Ionization Energy Loss of Charged Projectiles in Matter

Steve Ahlen
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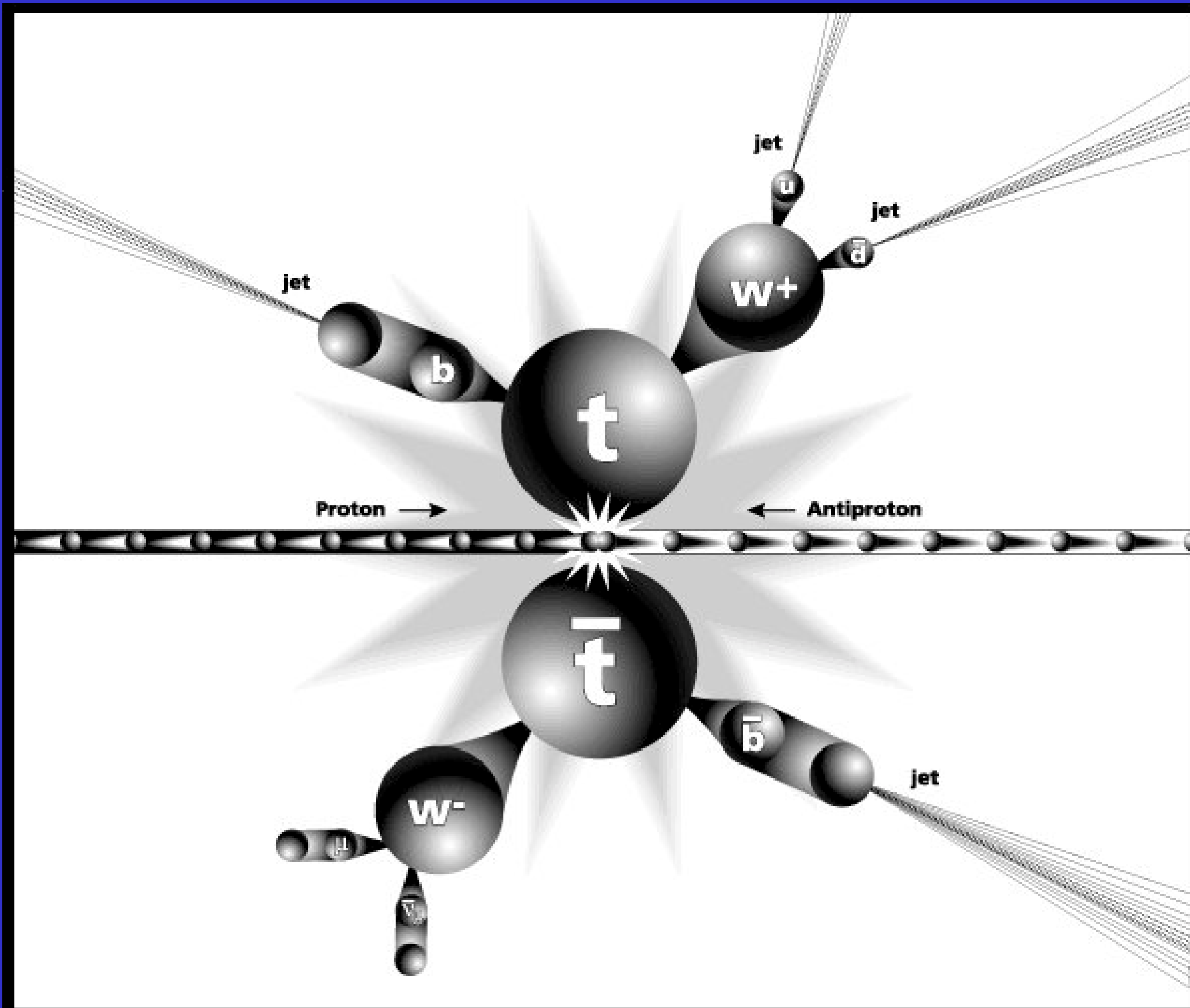
Almost all particle detection and measurement techniques in high energy physics are based on the energy deposited by energetic charged particles in matter by means of atomic excitation and ionization.


jet
u
jet
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Proton →
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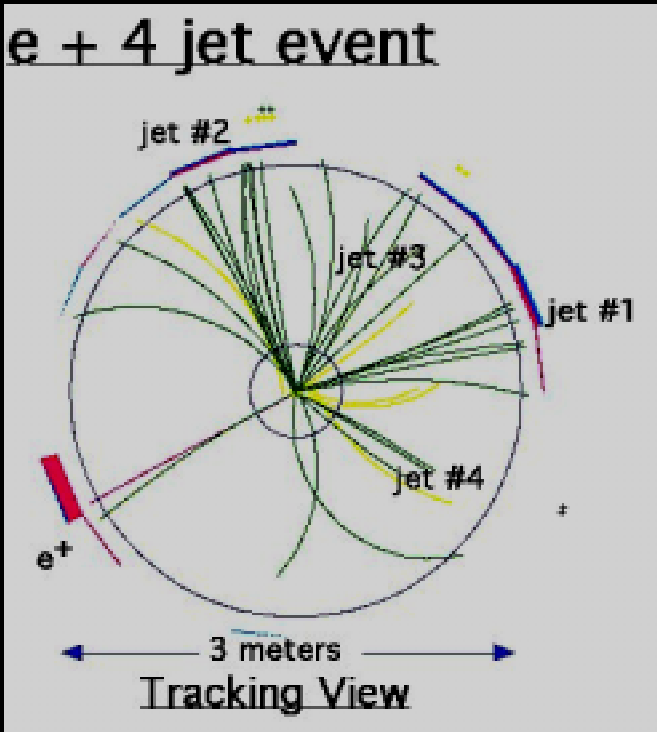
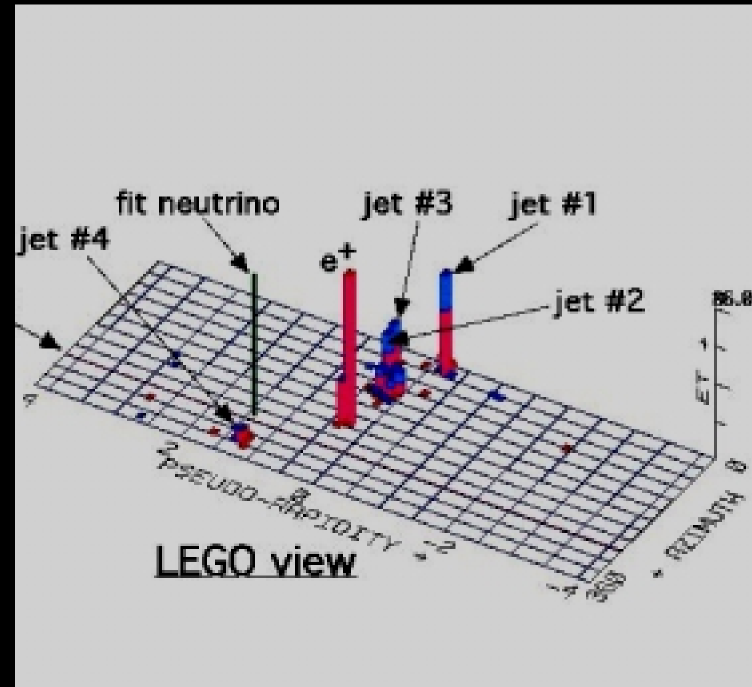
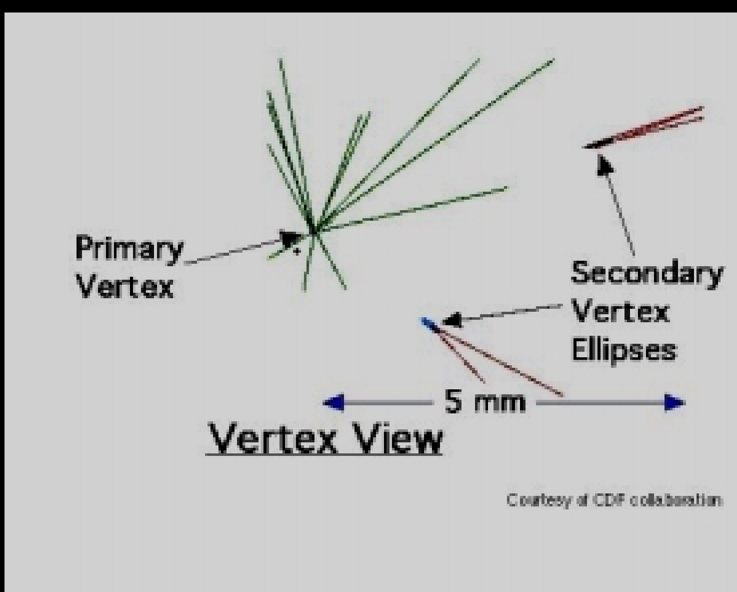
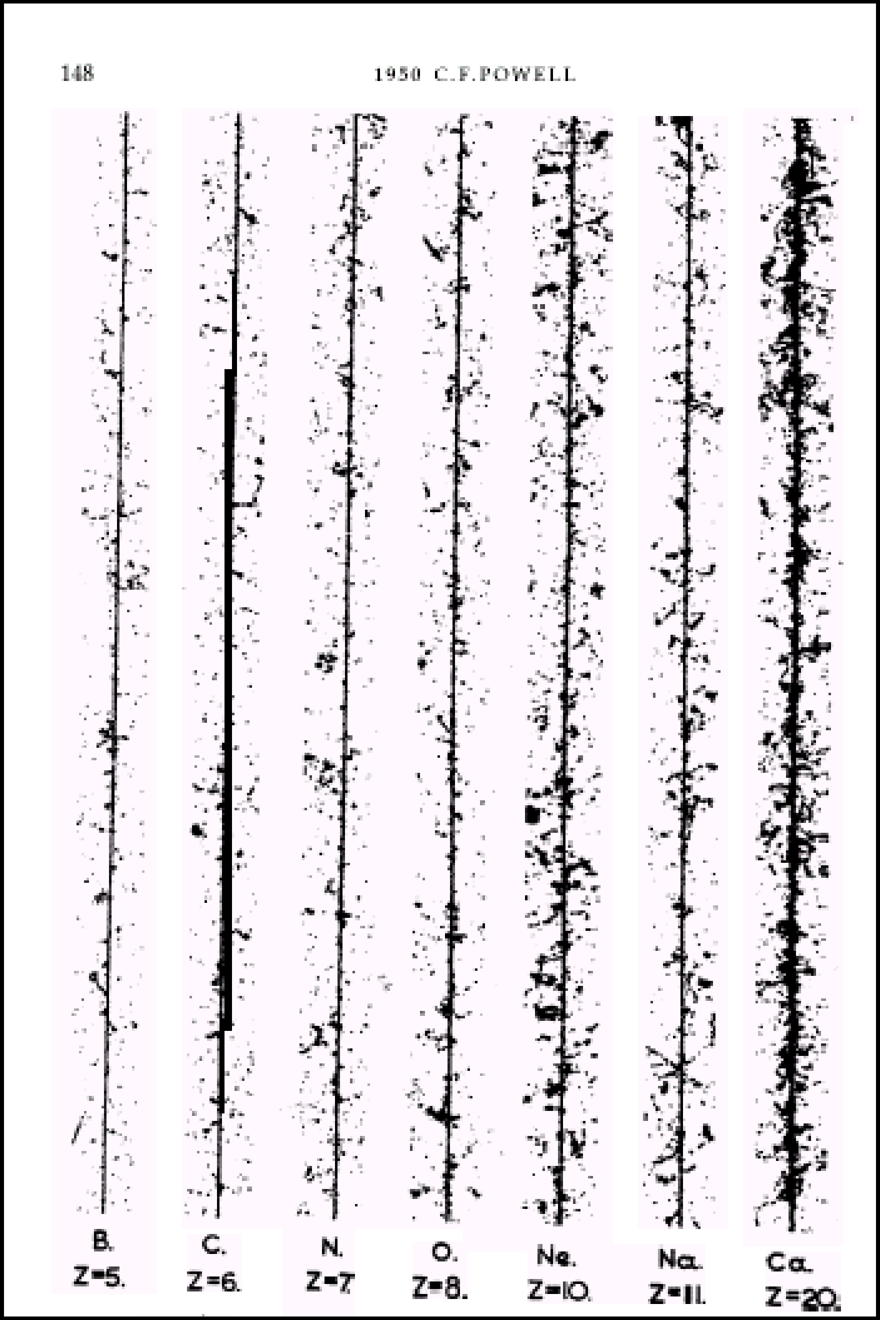


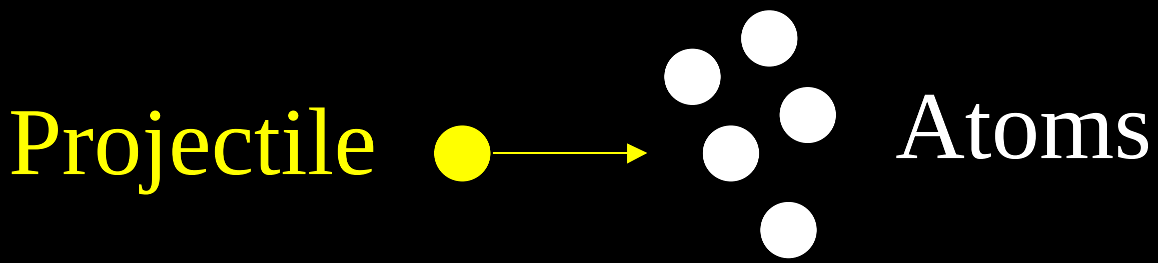




- Drift chambers for tracking
- Silicon detectors for vertex
- Electromagnetic calorimeters
- Hadronic calorimeters

