



# Superconducting RF Accelerators: Why all the interest?

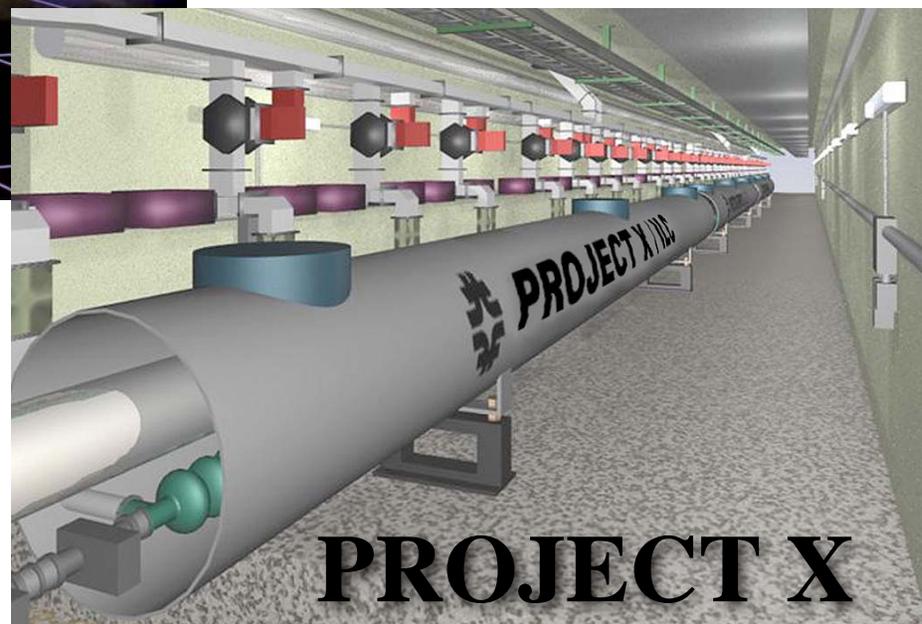
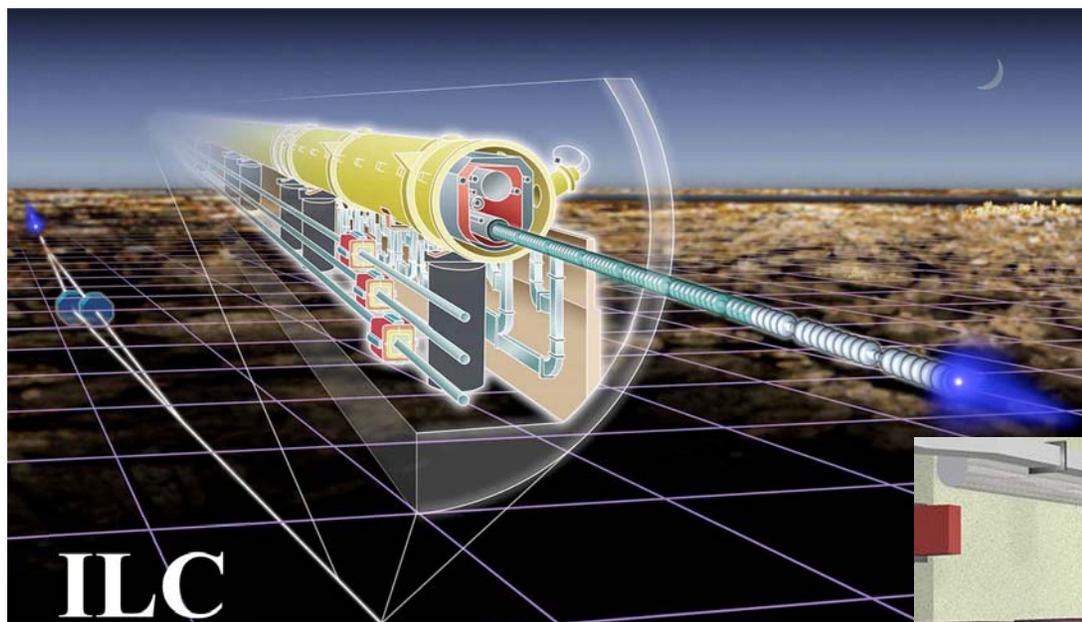
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Dept. of Physics, MIT



# The HEP prespective

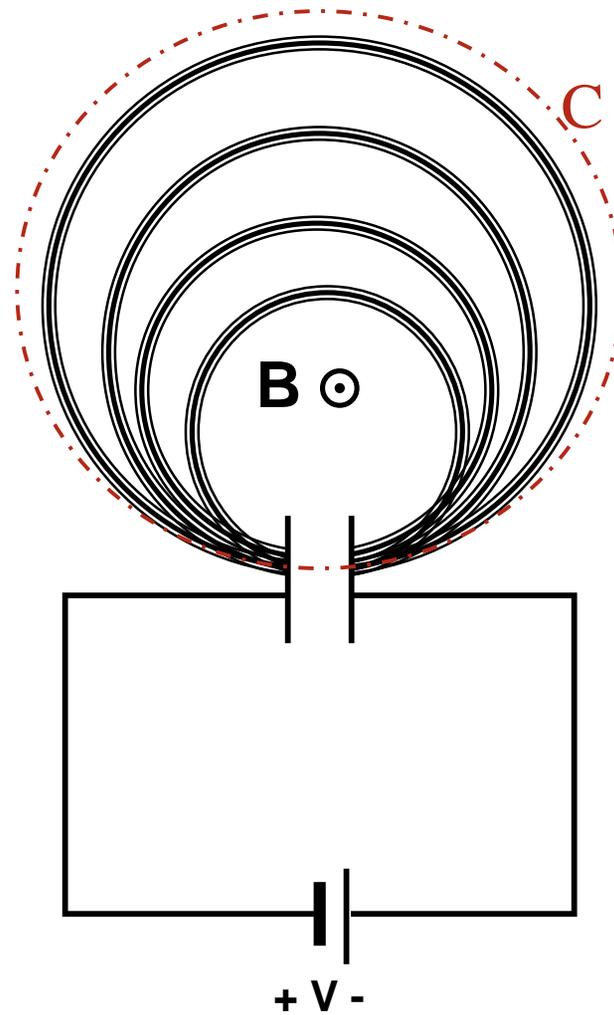




## Why do we need RF structures & fields?

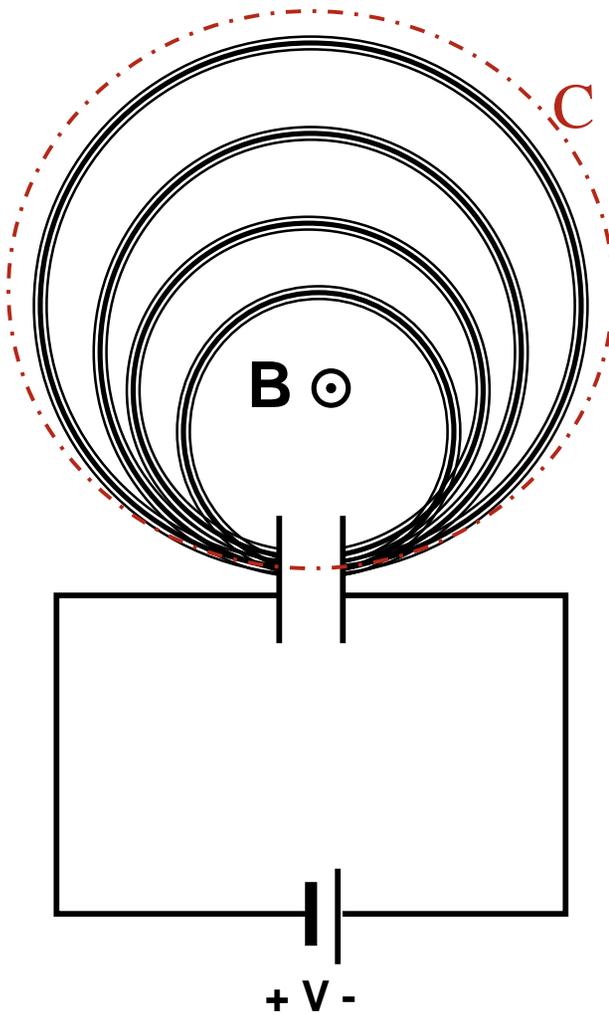


# Possible DC accelerator?





## Maxwell forbids this!



$$\nabla \times \mathbf{E} = -\frac{d\mathbf{B}}{dt}$$

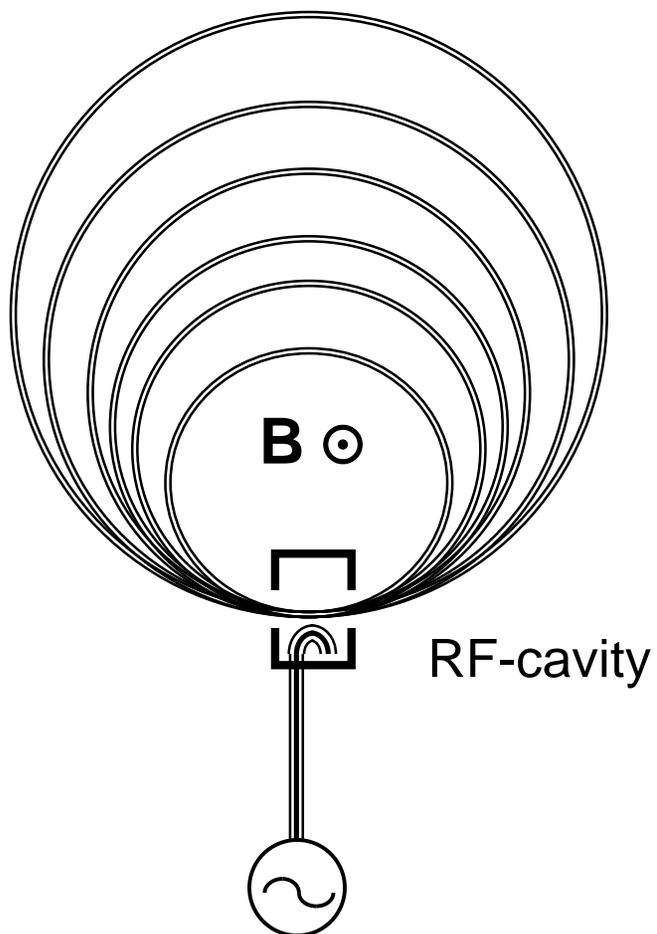
or in integral form

$$\oint_C \mathbf{E} \cdot d\mathbf{s} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot \mathbf{n} da$$

