \left[\ln \left(\frac{2\gamma^2 \beta^2 mc^2}{\epsilon} \right) - \beta^2 \right]$$

Observations:

1. Goes like $1/\beta^2$ \rightarrow very large loss at low β
2. Goes like $\ln \gamma$ as $\beta \rightarrow 1$, but density effect
3. Independent of particle mass \rightarrow can measure β
4. Distant collisions (Bethe 1930), $\epsilon \rightarrow \hbar \langle \omega \rangle / 2\pi$

- Emulsion pictures of π - μ - e decays

- μ all have same length \rightarrow two body π decay

- μ track gets wider near end $\rightarrow 1/\beta^2$ energy loss



Minimum ionizing

Leading factor $\frac{2\pi NZe^4}{mc^2}$ gives characteristic energy loss.

For most materials:

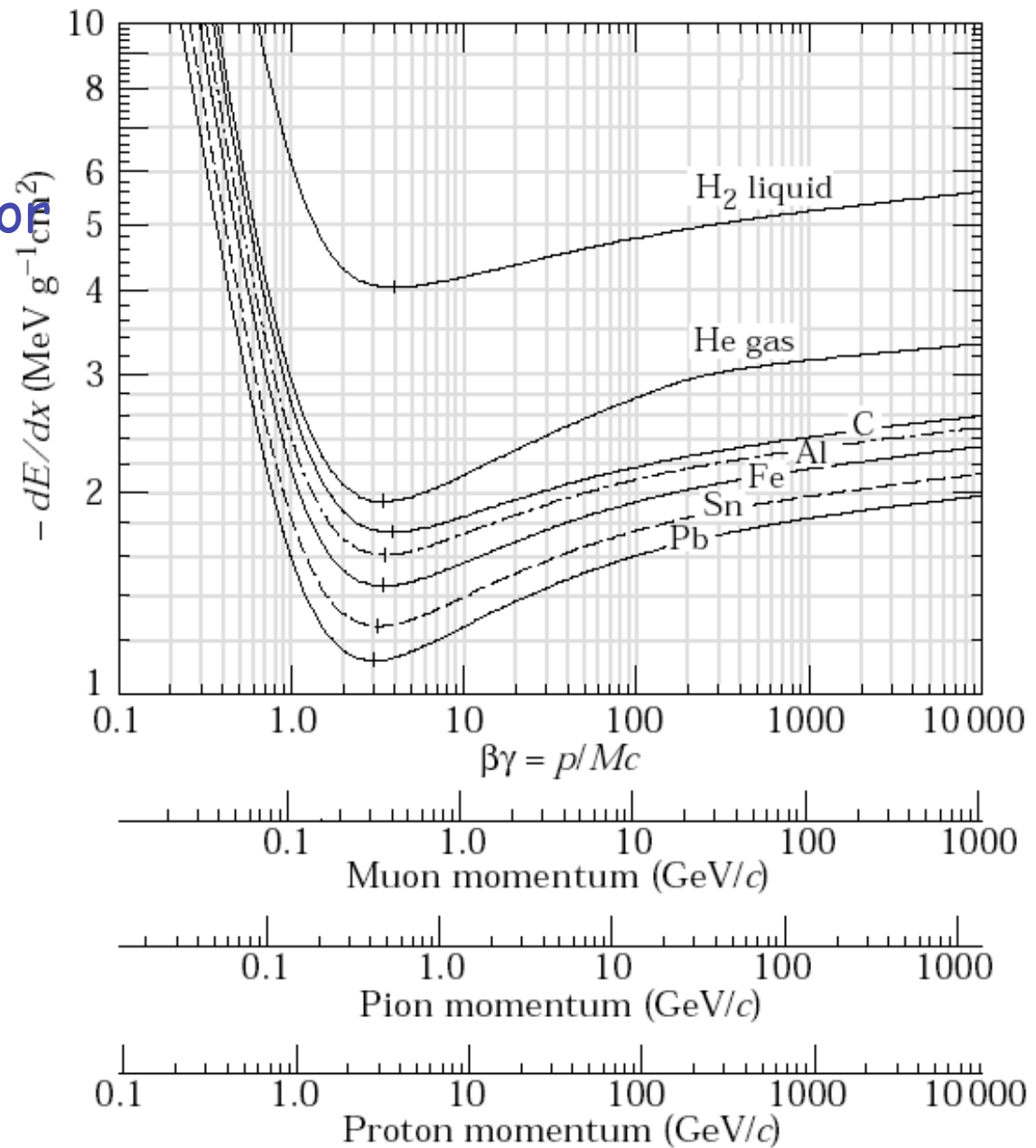
$$\begin{aligned} \bullet N &= \rho / Am_N \\ \bullet Z &= A/2 \end{aligned} \quad \frac{2\pi\alpha^2(\hbar c)^2}{mc^2} \frac{A}{2} \frac{\rho}{Am_N} \approx 0.08 \frac{\text{MeV}}{\text{g/cm}^2} \rho$$

Minimum occurs at $\gamma \sim 4$:

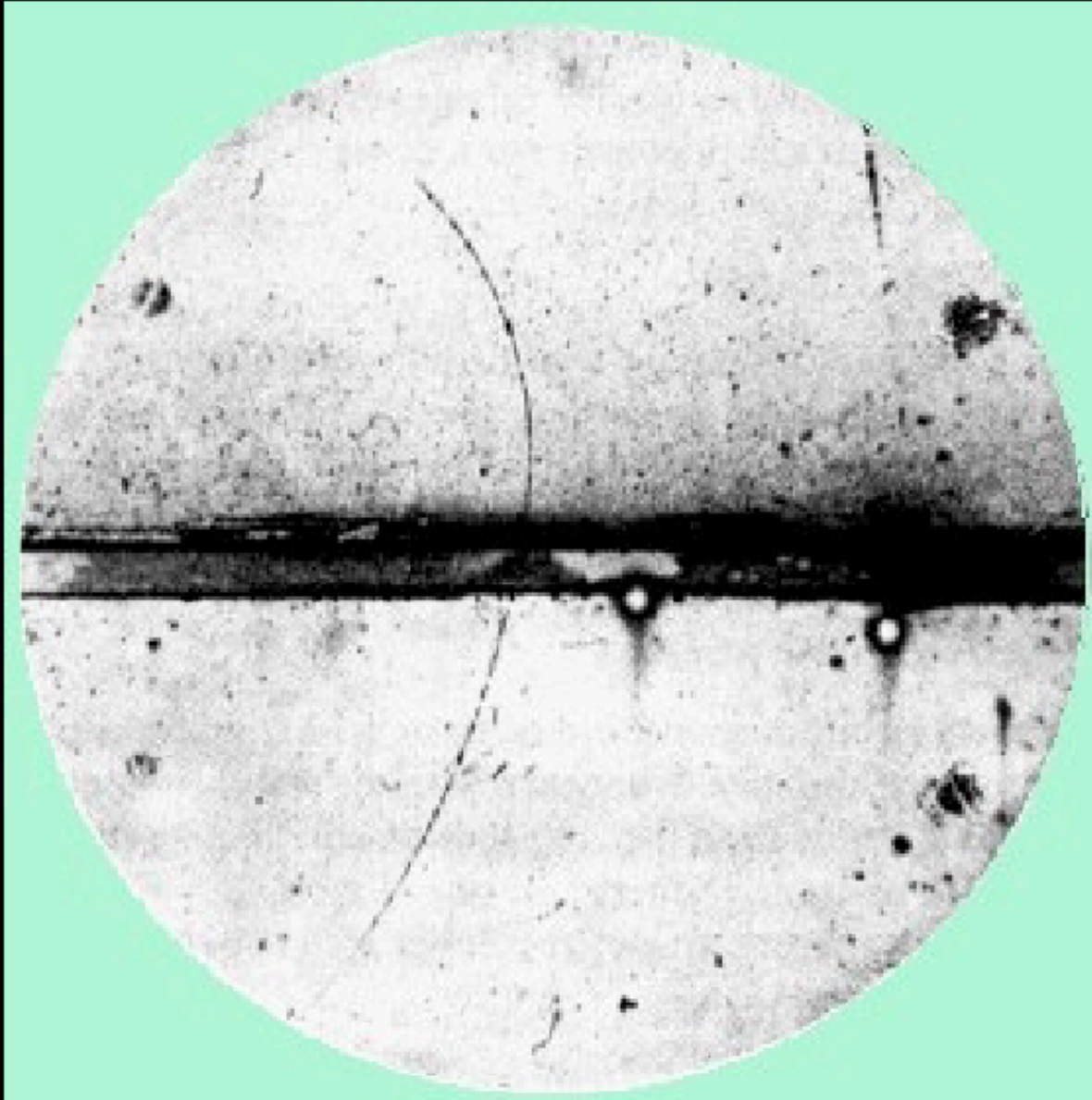
$$\left(\frac{dE}{dx} \right)_{\min} \approx 1.8 \frac{\text{MeV}}{\text{g/cm}^2}$$

From PDG - dE/dx for various materials

- $(dE/dx)_{\min} = 1-2 \text{ MeV/g-cm}^2$
- Minimum occurs near $E=4m$



Discovery of positron in cloud chamber



Lead plate thickness = 6 mm

Magnetic field = 24000 gauss
into picture

Dimensions: 17 x 17 x 3 cm

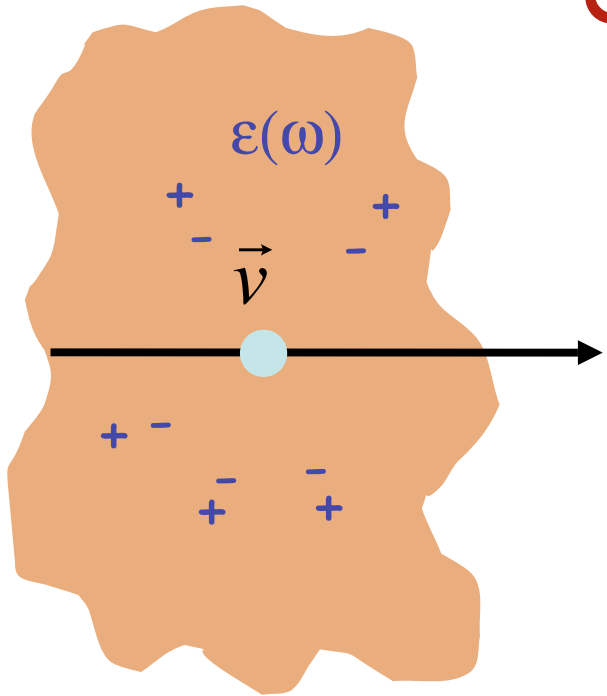
Change of curvature implies
particle moving up, so charge
is positive

Track density $\Rightarrow Z/\beta = 1$

$A\beta\gamma/Z = 0.07$ below plate,
 0.02 above plate: so $A < 0.02$

For very low energy particles
like these, mass and charge
can be measured, showing
them to be positrons.

1b. Single collision in dielectric



source terms:

$$\rho(\vec{x}, t) = ze\delta(\vec{x} - \vec{v}t)$$

$$\vec{J}(\vec{x}, t) = ze\vec{v}\delta(\vec{x} - \vec{v}t)$$

Solve for potentials with source terms:

$$\left(k^2 - \frac{\omega^2}{c^2} \epsilon(\omega)\right) \Phi = \frac{4\pi}{\epsilon(\omega)} \rho$$

$$\left(k^2 - \frac{\omega^2}{c^2} \epsilon(\omega)\right) \vec{A} = \frac{4\pi}{c} \vec{J}$$

Calculation of energy loss

$$E_{\parallel} = -\frac{ize\omega}{v^2} \left(\frac{2}{\pi}\right)^{1/2} \left[\frac{1}{\epsilon(\omega)} - \beta^2 \right] K_0(\lambda b)$$

$$E_{\perp} = \frac{B_{\perp}}{\beta\epsilon(\omega)} = \frac{ze}{v} \left(\frac{2}{\pi}\right)^{1/2} K_1(\lambda b) \quad \Rightarrow \quad \Delta E = -\int_{-\infty}^{\infty} \vec{v} \cdot \vec{F} dt = -e \int_{-\infty}^{\infty} \vec{v} \cdot \vec{E} dt$$

$$\lambda^2 = \frac{\omega^2}{c^2} (1 - \beta^2 \epsilon(\omega)) \quad = -\frac{ca}{2} e \int_{-\infty}^{\infty} B_{\perp}(t) E_{\parallel}(t) dt$$

λ is a characteristic length of the fields

For $\beta \sim 1$ and $\lambda\beta \ll 1$ (near field region) $\frac{dE}{dx} = \frac{(ze)^2 \omega_p^2}{c^2} \ln\left(\frac{1.123c}{a\omega_p}\right)$

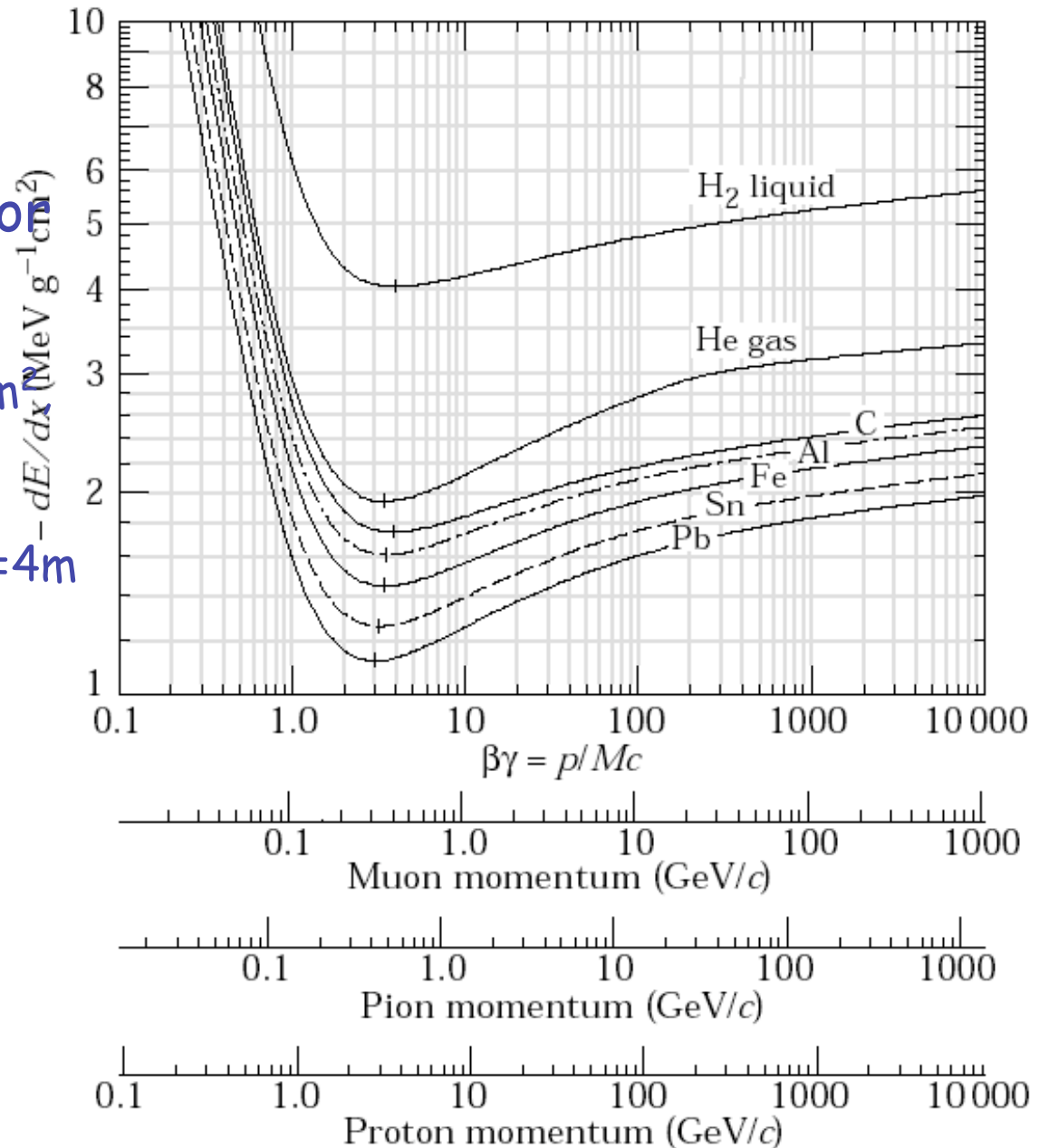
Energy loss becomes independent of energy at $\beta \sim 1$