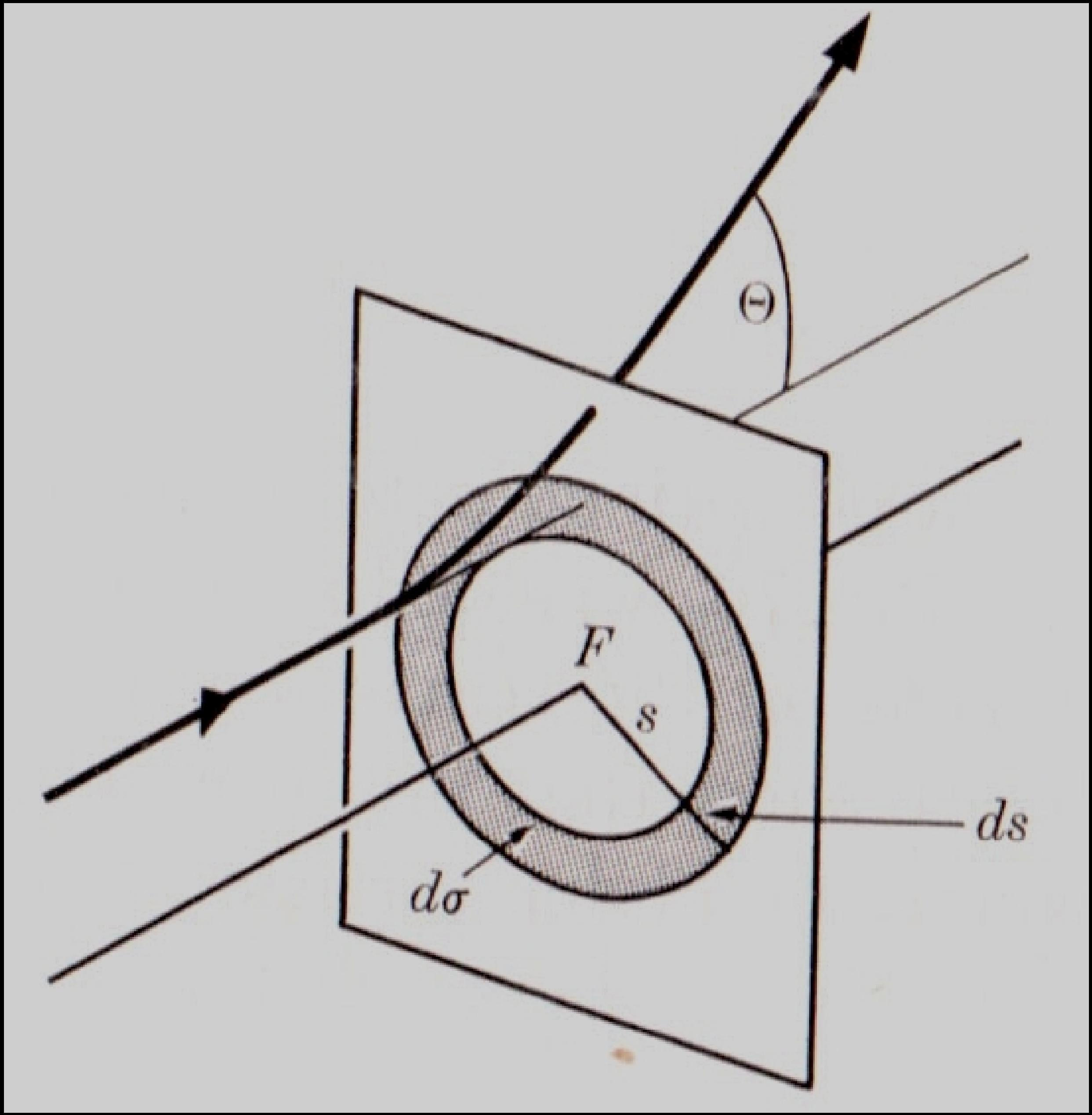
Examples of the tracks in photographic emulsions of primary nuclei of the cosmic radiation moving at relativistic velocities. Taken from Powell’s Nobel Prize lecture.





- Projectile mass = M = Am_u
- Electron mass = m << M (we assume)
- Electron charge = –e
- Projectile charge = Z_1e
- Target atomic number = Z_2
- Target atomic mass = A_2
- Projectile velocity = v = βc
- Projectile Lorentz factor = γ
- N = number density of electrons

Electrons scatter off projectile in center of mass frame



Rutherford scattering cross section:

$$\frac{d\sigma}{d\Omega} = \frac{q_1^2 q_2^2}{4m^2 v^4 \sin^4(\Theta/2)}$$

q_2 = electron charge = –e

q_1 = projectile charge = Z_1e

m = electron mass

v = relative speed

Θ = center of mass scattering angle

Mott scattering cross section for electrons off nuclei (lowest order):

$$\frac{d\sigma}{d\Omega} = \frac{Z_1^2 e^4}{4p^4 c^2 \sin^4(\Theta/2)}\left(m^2c^2 + p^2 \cos^2 \frac{\Theta}{2} \right)$$

$$\frac{d\sigma}{d\Omega} = \frac{Z_1^2 e^4 m^2 \gamma^2}{4p^4 \sin^4(\Theta/2)}\left(1 - \beta^2 \sin^2 \frac{\Theta}{2} \right)$$

Some kinematics

Energy transfer to electron in lab frame:

$$\varepsilon = 2mv^2\gamma^2 \sin^2\frac{\Theta}{2} = \varepsilon_m \sin^2\frac{\Theta}{2}$$

Scattered electrons produced in thickness dx in lab:

$$dn = NAdx\frac{d\sigma}{A}$$

Scattered electron production per unit energy transfer:

$$\frac{dn}{d\varepsilon dx} = N\frac{d\sigma}{d\varepsilon} = N\frac{d\sigma}{d\Omega}\frac{2\pi\sin\Theta}{\varepsilon_m\left(\sin\frac{\Theta}{2}\cos\frac{\Theta}{2}\right)} = \frac{4\pi N}{\varepsilon_m}\frac{d\sigma}{d\Omega}$$

$$\frac{dn}{d\varepsilon dx} = \frac{4\pi N}{\varepsilon_m} \frac{Z_1^2 e^4 m^2 \gamma^2 \varepsilon_m^2}{4p^4 \varepsilon^2} \left(1 - \beta^2 \sin^2 \frac{\Theta}{2}\right)$$

$$\frac{dn}{d\varepsilon dx} = \frac{2\pi N Z_1^2 e^4}{m v^2 \varepsilon^2} \left(1 - \beta^2 \frac{\varepsilon}{\varepsilon_m}\right)$$

Energy loss per path length for large energy transfers (> ε_1):

$$\left.\frac{dE}{dx}\right|_{>\varepsilon_1} = {}_{\varepsilon_1}^{\varepsilon_m} \varepsilon \frac{dn}{d\varepsilon dx} d\varepsilon \approx \frac{2\pi N Z_1^2 e^4}{mv^2}\left(n \frac{\varepsilon_m}{\varepsilon_1} - \beta^2 \right)$$

ELECTRODYNAMICS OF CONTINUOUS MEDIA

by

L. D. LANDAU AND E. M. LIFSHITZ

INSTITUTE OF PHYSICAL PROBLEMS, U.S.S.R. ACADEMY OF SCIENCES

Volume 8 of *Course of Theoretical Physics*

Translated from the Russian by

J. B. SYKES AND J. S. BELL

§85. Ionisation losses by fast particles in matter: the relativistic case

At velocities comparable with that of light, the effect of the polarisation of the medium on its stopping power with respect to a fast particle may become very important even in gases.††

To derive the appropriate formulae, we use a method analogous to that used in §84, but it is now necessary to begin from the complete Maxwell's equations. When extraneous charges are present with volume density ρ_{ex}, and extraneous currents with density $\mathbf{j}_{ex}$, these equations are‡‡

$$\operatorname{div}\mathbf{H} = 0, \qquad \mathbf{curl}\,\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}, \tag{85.1}$$

$$\operatorname{div}\hat{\epsilon}\mathbf{E} = 4\pi\rho_{ex}, \qquad \mathbf{curl}\,\mathbf{H} = \frac{1}{c}\frac{\partial \hat{\epsilon}\mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{j}_{ex}. \tag{85.2}$$

In the present case the extraneous charge and current distribution are given by

$$\rho_{ex} = e\delta(\mathbf{r} - \mathbf{v}t), \qquad \mathbf{j}_{ex} = e\mathbf{v}\,\delta(\mathbf{r} - \mathbf{v}t). \tag{85.3}$$

dE/dx = force exerted on projectile by medium = Z_1eE

$$\left.\frac{dE}{dx}\right|_{>\varepsilon_1} = \frac{2\pi N Z_1^2 e^4}{mv^2}\left(n\frac{\varepsilon_m}{\varepsilon_1} - \beta^2 \right)$$

$$\left.\frac{dE}{dx}\right|_{<\varepsilon_1} = \frac{2\pi N Z_1^2 e^4}{mv^2}\left(n\frac{2mv^2\gamma^2\varepsilon_1}{I^2} - \beta^2 \right)$$

(if below Cherenkov threshold, $v < c/n$)

I = logarithmic average of atomic excitation energies

Bethe Formula:

Ann. Physik **5**, 325 (1930); Z. Phys. **76**, 293 (1932)

$$\frac{dE}{dx} = \frac{4\pi N Z_1^2 e^4}{mv^2}\left(n\frac{\varepsilon_m}{I} - \beta^2 \right)$$

Half the energy is lost in *close collisions* (small number of events, large energy transfer - *delta rays*)

Half lost in *distant collisions* (large number, energy lost per collision is about same as ionization energy)

Energy loss expressed per *grammage*

$X = \rho x \qquad \rho = \text{mass density}$

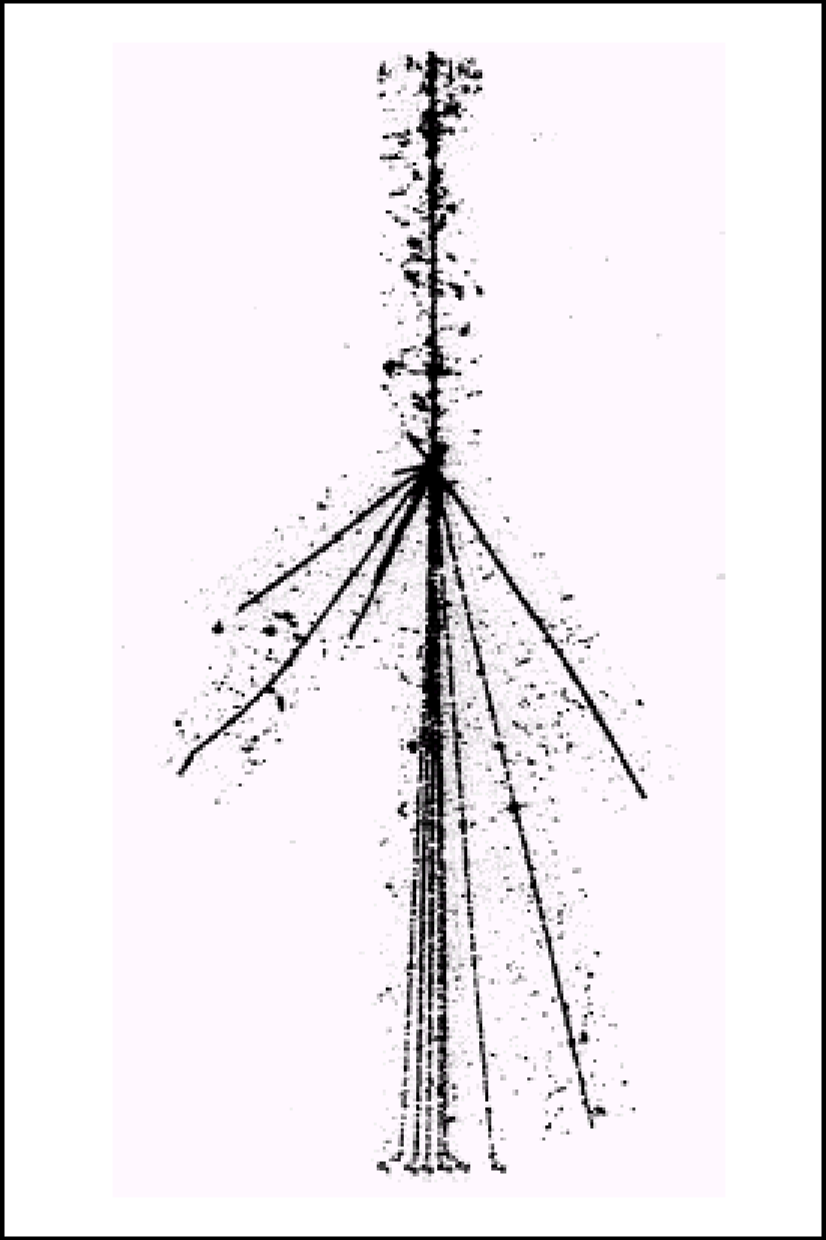
$$\frac{dE}{dX} = \frac{4\pi N Z_1^2 e^4}{mv^2 \left(m_u \; NA_2 / Z_2\right)} \left(n \frac{\varepsilon_m}{I} - \beta^2 \right)$$

Minimum ionization rate ($\gamma \sim 3$):

$$\left.\frac{dE}{dX}\right|_{min} \approx 2 \frac{\text{MeV}}{\text{g/cm}^2}$$

Material	I (eV)
Plastic scintillator	63
Aluminum	163
Iron	285
Lead	825

A nucleus of magnesium or aluminium moving with great velocity, collides
with another nucleus in a photographic emulsion. The incident nucleus splits up into six alpha - particles of the same speed and the struck nucleus is shattered.