**$\therefore$  There is no acceleration  
without time-varying magnetic flux**

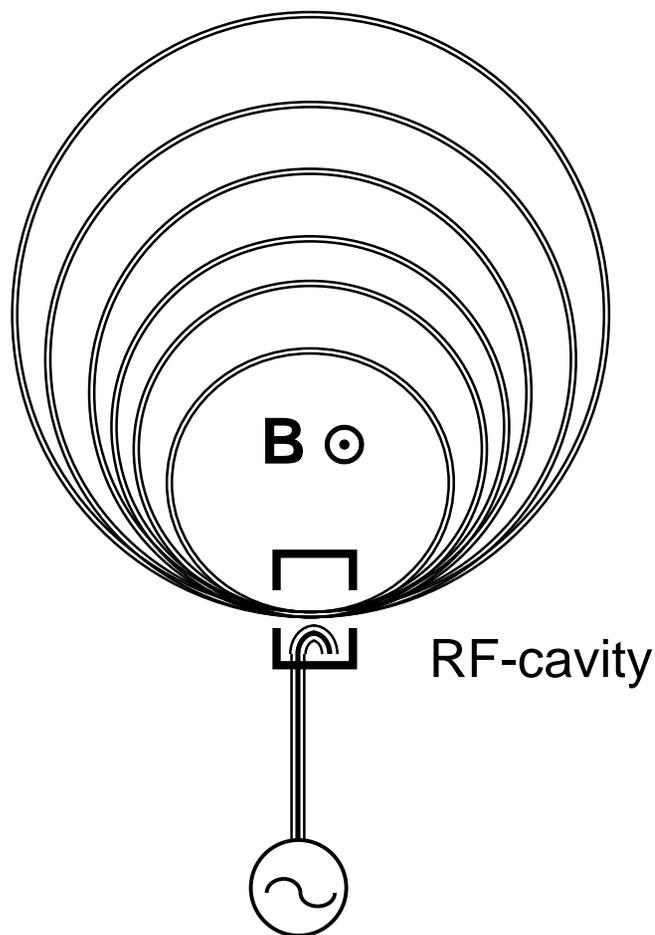


# Will this work?





## We can vary B in an RF cavity



**Note that inside the cavity**  
 $\frac{dB}{dt} \neq 0$

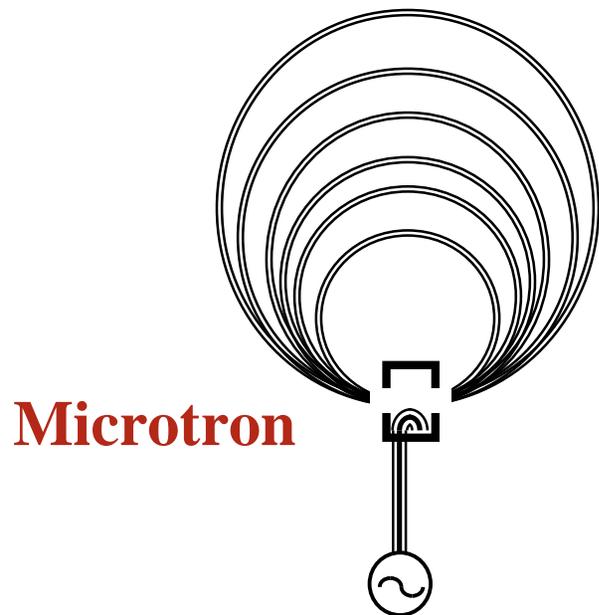
**However,**

**Synchronism condition:**

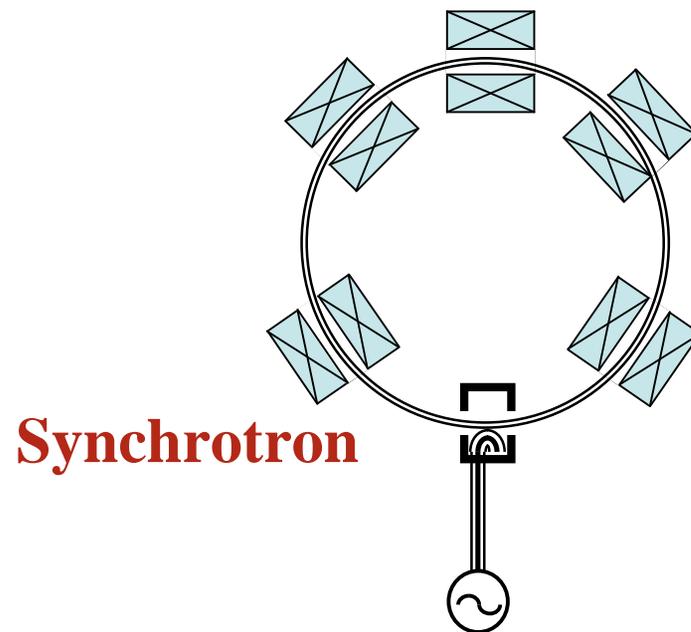
$$\Delta\tau_{\text{rev}} = N/f_{\text{rf}}$$



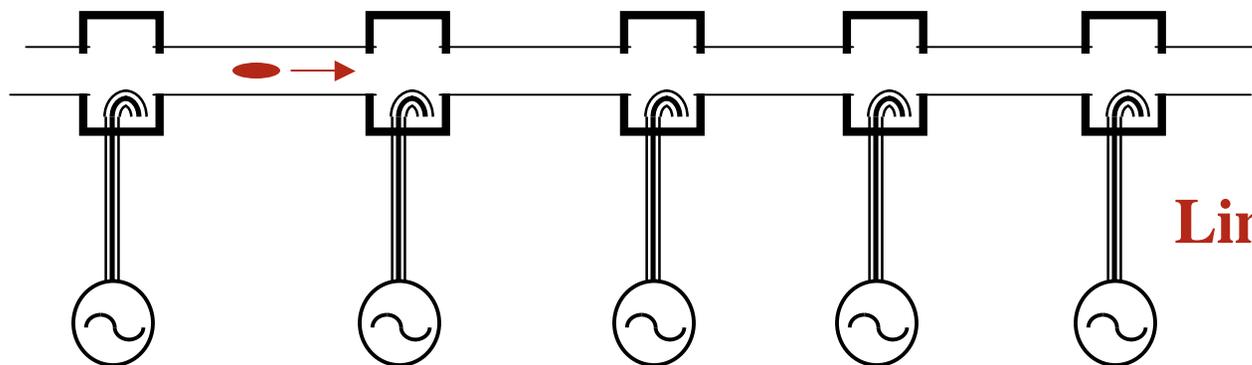
# We can arrange rf-cavities in many ways



**Microtron**



**Synchrotron**



**Linac**

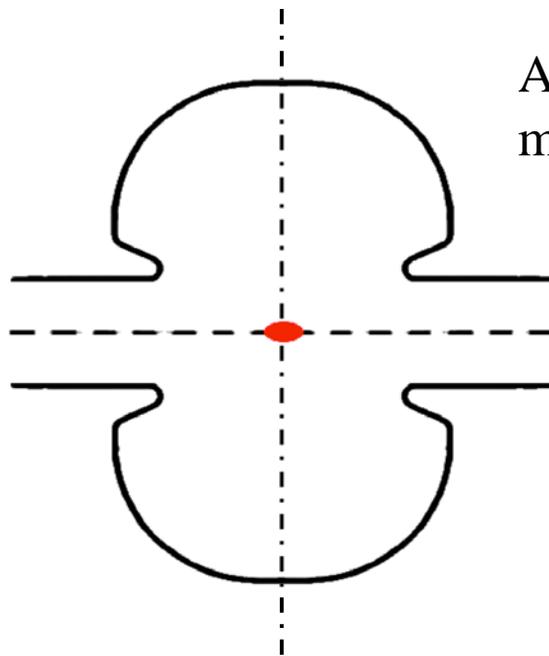


## RF cavities: Basic concepts



\* Fields and voltages are complex quantities

$$\tilde{V} = V e^{i\omega t + \vartheta_0}$$



At  $t = 0$  particle receives maximum voltage gain

$Z_0$  is the reference plane



## Basic principles: Reciprocity & superposition

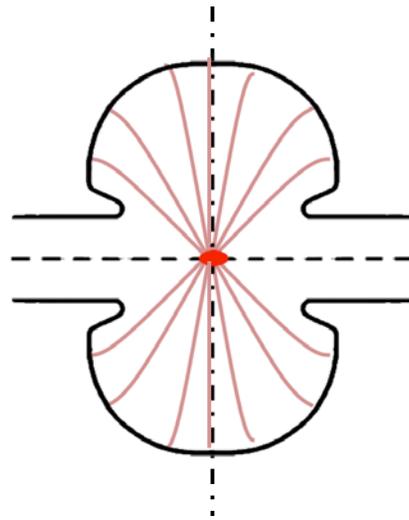


✱ If you can kick the beam, the beam can kick you

==>

$$\text{Total cavity voltage} = V_{\text{generator}} + V_{\text{beam-induced}}$$

$$\text{Fields in cavity} = \mathbf{E}_{\text{generator}} + \mathbf{E}_{\text{beam-induced}}$$

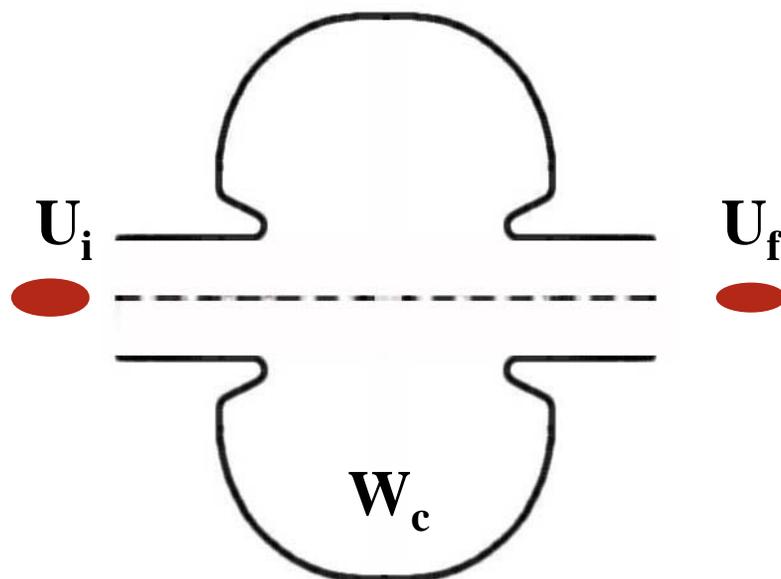




## Basic principles: Energy conservation



- ✱ Total energy in the particles and the cavity is conserved
  - Beam loading



$$\Delta W_c = U_i - U_f$$



# Basic components of an RF cavity



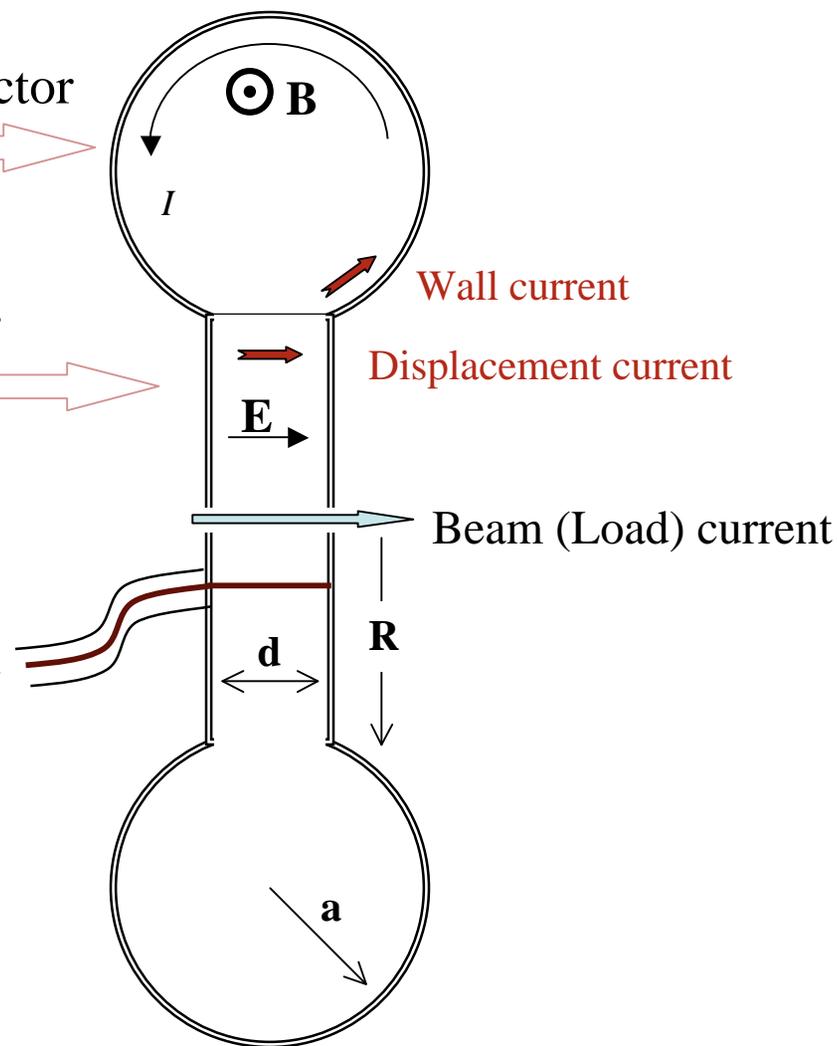
Outer region: Large, single turn Inductor



Central region: Large plate Capacitor

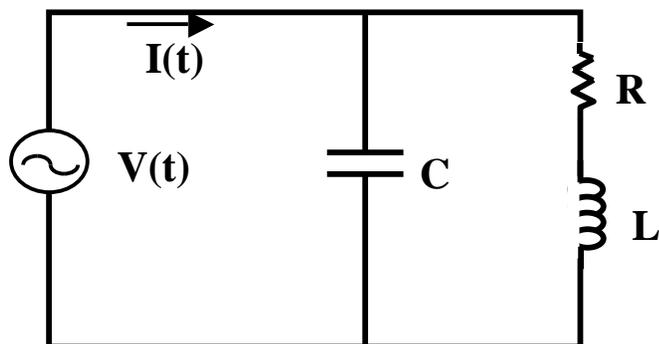


Power feed from rf - generator





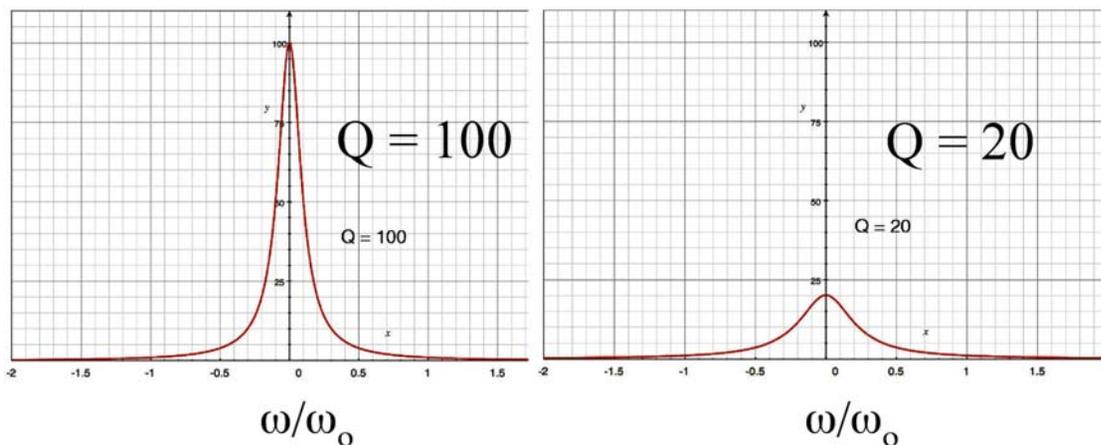
# Lumped circuit analogy of resonant cavity



$$Z(\omega) = [j\omega C + (j\omega L + R)^{-1}]^{-1}$$

$$|Z(\omega)| \sim \left[ \left( 1 + \frac{\omega^2}{\omega_0^2} \right) + (\omega RC)^2 \right]^{-1}$$