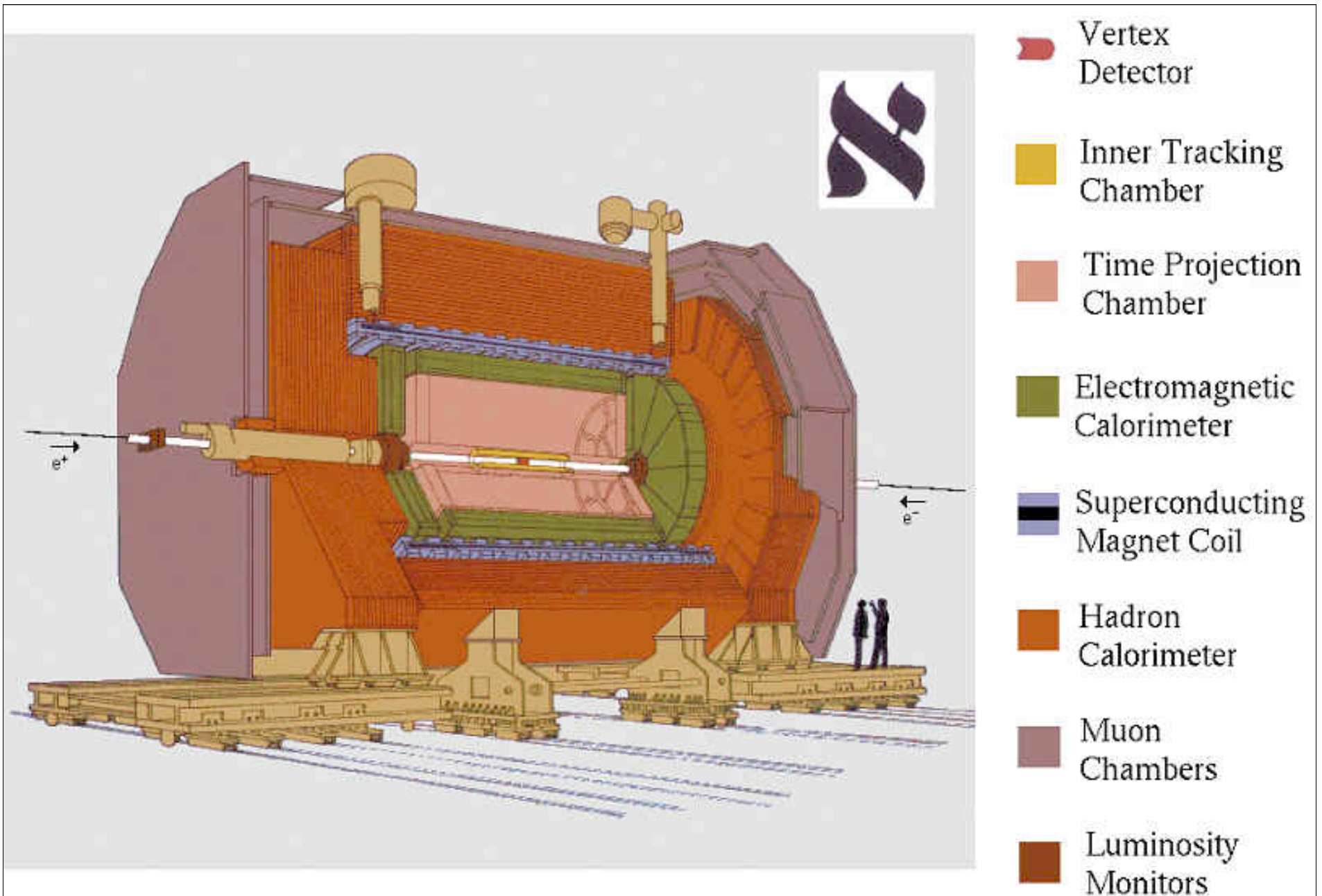
From PDG - dE/dx for various materials

• $(dE/dx)_{\min} = 1-2 \text{ MeV/g-cm}^2$
depends on Z/A .

• Minimum occurs near $E=4m$

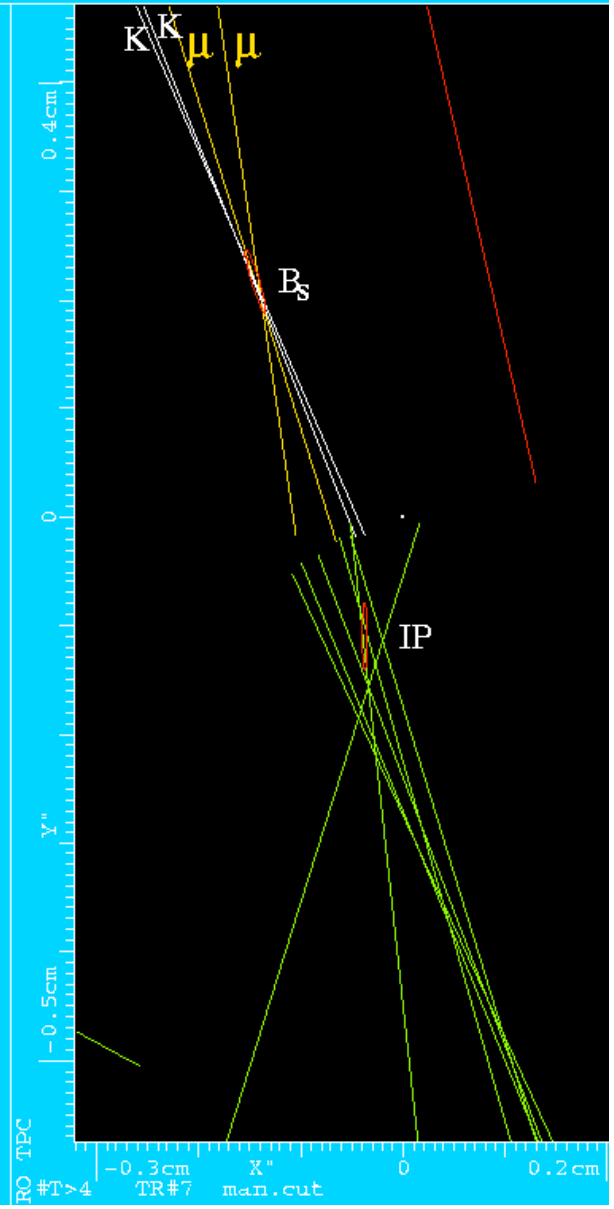
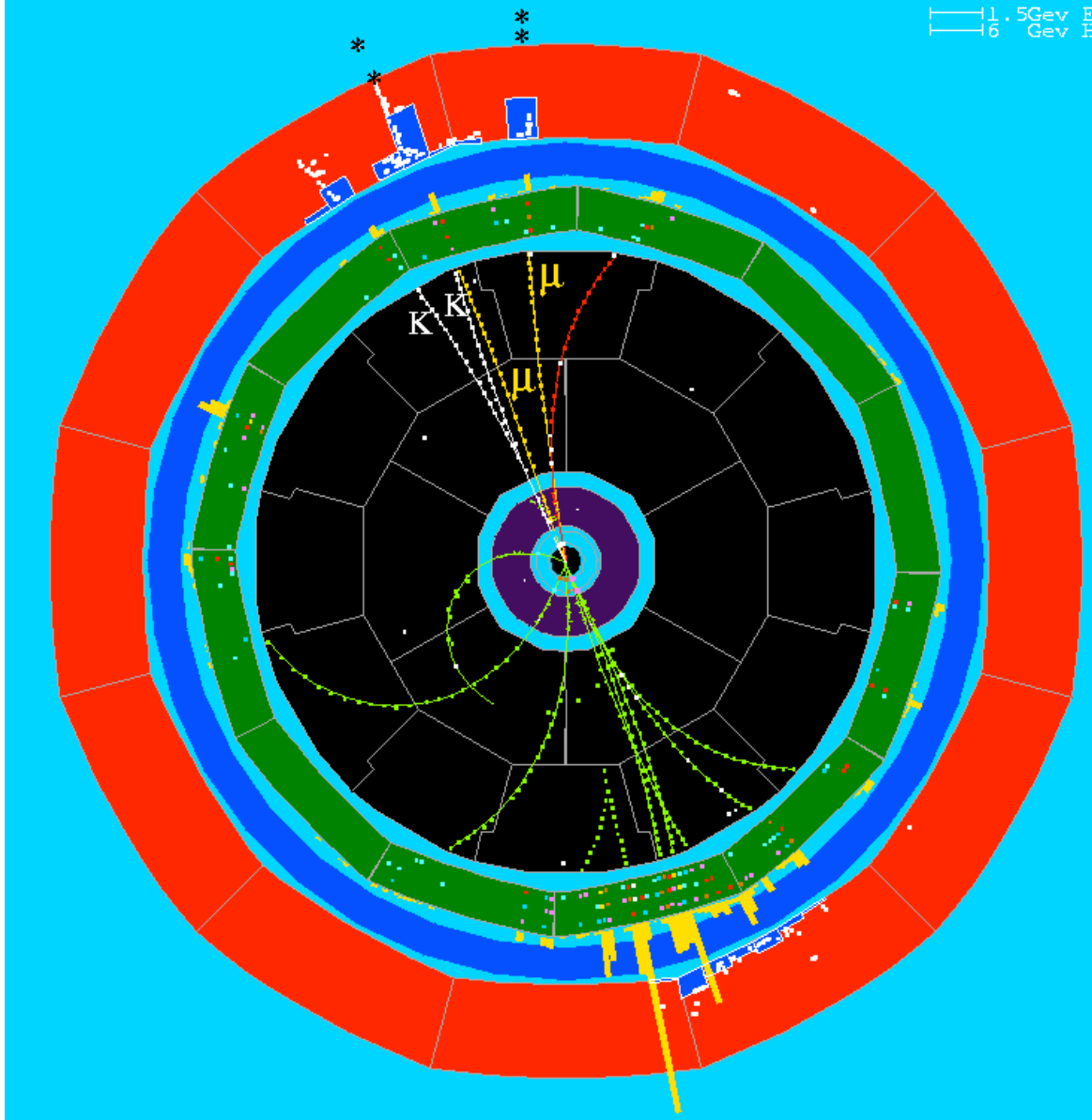
• "Relativistic rise" by 10-20% to $\gamma=10,000$



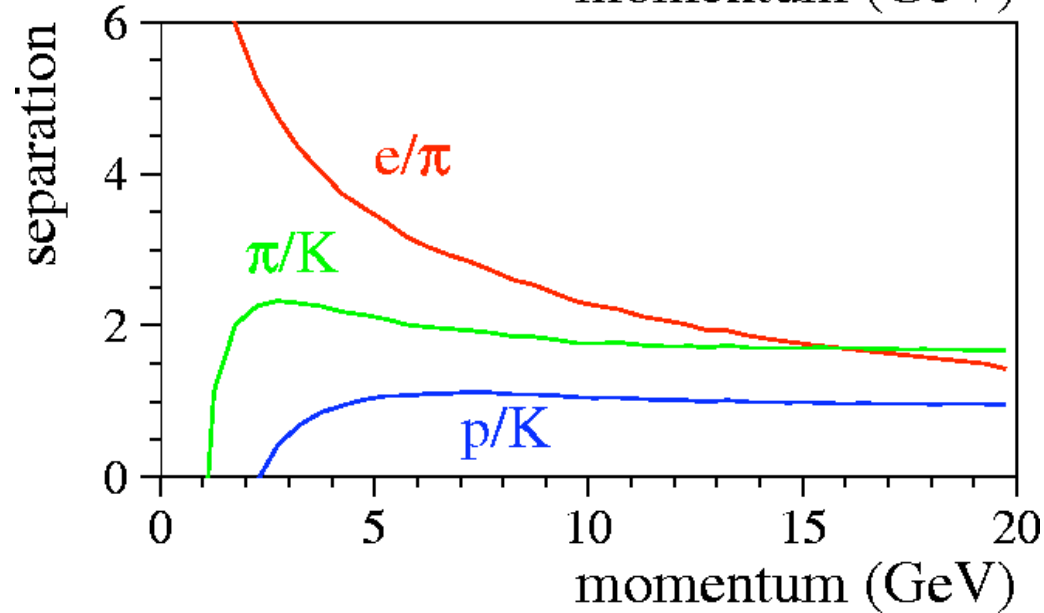
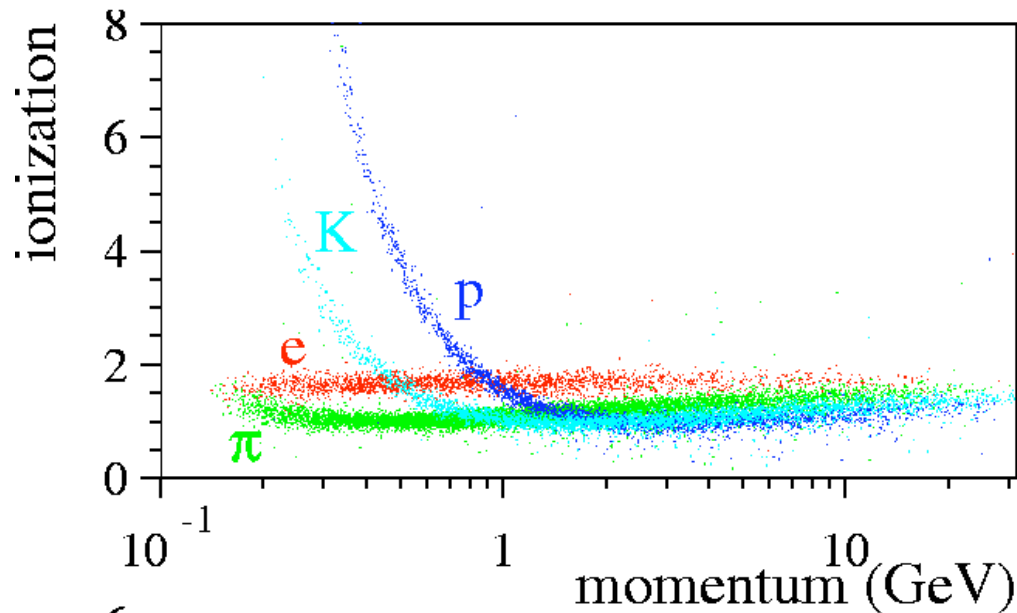


The ALEPH Detector

1.5Gev EC
6 Gev HC



ALEPH



• Masses:

• $M_e = 0.0005 \text{ GeV}$

• $M_\pi = 0.14 \text{ GeV}$

• $M_K = 0.49 \text{ GeV}$

• $M_p = 0.938 \text{ GeV}$

• Thickness of bands due to statistical fluctuations in energy loss

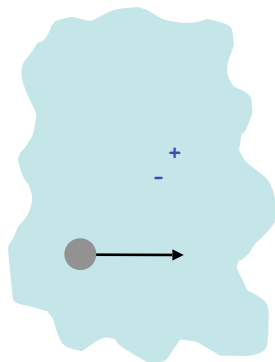
• Separation up to $\sim 10 \text{ GeV}$

Further topics

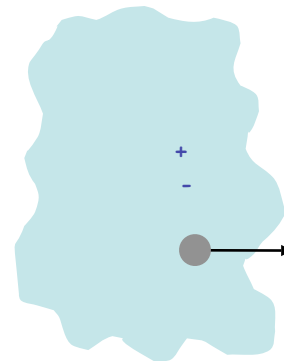
- High energy secondary electrons ("δ-rays")
- Radiation from collisions (bremstrahlung)
- Electromagnetic showers (cascade of pair production from initial e or γ).
- Hadronic showers

2. Cerenkov radiation - the radiation field in a homogeneous medium

- Energy lost in photons \rightarrow radiation field from charged particle
- Mechanism - polarization of medium changes as particle passes

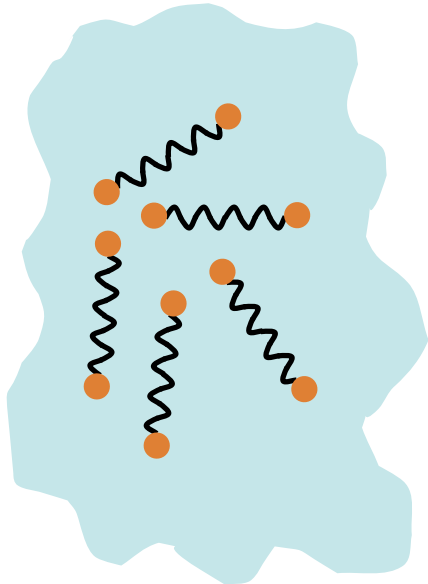


t_1



t_2

Picture of matter



View atoms in matter as "harmonically bound", which gives a simple expression for dielectric constant:

$$m(\ddot{\vec{x}} + \gamma\dot{\vec{x}} + \omega_o^2\vec{x}) = -e\vec{E} \Rightarrow$$

$$\vec{p} = -e\vec{x} = \frac{e}{m} \frac{\vec{E}}{\omega_o^2 - \omega^2 - i\omega\gamma} = \epsilon(\omega)\vec{E} \Rightarrow$$

$$\epsilon(\omega) = \epsilon_o \left(1 + \frac{Ne^2}{m\epsilon_o} \sum_i \frac{f_i}{\omega_o^2 - \omega^2 - i\omega\gamma} \right) \quad \text{and} \quad \sum_i f_i = Z$$

Plasma frequency: $\omega_p = \frac{NZe^2}{\epsilon_o m}$ Oscillation frequency of a fully ionized plasma

Calculation of energy loss

$$\begin{aligned}
 E_{\parallel} &= -\frac{ize\omega}{v^2} \left(\frac{2}{\pi}\right)^{1/2} \left[\frac{1}{\epsilon(\omega)} - \beta^2 \right] K_0(\lambda b) \\
 E_{\perp} &= \frac{B_{\perp}}{\beta\epsilon(\omega)} = \frac{ze}{v} \left(\frac{2}{\pi}\right)^{1/2} K_1(\lambda b) \\
 \lambda^2 &= \frac{\omega^2}{c^2} (1 - \beta^2\epsilon(\omega))
 \end{aligned}
 \Rightarrow
 \begin{aligned}
 \Delta E &= -\int_{-\infty}^{\infty} \vec{v} \cdot \vec{F} dt = -e \int_{-\infty}^{\infty} \vec{v} \cdot \vec{E} dt \\
 &= -\frac{ca}{2} e \int_{-\infty}^{\infty} B_{\perp}(t) E_{\parallel}(t) dt
 \end{aligned}$$