Delta Ray production:

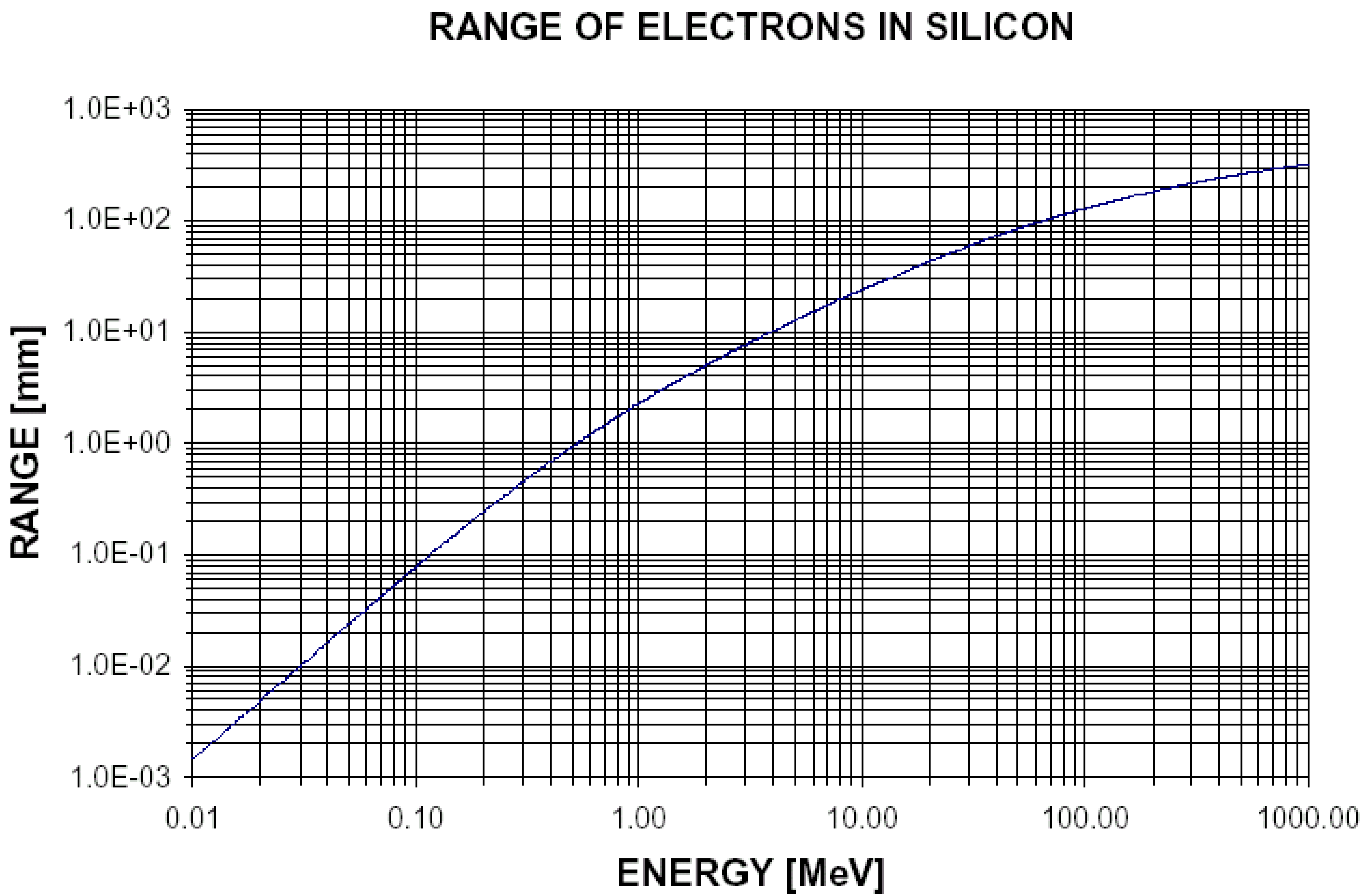
$$\frac{dn}{d\varepsilon dX} = \frac{2\pi N Z_1^2 e^4}{m v^2 \varepsilon^2 \rho}\left(1 - \beta^2 \frac{\varepsilon}{\varepsilon_m}\right) \approx \frac{1}{2}\frac{dE}{dX}\frac{1}{10}\frac{1}{\varepsilon^2}$$

$$\left.\frac{dn}{dX}\right|_{>\varepsilon_0} \approx \frac{0.1}{\varepsilon}\frac{\text{MeV}}{\text{g/cm}^2}$$

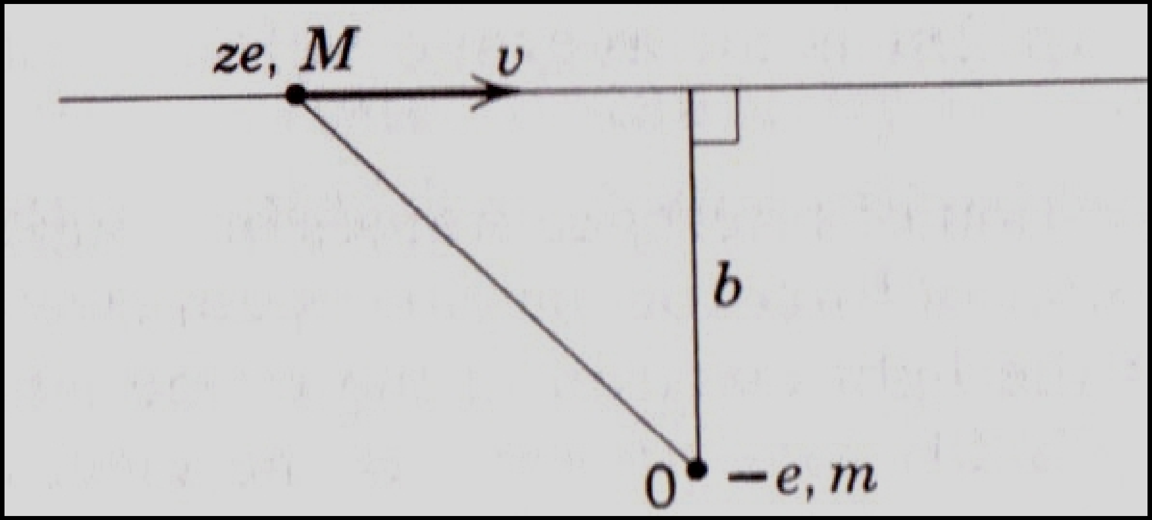
Electron range for electron energy ε:

$$R \approx 0.537\varepsilon \frac{\mathrm{g/cm^2}}{\mathrm{MeV}}\left(1-\frac{0.9815}{1+3.123\,\mathrm{MeV}/\varepsilon}\right)$$

But lots of *straggling* for electrons


RANGE OF ELECTRONS IN SILICON
RANGE [mm]
1.0E+03
1.0E+02
1.0E+01
1.0E+00
1.0E-01
1.0E-02
1.0E-03
0.01
0.10
1.00
10.00
100.00
1000.00
ENERGY [MeV]

Impact parameter approach for distant collisions



$$\Delta p_y = -\int_{-\infty}^{\infty} eE_y(t)\,dt = \int_{-\infty}^{\infty} e\frac{\gamma Z_1 eb}{\left(b^2+\gamma^2 v^2 t^2\right)^{3/2}}dt$$

Collision time: $t \approx \frac{b}{\gamma v}$

$$\gamma v t = b\tan\theta$$

$$\Delta p_y = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e\frac{\gamma Z_1 eb}{\left(b^2+b^2\tan^2\theta\right)^{3/2}}\frac{b}{\gamma v}\sec^2\theta\,d\theta = \frac{2Z_1 e^2}{bv}$$

$$\Delta p_y = \frac{2Z_1 e^2}{bv}$$

$$\left.\frac{dE}{dx}\right|_{distant} = \int_{b_{min}}^{b_{max}} \frac{(\Delta p_y)^2}{2m} 2\pi N b\,db = \frac{4\pi N Z_1^2 e^4}{mv^2} \ln \frac{b_{max}}{b_{min}}$$

$$b_{max} = \frac{\gamma v}{\omega}$$

Collision time equals oscillation time of electron

$$b_{min} = a_0$$

Impact parameter approach valid only for distances larger than size of atoms

$$\frac{dE}{dx} = \frac{4\pi N Z_1^2 e^4}{mv^2}\left(n\frac{2mv^2\gamma^2}{I} - \beta^2 \right)$$

Relativistic rise

One γ from increasing impact parameter;

The other γ from increase in kinematic limit to energy transfer

Relativistic rise does not continue indefinitely:

Density effect - polarization of material screens field of projectile at distant atoms.

$$\left.\frac{dE}{dx}\right|_{average} = \frac{4\pi N Z_1^2 e^4}{mv^2}\left(n\frac{2mv^2\gamma^2}{I} - \beta^2 - \frac{\delta}{2}\right)$$

Asymmetric energy loss fluctuations skew energy loss distribution (*Landau distribution*).

At high energies ($\gamma \sim 10$ for solids, 1000 for gases), the *most probable energy loss* in thickness x is:

$$\Delta_{mp}\left(\beta = 1\right) = \frac{2\pi N Z_1^2 e^4 x}{mc^2} \; n \frac{1.219 Z_1^2 x}{a_0} \approx 1.6 \frac{\text{MeV}}{\text{g}/\text{cm}^2}$$

$$\frac{\text{FWHM}}{\Delta_{mp}} = \frac{\left(4.02\right)}{n\left(\dfrac{1.219 Z_1^2 x}{a_0}\right)} \approx 20\%$$

If maximum energy delta ray is small compared to mean energy loss,

$$\varepsilon_m < \frac{1}{20}\frac{dE}{dx}x$$

energy loss distribution is nearly gaussian and square of standard deviation of energy loss is:

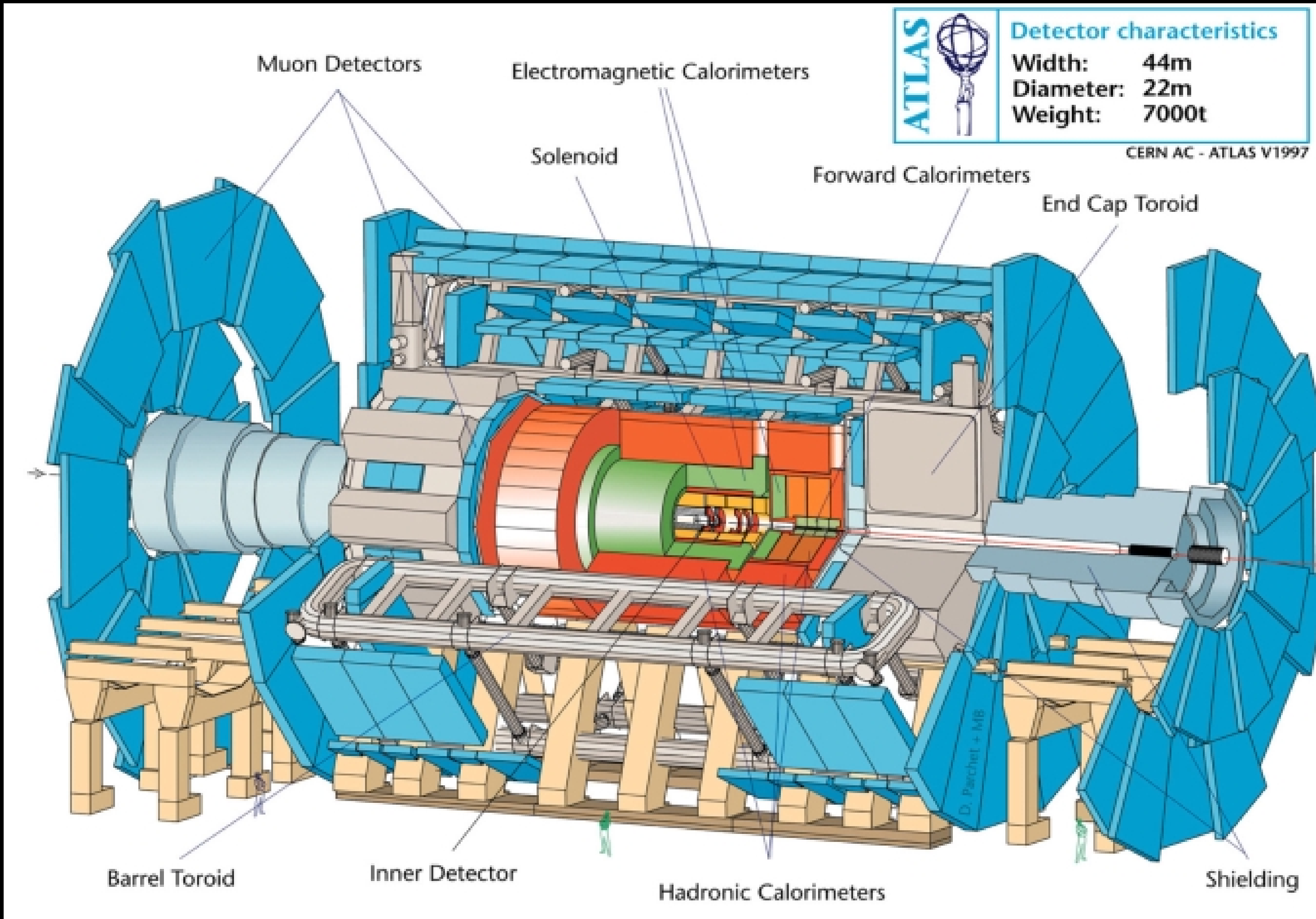
$$\sigma^2 = 4\pi N Z_1^2 e^4 x \frac{\left(1-\beta^2/2\right)}{\left(1-\beta^2\right)}$$

Some observational examples

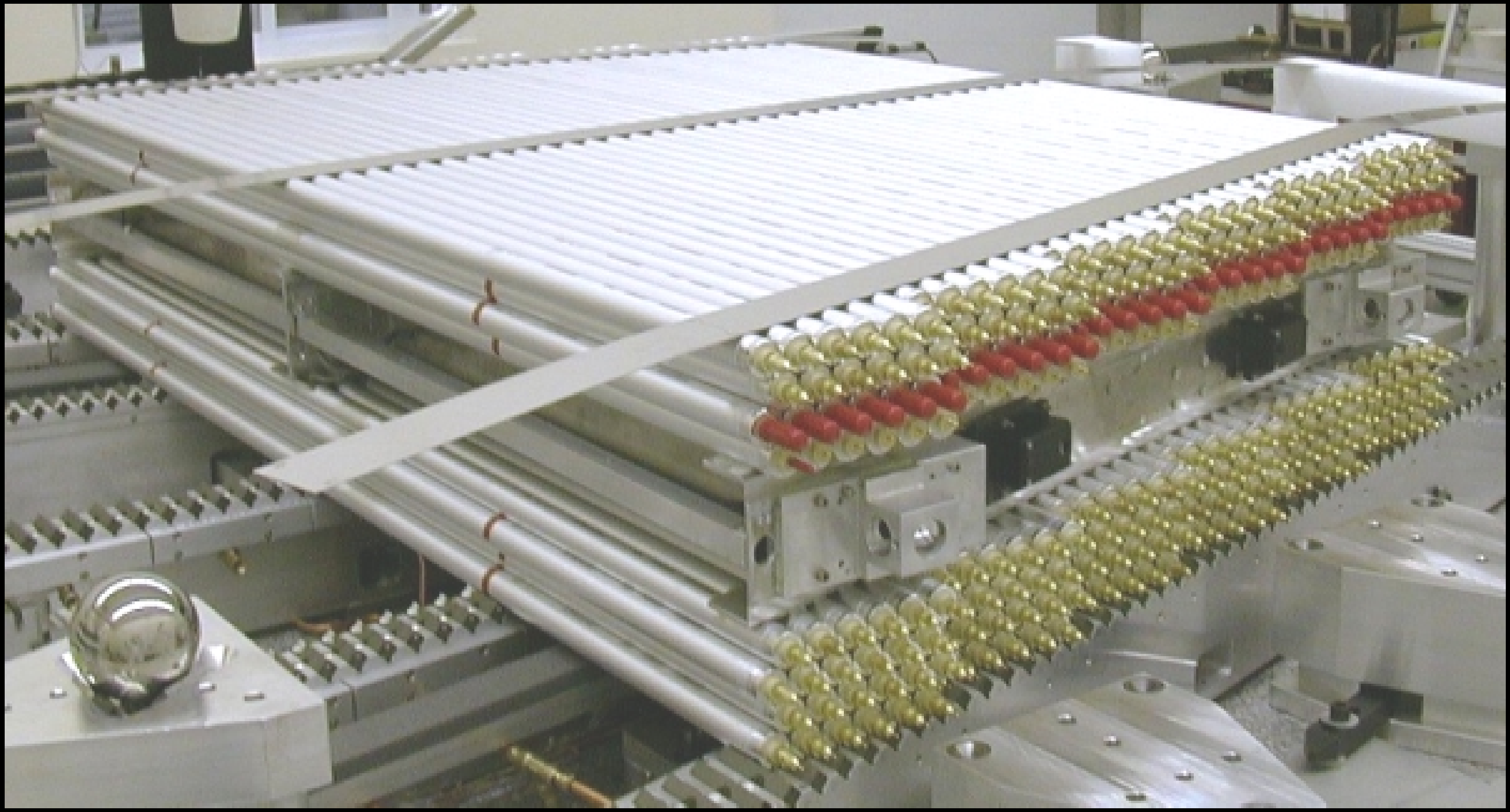
Large Hadron Collider (LHC) at Geneva, Switzerland