The resonant frequency is  $\omega_0 = \frac{1}{\sqrt{LC}}$



Width to reduce

Z by  $e^2$

$$\frac{\Delta\omega}{\omega_0} = \frac{R}{\sqrt{LC}} = \frac{1}{Q}$$

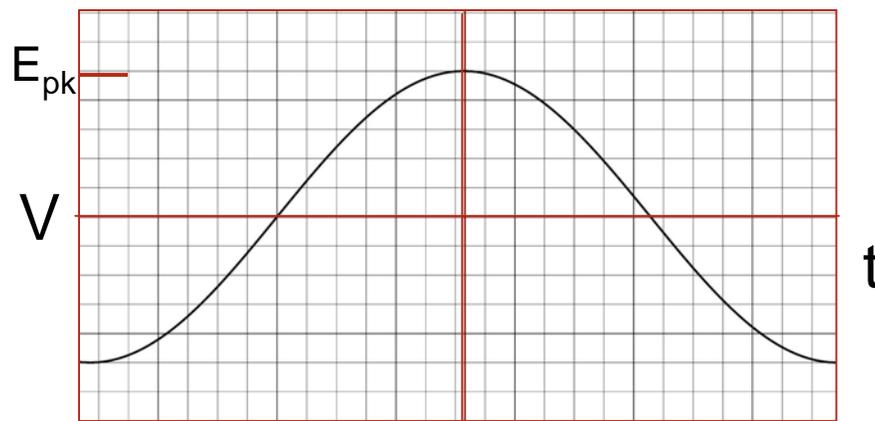
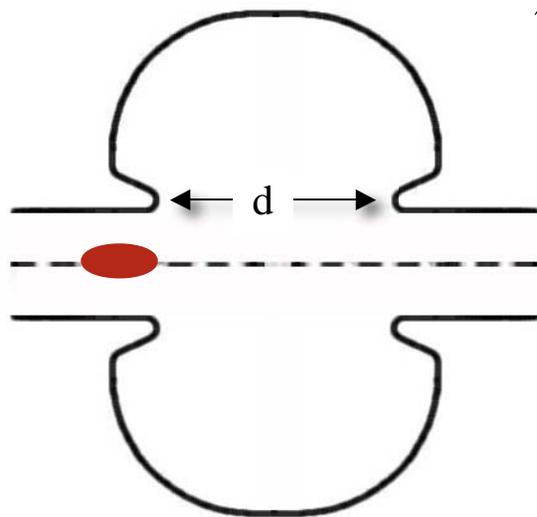


## Figure of Merit: Accelerating voltage



- \* The voltage varies during the time that bunch takes to cross the gap  
→ reduction of the peak voltage by  $\Gamma$

$$\Gamma = \frac{\sin(\vartheta/2)}{\vartheta/2} \quad \text{where } \vartheta = \omega d / \beta c$$



For maximum acceleration  $T_{\text{cav}} = \frac{d}{c} = \frac{T_{\text{rf}}}{2} \implies \Gamma = 2/\pi$



## Figure of merit from circuits - Q



$$Q = \frac{\omega_o \circ \text{Energy stored}}{\text{Time average power loss}} = \frac{2\pi \circ \text{Energy stored}}{\text{Energy lost per cycle}}$$

$$\mathcal{E} = \frac{\mu_o}{2} \int_v |H|^2 dv = \frac{1}{2} L I_o I_o^*$$

$$\langle \mathcal{P} \rangle = \frac{R_{surf}}{2} \int_s |H|^2 ds = \frac{1}{2} I_o I_o^* R_{surf}$$

$$R_{surf} = \frac{1}{\text{Conductivity} \circ \text{Skin depth}} \sim \omega^{1/2}$$

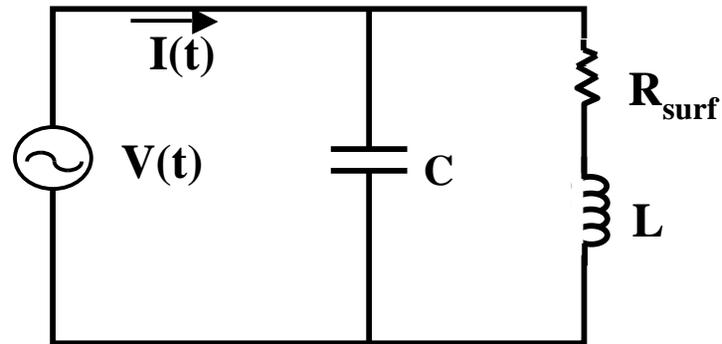
$$\therefore Q = \frac{\sqrt{L/C}}{R_{surf}} = \left( \frac{\Delta\omega}{\omega_o} \right)^{-1}$$



## What makes SC RF attractive?



## Recall the circuit analog



As  $R_{surf} \implies 0$ , the  $Q \implies \infty$ .

In practice,

$$Q_{nc} \sim 10^4$$

$$Q_{sc} \sim 10^{11}$$



## Figure of merit for accelerating cavity: power to produce the accelerating field



Resistive input (shunt) impedance at  $\omega_0$  relates power dissipated in walls to accelerating voltage

$$R_{in} = \frac{\langle V^2(t) \rangle}{\mathcal{P}} = \frac{V_o^2}{2\mathcal{P}} = Q\sqrt{L/C}$$

Linac literature more commonly defines “shunt impedance” without the “2”

$$\mathcal{R}_{in} = \frac{V_o^2}{\mathcal{P}} \sim \frac{1}{R_{surf}}$$

For SC-rf  $\mathcal{P}$  is reduced by orders of magnitude

**BUT, it is deposited @ 2K**



# Translate circuit model back to directly driven, re-entrant RF cavity model



Outer region: Large, single turn Inductor

$$L = \frac{\mu_o \pi a^2}{2\pi(R + a)}$$



Central region: Large plate Capacitor

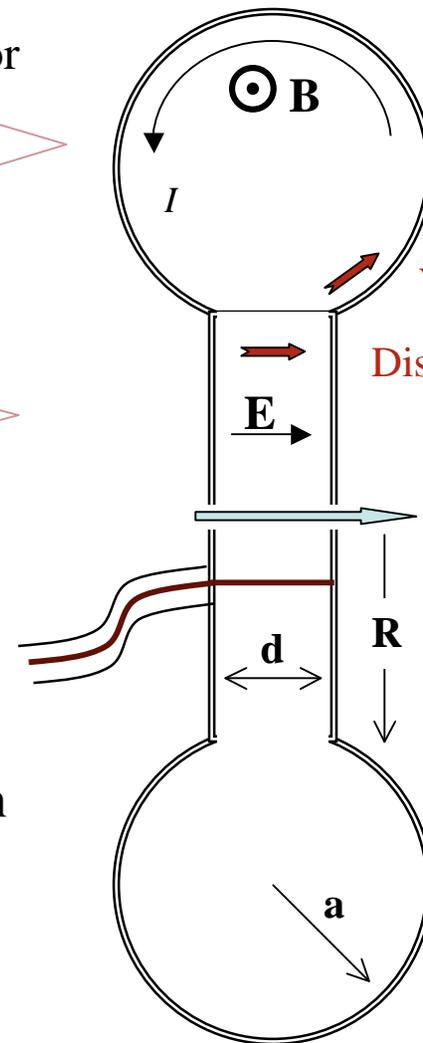
$$C = \epsilon_o \frac{\pi R^2}{d}$$



$$\omega_o = \frac{1}{\sqrt{LC}} = c \left[ \frac{2((R + a)d)}{\pi R^2 a^2} \right]^{1/2}$$

$Q$  – set by resistance in outer region

$$Q = \sqrt{L/C} / \mathcal{R}_{surface} = \omega L / \mathcal{R}_{surface}$$



*Expanding outer region raises  $Q$*

Wall current

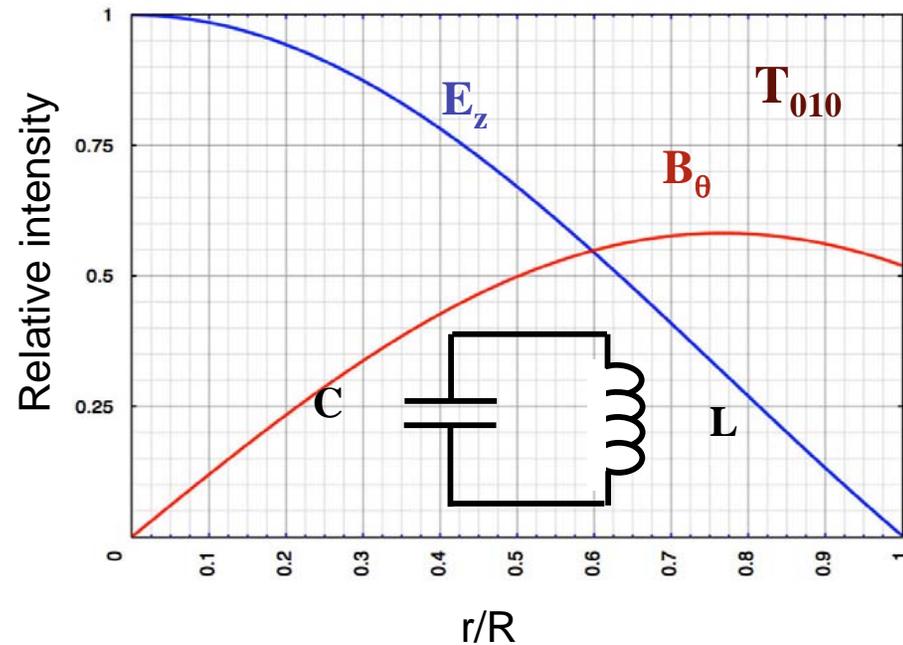
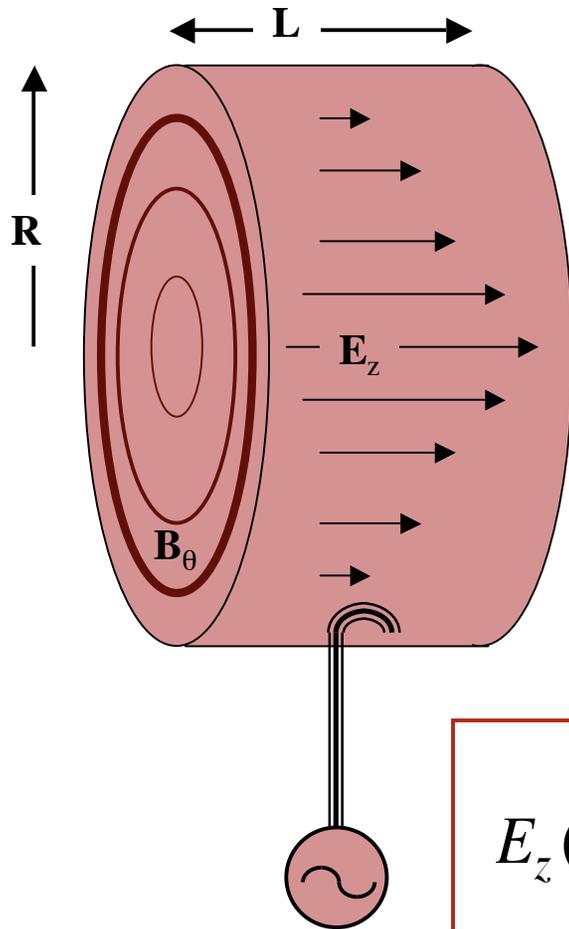
Displacement current