λ is a characteristic length of the fields

For $\beta \sim 1$ and $\lambda\beta \ll 1$ (near field region) $\frac{dE}{dx} = \frac{(ze)^2 \omega_p^2}{c^2} \ln\left(\frac{1.123c}{a\omega_p}\right)$

Energy loss becomes independent of energy at $\beta \sim 1$

Fields (we have already done the work)

Cerenkov involves collisions far from the particle trajectory, $\lambda b \gg 1$, asymptotic form for K_0, K_1 :

$$E_{\parallel} = i \frac{ze\omega}{c^2} \left[1 - \frac{1}{\beta^2 \epsilon(\omega)} \right] \frac{e^{-\lambda b}}{\sqrt{\lambda b}}$$

$$B_{\perp} = \frac{\beta ze}{v} \sqrt{\frac{\lambda}{b}} e^{-\lambda b}$$

$$E_{\perp} = \frac{ze}{v \epsilon(\omega)} \sqrt{\frac{\lambda}{b}} e^{-\lambda b}$$

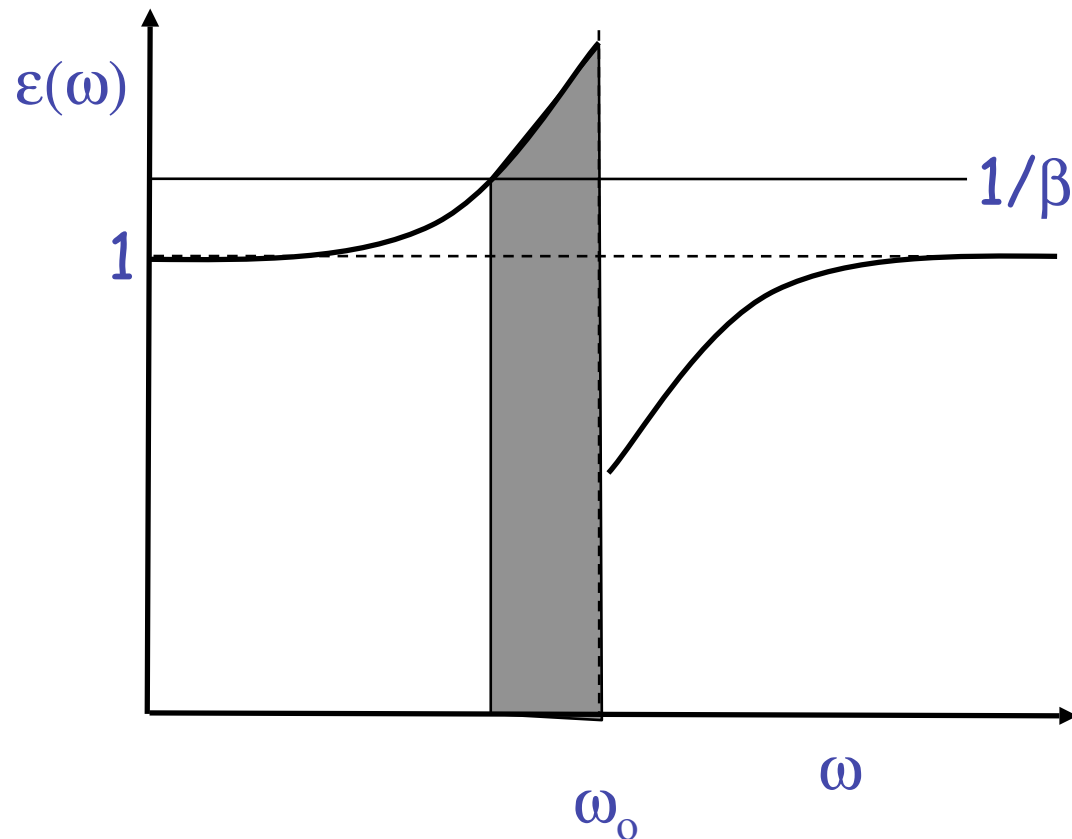
λ imaginary \rightarrow propagating fields

$$\rightarrow \lambda^2 = \omega^2 / c^2 (1 - \beta^2 \epsilon(\omega)) < 0 \rightarrow \beta > 1 / \epsilon(\omega)^{1/2}$$

$$\rightarrow v > c/n$$

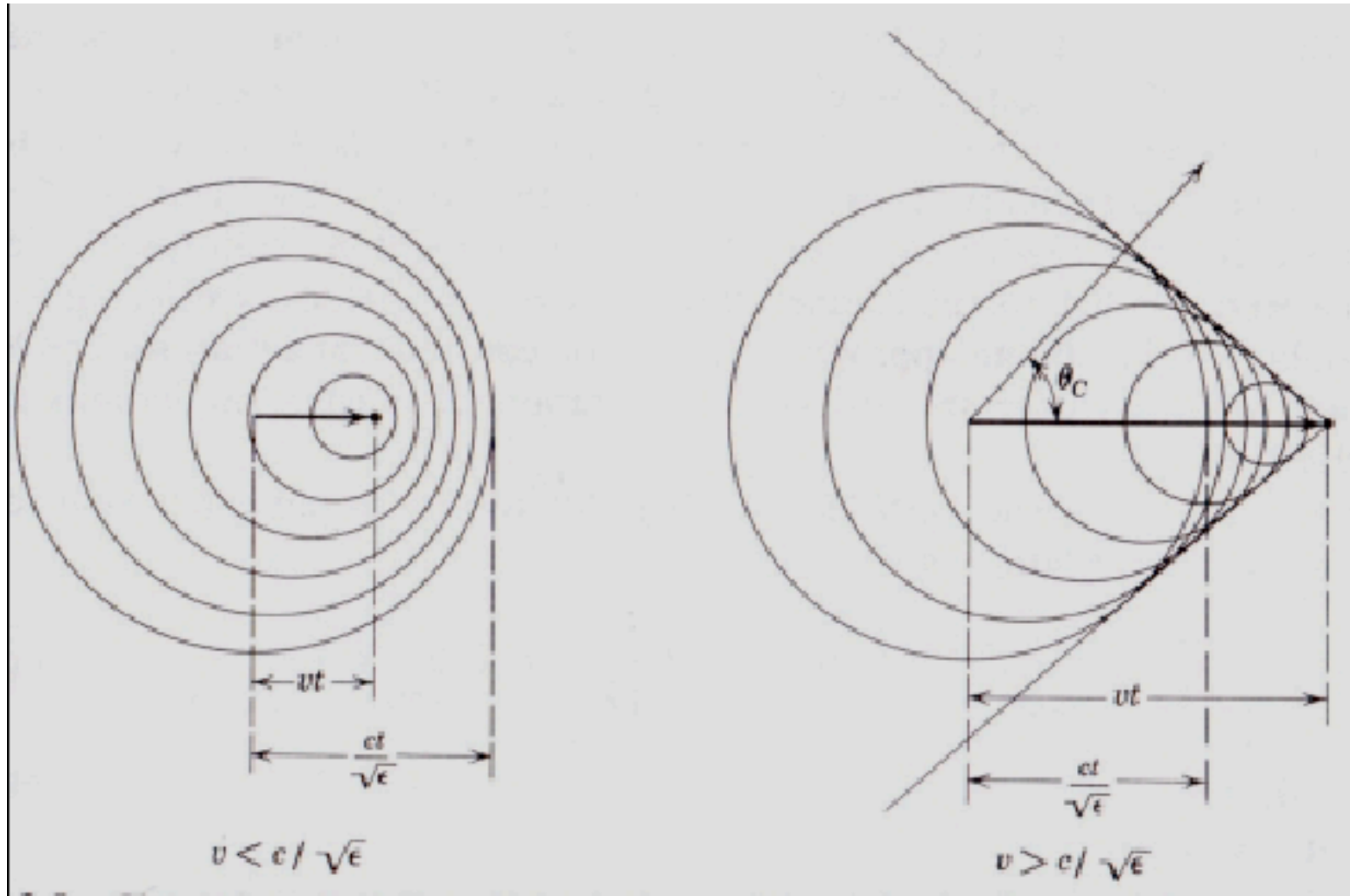
Radiation

Direction: $\tan \theta_c = -\frac{E_{\parallel}}{E_{\perp}} \Rightarrow \cos \theta_c = \frac{1}{\beta \sqrt{\epsilon(\omega)}}$



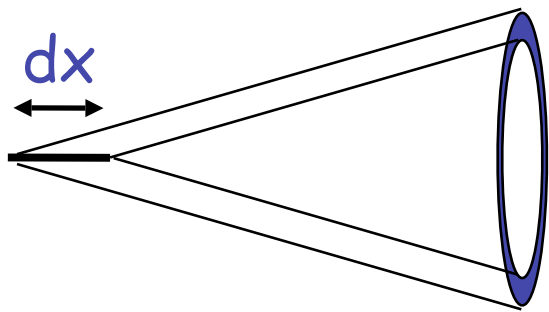
Cerenkov radiation occurs near atomic resonances \rightarrow radiation in optical at angle θ_c

The usual picture....



Intensity of Radiation

Photon flux into Cerenkov cone:

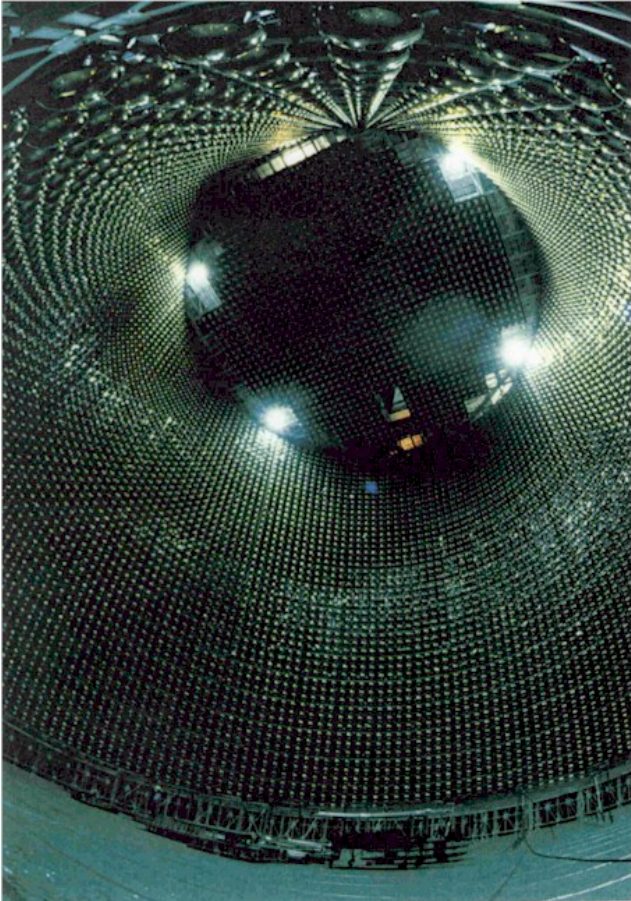


$$\frac{d^2N}{d\omega dx} = \int_{\text{cone}} \frac{\vec{S} \cdot d\vec{a}}{\hbar\omega} = \frac{c}{4\pi} \int_{\text{cone}} \frac{(\vec{E} \times \vec{B}) \cdot d\vec{a}}{\hbar\omega}$$

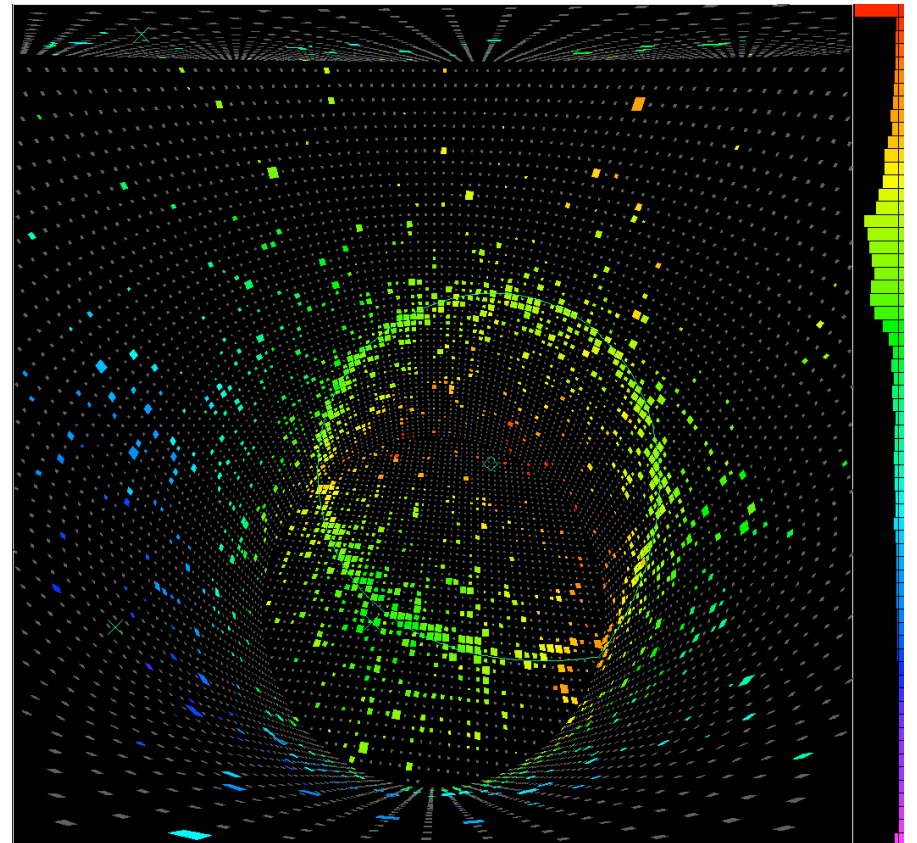
$$\begin{aligned} \frac{d^2N}{dE dx} &= \frac{\alpha z^2}{\hbar c} \sin^2 \theta_c = \frac{\alpha^2 z^2}{r_e m_e c^2} \left(1 - \frac{1}{\beta^2 n^2(E)} \right) \\ &\approx 370 \sin^2 \theta_c(E) \text{ eV}^{-1} \text{ cm}^{-1} \quad (z = 1), \end{aligned}$$

Integrate over PMT response: $dN_\gamma/dx = (90/\text{cm}) \sin^2 \theta_c$

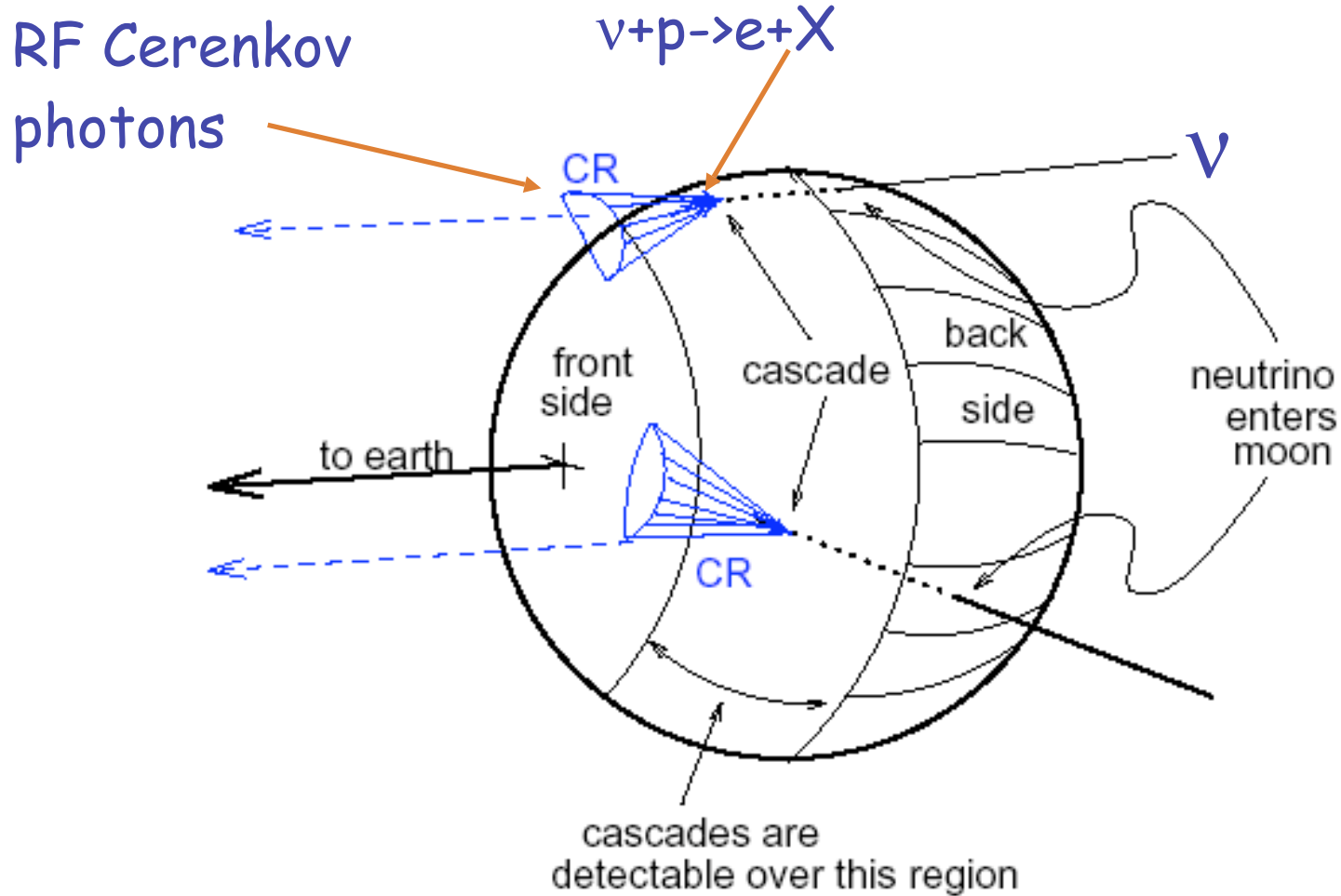
Superkamiokande - $n_{\text{water}}=1.33$



