Introduction
to
Collider Physics

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The Very Big Picture

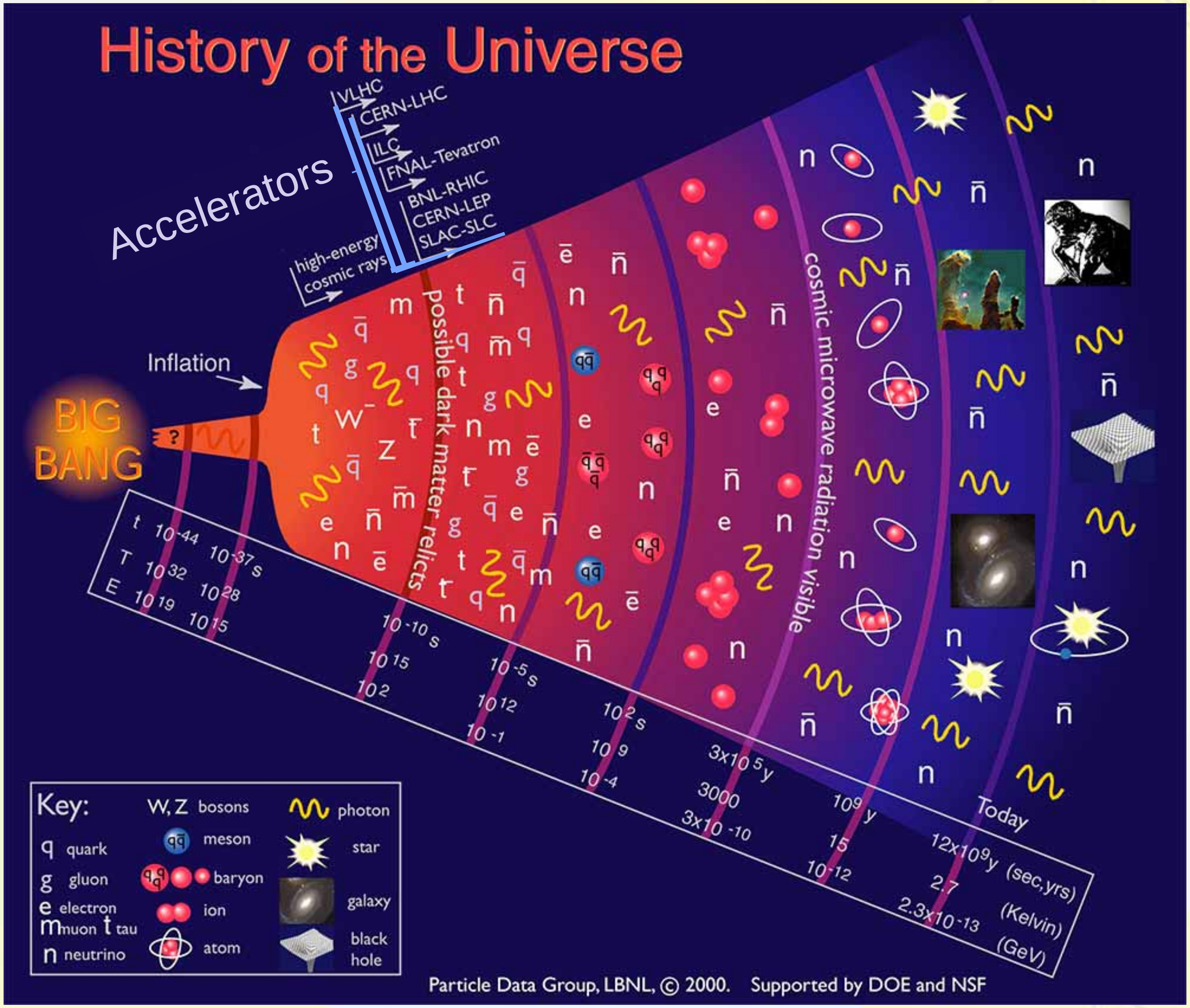

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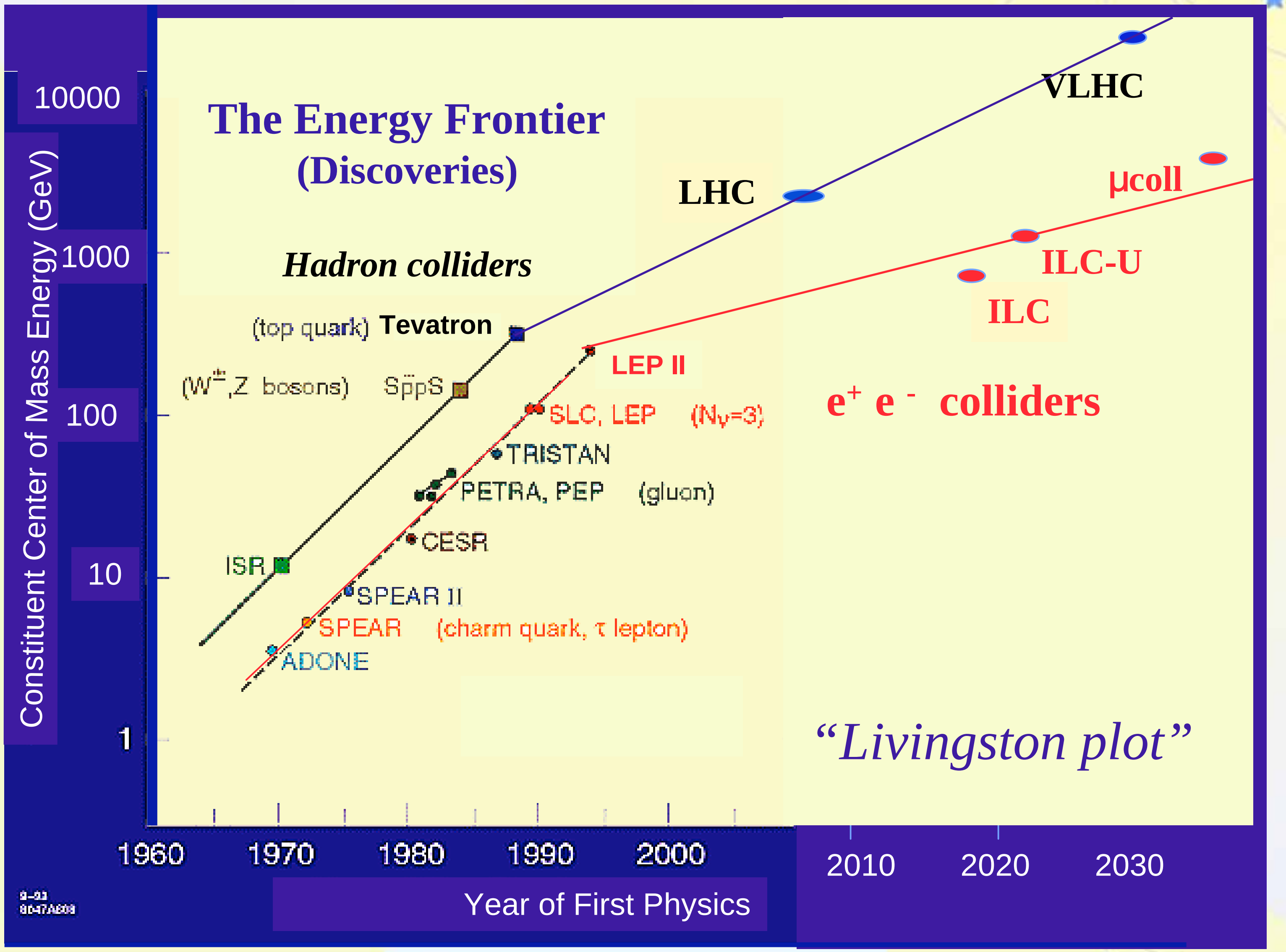
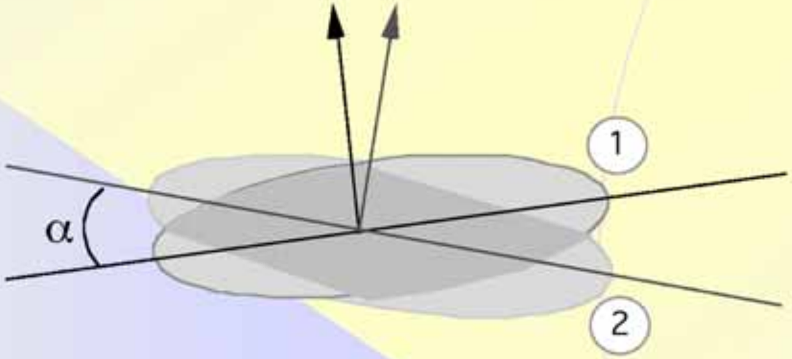
Figure of Merit 1: Accelerator energy ==> energy frontier of discovery



Physics figure of Merit 2: Number of events