Beam (Load) current

*Narrowing gap raises shunt impedance*



In an ideal pillbox,  $\omega_0$  is independent of  $L$

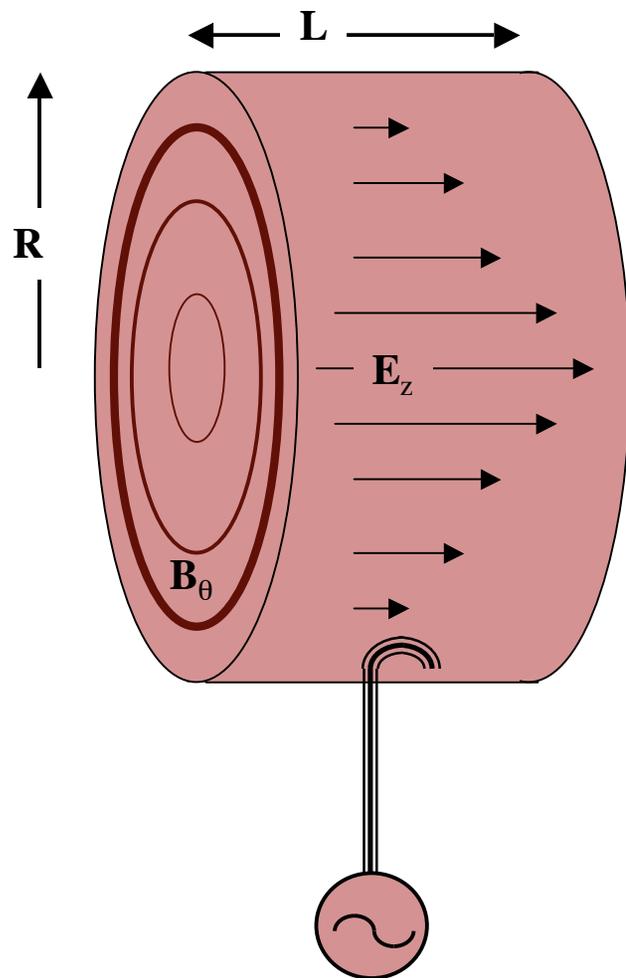


$$E_z(r) = E_0 J_0\left(\frac{\omega}{c} r\right)$$

$$\implies \omega = 2.405 \frac{c}{R}$$



## Simple consequences of pillbox model



- \* Increasing R lowers frequency

==> Stored Energy,  $\mathcal{E} \sim \omega^{-2}$

- \*  $\mathcal{E} \sim E_z^2$

- \* Beam loading lowers  $E_z$  for the next bunch

- \* Lowering  $\omega$  lowers the fractional beam loading

- \* Raising  $\omega$  lowers  $Q \sim \omega^{-1/2}$

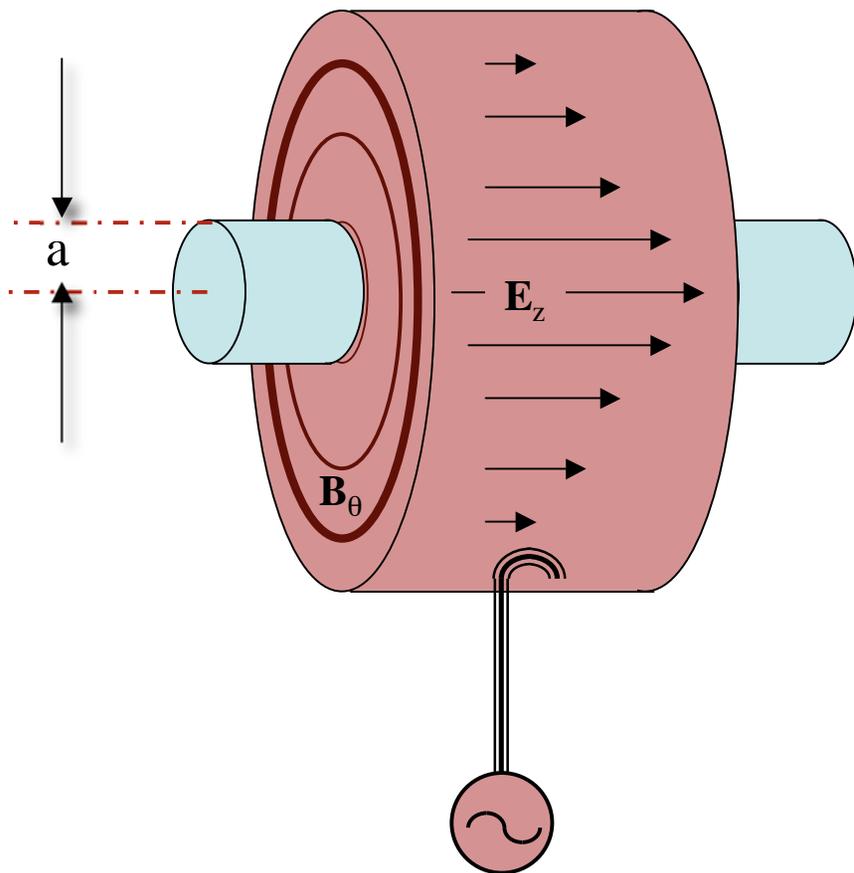
- \* If time between beam pulses,

$$T_s \sim Q/\omega$$

almost all  $\mathcal{E}$  is lost in the walls



## The beam tube makes the field modes (& cell design) more complicated



- \* Peak E no longer on axis
  - $E_{pk} \sim 2 - 3 \times E_{acc}$
  - $FOM = E_{pk}/E_{acc}$
- \*  $\omega_0$  sensitive to cavity length
  - Mechanical tuning & detuning
- \* Beam tubes add length & \$'s w/o acceleration
- \* Beam induced voltages  $\sim a^{-3}$ 
  - Instabilities



## Comparison of SC and NC RF



### Superconducting RF

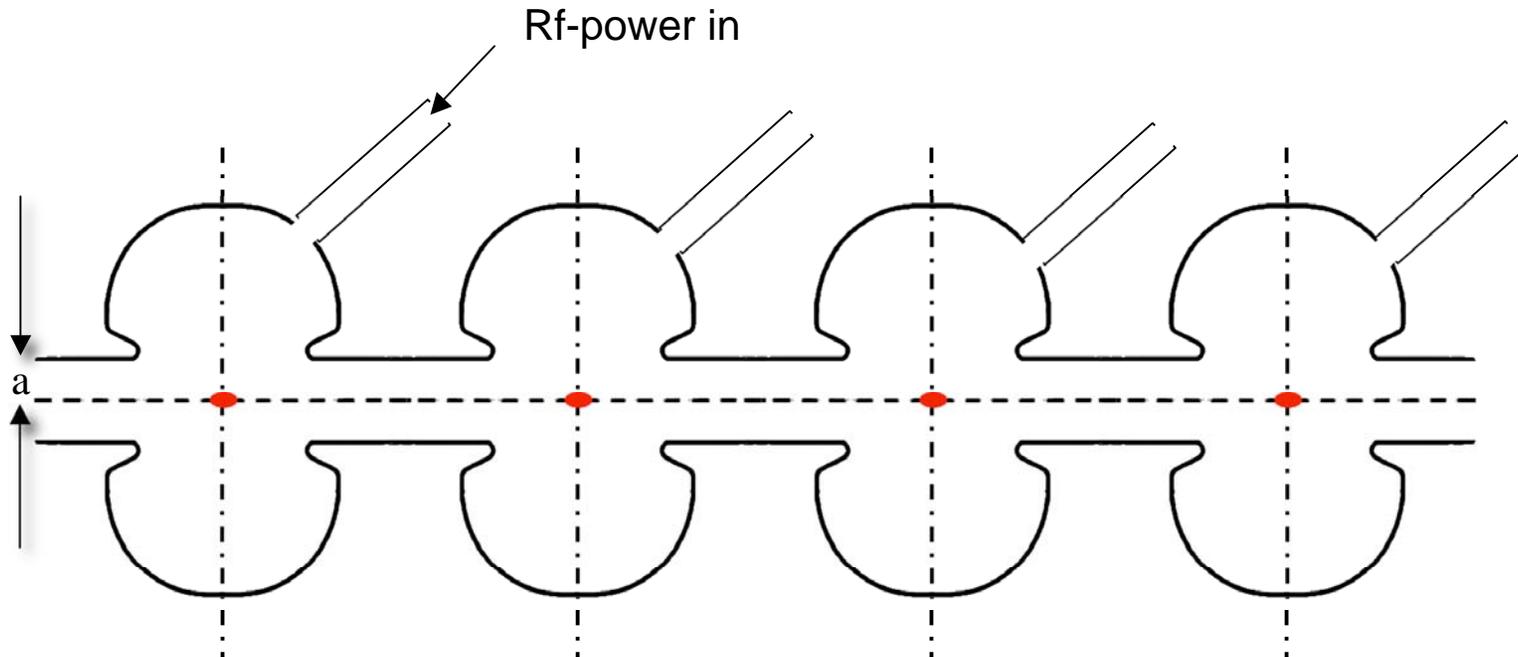
- \* High gradient  
==> 1 GHz, meticulous care
- \* Mid-frequencies  
==> Large stored energy,  $\mathcal{E}_s$
- \* Large  $\mathcal{E}_s$   
==> very small  $\Delta E/E$
- \* Large Q  
==> high efficiency

### Normal Conductivity RF

- \* High gradient  
==> high frequency (5 - 17 GHz)
- \* High frequency  
==> low stored energy
- \* Low  $\mathcal{E}_s$   
==> ~10x larger  $\Delta E/E$
- \* Low Q  
==> reduced efficiency



# Linacs can be considered as a series of distorted pillbox cavities...

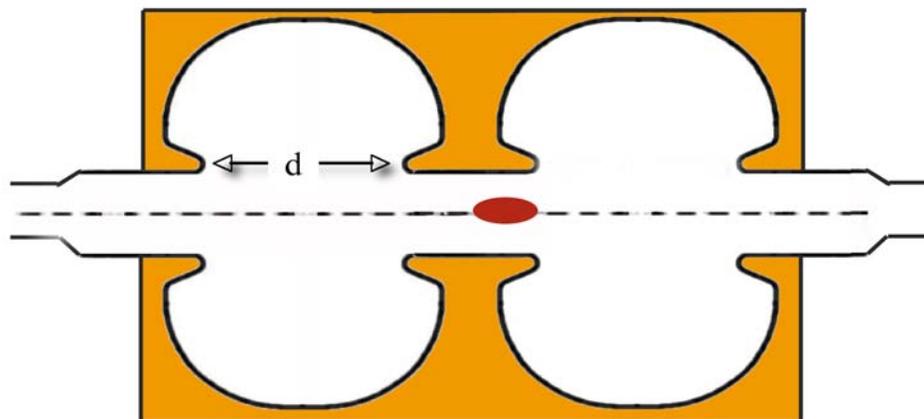


In warm linacs “nose cones” optimize the voltage per cell with respect to resistive dissipation

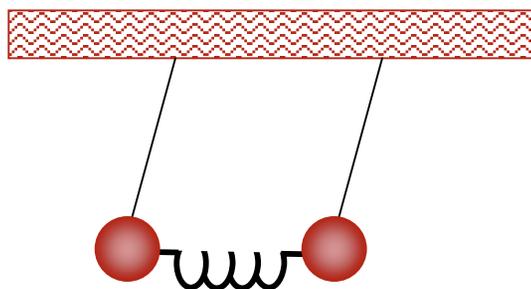
$$Q = \sqrt{L/C} / \mathcal{R}_{surface}$$



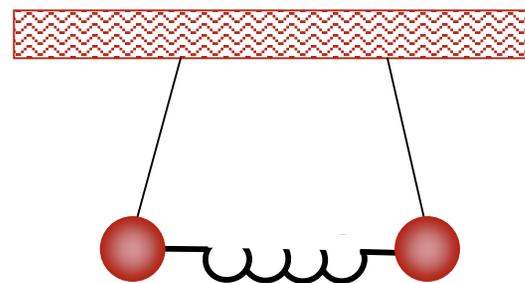
# Linacs cells are linked to minimize cost



==> coupled oscillators ==> multiple modes



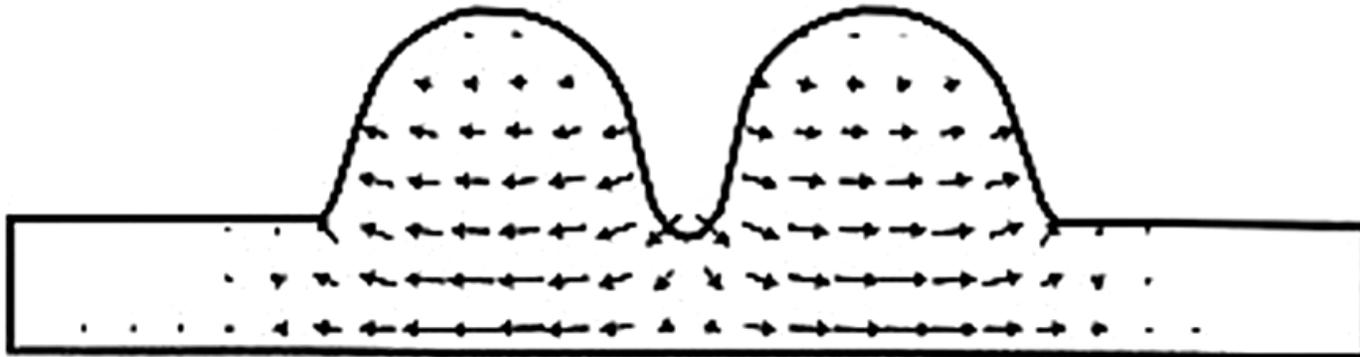
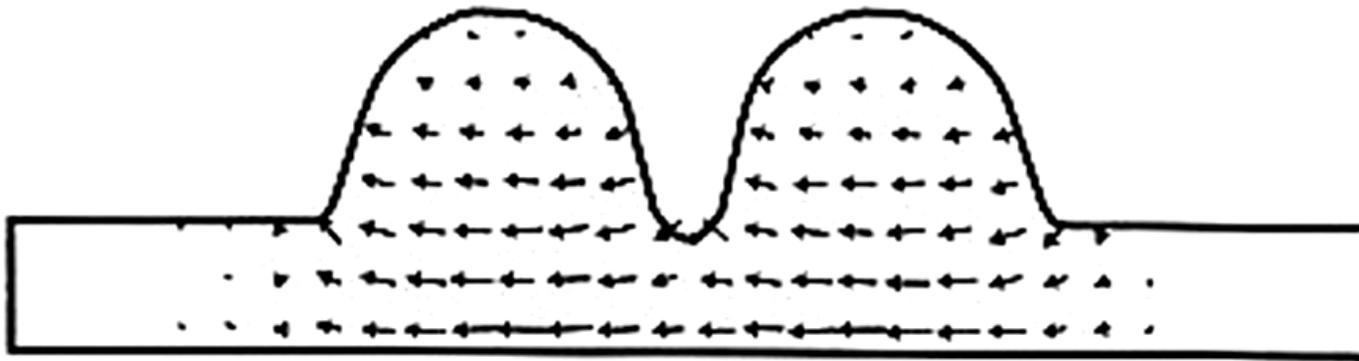
Zero mode