Single electron event

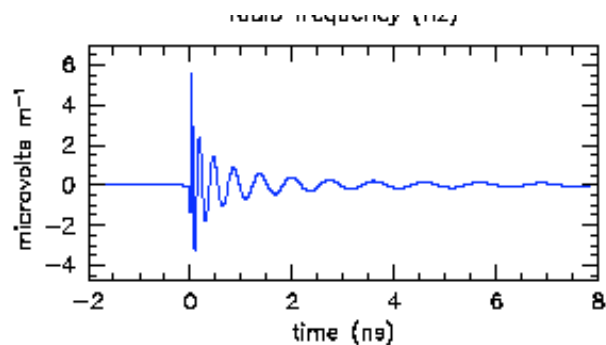


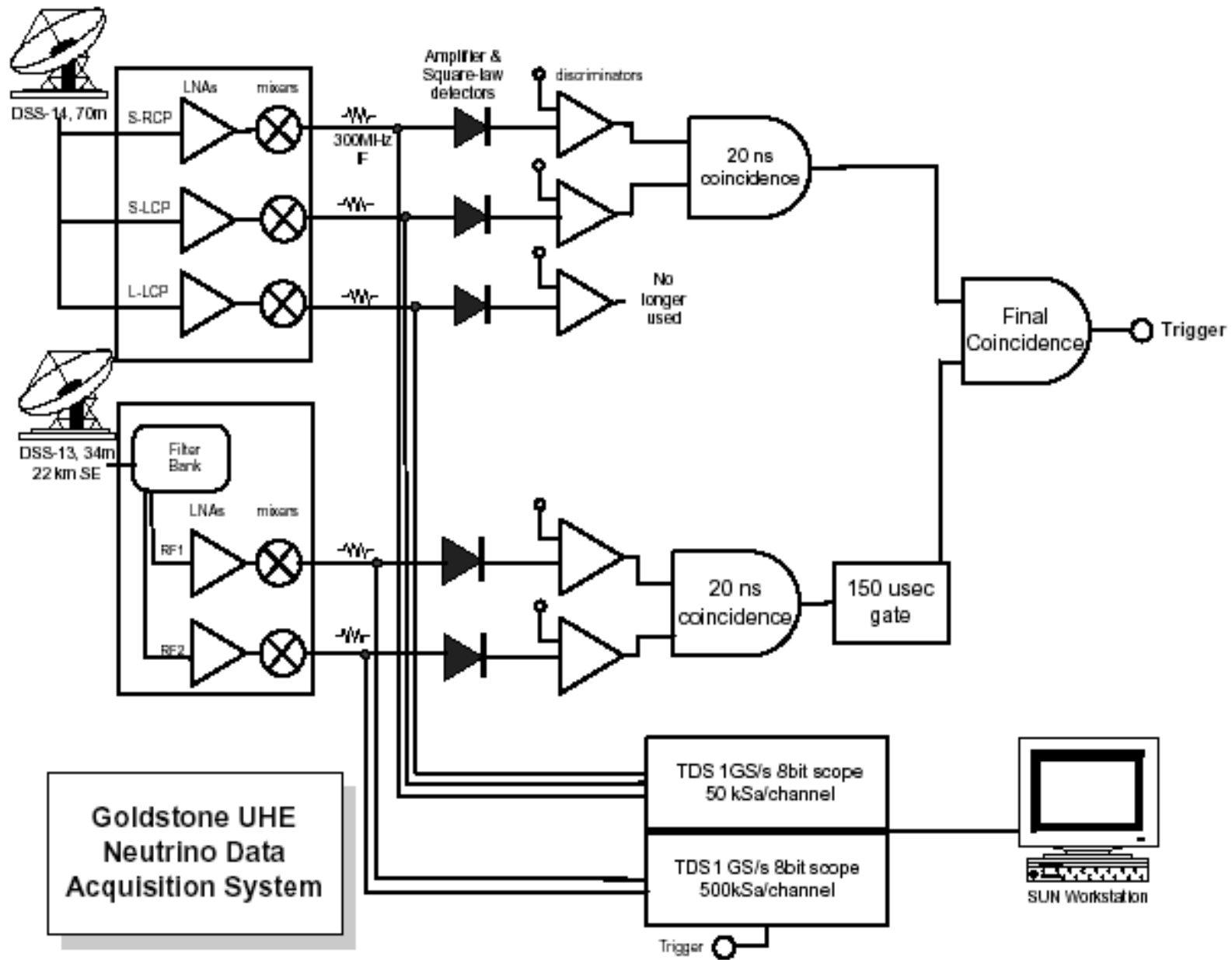
Very high energy neutrinos



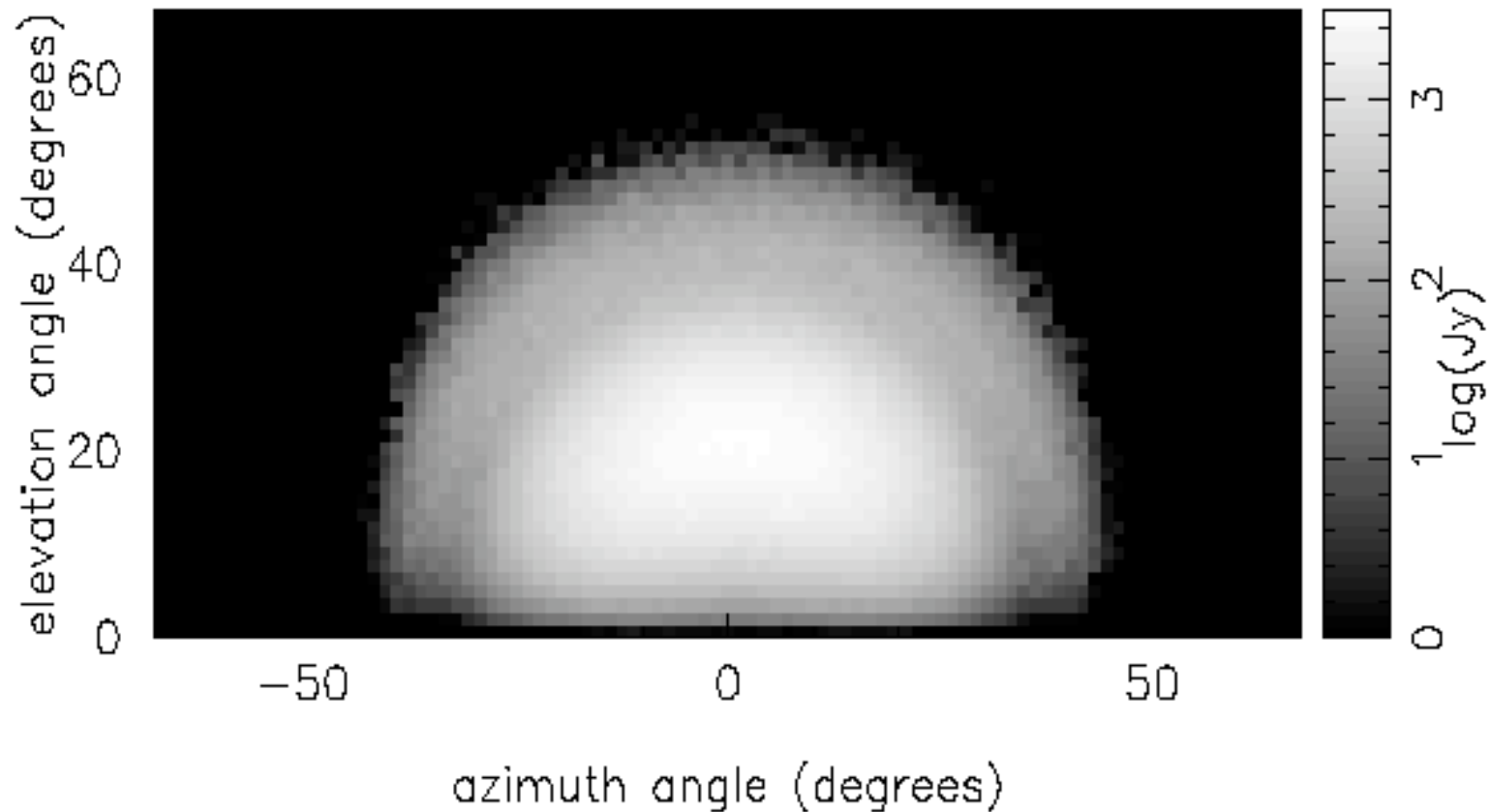
Goldstone DSN receiver

- Able to image the entire moon
- Power sensitivity - pW
- Large backgrounds - operate two dishes in coincidence
- Typical signal:



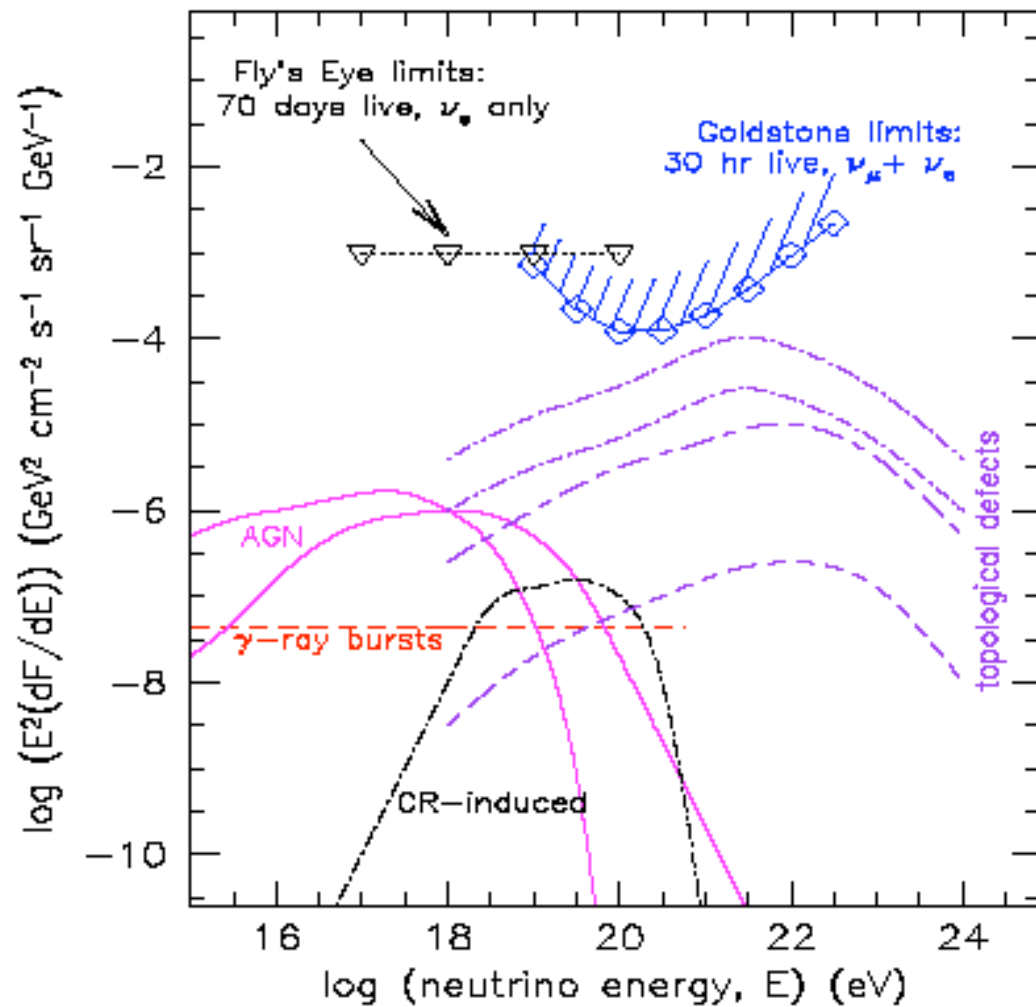


Moon imaged by RF Cerenkov from EHE neutrinos

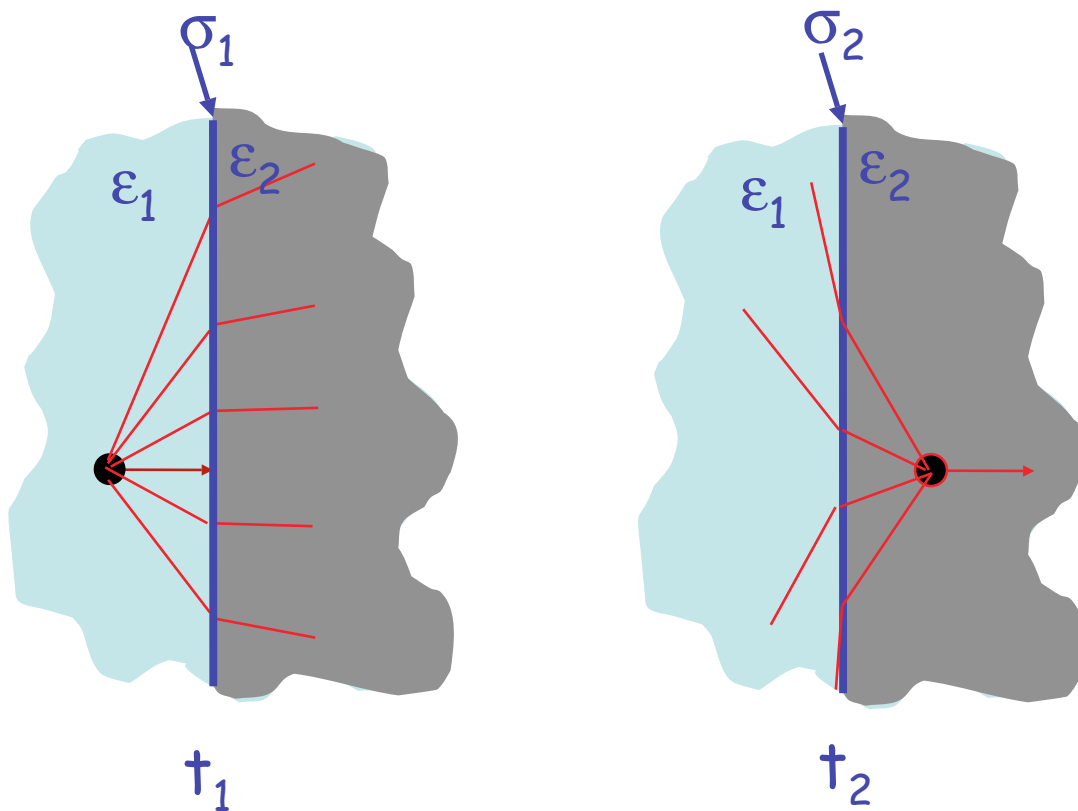


Results

30 h data - unique results within factor of 100 of models



3. Transition radiation - radiation field in a heterogeneous medium



Particle crossing
interface causes
coherent change in
surface charge
density \rightarrow radiation

Coherent radiation region

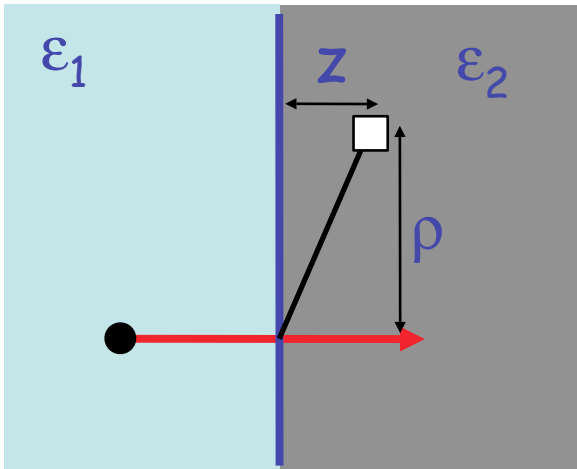
Fields of a moving point charge:

$$E_{\parallel} = \frac{qv\gamma t}{(\rho^2 + \gamma^2 v^2 t^2)^{3/2}} = \int E_{\parallel}(\omega) d\omega$$

Fields have radial range

$$E_{\perp} = \frac{q\gamma\rho}{(\rho^2 + \gamma^2 v^2 t^2)^{3/2}} = \int E_{\perp}(\omega) d\omega$$

$$\rho_{\max} \sim \gamma v / \omega$$



For coherent radiation, phase factor

$$E \propto \exp\left(i\frac{\omega}{v}z\right) \exp\left(-i\frac{\omega}{v}n(\omega)\cos\theta z\right) \\ \times \exp\left(-i\frac{\omega}{v}n(\omega)\rho\sin\theta\cos\phi\right)$$

Total phase should be close to one.