proton-proton collisions with 14 trillion electron volts energy

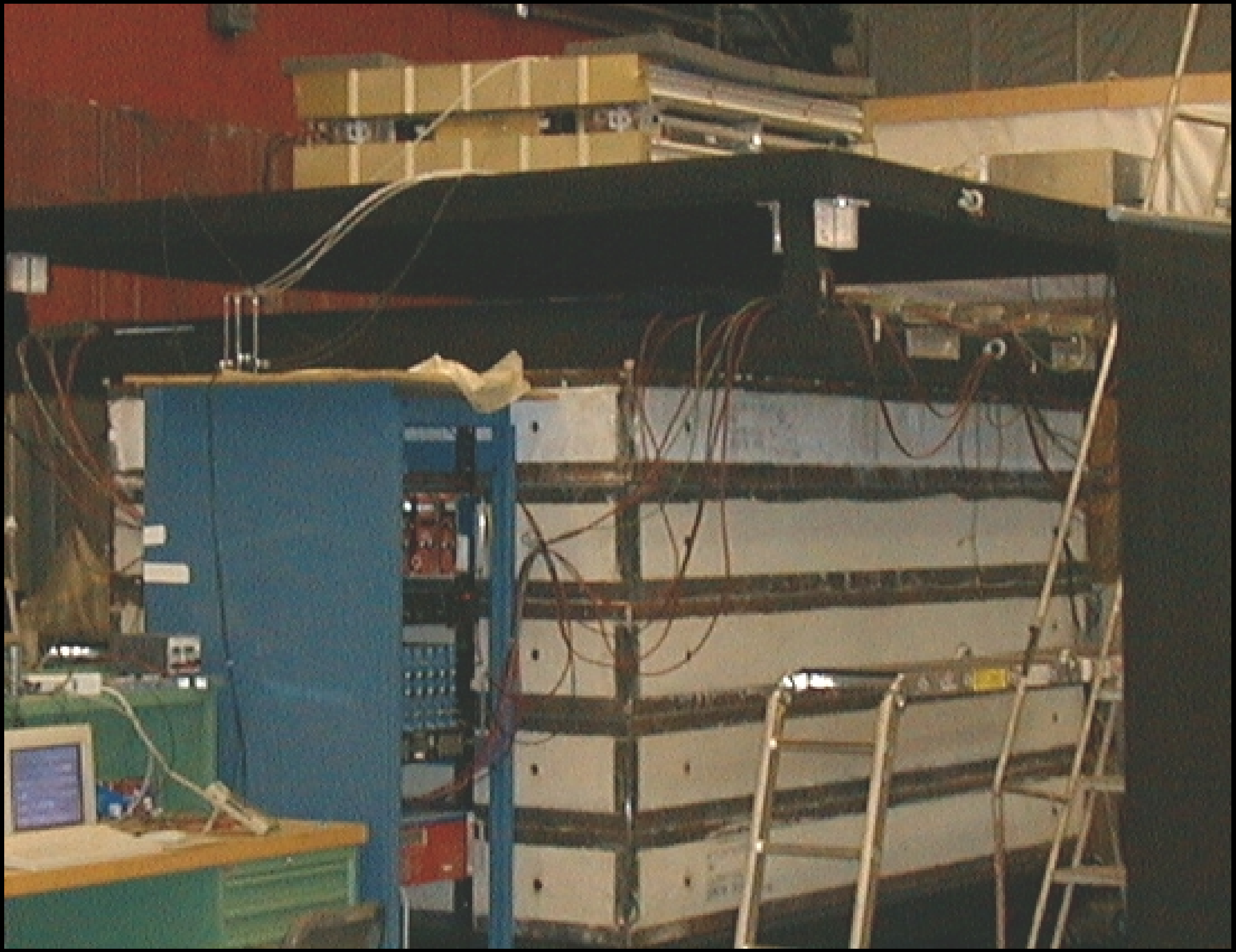
ATLAS


Detector characteristics
Width: 44m
Diameter: 22m
Weight: 7000t
ATLAS
CERN AC - ATLAS V1997
Muon Detectors
Electromagnetic Calorimeters
Solenoid
Forward Calorimeters
End Cap Toroid
Barrel Toroid
Inner Detector
Hadronic Calorimeters
Shielding
D. Parchet + MB