$$Events = Cross\text{-}section \times \langle Collision\ Rate \rangle \times Time$$

Beam energy: sets scale of physics accessible



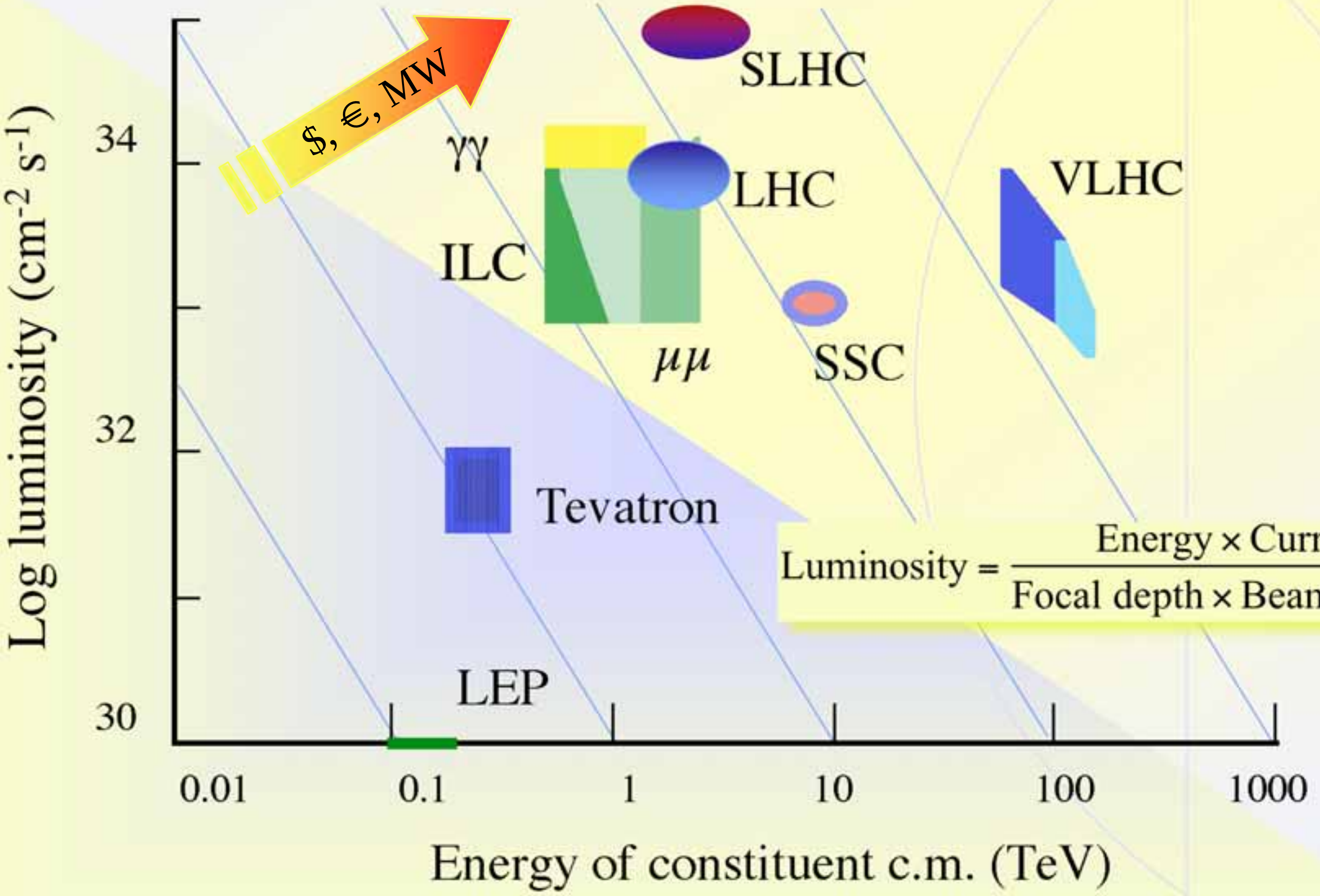
$$\text{Luminosity} = \frac{\mathrm{N}_1 \times \mathrm{N}_2 \times frequency}{Overlap\ Area} = \frac{\mathrm{N}_1 \times \mathrm{N}_2 \times f}{4\pi\sigma_x\sigma_y} \times \text{Disruption} \times \alpha\ \text{correction}$$