Conditions for radiation

Two conditions for coherence:

1. $n(\omega)\gamma\beta\sin\theta \sim n(\omega)\gamma\theta < 1 \rightarrow$ collinear, for γ large (~ 1000)
2. $(\omega/c)[1/\beta - n(\omega)\cos\theta]d(\omega) < 1 \rightarrow d_{\max} \sim \gamma c / \omega_p \sim 10^{-6} \text{ m}$

Transition radiation:

Emitted at the interface

Propagates along with charged particle

Requires $\gamma > 1000$

At typical collider energies, TR is a signal for electrons

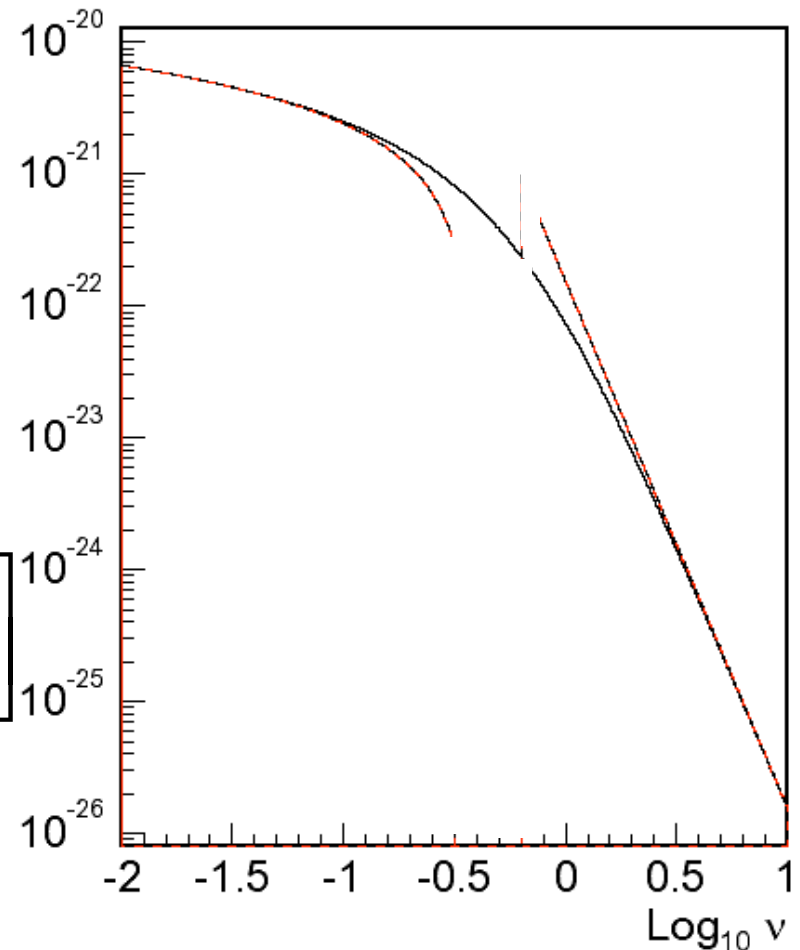
Radiation spectrum

- Must compute fields, complicated!
- Dimensionless frequency:

$$\nu = \frac{\omega}{\gamma\omega_p}$$

- Peak around $\nu=1 \rightarrow 10$ keV

$$\begin{aligned} \frac{dI}{d\nu} &= \frac{z^2 e^2 \gamma \omega_p}{\pi c} \left[(1 + 2\nu^2) \ln\left(1 + \frac{1}{\nu^2}\right) - 2 \right] \\ &= \frac{z^2 e^2 \gamma \omega_p}{\pi c} 2 \ln\left(\frac{1}{e\nu}\right) \quad \text{for } \nu \ll 1 \\ &= \frac{z^2 e^2 \gamma \omega_p}{\pi c} \frac{1}{6\nu^4} \quad \text{for } \nu \gg 1 \end{aligned}$$



Total intensity

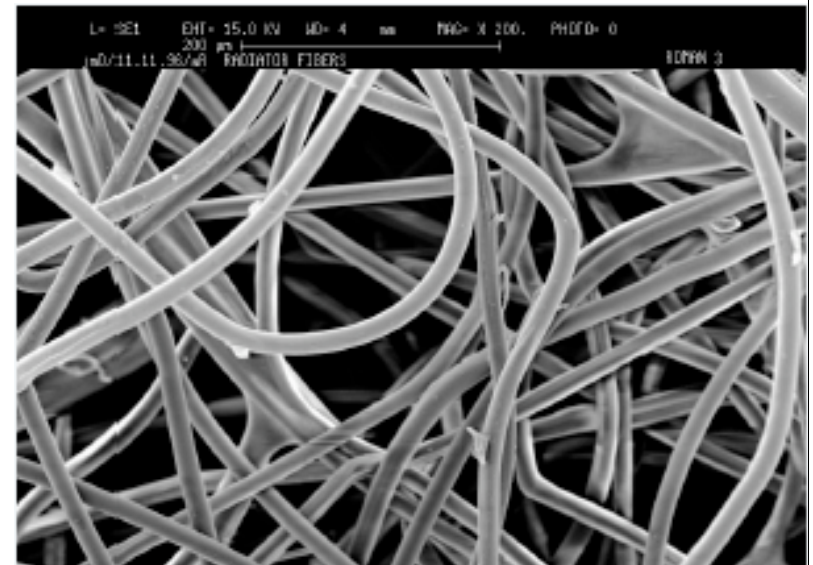
Total intensity:

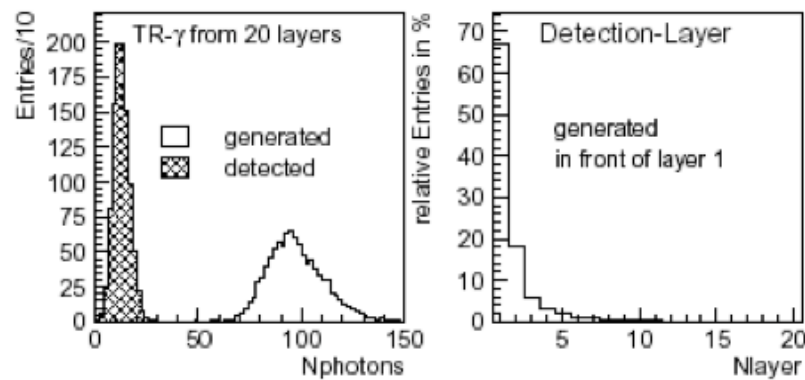
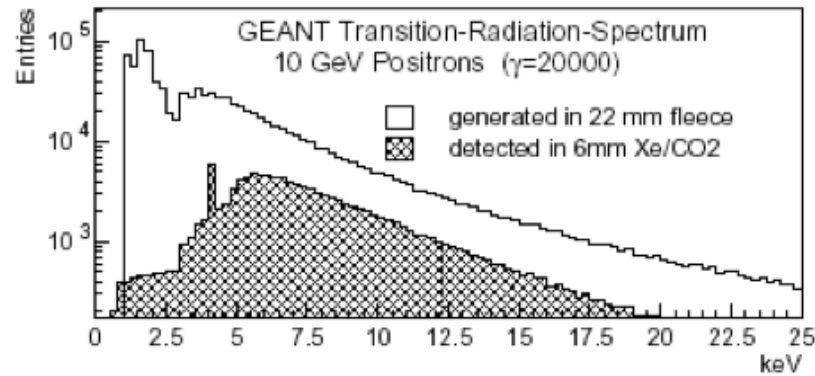
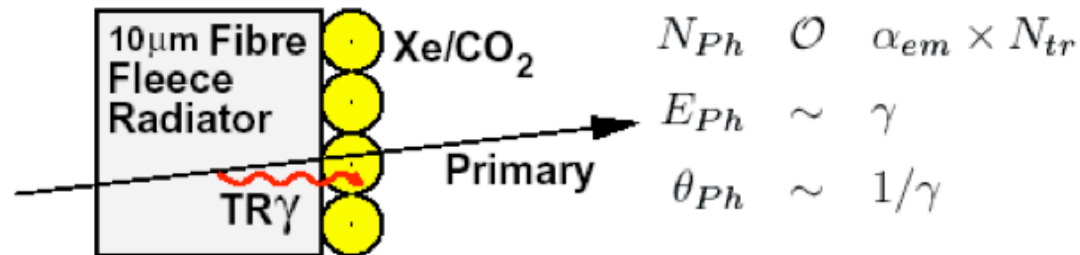
$$I = \frac{z^2 \alpha}{3} \gamma \hbar \omega_p$$

Only 10^{-3} photons per interface;
need many interfaces ->

- Stack up foils
- Polypropylene fleece

Need to detect ~ 10 keV photons



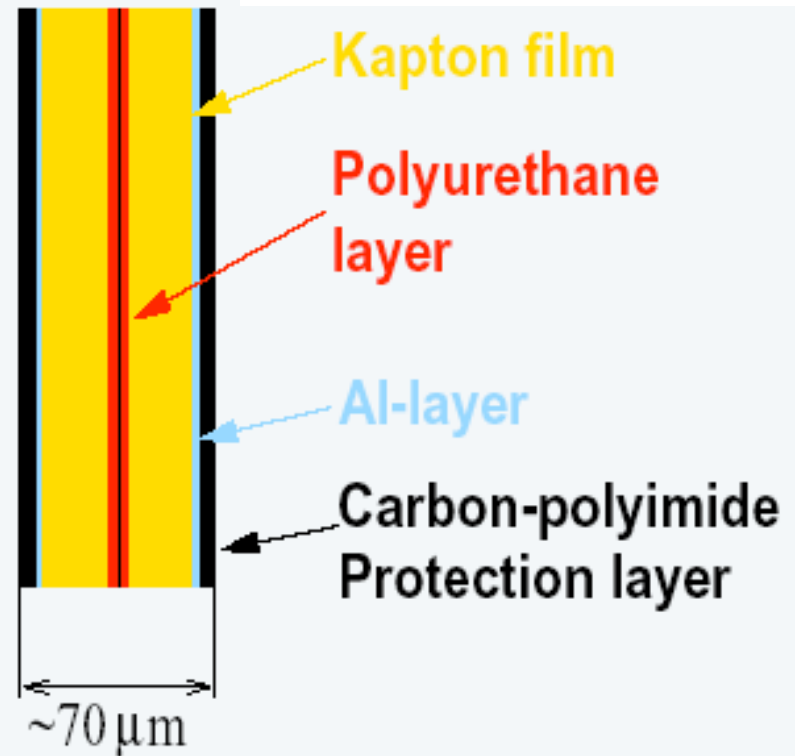


TR photon detection:

- Thin walls
- High Z absorber (Xe)
- Proportional amplification
- Timing for tracking, better discrimination

Straw

Straw wall



Large acceptance $0.5 \text{ m}^2\text{sr}$ in orbit for 3 years

TRD Particle ID & 3D tracking

20 layers fleece + Xe/CO₂

5248 channels 6mm straw-tubes

$$p^+/e^+ < 10^{-2} \text{ (10 - 300 GeV)}$$

TOF 1,2 Trigger $\sigma_t \approx 125\text{ps}$

Anticoincidence (Veto) counter

Silicon strip tracker

with internal laser alignment

6 m^2 in 3 double + 2 single xy layers

1σ charge separation up to 1TV

Superconducting Magnet (ETH)