Endcap Muon Chambers made at Harvard (42 so far)
Mod-0 - May 17, 2000

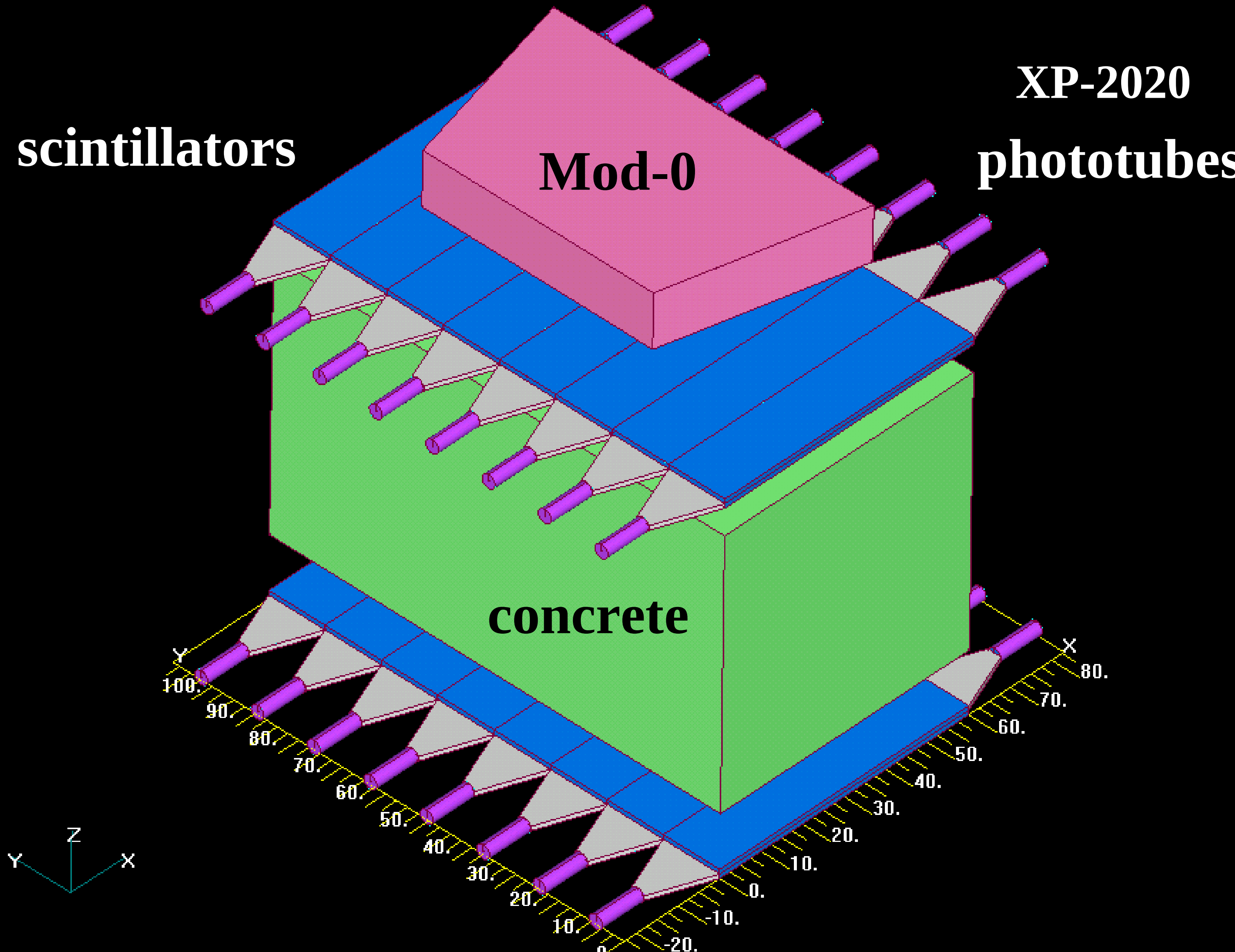


Mod-0 on Test Stand

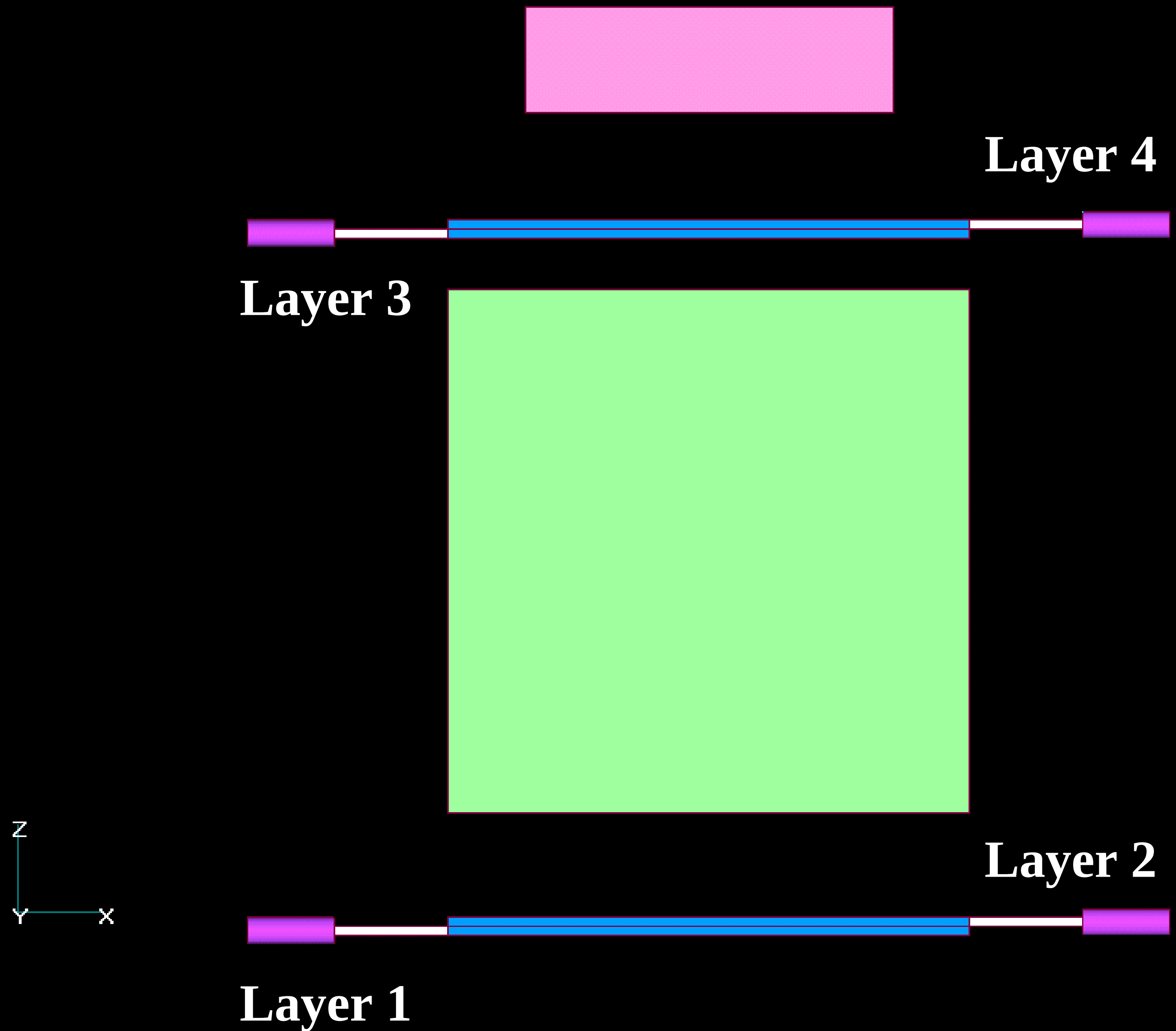


Scintillators and Phototubes

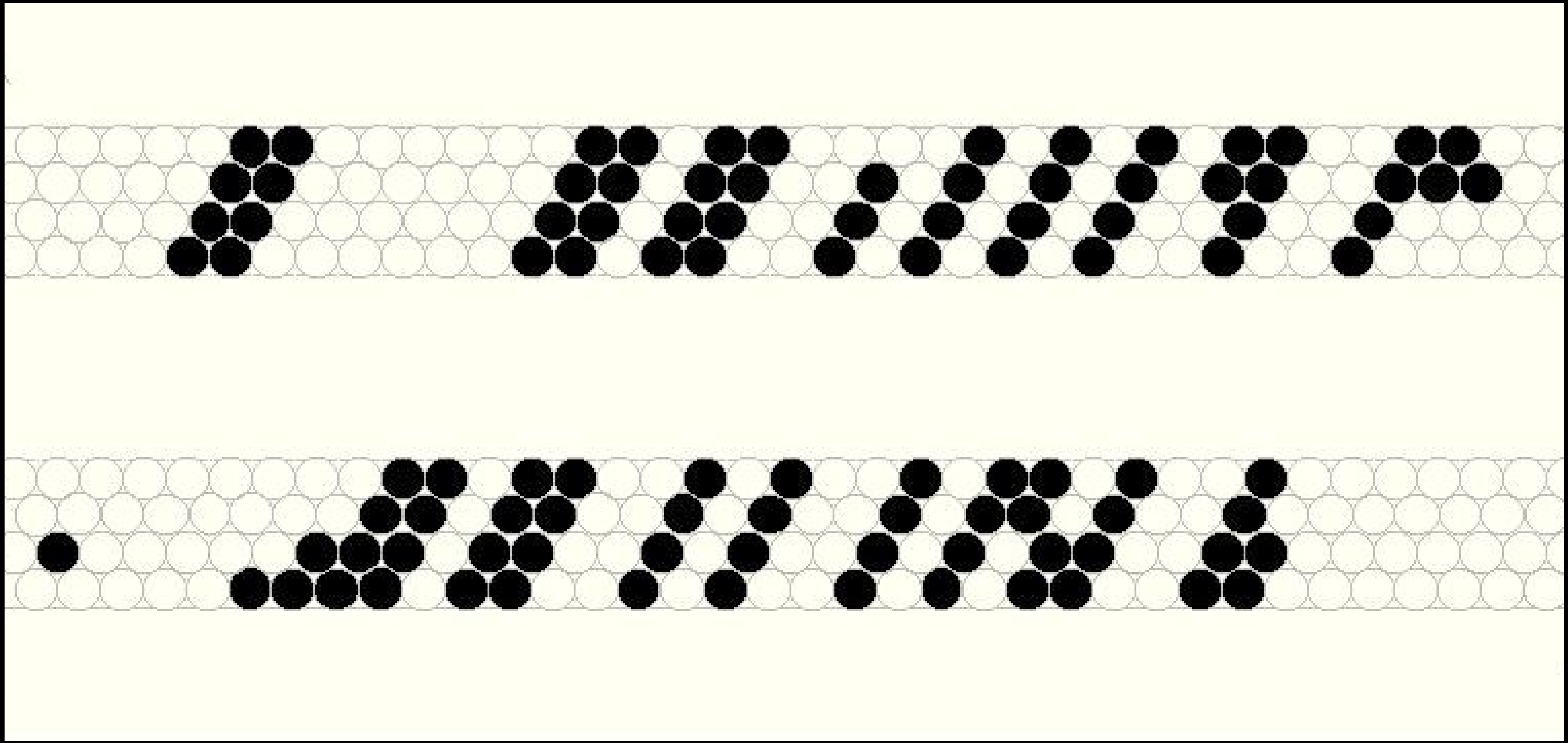
BMC Test Stand



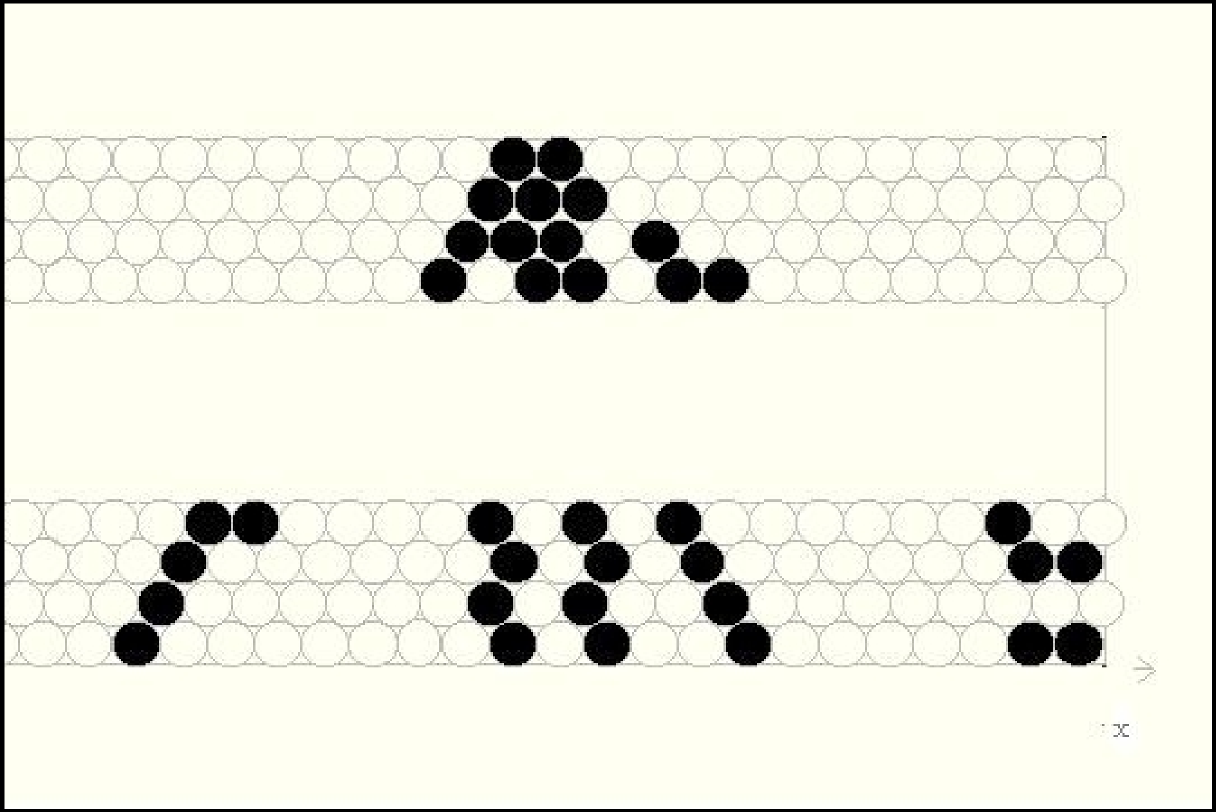
Side View



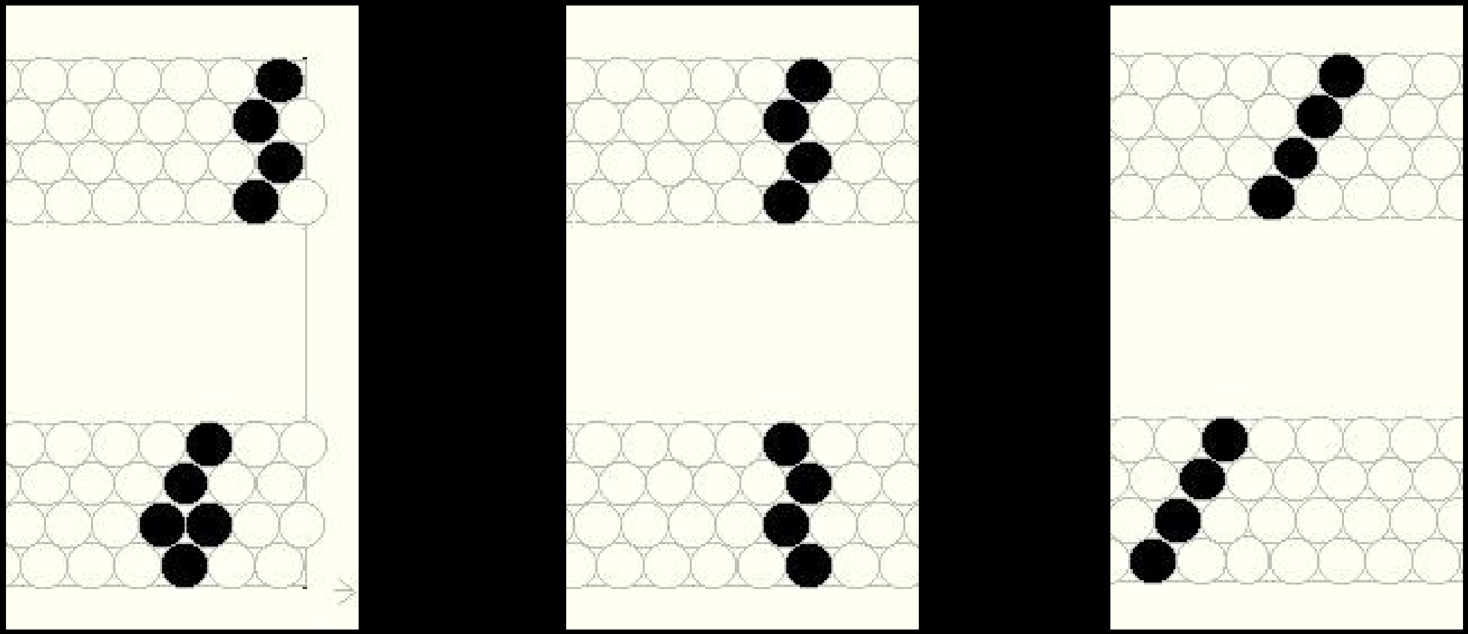
Shower event if trigger on coincidence of top two scintillators - these are electrons and positrons



Another shower, this one made locally

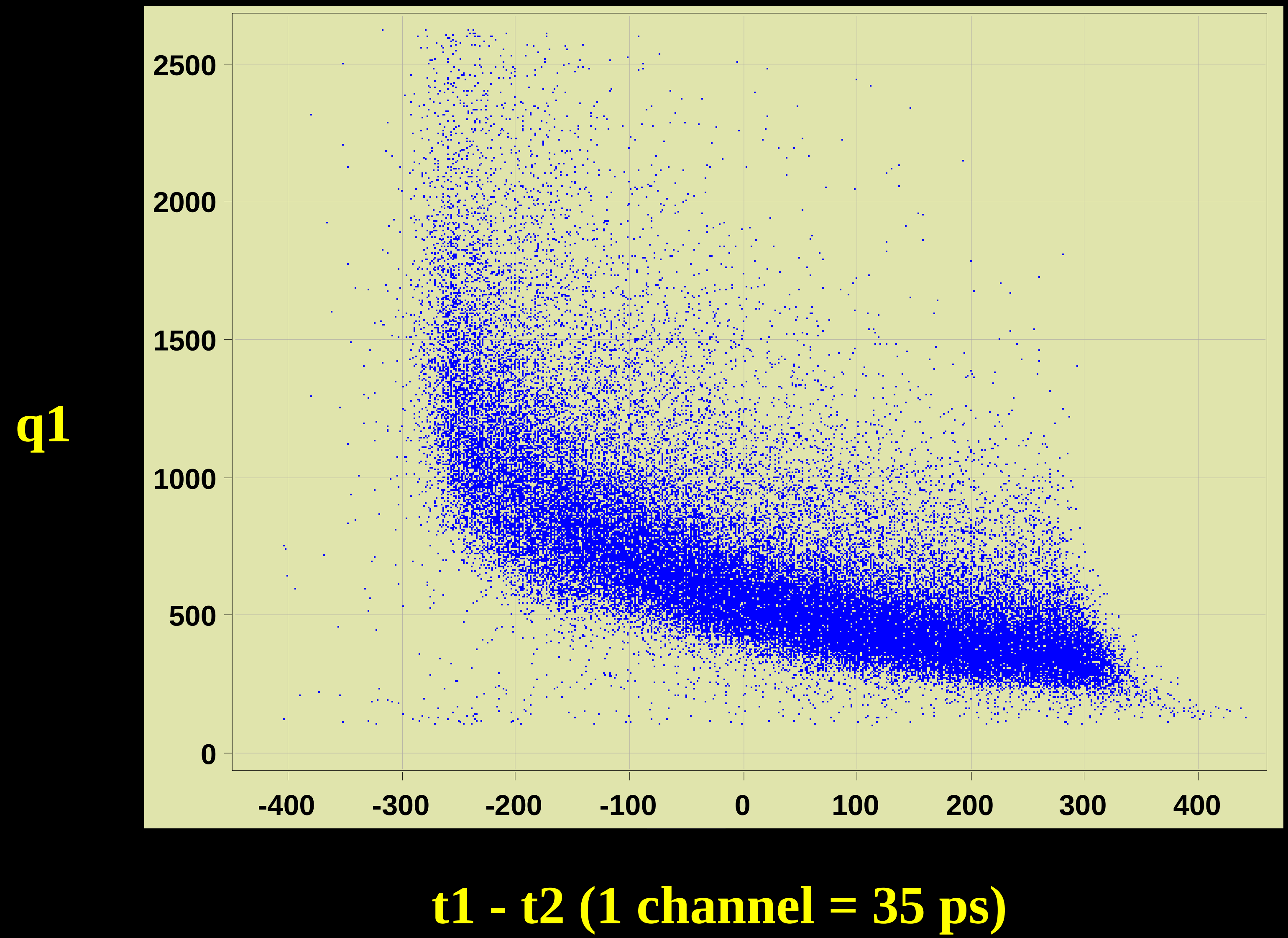


Single tracks for trigger that includes scintillators above AND below the concrete - these are muons



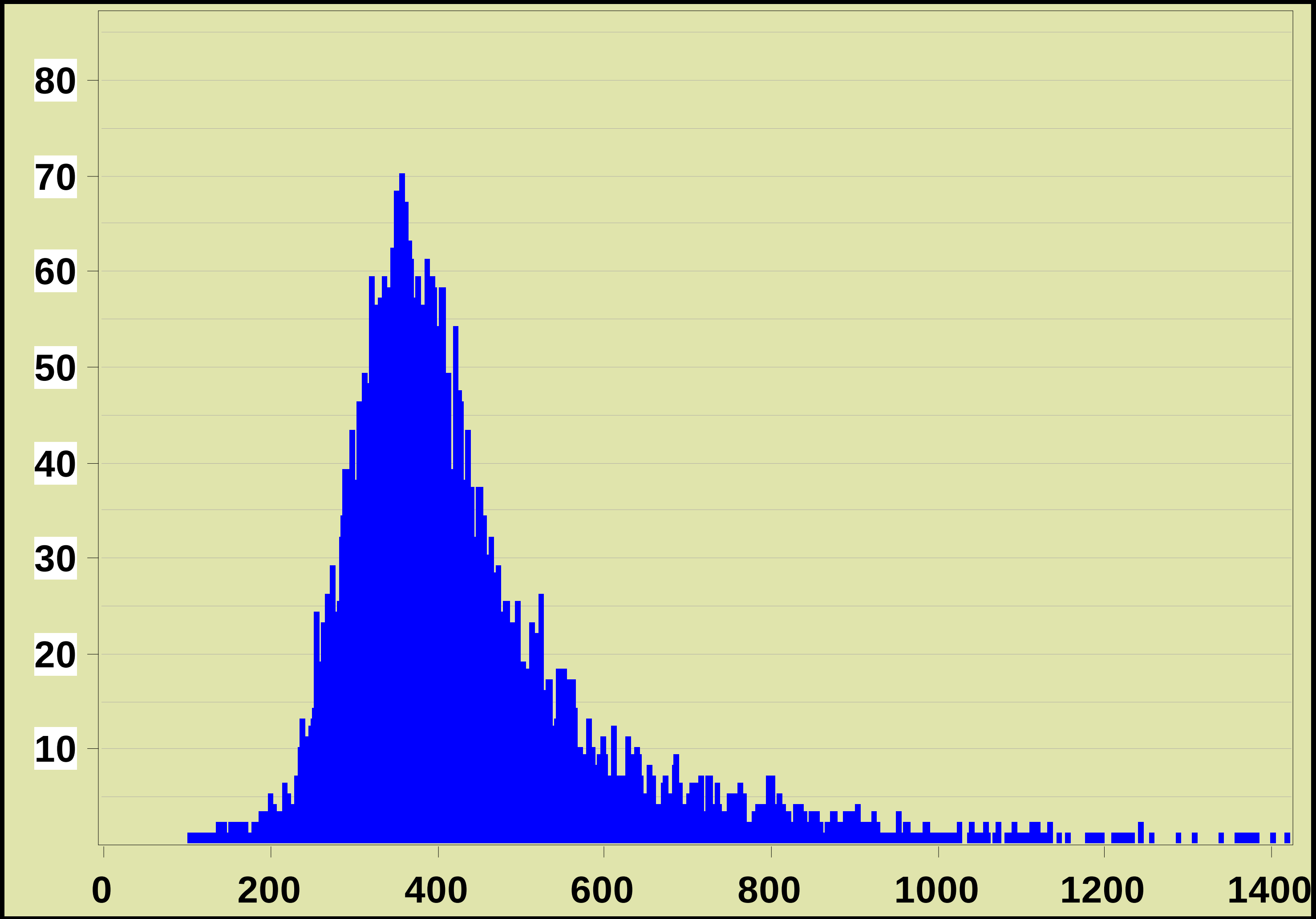
Scintillator signal vs position from timing