$\pi$  mode



## Modes of a two-cell cavity



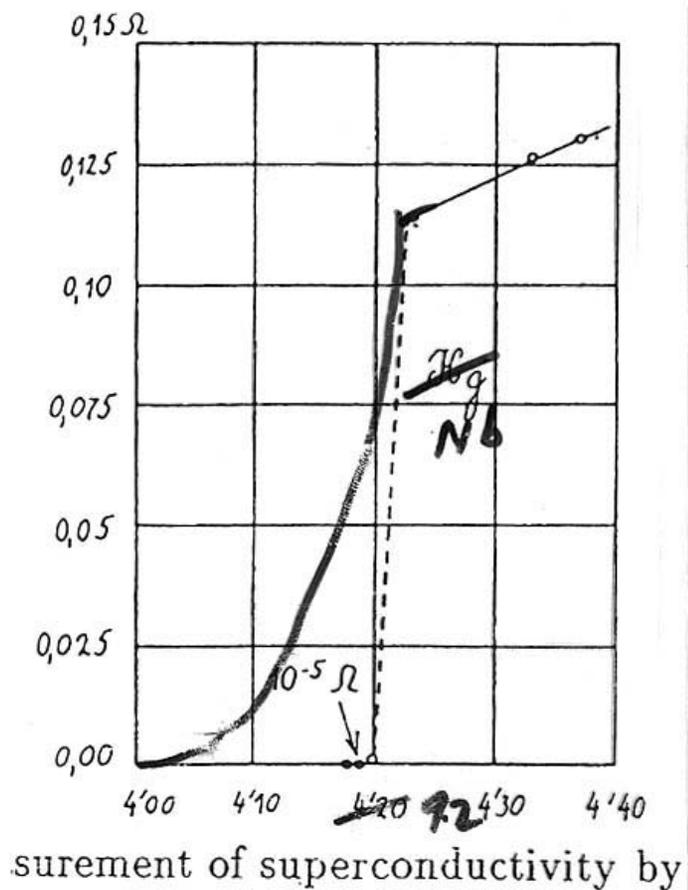


## 9-cavity TESLA cell





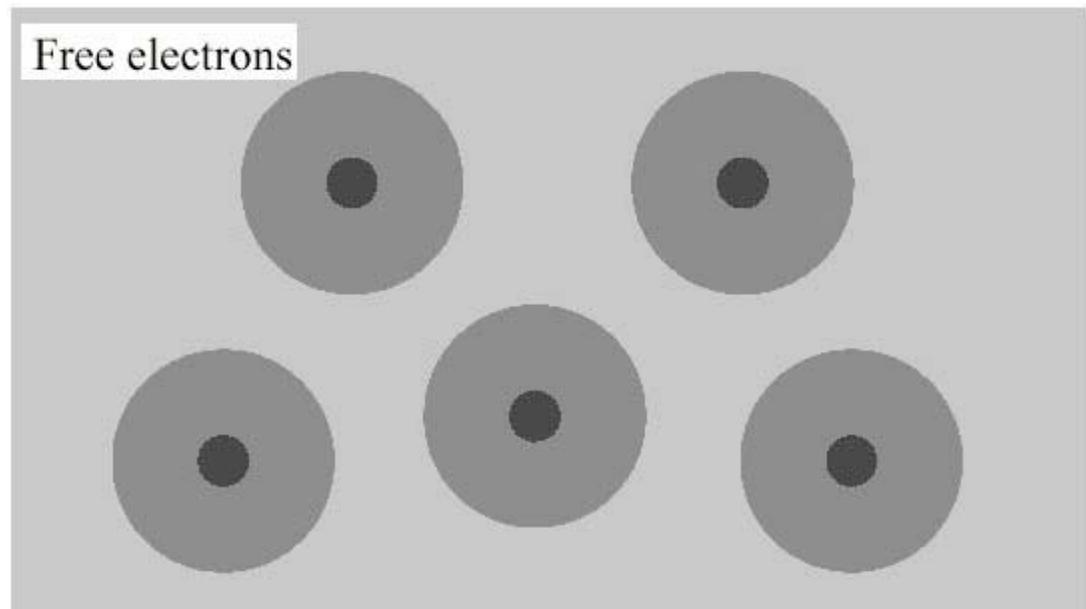
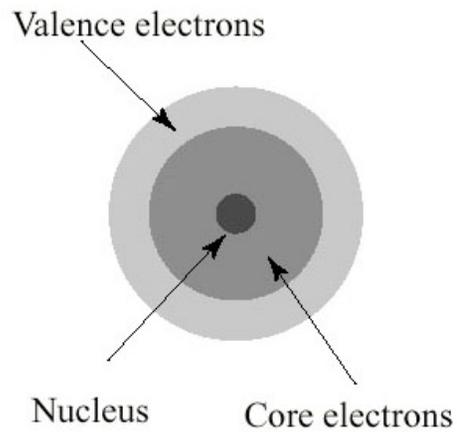
# Enter Superconductivity



1911

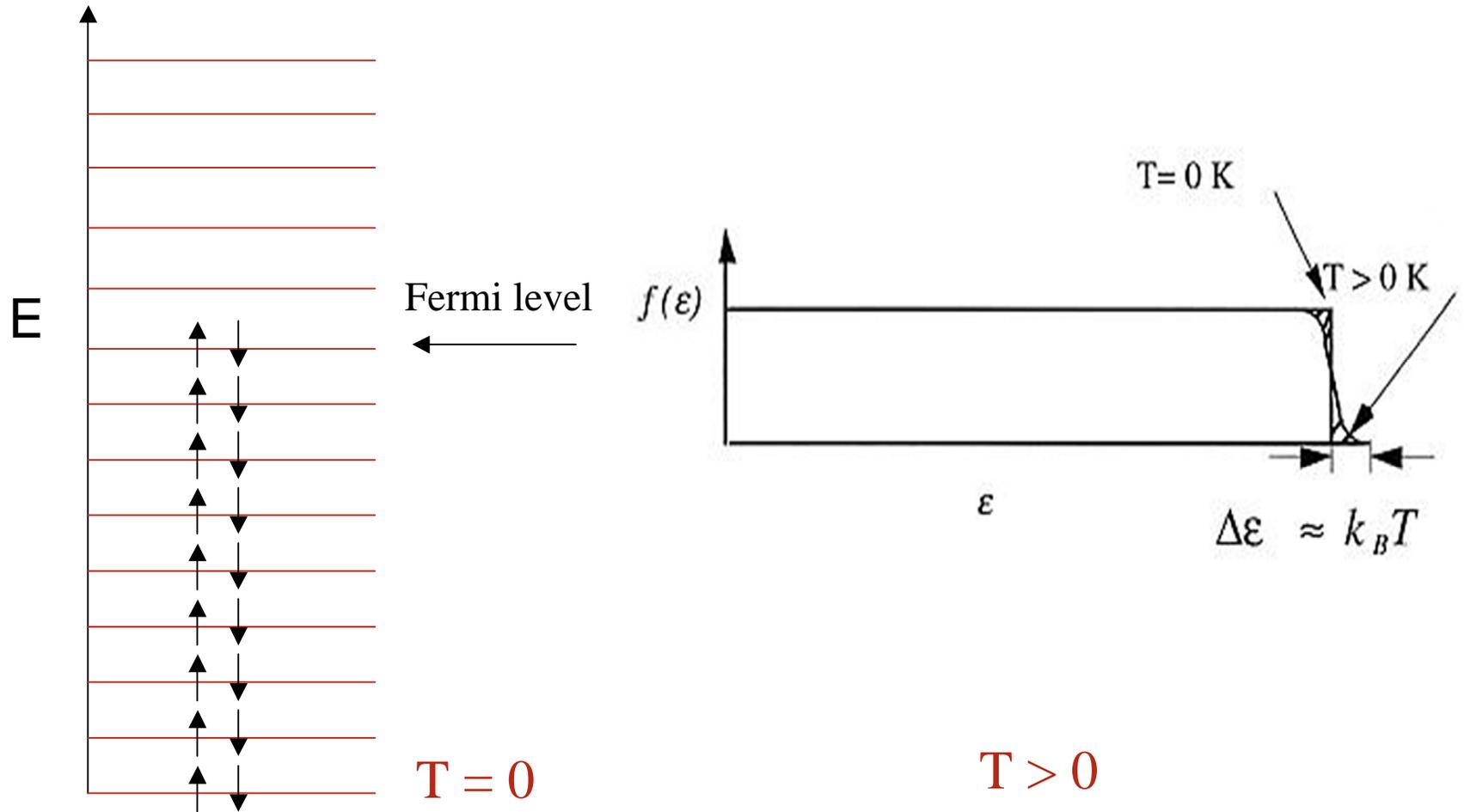


# Electrons in Solids - naïve picture



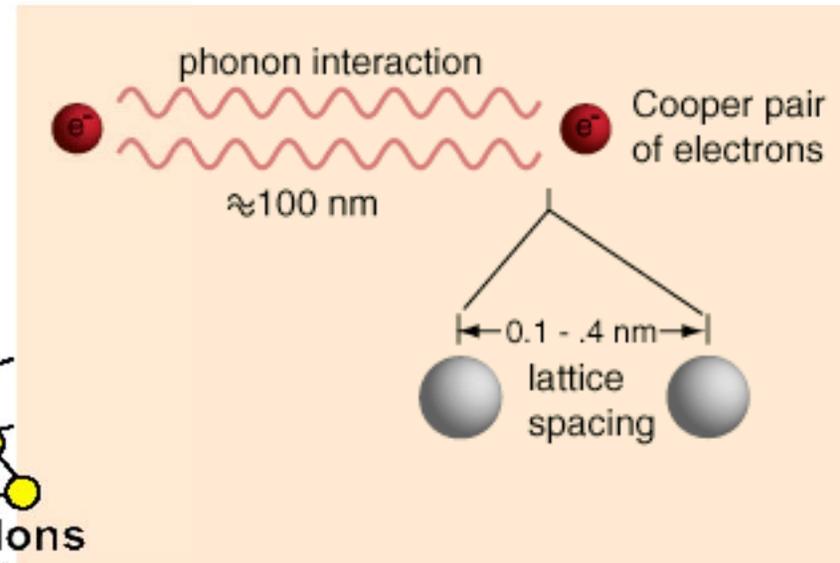
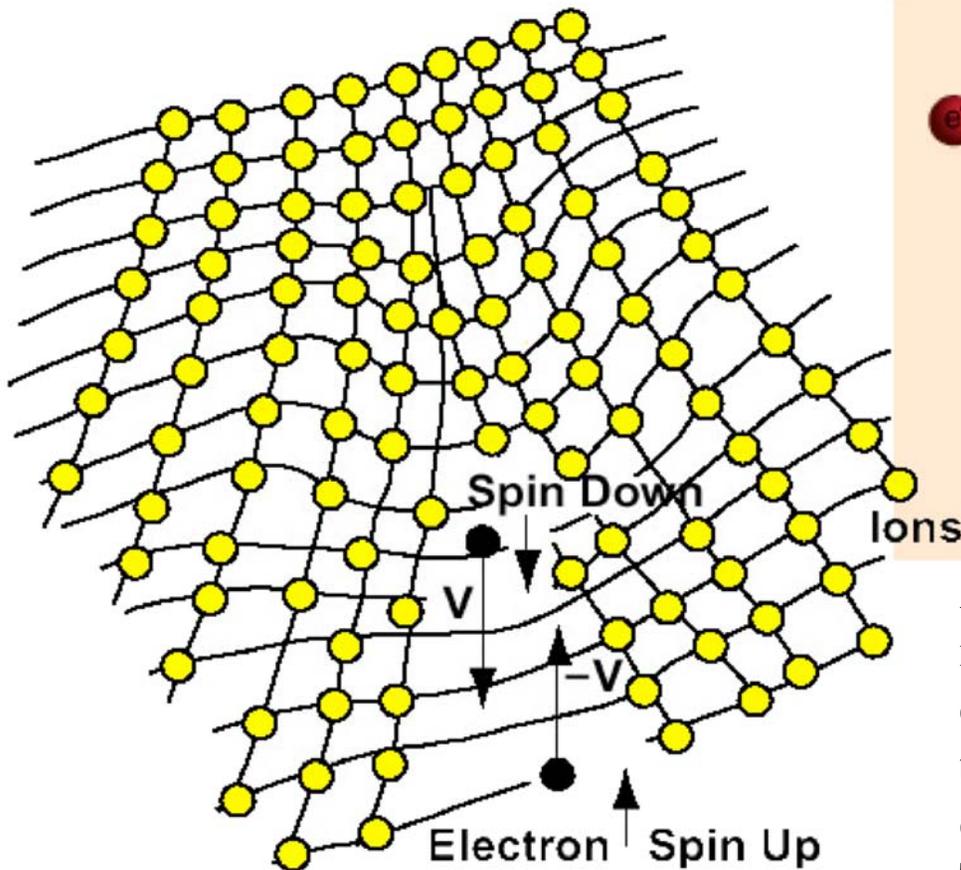


# Energy distribution of electrons in normal conductors





# Electron-Phonon interaction $\implies$ electron pairs - BCS theory



An  $e^-$  moving thru a conductor attracts nearby ions. The lattice deformation causes another  $e^-$ , with opposite "spin", to move into region of higher + charge density.