$$B = 0.9\text{T} \quad V = 0.6\text{m}^3$$

TOF 3,4 1.3m distance to TOF 1,2

$$p^+/e^+ > 3\sigma \text{ below } 2 \text{ GeV}$$

PERICH AGL(+NaF) Radiator

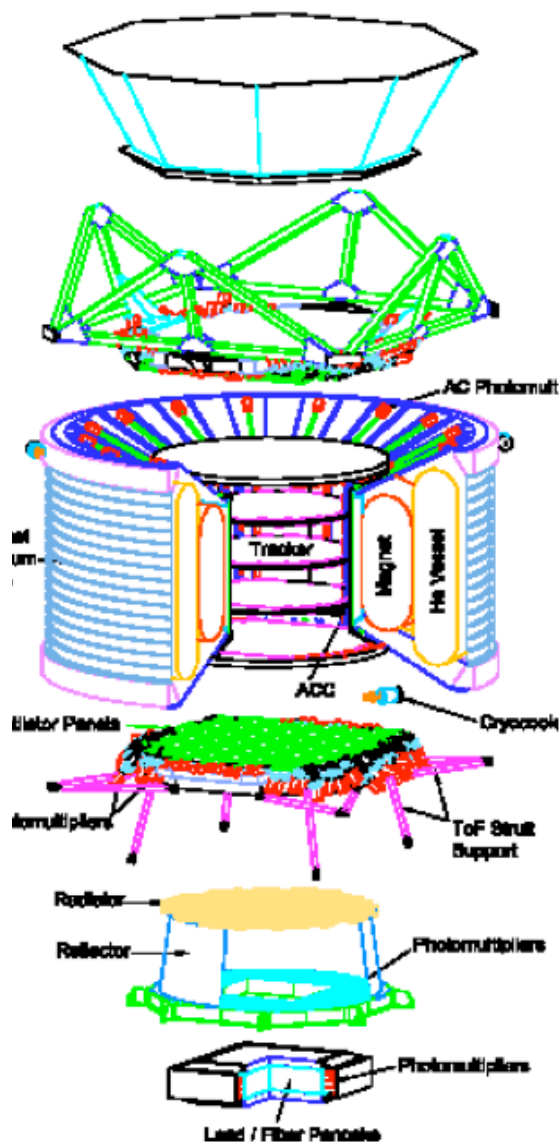
for $A \leq 27$ and $Z \leq 28$

separation $> 3\sigma$ from 1-12 GeV

ECAL 3D sampling lead/scint.-fibre

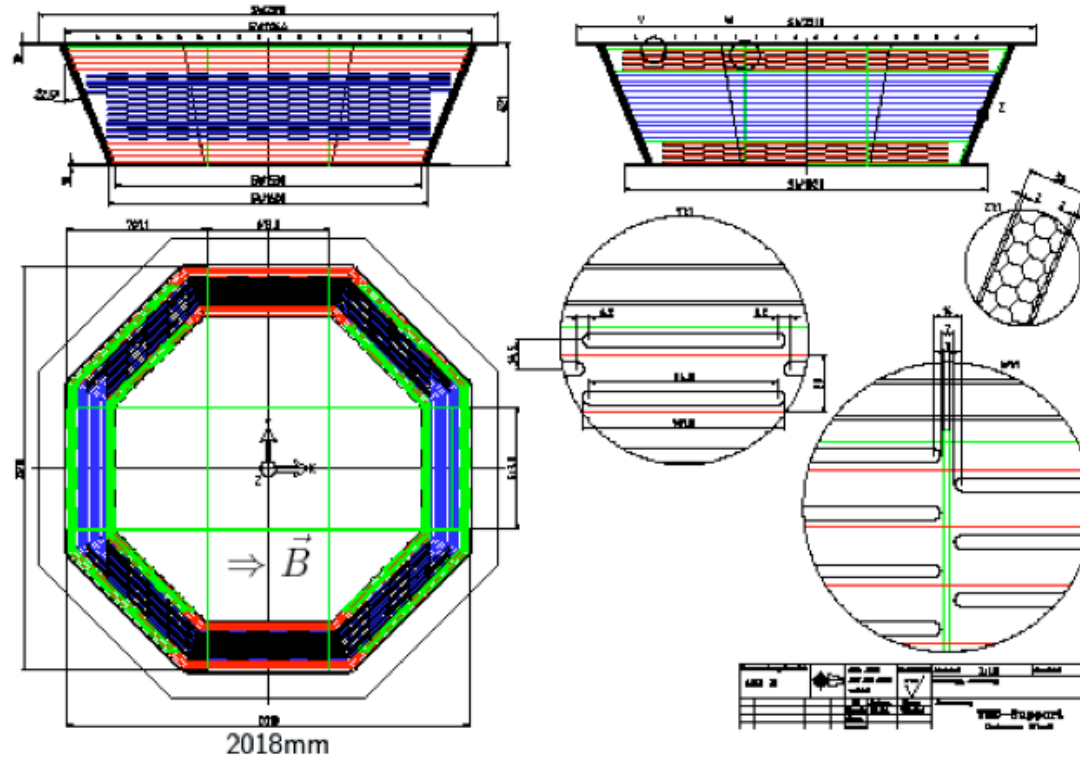
with p-E matching and shower-shape

$$p^+/e^+ < 10^{-4} \text{ (10 - 300 GeV)}$$



TRD Octagon Construction

20 layers with 22mm radiator 6mm straw tube modules

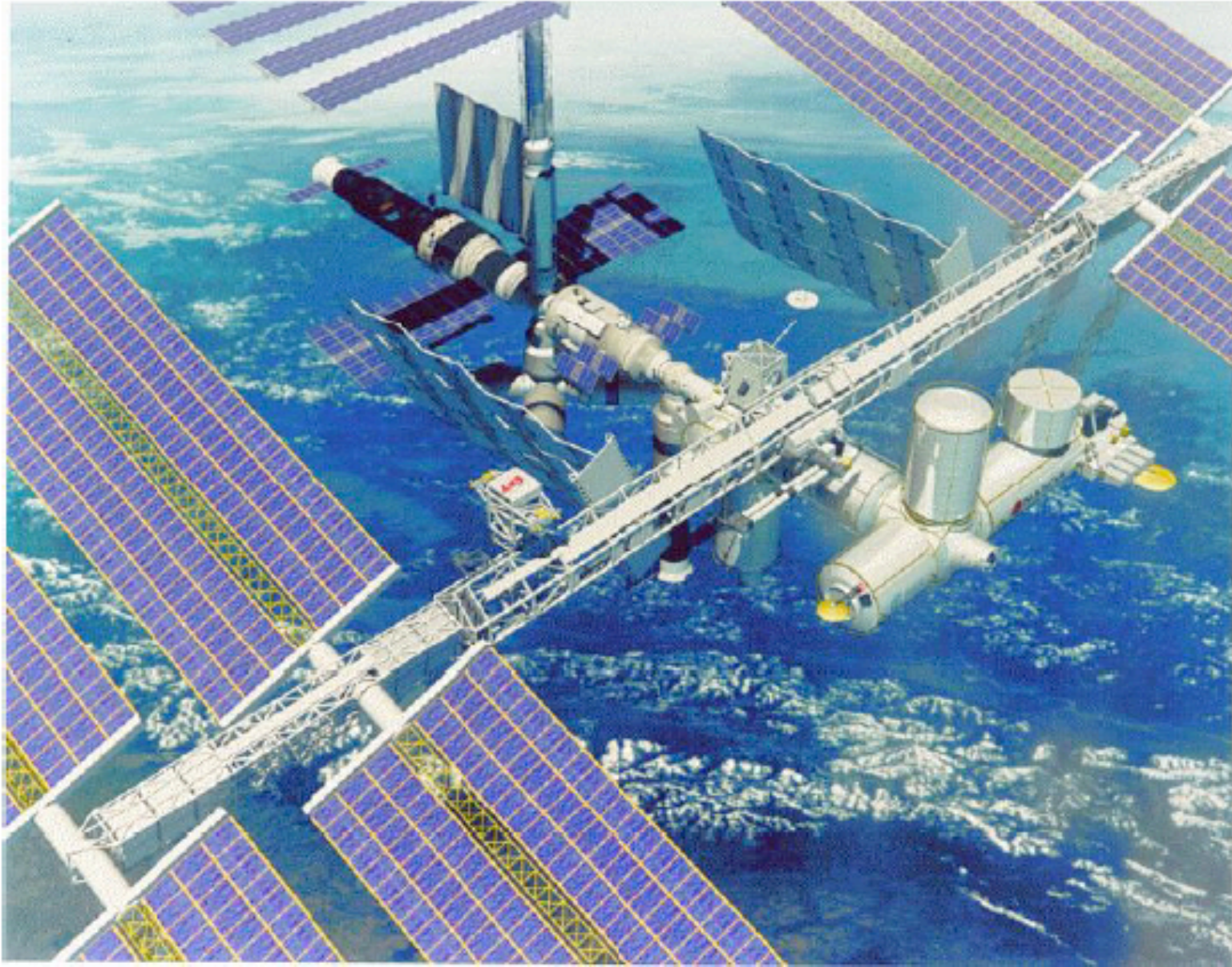


Upper/lower 4 layers measure in bending plane

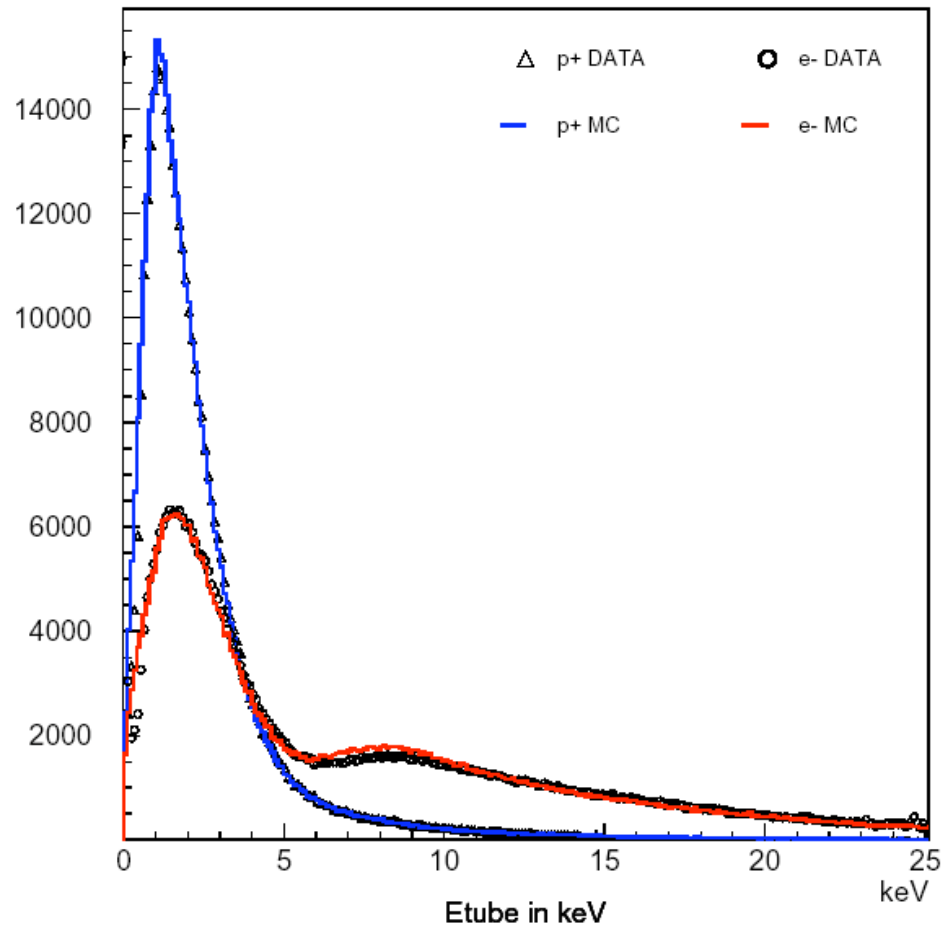
Middle 12 layers measure in perpendicular plane

328 Modules, supported with 100 μm mech. accuracy

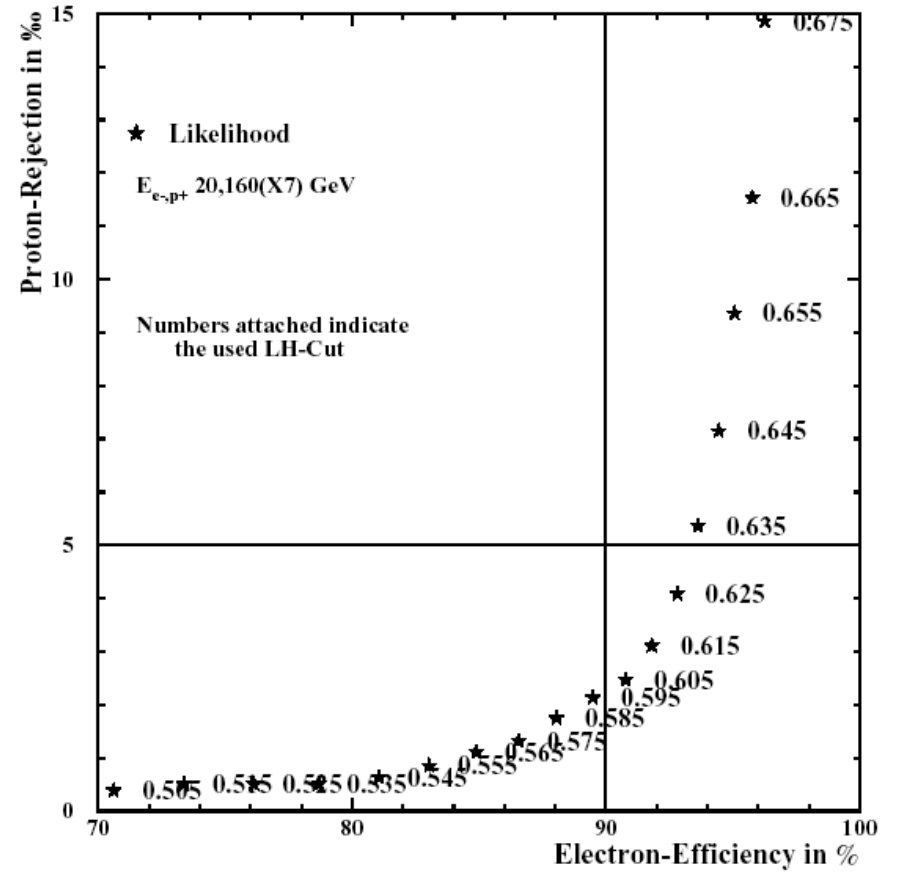
AMS - detect cosmic rays, must discriminate high energy e^+ / p



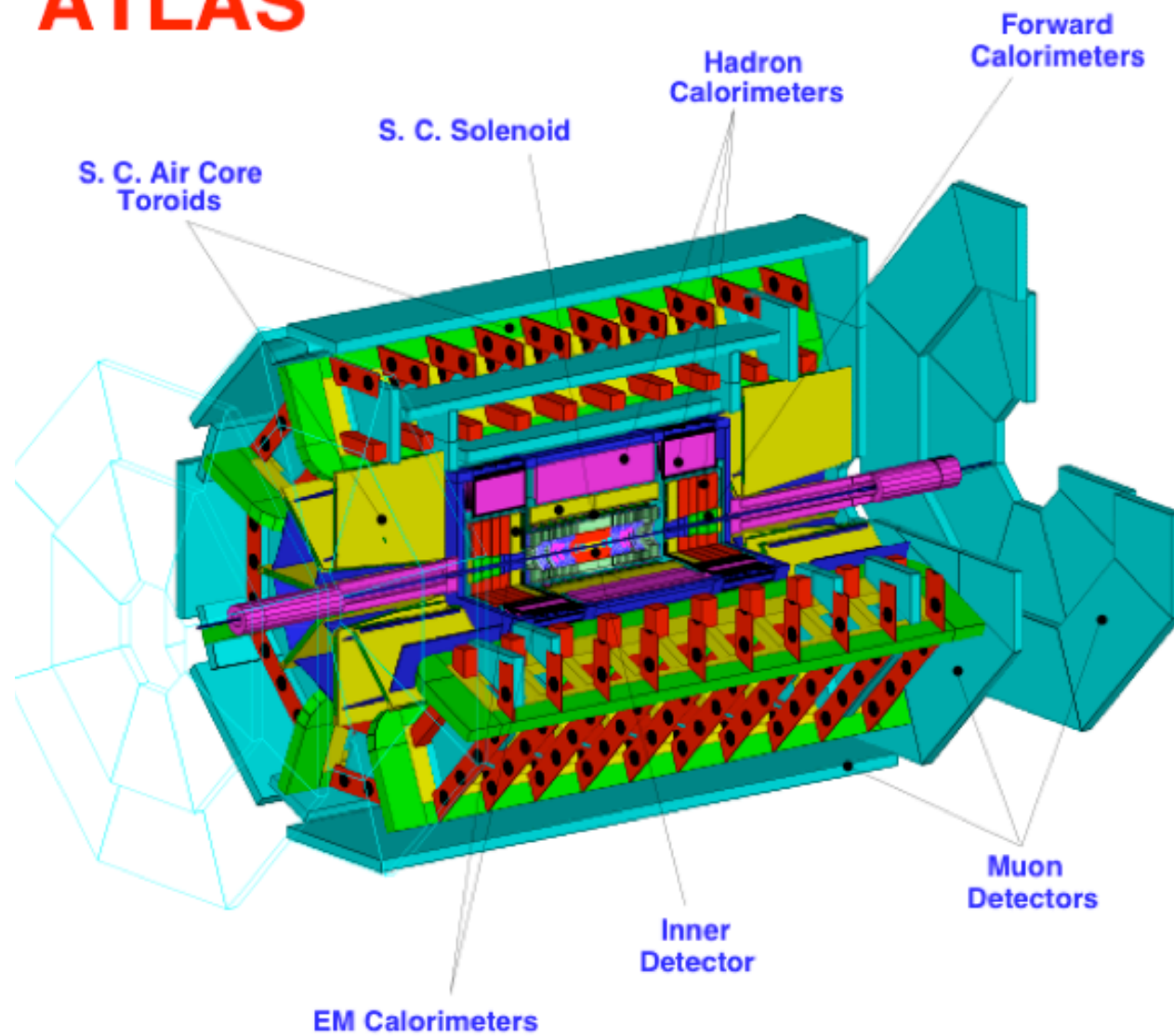
20 GeV Tube Spectra



X7 Beamtest: Proton-Rej. vs. Electron-Eff.