mu062901a_f; y_12=115



Landau distribution for one scintillator

mu062901a_f; y_12=115; deltb:(190,300)



q1

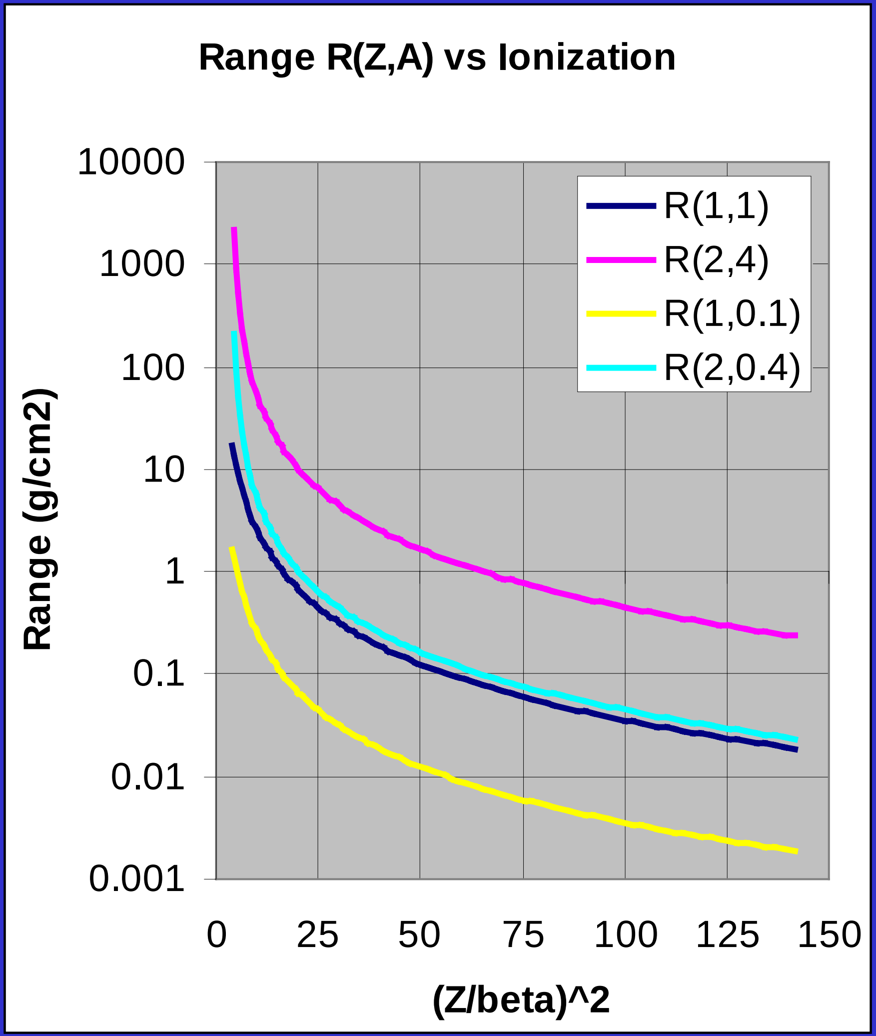
Ionization Measurements and Particle Identification

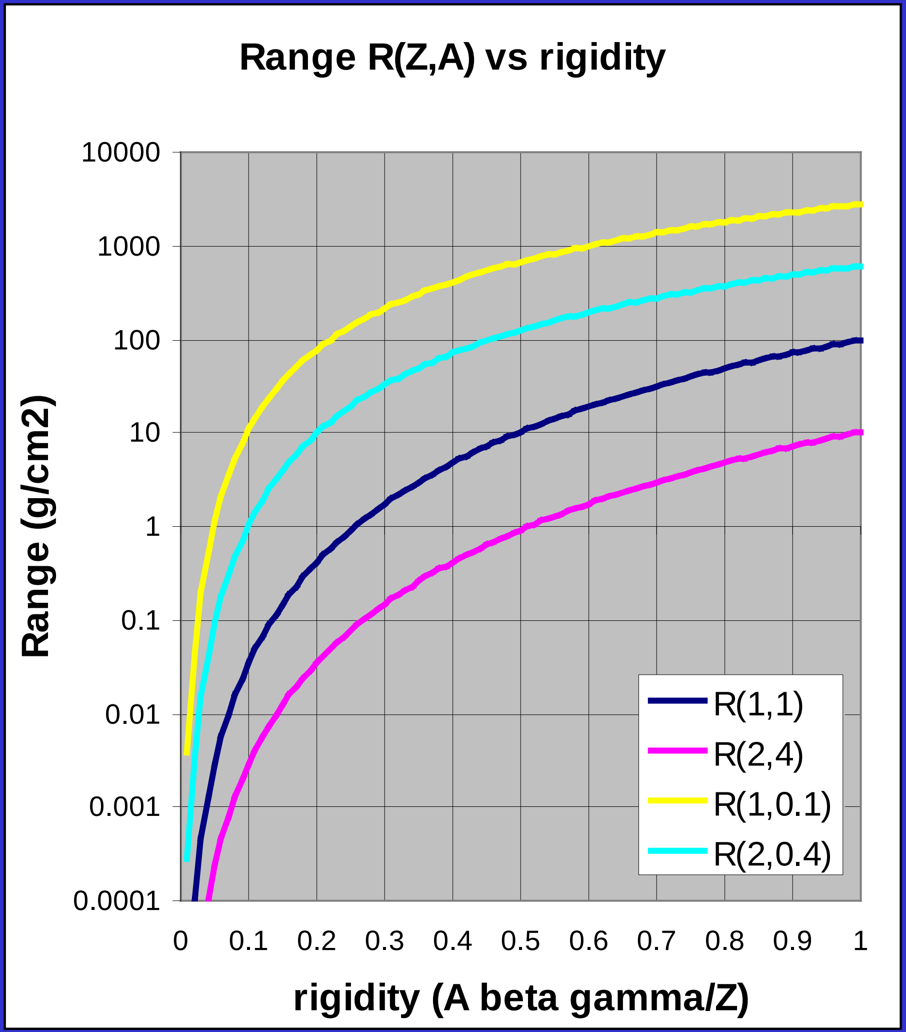
Range is well defined for muons, pions, nuclei, if energy is not too high

$$R(\beta) = \frac{M}{Z_1^2 m_p}\left(R_{proton}(\beta) + B_{Z_1}(\beta)\right)$$

2nd term is *range extension* due to electron capture: this is small for small Z_1 or large velocity

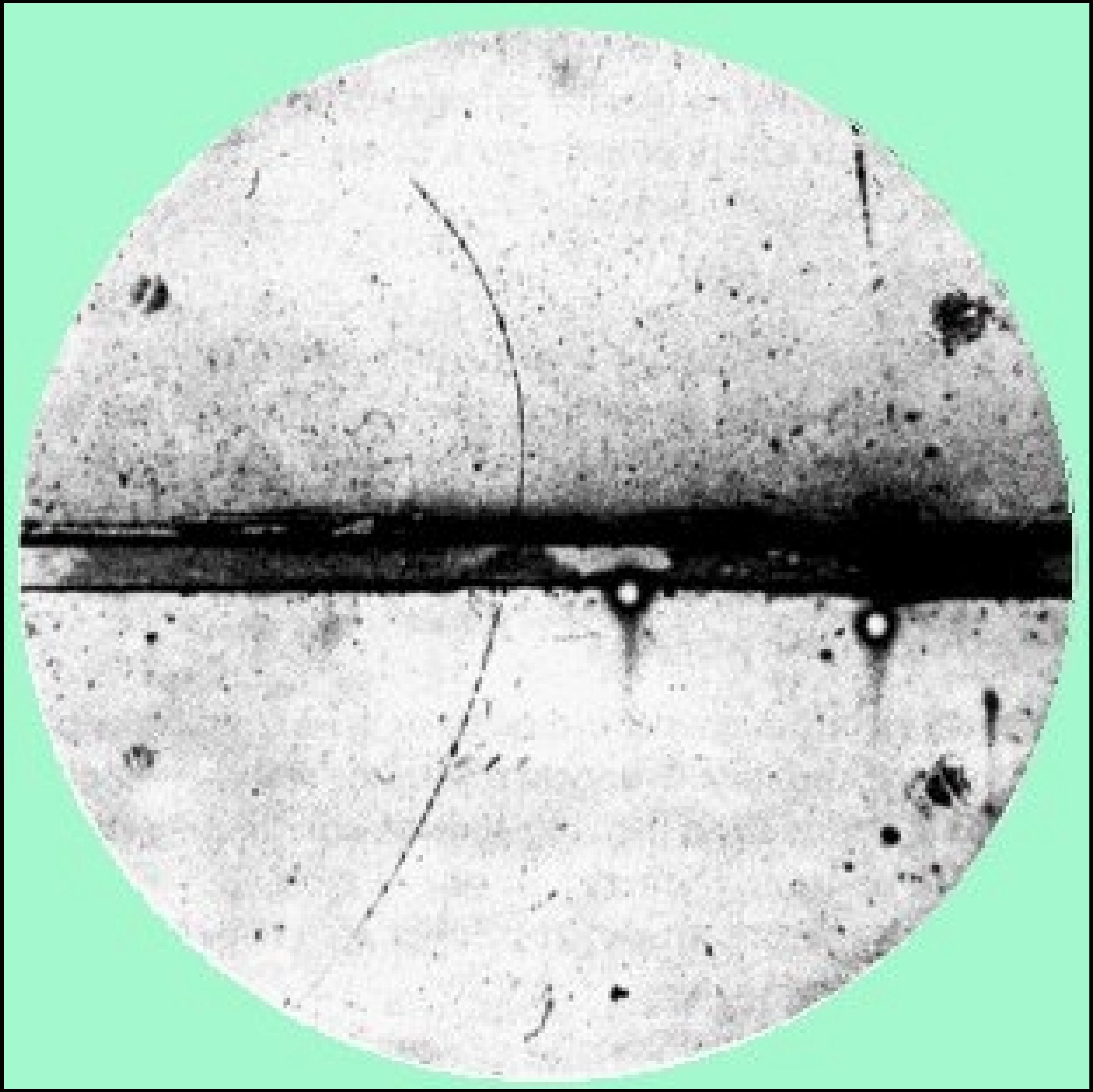
$$\frac{\sigma_R}{R} \approx \frac{1\%}{\sqrt{M/m_p}}$$





Ranges are for air

Discovery of positron in cloud chamber



Lead plate thickness = 6 mm

Magnetic field = 24000 gauss into picture

Dimensions: 17 x 17 x 3 cm

Change of curvature implies particle moving up, so charge is positive

Track density => $Z/\beta = 1$

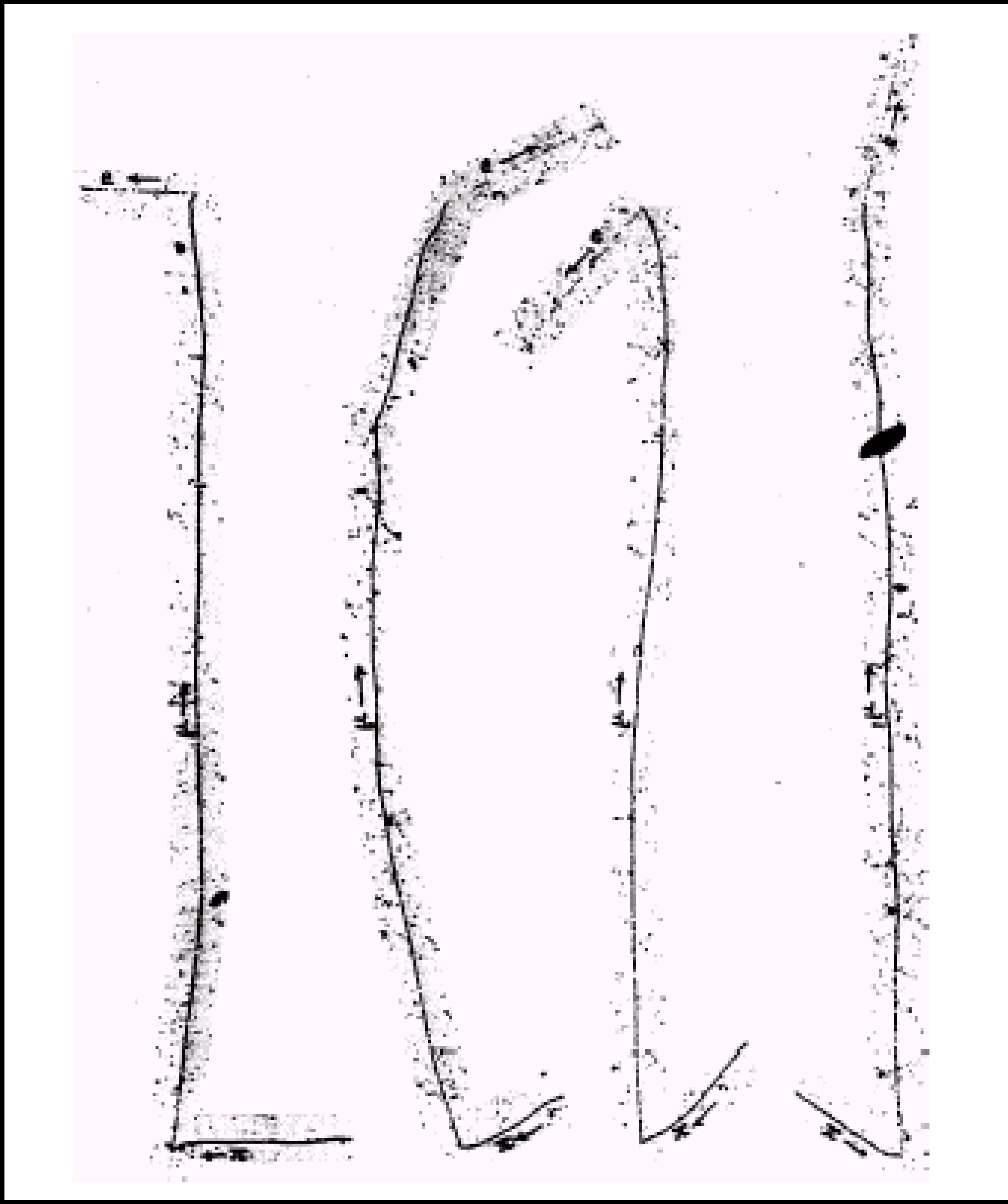
$A\beta\gamma/Z = 0.07$ below plate, 0.02 above plate: so $A < 0.02$

For very low energy particles like these, mass and charge can be measured, showing them to be positrons.