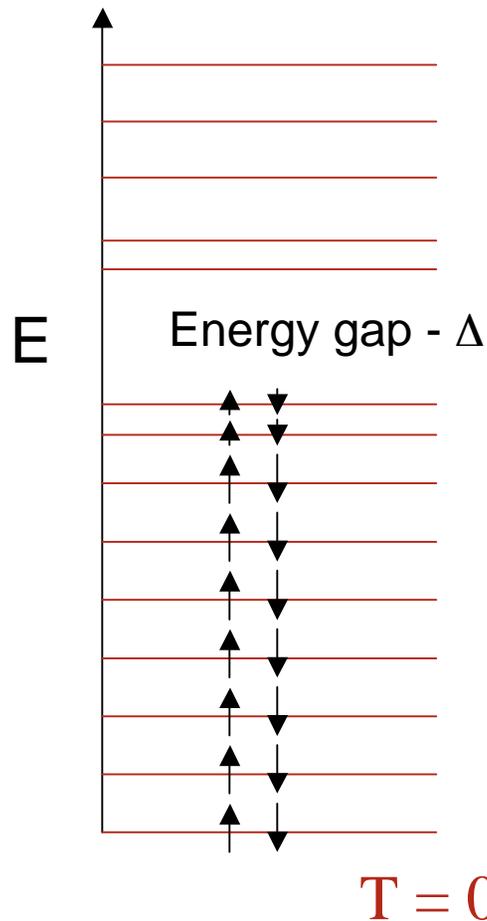
The two  $e^-$  are held together with a binding energy,  $2\Delta$



# In superconductors interaction of electrons with lattice phonons ==> pairs (bosons)



✱ Possibility of Bose condensate at  $T_{\text{critical}}$



Two fluid model:

For  $T_c > T > 0$ , excitation of unpaired electrons

$$n_{\text{normal}} \propto \exp\left(-\frac{\Delta}{k_B T}\right)$$

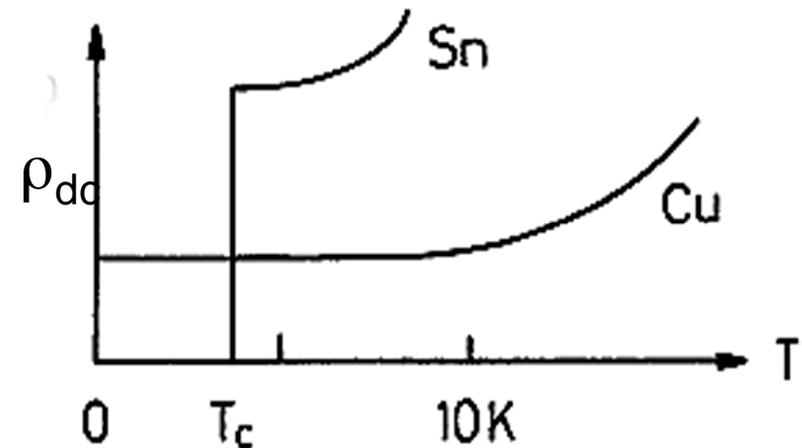
where  $2\Delta$  is the energy to break apart the Cooper pairs, until no electrons are paired above  $T_c$



## DC conductivity in superconductors



- ✱ DC resistance = 0  
because unpaired electrons  
are shorted out by Cooper  
pairs.





# RF Resistance in Superconductors

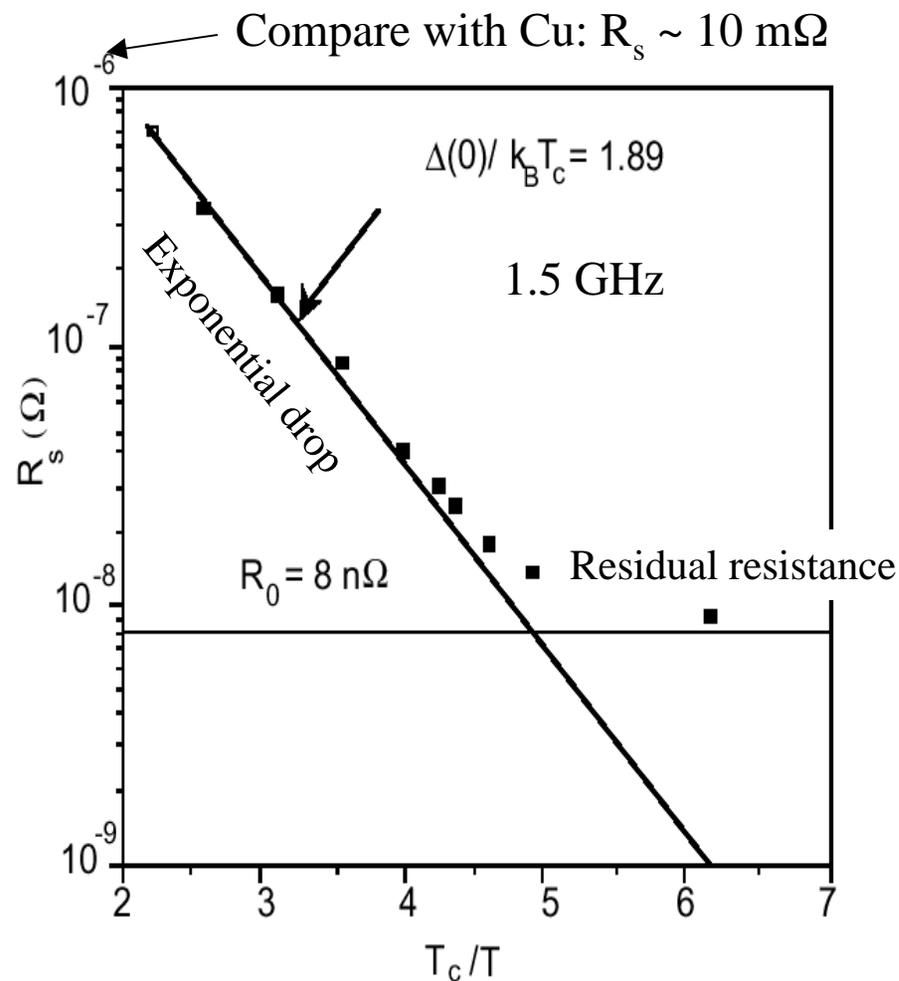


- \* RF resistance is finite because Cooper pairs have inertia  $\rightarrow$  unpaired electrons “see” an electric field.

$$R_s = A_s \omega^2 \exp\left(-\frac{\Delta(0)}{k_B T}\right)$$

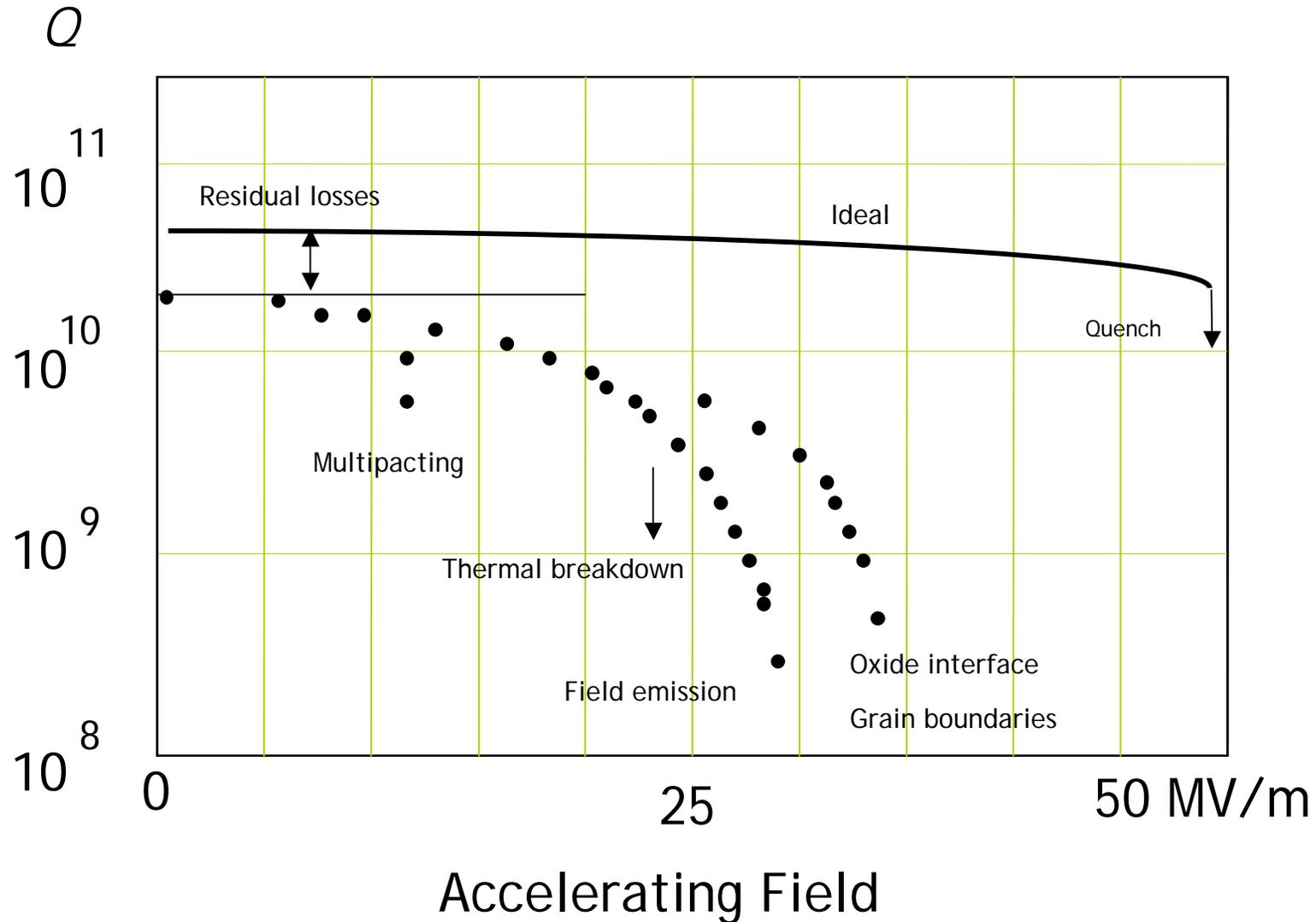
More resistance the more the sc pairs are jiggled around

More resistance the more unpaired electrons are excited





# In practice several effects limit the most important measure of cell performance

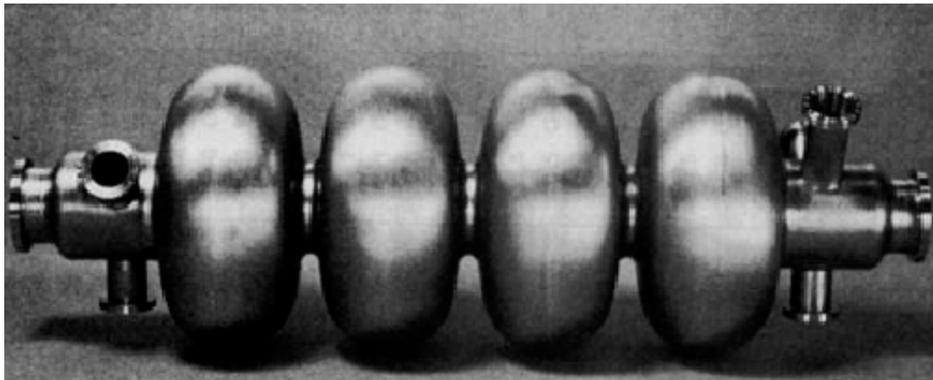




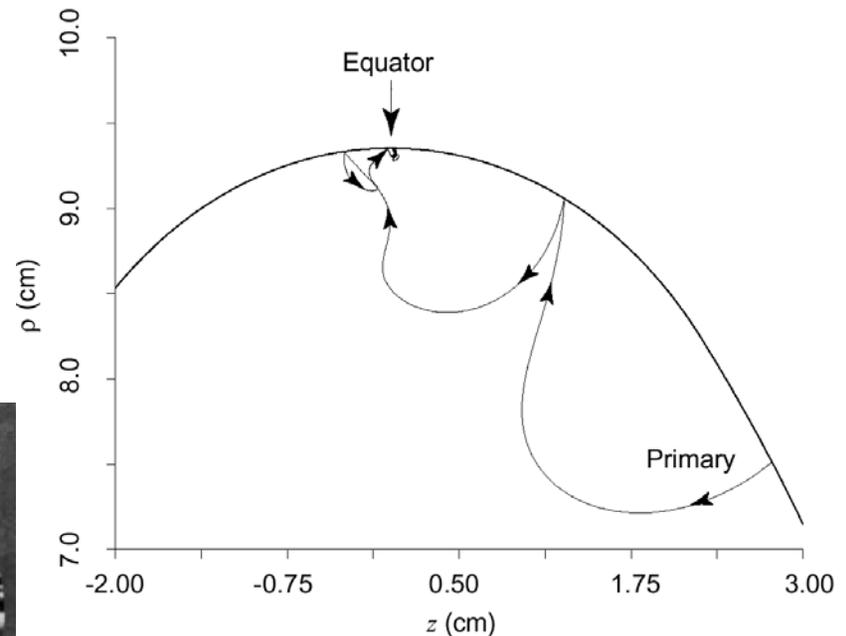
# Multipacting Solution



- ✱ First spherical, later elliptical shape cells.