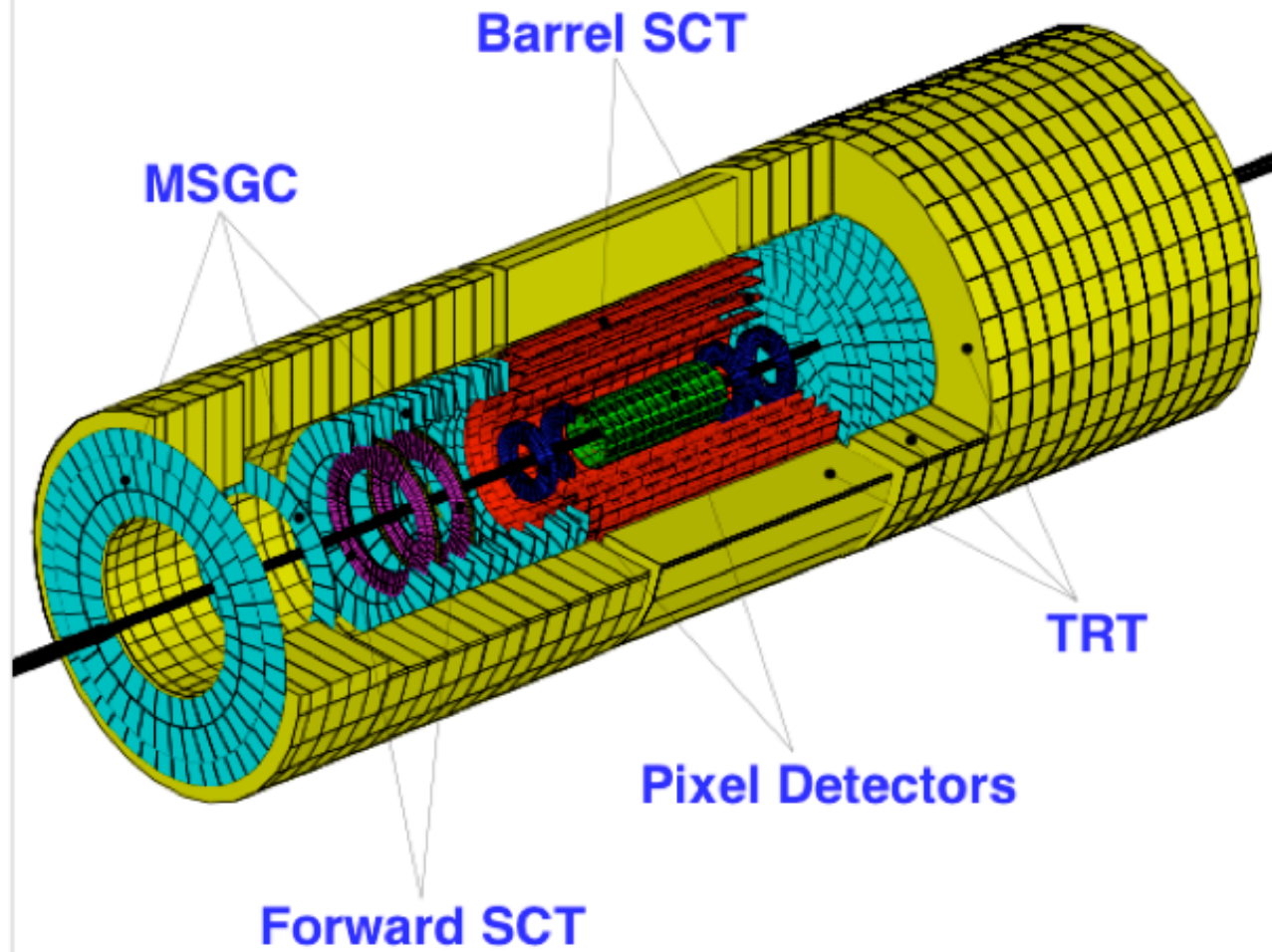


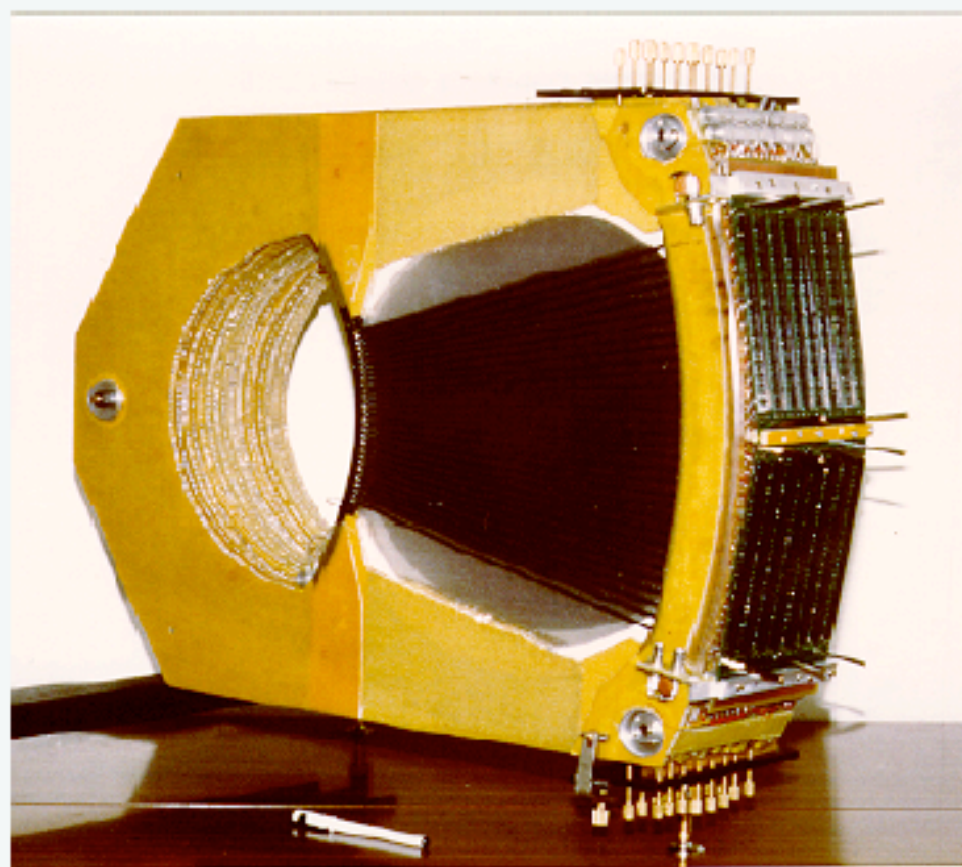
ATLAS



ATLAS

Inner Detector





**30° sector prototype of the
end-cap TRT at $\eta \sim 1.4$**

5 blocks, straws in total **2560**

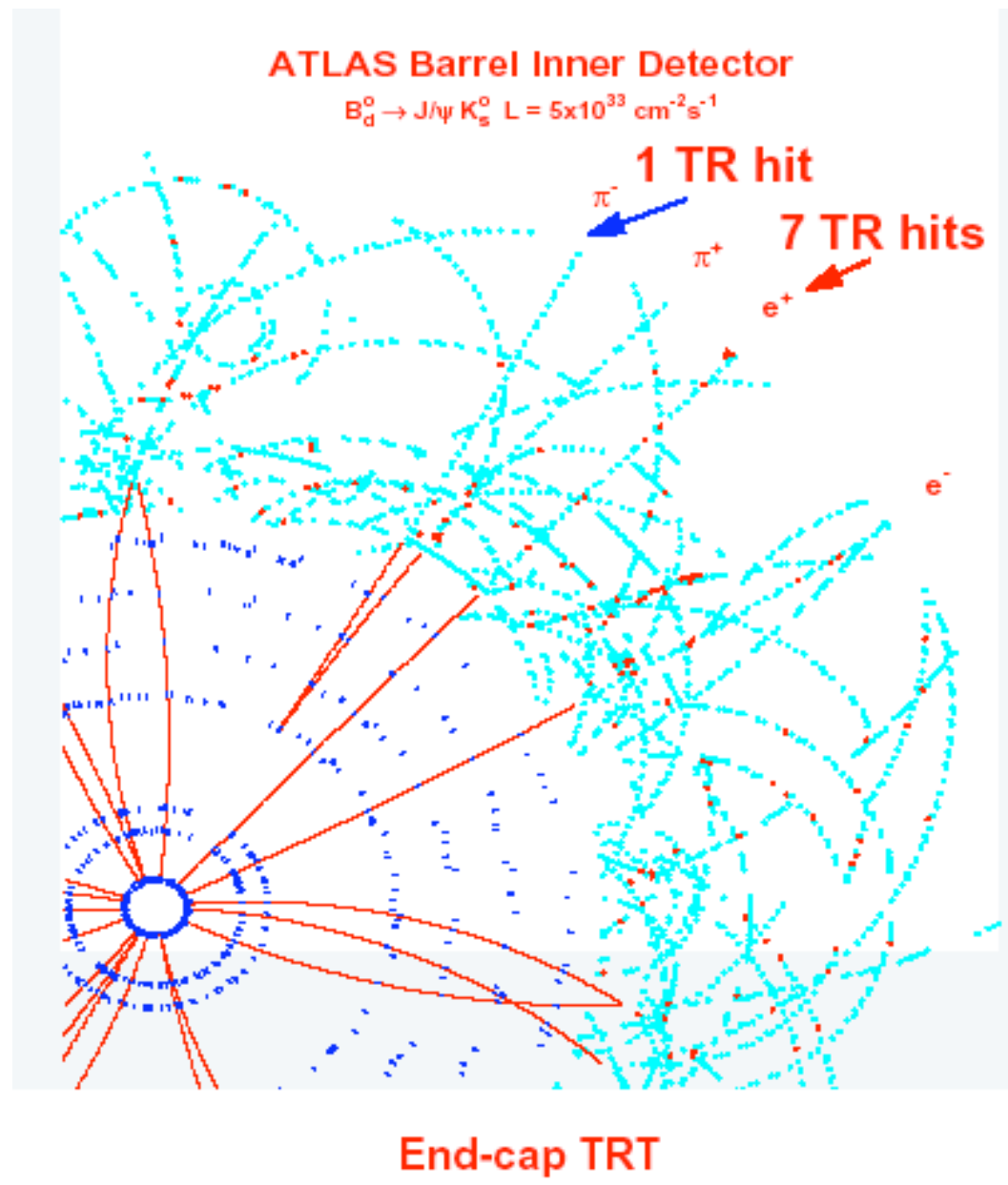
Wire offset **< 300 μm (97%)**

Radiation length: **10% X_0**

Wire-positioning accuracy: **50 μm**

Number of crossed straws: **35-40**

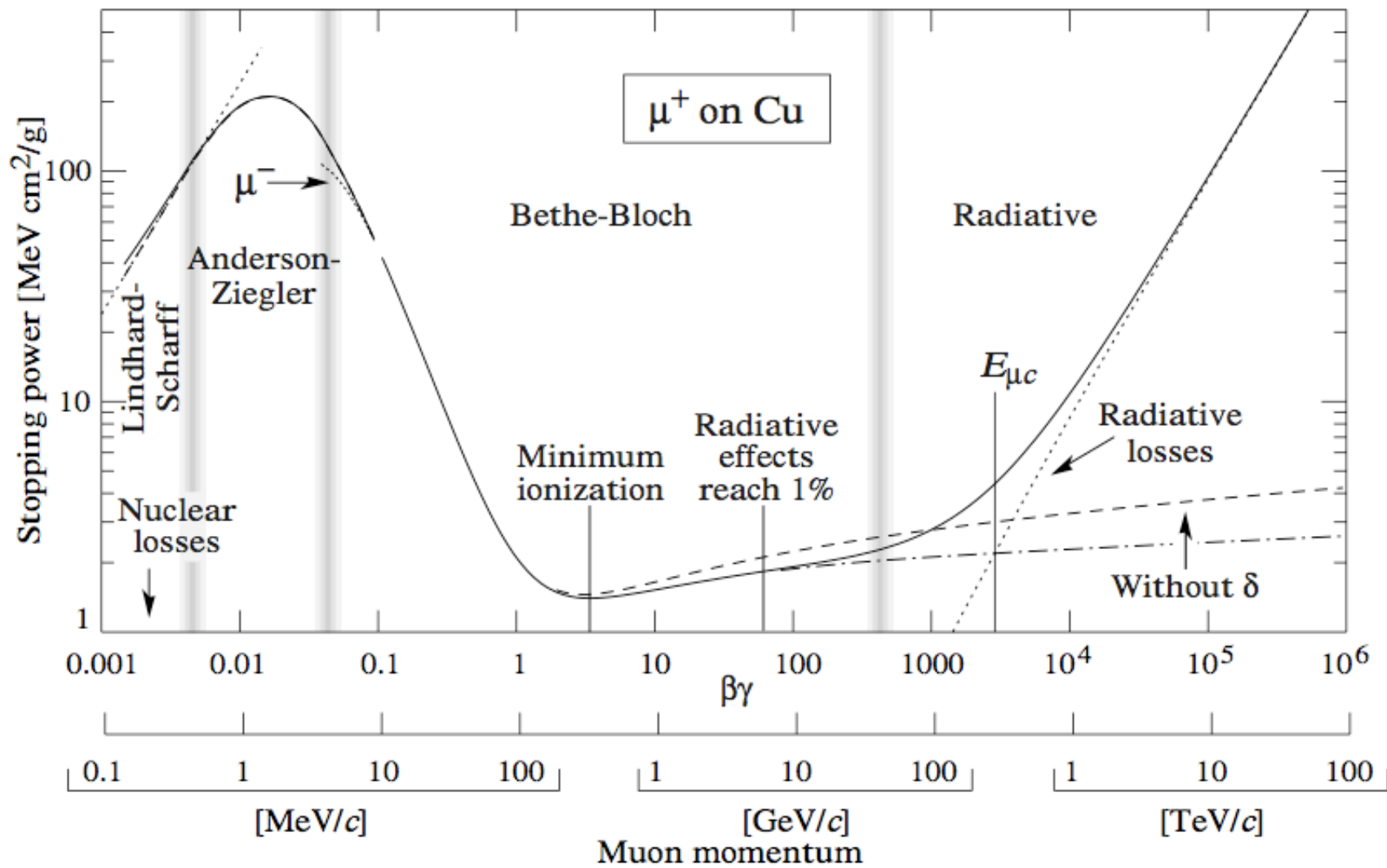
Simulated LHC event
for B decays

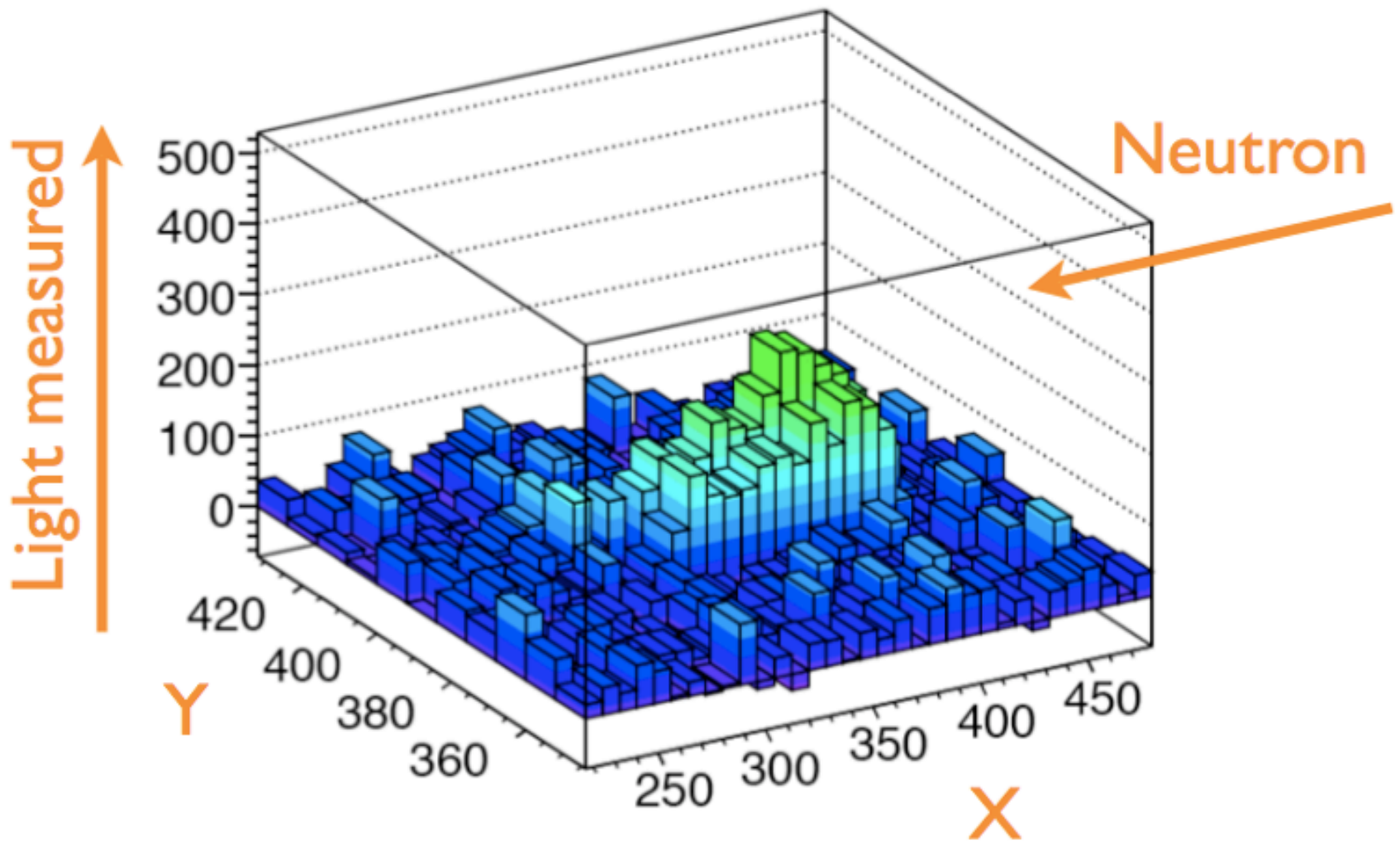


4. Nuclear recoil - single collisions in a lattice

- Galactic dynamics \rightarrow dark matter $\rho=0.3\text{GeV}/\text{cm}^3$, $v=270\text{ km/s}$ ($\beta=0.0008$)
- $m_{\text{DM}}=10\text{-}1000\text{ GeV}$ (accelerator limits)
- May weakly interact via neutral current:
 - $\lambda=h/p\sim 20\text{-}2000\text{ fm}$ \rightarrow coherently interacts with all nucleons (mostly neutrons):

$$\frac{d\sigma}{dT} = \frac{G_F^2 A m_N c^2}{8\pi v^2} N^2 \underbrace{\exp\left(-\frac{A m_N 2TR^2}{3\hbar^2}\right)}_{\text{Coherence factor}} \quad \begin{array}{l} \text{For elastic} \\ \text{scattering off a} \\ \text{nucleus} \end{array}$$





Detection

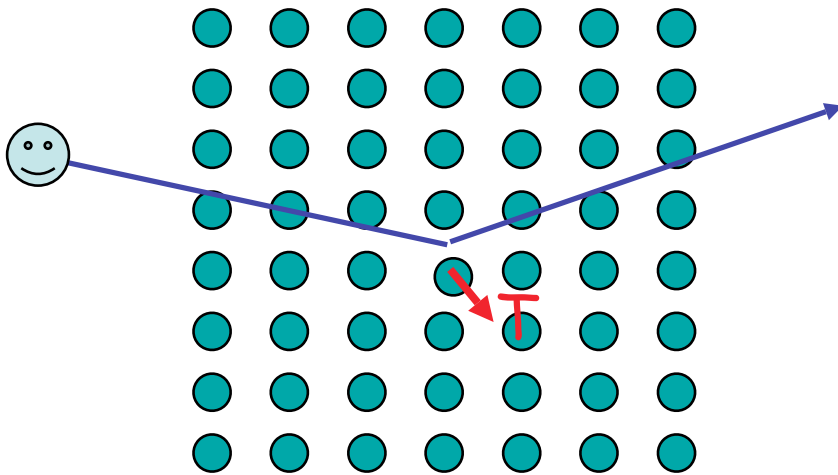
Must detect single interaction with a nucleus, $T \sim 5$ keV

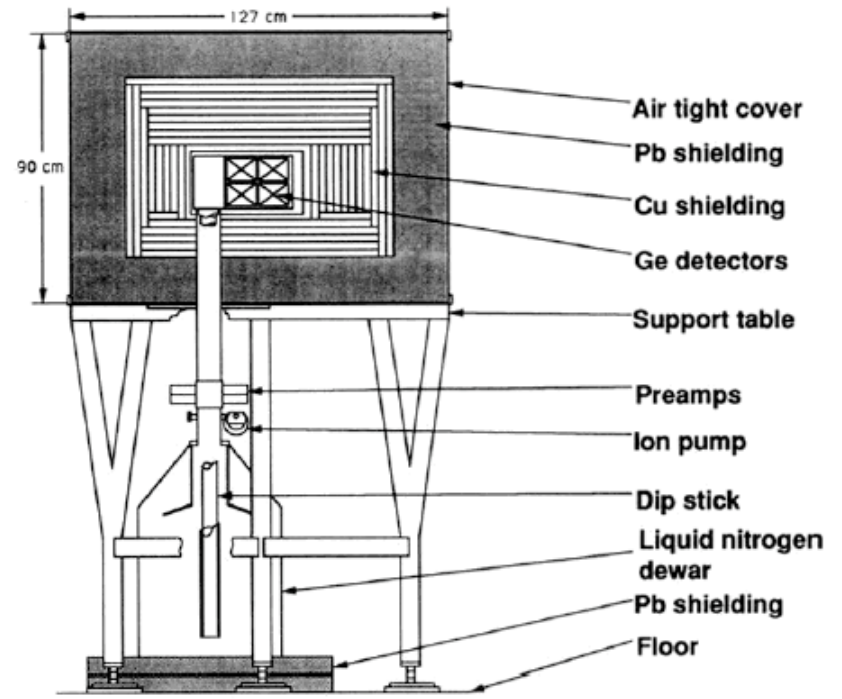
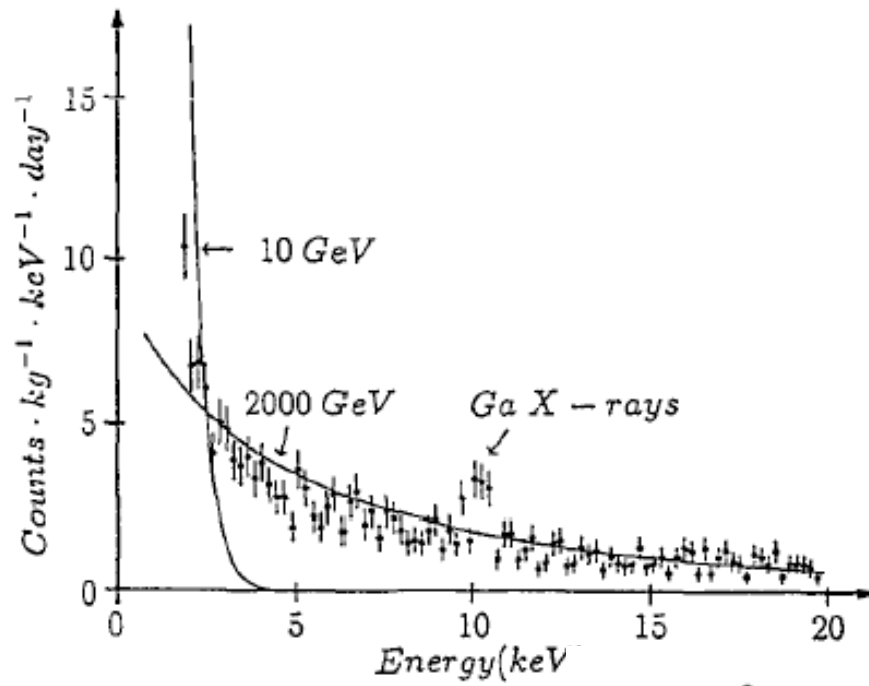
-> use nucleus in a semiconductor crystal lattice