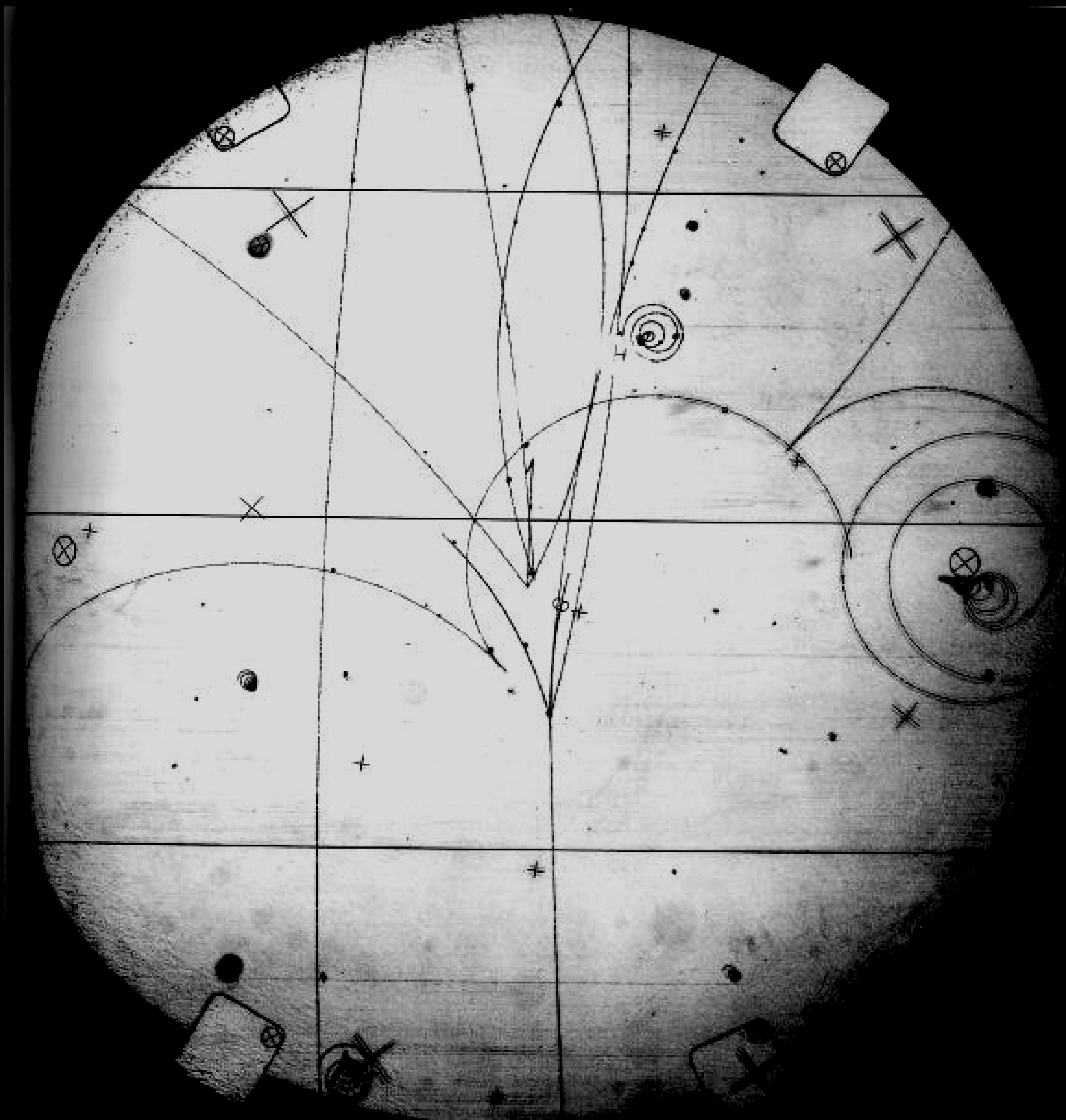
Pion decays in nuclear emulsion

Photo-micrographs of four examples of the successive decay π - μ - e as recorded in photographic emulsions.

Note that the muon ranges are nearly the same, so they must have same energy when pion decays at rest - a two body decay.



event in bubble chamber



$$\bar{p}\,p \rightarrow p\,\pi^{+}\,K^{-}\,\pi^{-}\,\pi^{0}\,K^{0}\,\bar{n}$$

* the slow proton is identifed by its heavier ionisation,

* the K0 subsequently decays into a pair of charged pions,

* the antineutron annihilates with a proton a short distance downstream from the primary interaction, to produce three charged pions,

* the neutral pion decays into two photons, which (unusually for a hydrogen chamber) both convert into e+e- pairs,

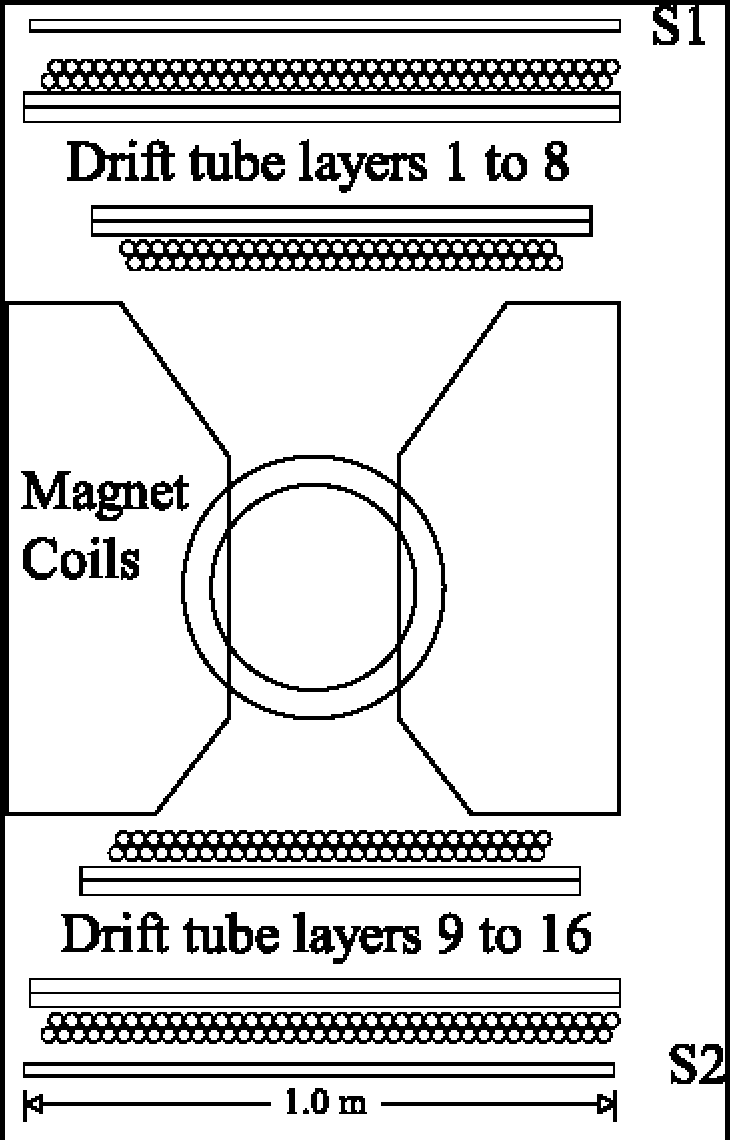
* (external particle detectors were used to identify the charged kaon).

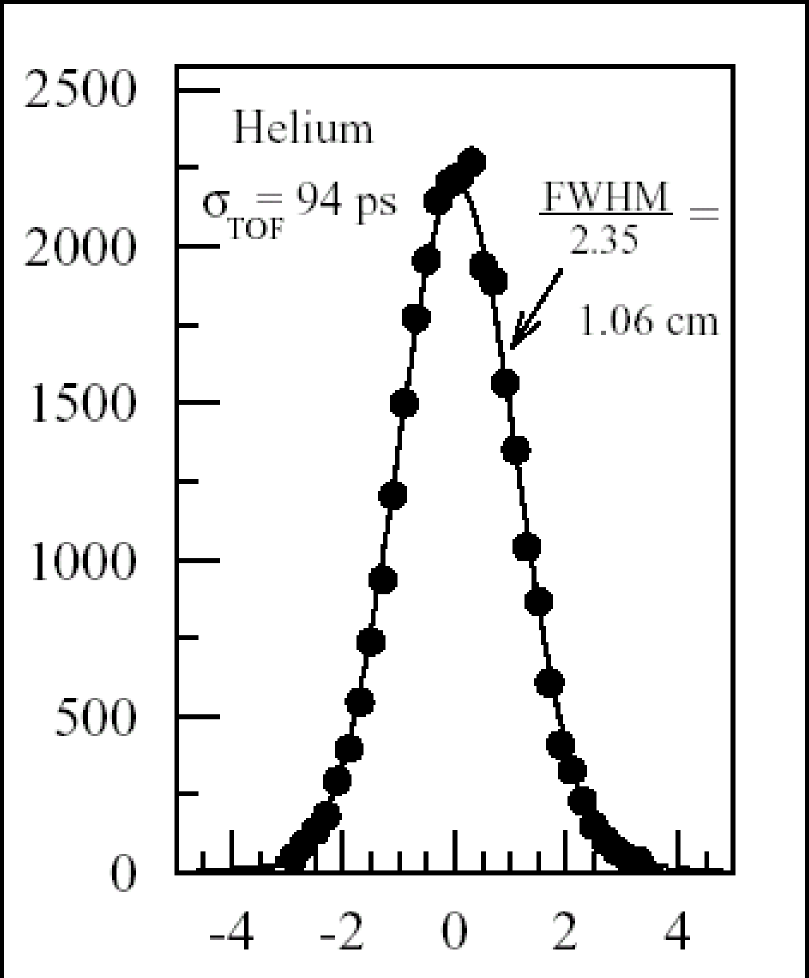
Velocity Measurements are the Most Effective Way to Identify Particles

$$\frac{dE}{dx} \approx K \frac{Z_1^2}{\beta^2}$$ Z_1 if velocity known

$$\text{Rigidity} = \frac{Mc\beta\gamma}{Z_1 e}$$ M if velocity and charge are known

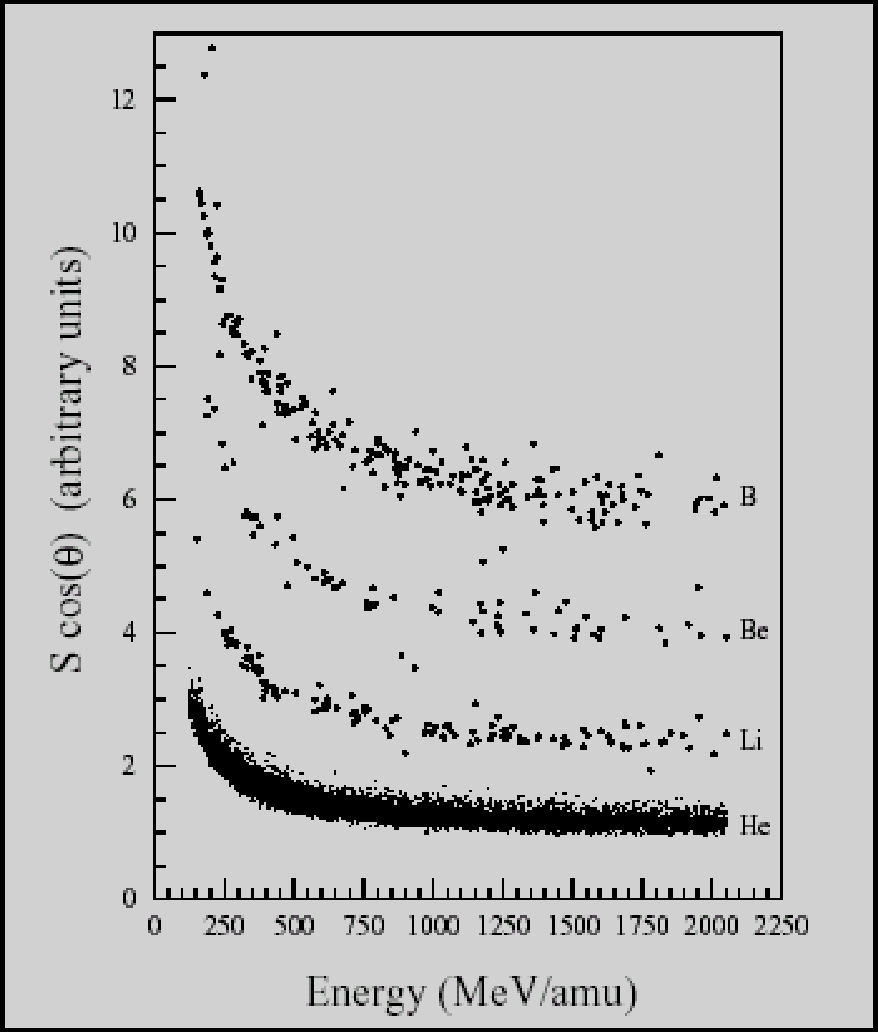
Scintillator timing can measure time-of-flight to 200 ps