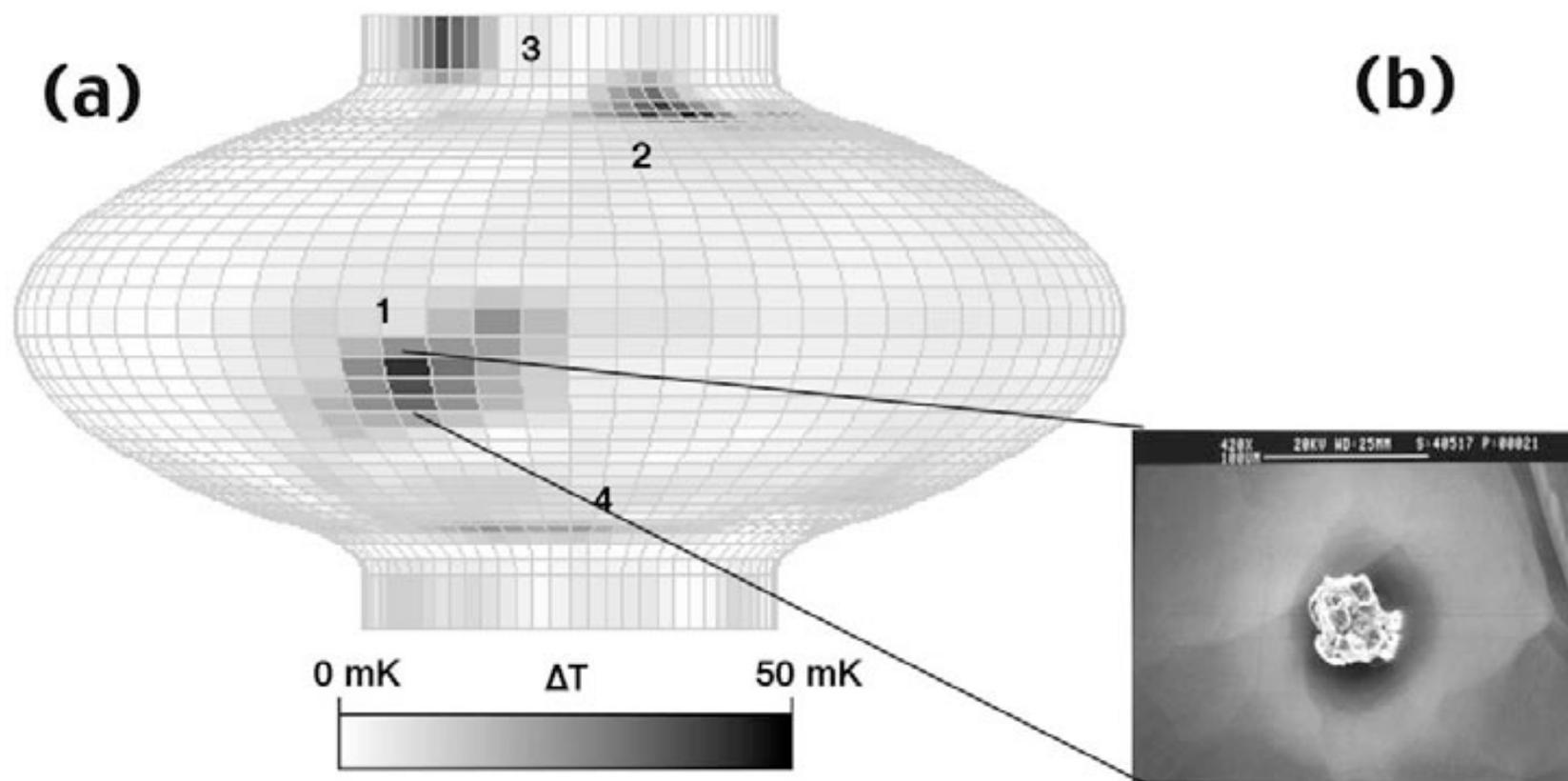
350-MHz LEP-II cavity (CERN)



Electrons drift to equator  
Electric field at equator is  $\approx 0$   
→MP electrons don't gain energy  
→MP stops



# Thermometry at a quench point

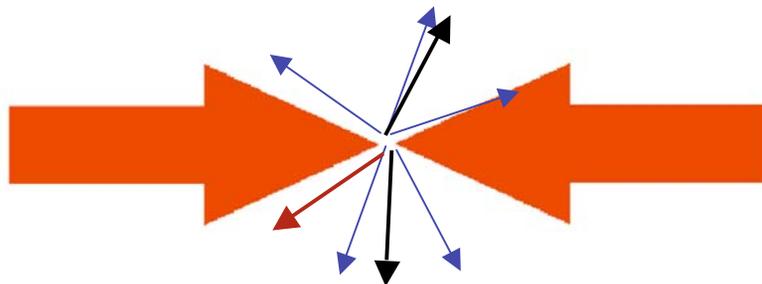




# Why do we need beams?

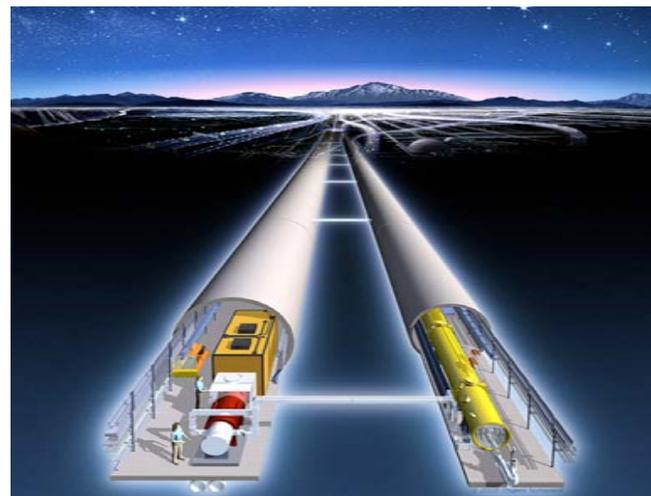


## Collide beams



FOMs: Collision rate, energy stability, Accelerating field

Examples: LHC, ILC, RHIC





## In LHC storage rings...

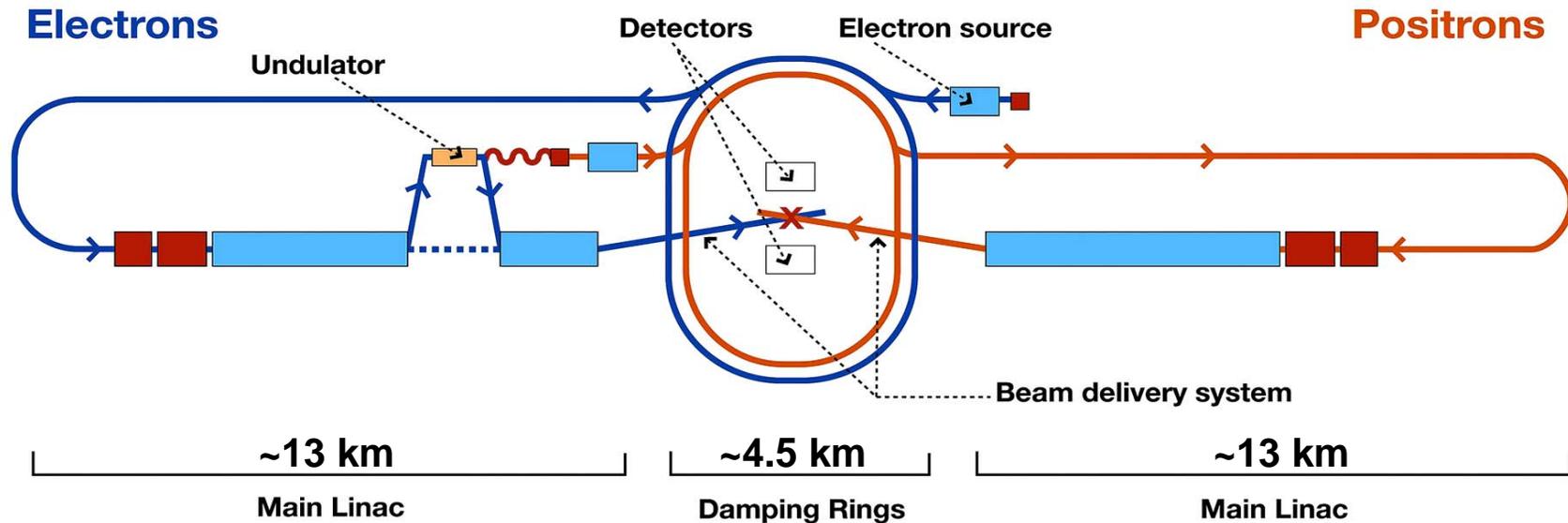


- ✱ Limited space & Large rf trapping of particles
  - $V/\text{cavity}$  must be high
- ✱ Bunch length must be large ( $\leq 1$  event/cm in luminous region)
  - RF frequency must be low
- ✱ Energy lost in walls must be small
  - $R_{\text{surf}}$  must be small

*SC cavities were the only practical choice*



# For ILC SC rf provides high power, high quality beams at high efficiency



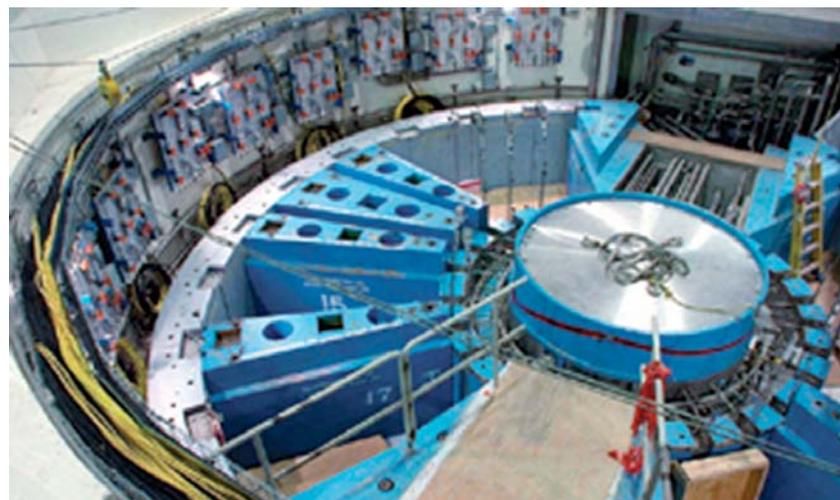
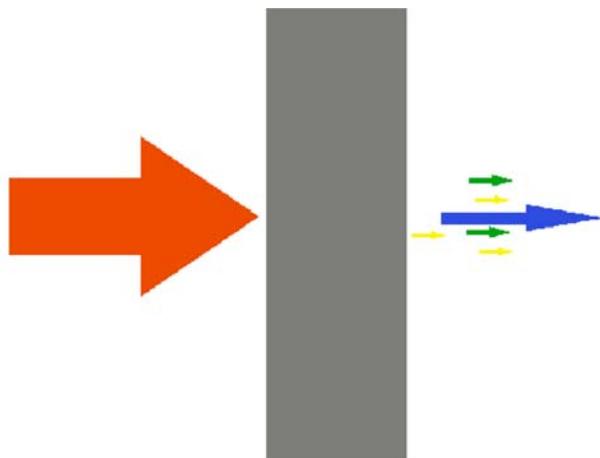
- **To deliver required luminosity ( $500 \text{ fb}^{-1}$  in 4 years) ==>**
  - powerful polarized electron & positron beams (11 MW /beam)
  - tiny beams at collision point ==> minimizing beam-structure interaction
- **To limit power consumption ==> high “wall plug” to beam power efficiency**
  - Even with SC rf, the site power is still 230 MW !