We want large charge/bunch, high collision frequency & small spot size

Note: L changes in time as collisions deplete the bunches

==> Luminosity lifetime

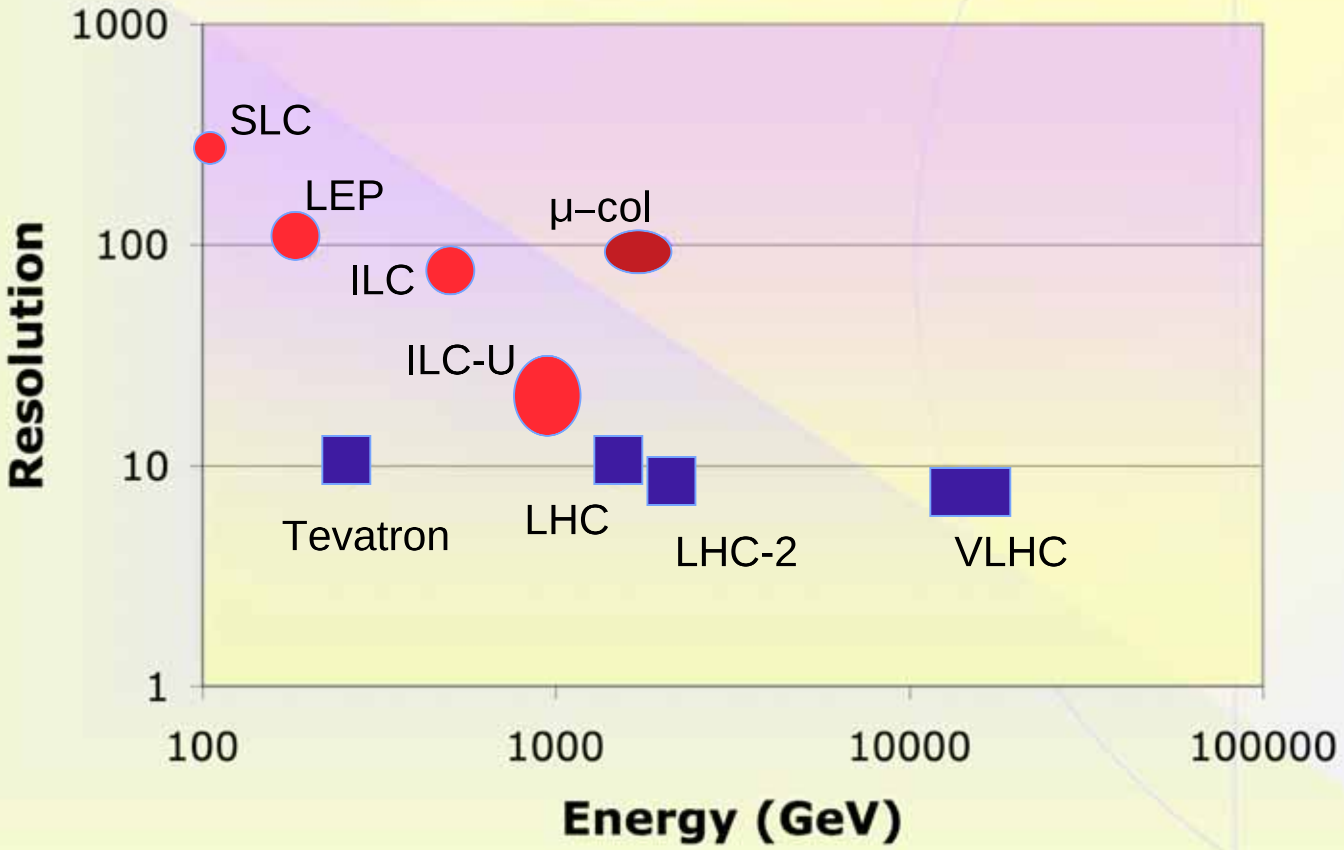
Discovery space for future accelerators



$$\text{Luminosity} = \frac{\text{Energy} \times \text{Current}}{\text{Focal depth} \times \text{Beam quality}}$$

FoM 3: Resolution (Energy/ΔE)

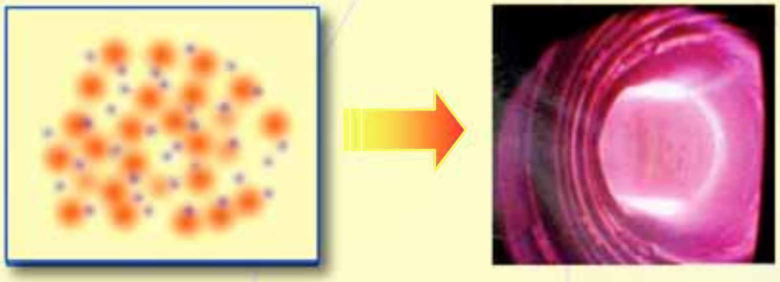
- Intertwined with detector & experiment design
  - → In hadron colliders: production change, parton energy distribution
  - → In lepton colliders: energy spread of beams (synchrotron radiation)