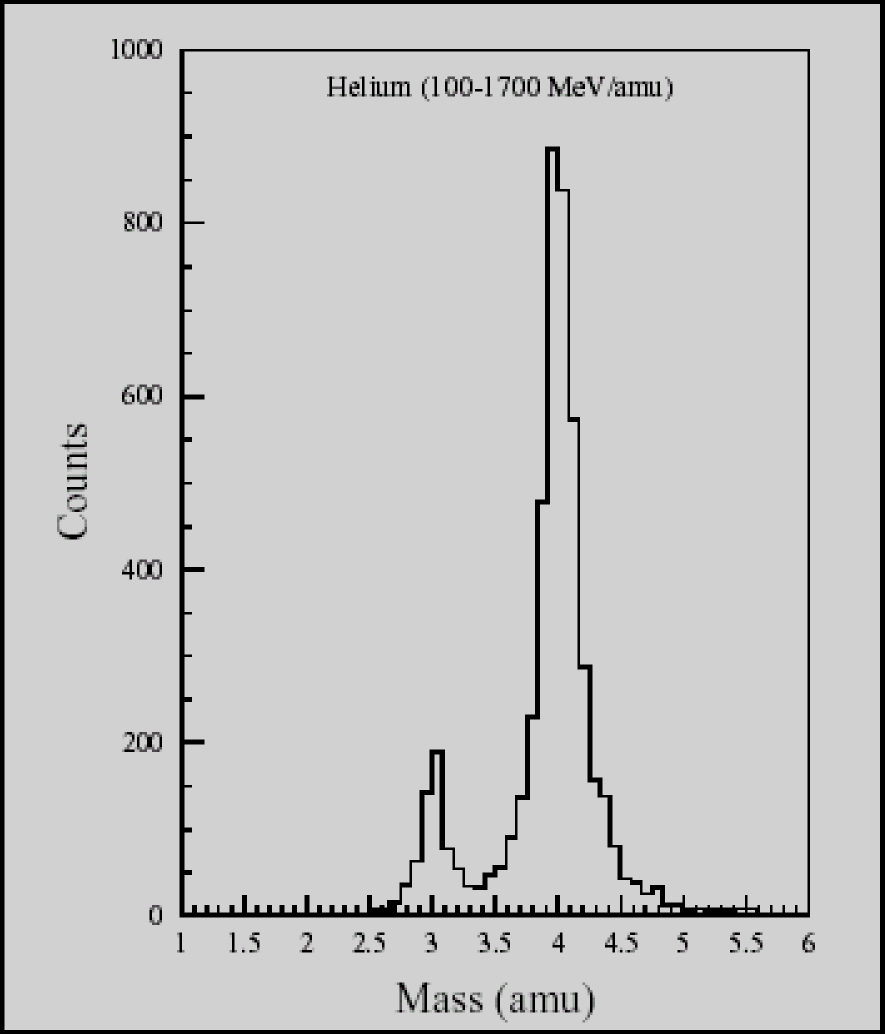


Position from scintillator timing compared to drift tube position

Charge

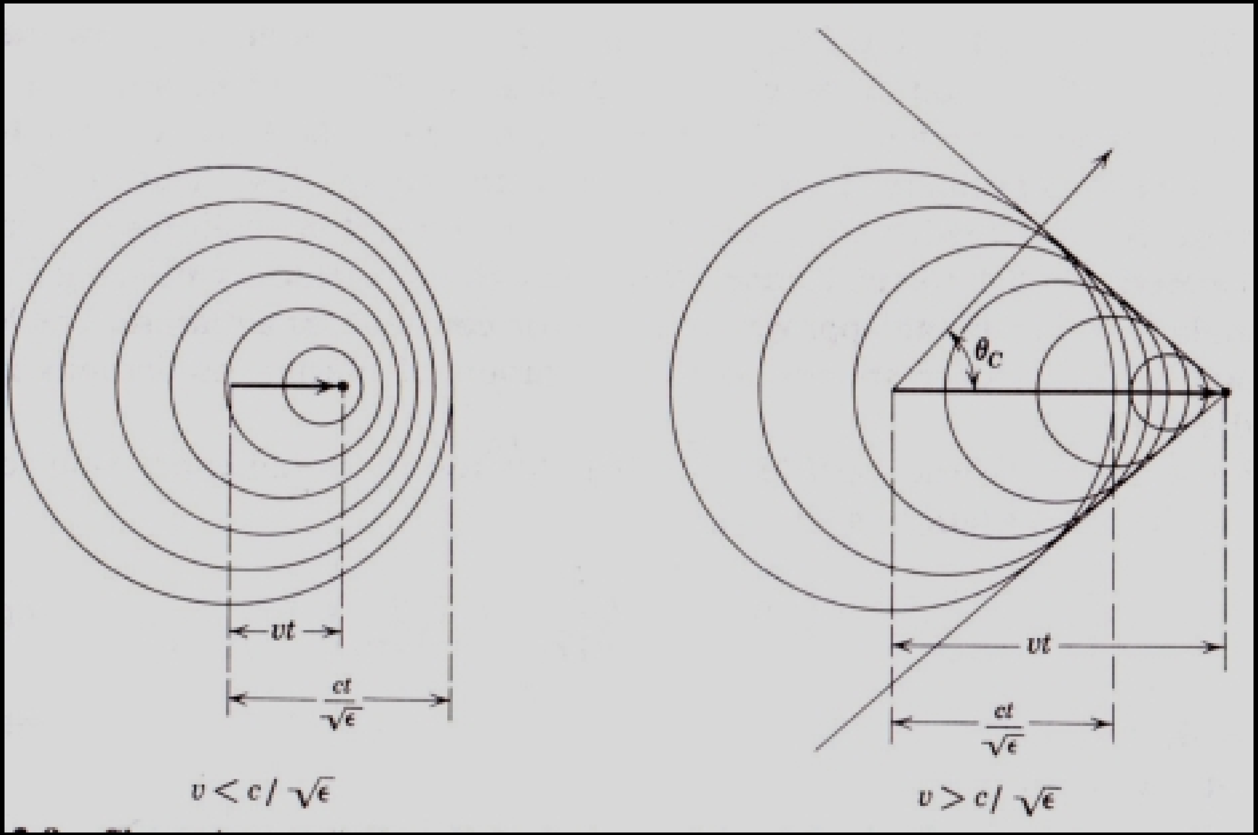


Mass



$$E(\mathrm{MeV/amu}) = 938\left(1/\sqrt{1-\beta^2} - 1\right)$$

Cherenkov Radiation if v > c/n:



$$\left.\begin{aligned} p_i &= p_f + \hbar k \\ E_i &= E_f + \omega \end{aligned}\right\} \qquad \cos\theta_C = \frac{c/n}{v}$$

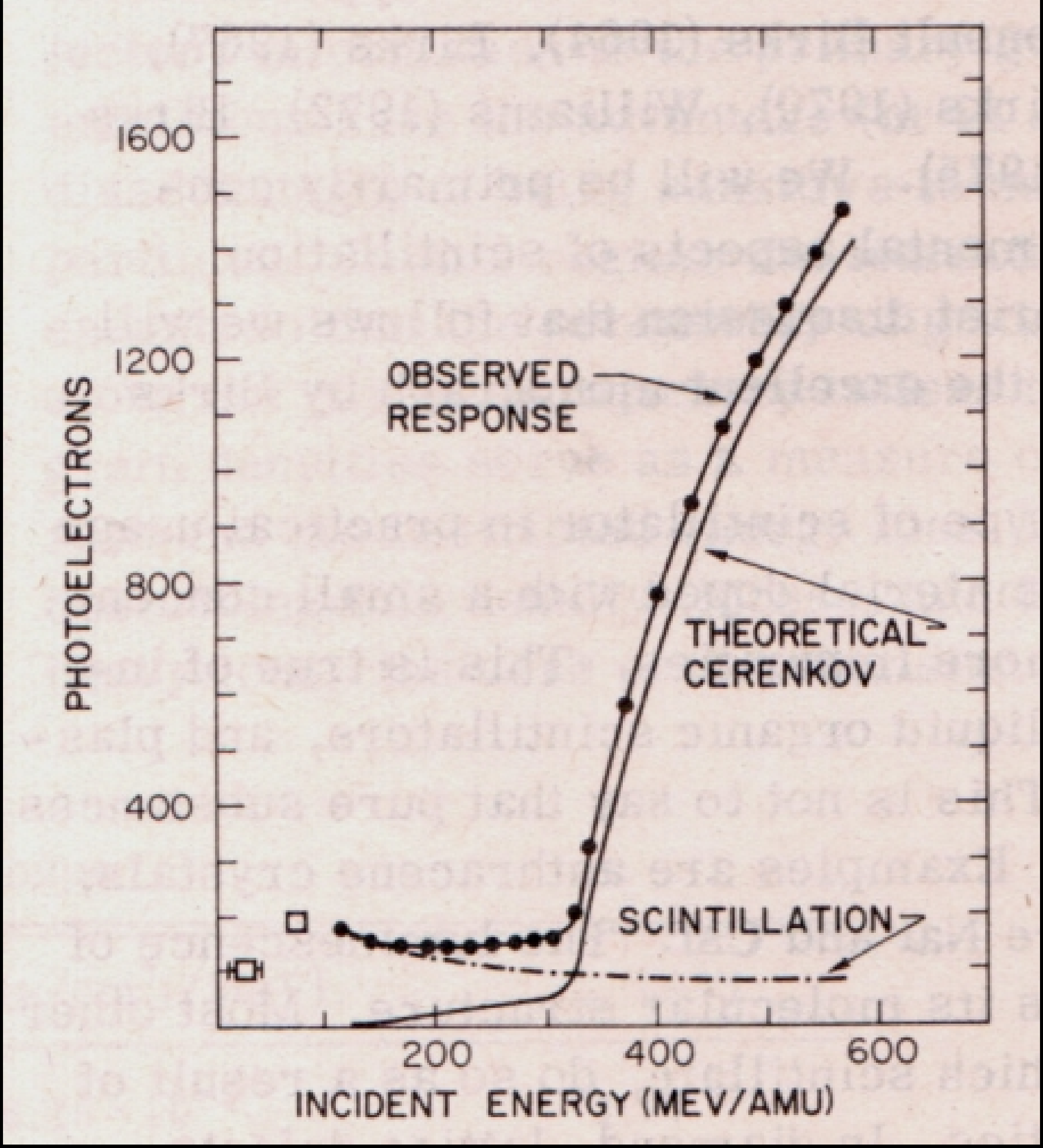
$$\omega = ck/n$$

Cherenkov Yield:

$$\left.\frac{dE}{dx}\right|_{Cherenkov} = \int_{\omega_1}^{\omega_2} \frac{Z_1^2 e^2}{c^2}\left(1 - \frac{1}{n^2\beta^2}\right)\omega\, d\omega$$

A few hundred photons per cm for n = 1.5 for typical photo-multiplier tubes

Cherenkov response for roughened 1.3 cm Pilot 425 radiator in white box for neon nuclei

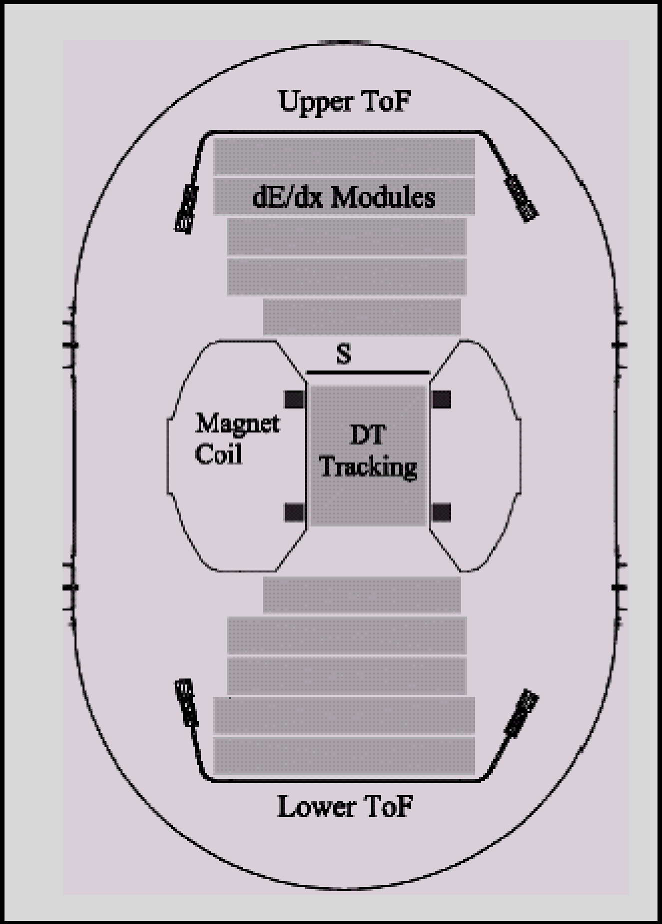


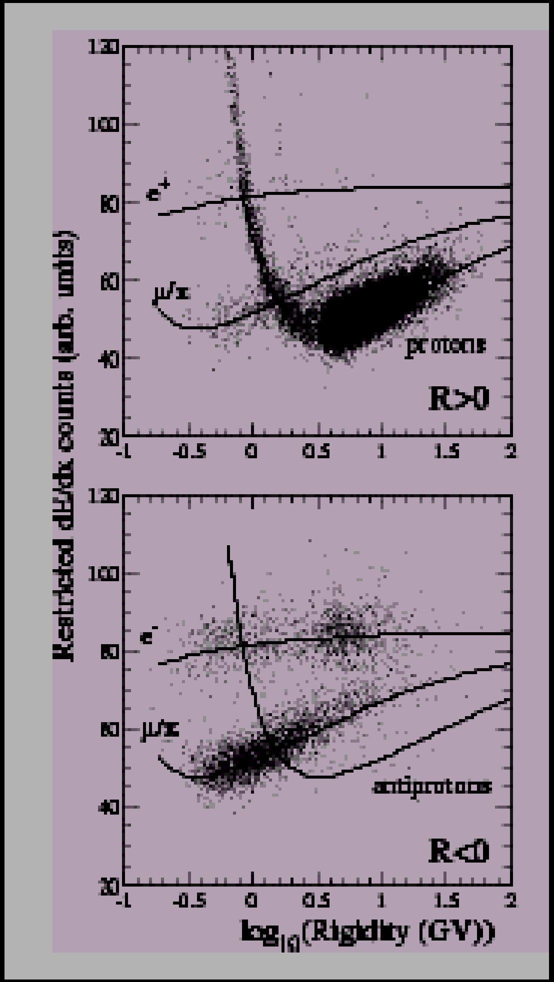
Ring imaging Cherenkov detectors using small light sensors provide best velocity resolution

Use of relativistic Rise in Gas Ionization Counters

From *Detection of cosmic-ray antiprotons with the HEAT-pbar instrument*

(Proceedings of ICRC 2001: 1691)





At very high energies, transition radiation can determine Lorentz factor