Energy loss at $T \sim 5$ keV:

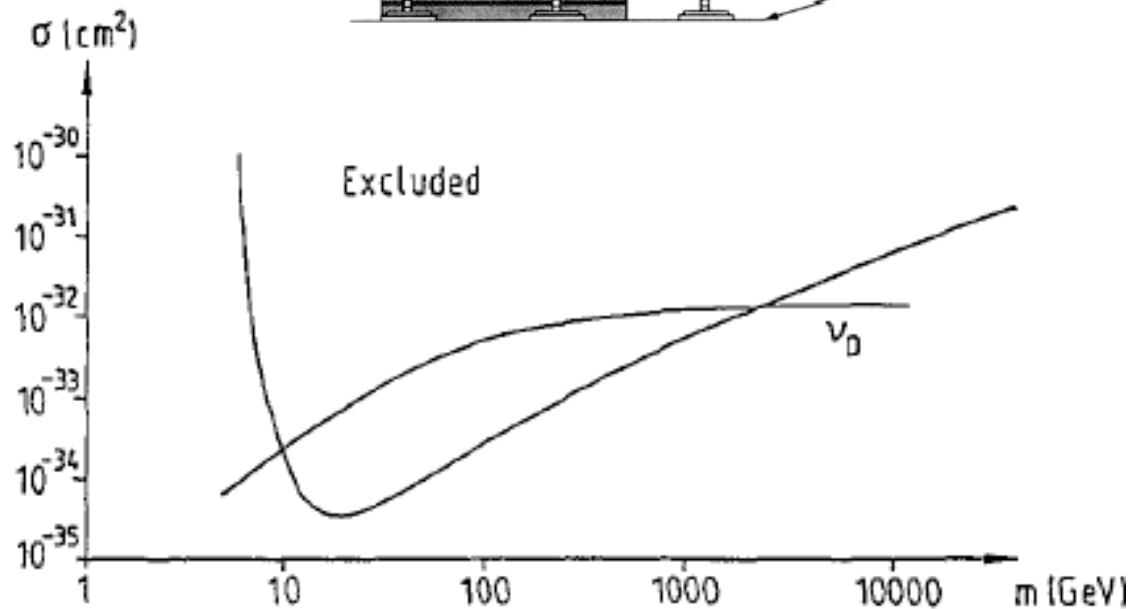
- 1/3 ionization (conduction electrons)
- 2/3 phonons (lattice vibrations)

Major background: γ rays -> ionization only



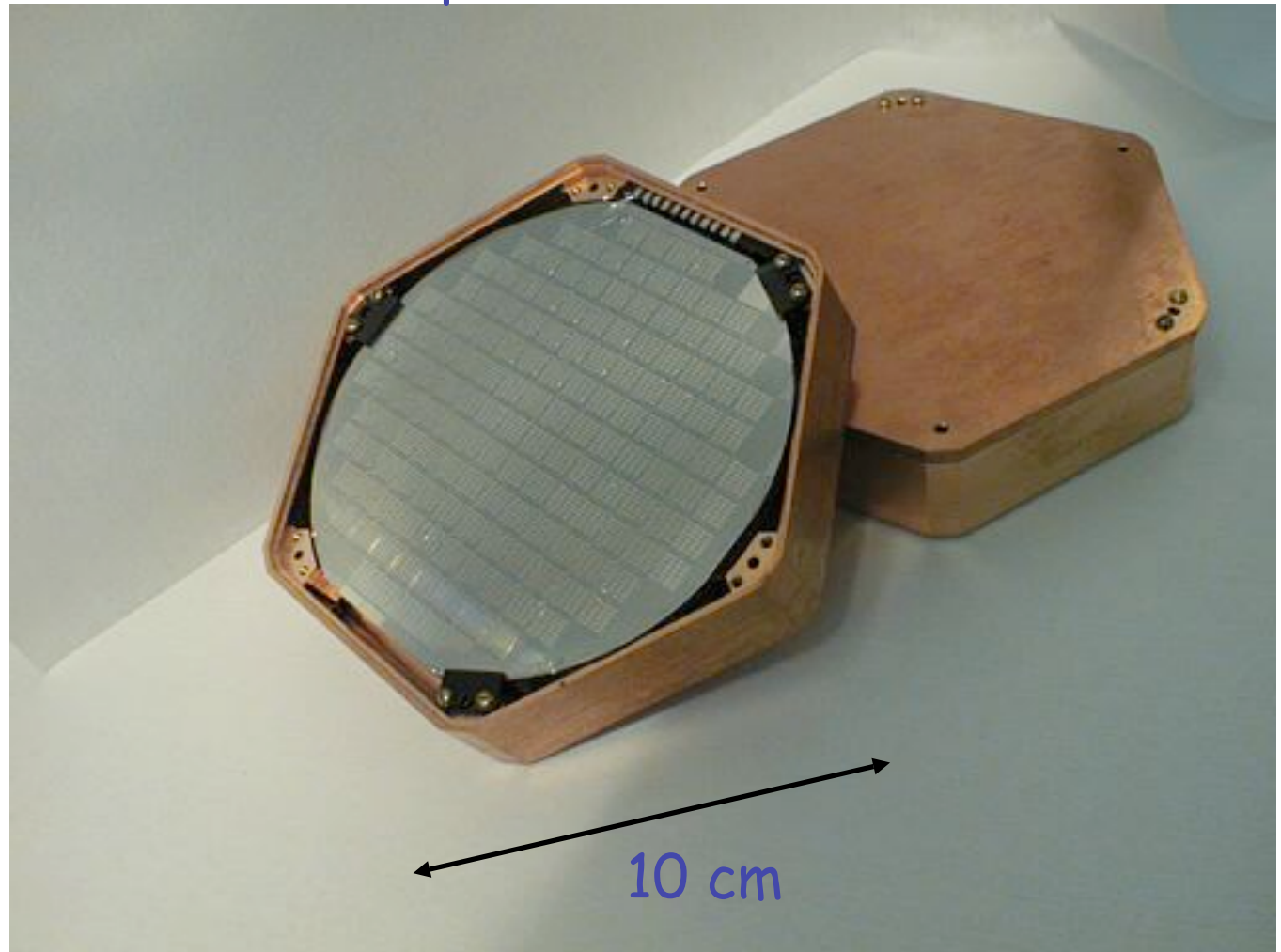


Early attempts: lower threshold on $\beta\beta$ decay experiments, detect ionization only

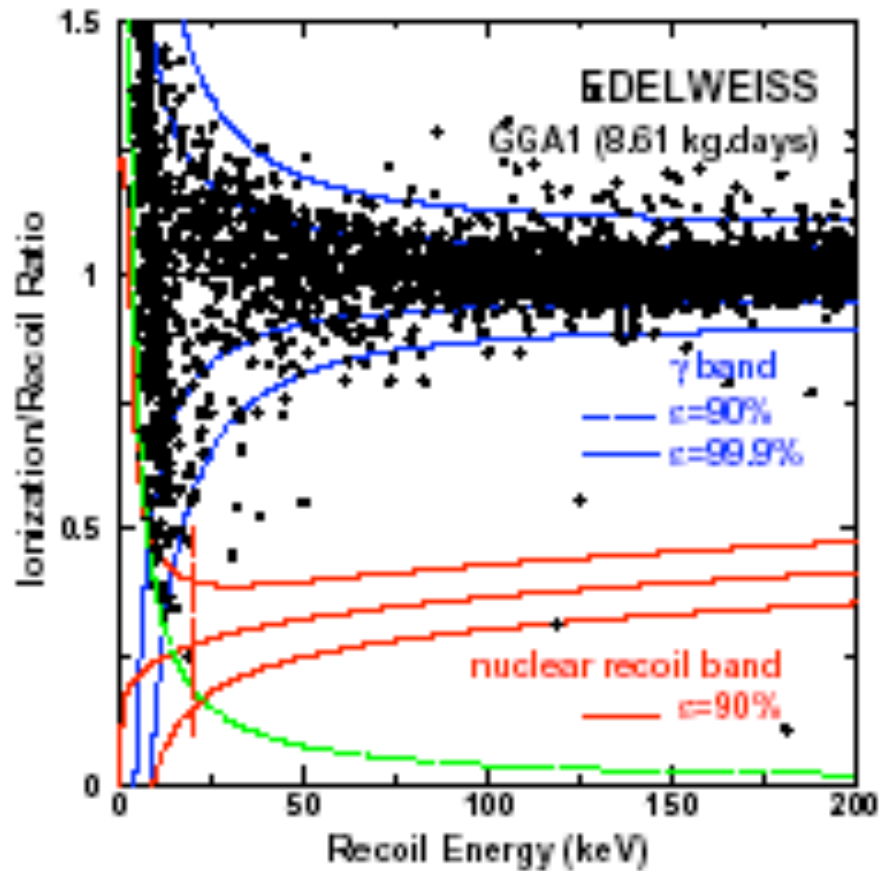


CDMS ZIP detector

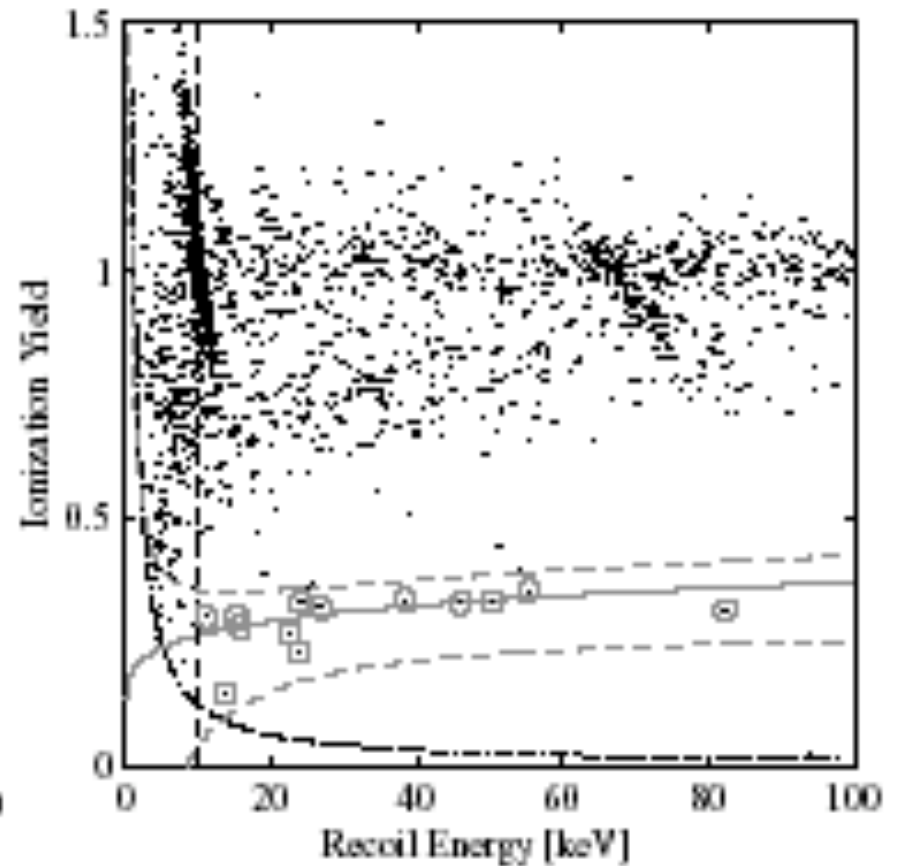
- One side collected ionization electrons
- Other side covered with s.c. phonon detections

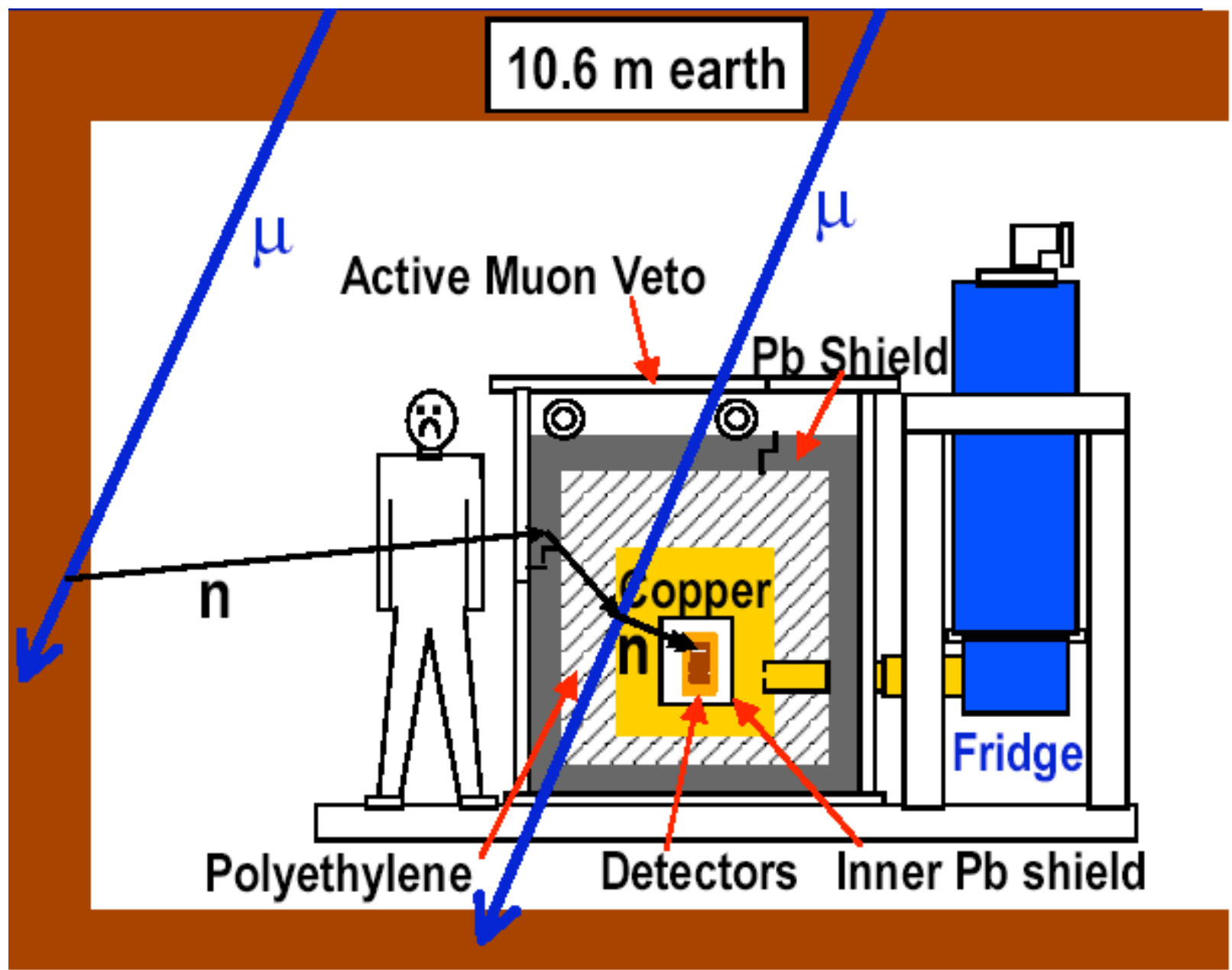


Rejection of γ backgrounds



Sensitivity improvement
by 10,000,000!
(in twenty years..._





Numbers

- $(dE/dx)_{\min} = 1.8 \text{ MeV/g/cm}^2$, $\gamma \sim 4$
- 30 eV/ion dense matter, $\sim 0.1 \text{ eV/e-h pair}$ in semiconductor, 100eV/photon in scintillator
- Cerenkov: $\cos\theta_c = 1/\beta n$, $N_\gamma = (90/\text{cm})\sin^2\theta$
- TR: 10^{-3} photons/interface, 10 keV
- Nuclear recoil: $T \sim 5 \text{ keV}$, 1/3 ionization, 2/3 phonons, all ionization for photons

References

- "Classical Electrodynamics", J.D. Jackson (esp. ch. 13)
- "Radiation Detection and Measurement", G.F. Knoll
- "Experimental Methods in High Energy Physics", T. Ferbel
- "High Energy Physics", Rossi (buy used)
- Particle Properties Data Book, <http://pdg.lbnl.gov>
- Original papers by Bethe, Fermi, Bloch, Ginzberg, Cerenkov, Landau, ...