Characteristics of Beams that Matter to Particle Physics

Beams: Bunches of particles with a directed velocity

- Ions - either missing electrons (+) or with extra electrons (-)
- Electrons or positrons
- Plasma - ions plus electrons



- Beams particles have random (thermal) ⊥ motion
- Beam must be confined against thermal expansion during transport

- Beam's have internal (self-forces)

Beam transverse self-fields

$$E_{sp}(V/cm) = \frac{60\ I_{beam}(A)}{R_{beam}(cm)}$$

$$B_\theta(gauss) = \frac{I_{beam}(A)}{5\ R_{beam}(cm)}$$

In vacuum:

Beam's transverse self-force scale as $1/\gamma^2$

→ Space charge repulsion: $E_{sp,\perp} \sim N_{beam}$

→ Pinch field: $B_\theta \sim I_{beam} \sim v_z N_{beam} \sim v_z E_{sp}$

$$\therefore F_{sp,\perp} = q\,(E_{sp,\perp} + v_z \times B_\theta) \sim (1-v^2)\, N_{beam} \sim N_{beam}/\gamma^2$$

Beams in collision are *not* in vacuum (beam-beam effect)

Envelope Equation: Evolution of beam during transport

Write equation of motion for each particle

$$\mathbf{F}_i = m\mathbf{a}_i = e\,(\mathbf{E} + \mathbf{v}_i \times \mathbf{B})$$

Energy equation for each particle

$$E_i = \gamma_i m c^2$$

Assume $v_\perp << v_z$

Take averages over the distribution of particles

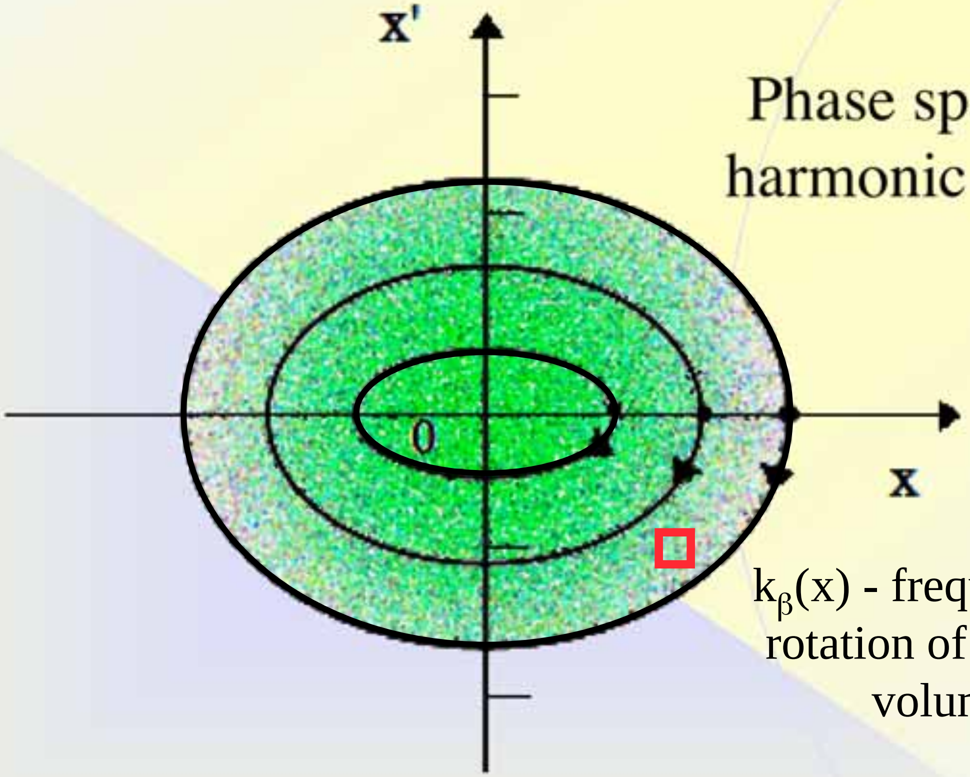
Apply Virial Theorem: K.E = -P.E./2

$$R'' + \frac{1}{\gamma}\gamma' R' + \frac{U_{self}}{R} + \frac{\left\langle k_\beta^2 r^2 \right\rangle}{R} - \frac{\varepsilon^2}{R^3} = \frac{1}{R^3}(\varepsilon^2)'$$