## Why do we need beams?



### Intense secondary beams



1 MW target at SNS

FOM: Secondaries/primary

Examples: spallation neutrons,  
neutrino beams



# The Spallation Neutron Source



✱ 1 MW @ 1 GeV (compare with ILC 11 MW at 500 GeV  
(upgradeable to 4 MW))

==> miniscule beam loss into accelerator

==> large aperture in cavities ==> large cavities

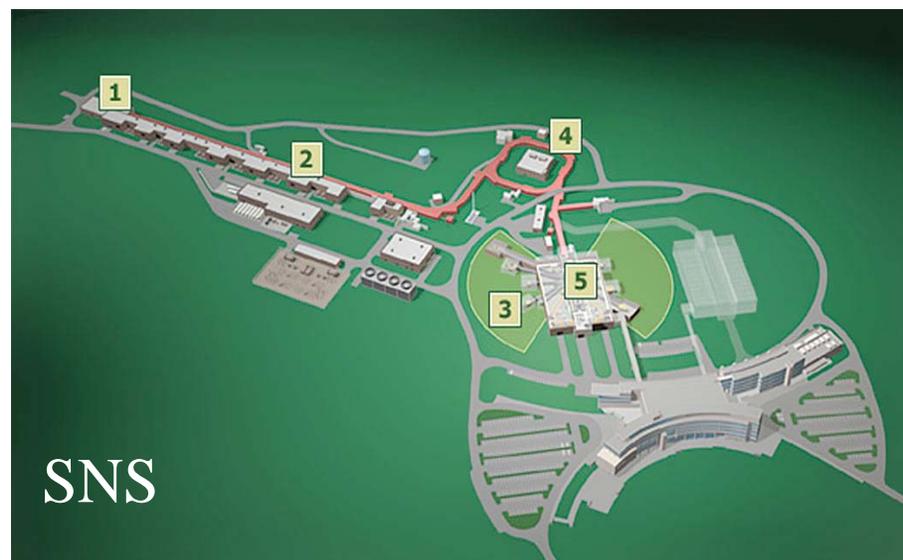
==> low frequency

==> high energy stability

==> large stored energy

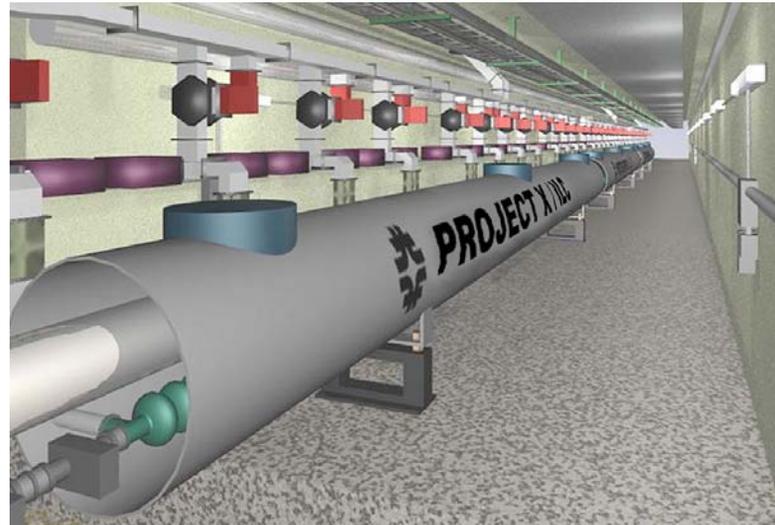
==> high efficiency at  $E_z$

==> **SC RF**





# Proton Intensity Frontier Option: Project X



Initially NOvA  
Possibly DUSEL later

200 kW  
at 8 GeV

>2 MW  
at 120 GeV

**Recycler**  
3 linac pulses / fill

**Main Injector**  
1.4 sec cycle

Stripping Foil

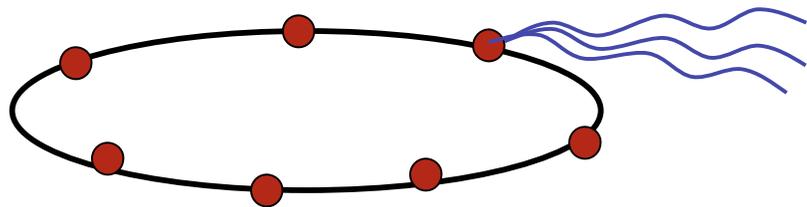
**ILC-style 8 GeV H<sup>-</sup> Linac**  
9mA x 1 msec x 5 Hz



# Matter to energy: Synchrotron radiation science



**Synchrotron light source**  
(pulsed incoherent X-ray emission)



FOM: Brilliance v.  $\lambda$

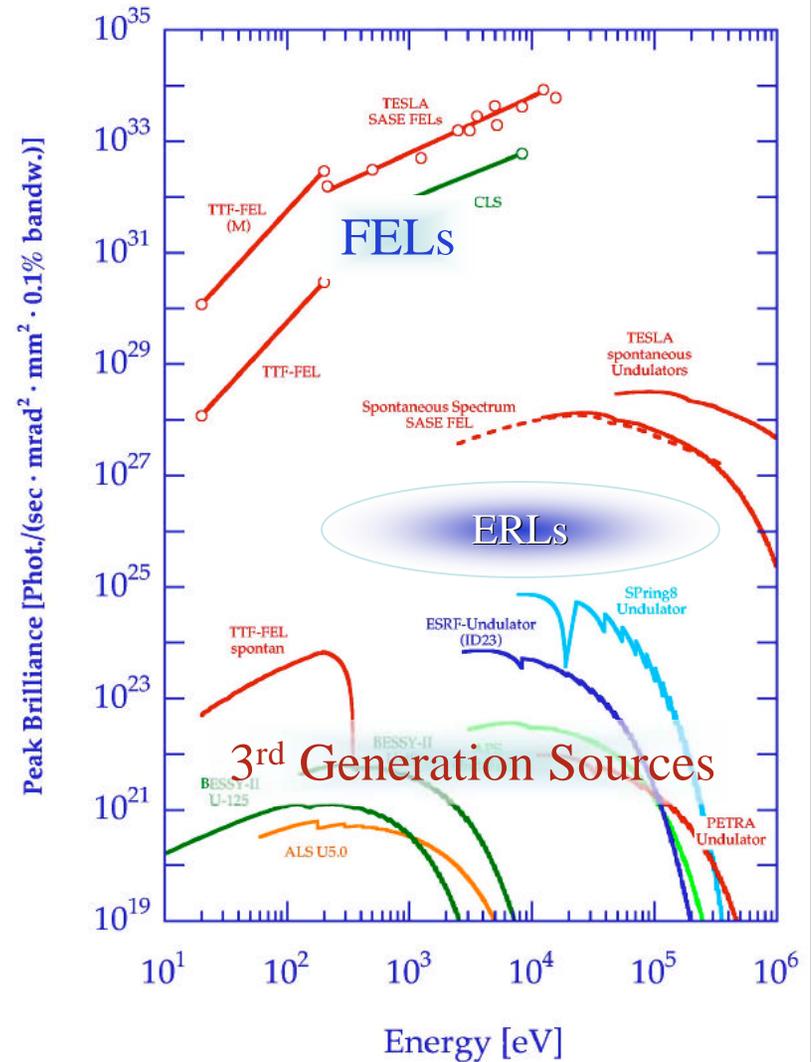
$$B = \text{ph/s/mm}^2/\text{mrad}^2/0.1\% \text{BW}$$

Pulse duration

Science with X-rays

Imaging

Spectroscopy





# Matter to energy: Energy Recovery Linacs

Hard X-rays  $\Rightarrow$   $\sim 5$  GeV



## Synchrotron light source

(pulsed incoherent X-ray emission)

Pulse rates – kHz  $\Rightarrow$  MHz

X-ray pulse duration  $\leq 1$  ps

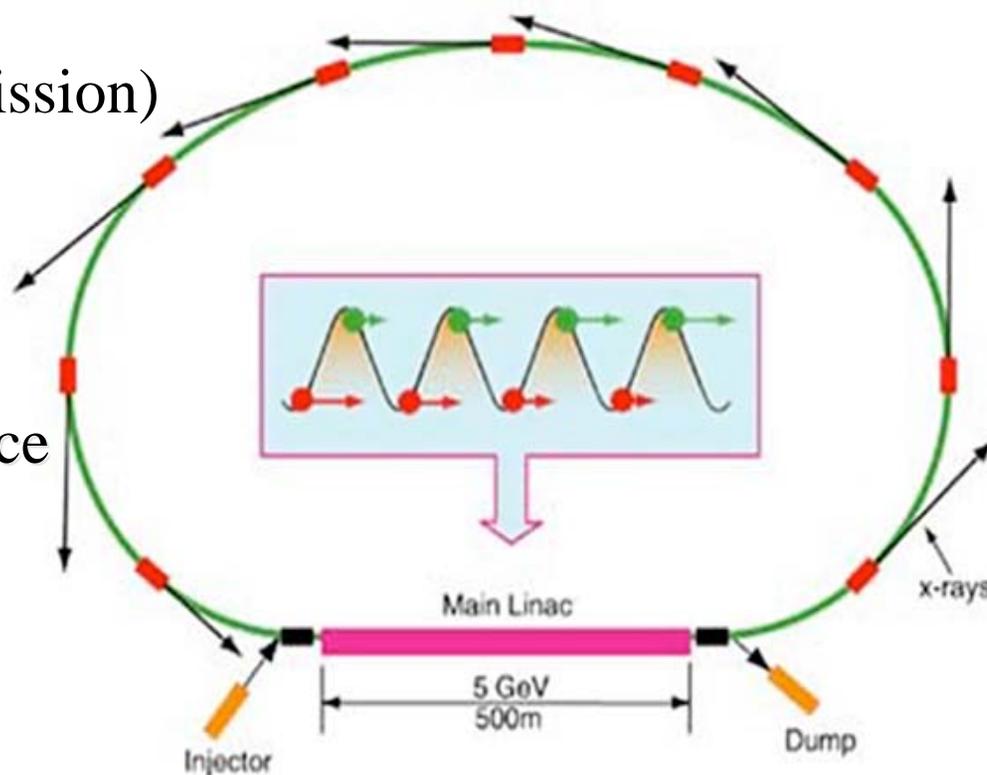
High average e-beam brilliance  
& e-beam duration  $\leq 1$  ps

$\Rightarrow$  One pass through ring

$\Rightarrow$  Recover beam energy

$\Rightarrow$  High efficiency

$\Rightarrow$  SC RF

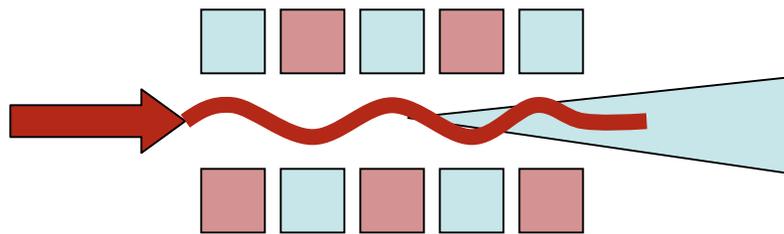




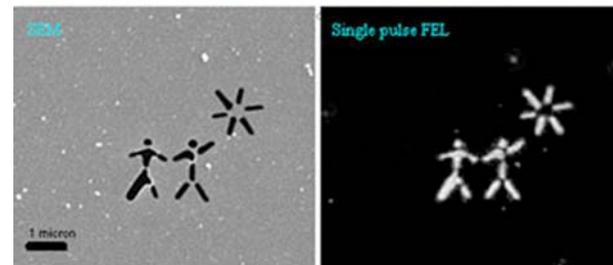
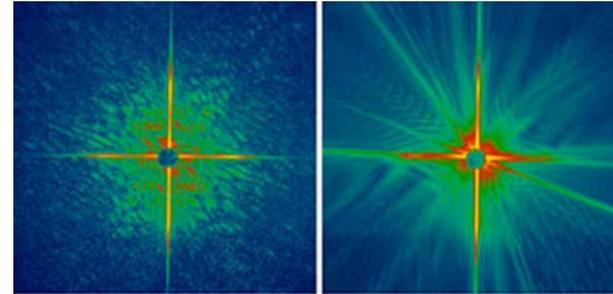
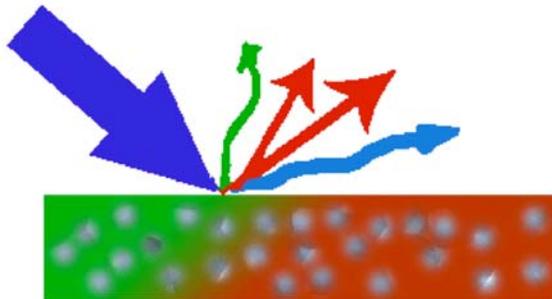
Even higher brightness requires  
coherent emission  $\implies$  FEL



## Free electron laser



FOM: Brightness v.  $\lambda$   
Time structure





## Full range of FEL-based science requires...



- ✱ Pulses rates 10 Hz to 10 MHz (NC limited to ~ 100 Hz)
  - High efficiency
  
- ✱ Pulse duration 10 fs - 1 ps
  
- ✱ High gain
  - Excellent beam emittance
    - ==> Minimize wakefield effect
    - ==> large aperture
    - ==> low frequency
  - Stable beam energy & intensity
    - ==> large stored energy in cavities
    - ==> high Q
  
- ====> SC RF



# 8 GeV Superconducting Linac

With X-Ray FEL, 8 GeV  $\nu$  & Spallation Sources, ILC &  $\nu$ -factory