$$\varepsilon^2 = R^2(V^2 - (R')^2)/c^2$$

Emittance describes the area in phase space of the ensemble of beam particles

Emittance - Phase space volume of beam



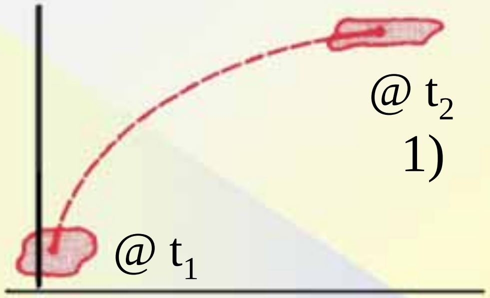
Phase space of an harmonic oscillator

$k_\beta(x)$ - frequency of rotation of a phase volume

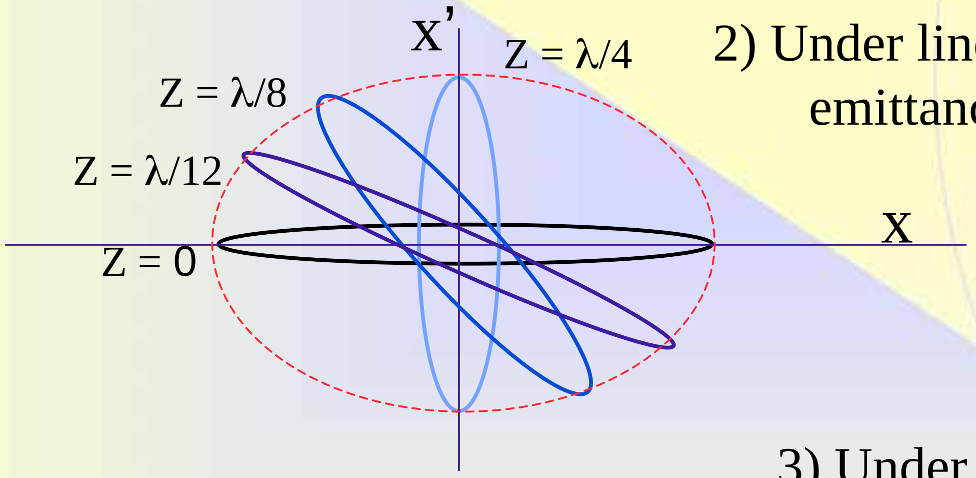
RMS emittance

$$\varepsilon^2 = R^2(V^2 - (R')^2)/c^2$$

Why is emittance an important concept



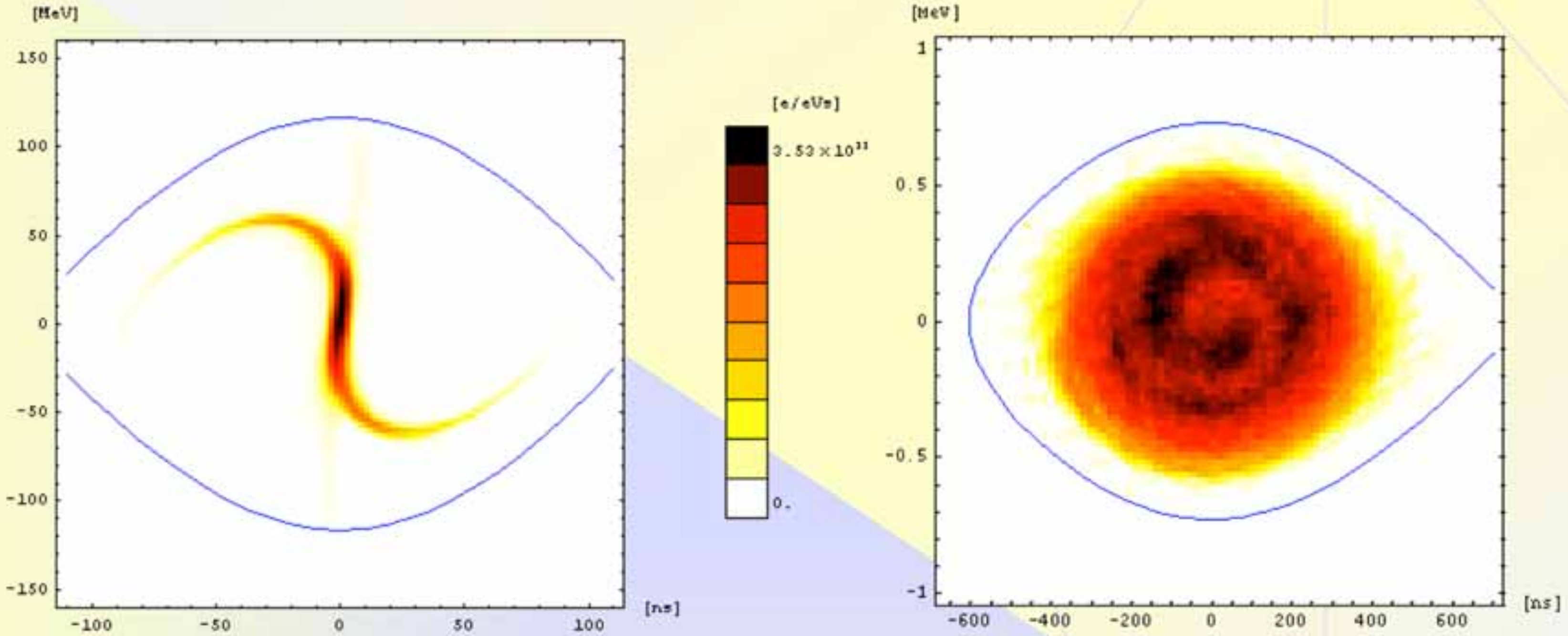
1) Liouville: Under conservative forces phase space evolves like an incompressible fluid



2) Under linear forces macroscopic emittance is preserved

3) Under acceleration $\gamma\varepsilon = \varepsilon_n$ is an adiabatic inavriant

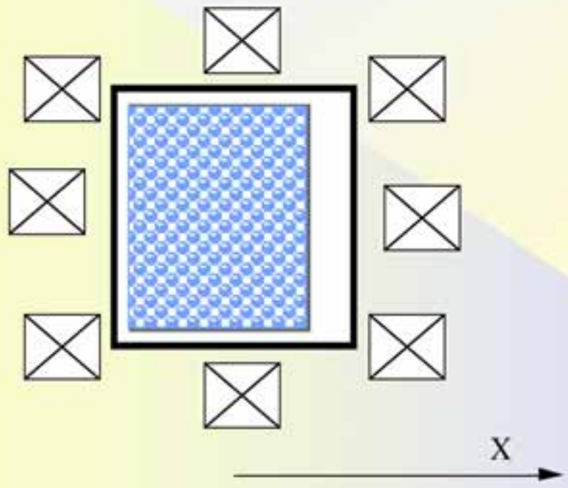
Nonlinear applied & space-charge fields lead to filamentation of phase space

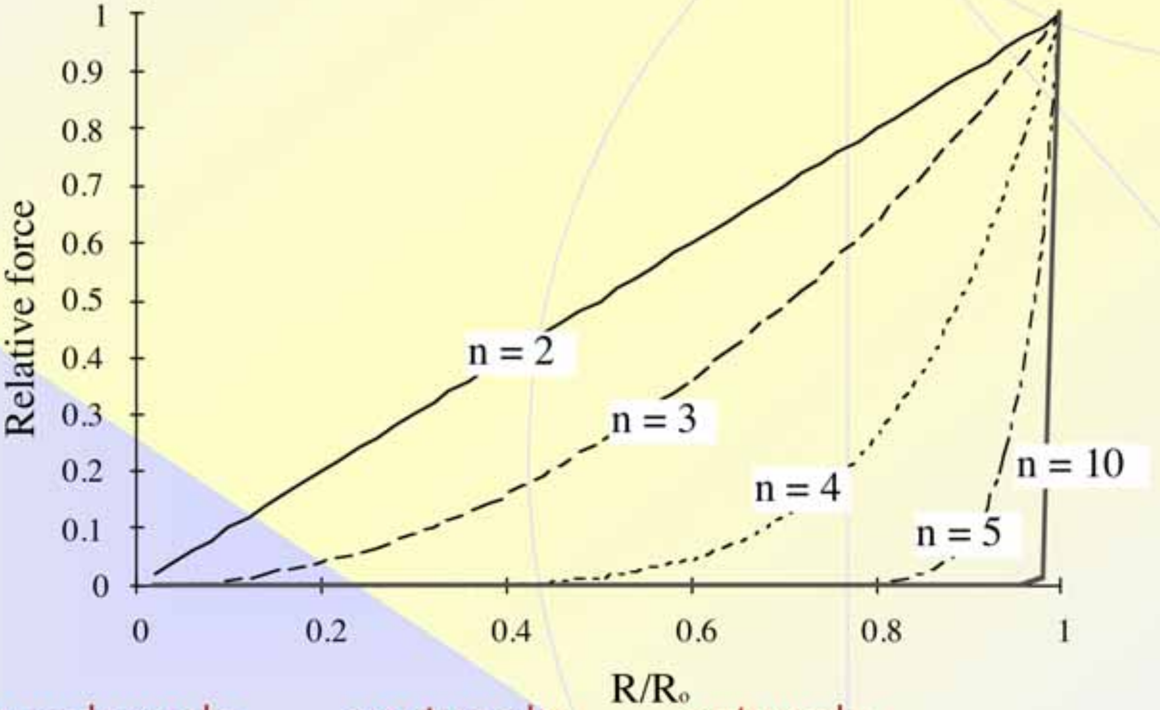


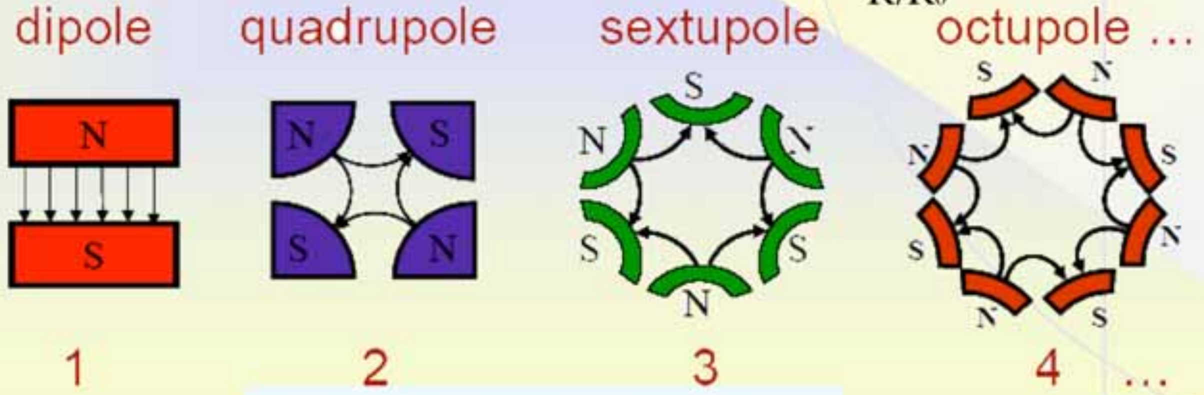
Data from CERN PS

Macroscopic (rms) emittance is not conserved

Damping of coherent beam displacement by frequency spread in non-linear elements







What sets spot size ?

- Strength (depth of focus) of lens at interaction point, β^*
- Distribution of positions & transverse momenta of beam particles (emittance), ε

$$\sigma_{x,y} = \sqrt{\varepsilon_{x,y}\beta^*}$$

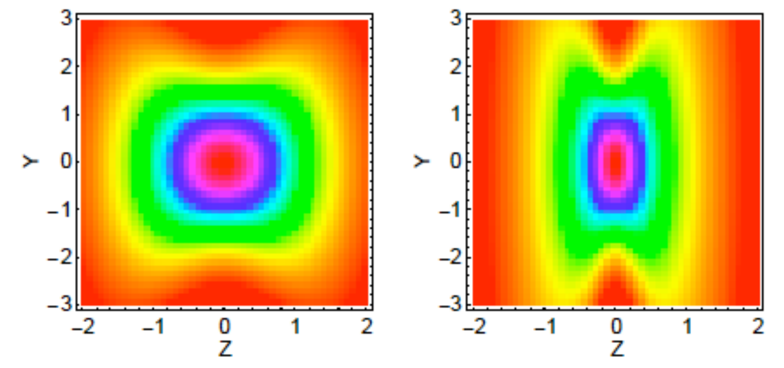
$$\text{Luminosity} = \frac{N_1 \times N_2 \times f}{4\pi\sqrt{\beta_x^*\beta_y^*\varepsilon_x\varepsilon_y}} \times \text{Pinch effect} \times \textit{a}\text{ngle correction}$$

- For simplicity say ε and β^* are equal for x and y

$$\text{Luminosity} = \frac{N_1 \times N_2 \times f}{4\pi\beta_x^*\varepsilon_x} \times H_D \times \textit{angle correction}$$

How can we maximize luminosity?

- If $\beta^* <$ bunch length, σ_z, bunch has hour glass shape
  - → Correction factor lowers luminosity

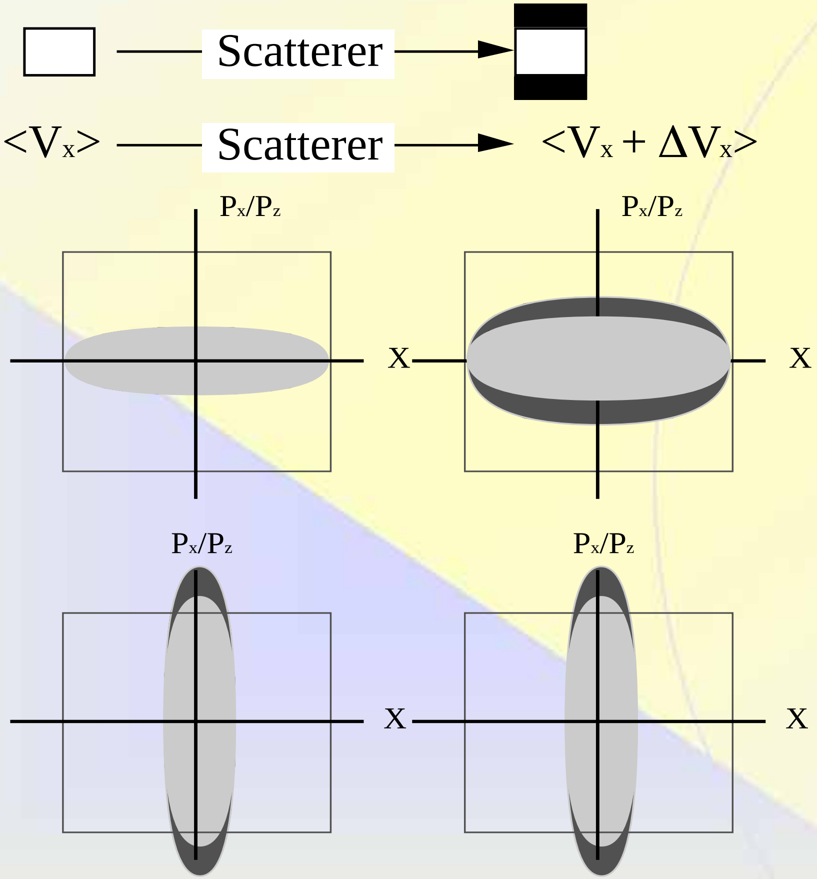


- Raise N: instability limits
- Raise f: Detector issues
- Can we change the emittance?

==> Non-conservative forces acting on beams

==> Beam cooling

Emittance increase from non-conservative forces (scattering) depends on beam size



Oops, we got the sign wrong! We wanted to **cool** the beam

So, remove energy from particles with largest v⊥

Synchrotron radiation (for electrons)

$$U_o = \frac{4\pi r_p m_p c^2}{3} \frac{\gamma^4}{\rho} = 6.03 \times 10^{-18} \frac{\gamma^4}{\rho\,(m)} \text{ GeV}$$

$$N_\gamma \sim 4\pi\alpha \text{ per turn}$$

$$P_{sr} = 26.5 \text{ kW } E^3_{GeV} I_A B_T \text{ (for electrons)}$$

$$E_{crit}(keV) = 0.66 E^2_{GeV} B_T$$

Implications: Power deposition on vacuum chamber

Beam-beam focusing at IP

Damping of coherent beam properties

Energy damping - $U \sim \gamma^4$

Equilibrium energy spread in ring

$$\Delta E \approx \sqrt{E_{beam} E_{crit}}$$

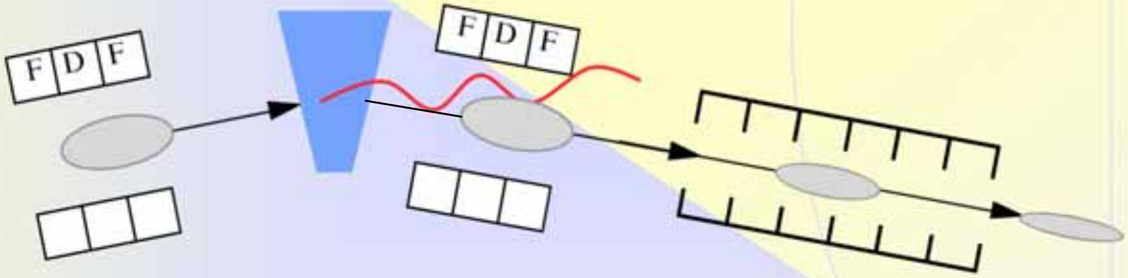
Radiation damping time ~ E_{beam}/U_o

Effects of synchrotron radiation: Cooling the particles in rings

- Particles radiate (loose energy) along instantaneous velocity.
- Energy is added along beam axis.

$$\frac{d\,\varepsilon}{\varepsilon} = \frac{d\,p_\perp}{p_\perp} = \frac{d\,p_\parallel}{p_\parallel}$$

- E-folding emittance requires replacing beam energy

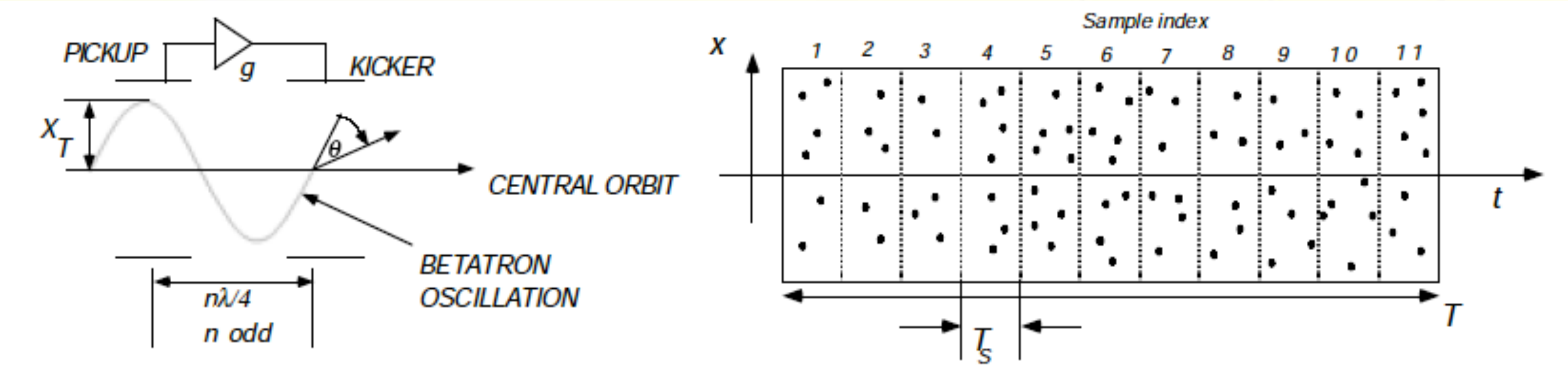


- Rate is independent of number of particles per bunch
- Radiation cooling of emittance limited by quantum nature of radiation
- Quantum spread of radiation energy increases longitudinal emittance

BUT, what if radiation is negligible?

Van der Meer's demon

- Stochastic cooling: Measures fluctuations in beam distribution



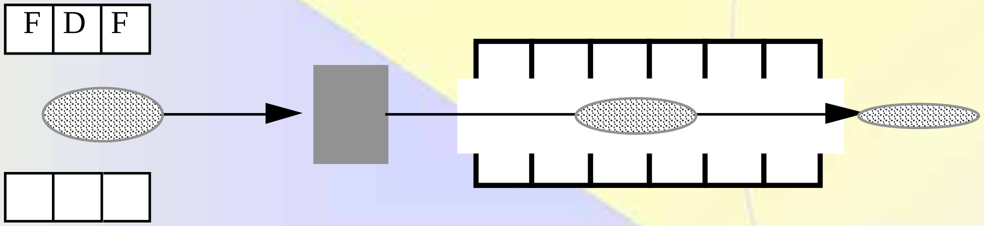
$$\frac{1}{\tau_{x,\max}} = \frac{f}{2N_s}$$

- The smaller the number of particles in the sample, & the higher the revolution frequency, the higher the cooling rate.
- For example, 1 GHz of bandwidth cools 10^9 particles at 1Hz.

Good for anti-protons; too slow for muons

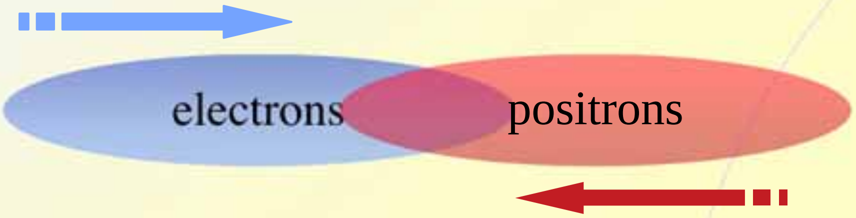
Basic components:
Ionization cooling of muons

- Radiation cooling & ionization cooling are analogous
- Strong lens to focus the beam
- Ionization medium in which the particles lose both transverse & longitudinal momentum,
- Accelerating structure to restore longitudinal momentum



- Rate is independent of number of particles per bunch
- Ionization cooling of transverse emittance limited by beam heating due to multiple Coulomb scattering
- Straggling increases longitudinal emittance $\frac{dE}{E} \approx 1.35 \times 10^{-3} \sqrt{E\ (\mathrm{MeV})}$

Beamstrahlung lowers energy resolution in linear collider



At IP space charge cancels; currents add

==> strong beam-beam focus

- --> Luminosity enhancement
- --> Strong synchrotron radiation

Consider 250 GeV beams with 1 kA focused to 100 nm

$B_{peak} \sim 40$ Mgauss ==> $E_{crit} \sim 166$ GeV

Quantum effects & radiation reaction must suppress synchrotron radiation

A correct calculation yields

$\delta E/E \sim 4\%$

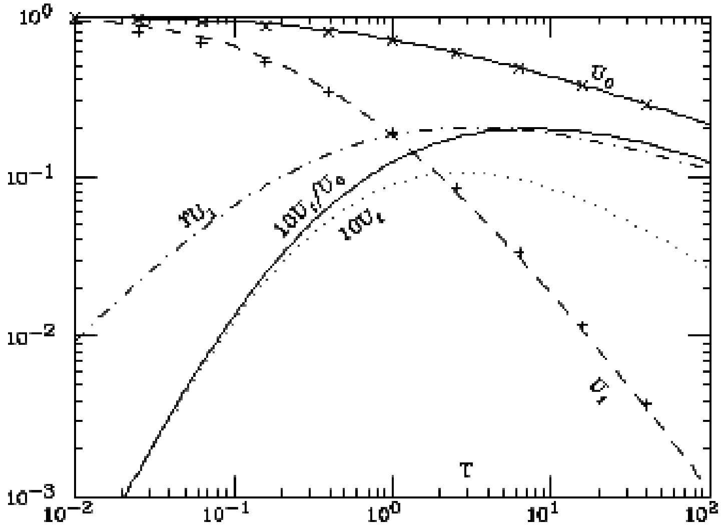
For details see: Yakoya & Chen
Beam-beam phenomena in linear colliders (1995)

$$\Upsilon \equiv \frac{2}{3}\frac{\hbar\omega_c}{E} = \frac{\lambda_e \gamma^2}{\rho} = \gamma\frac{B}{B_c}$$

$$\Upsilon_{av} \approx \frac{5}{6}\frac{N r_e^2 \gamma}{\alpha\sigma_z(\sigma_x+\sigma_y)}$$

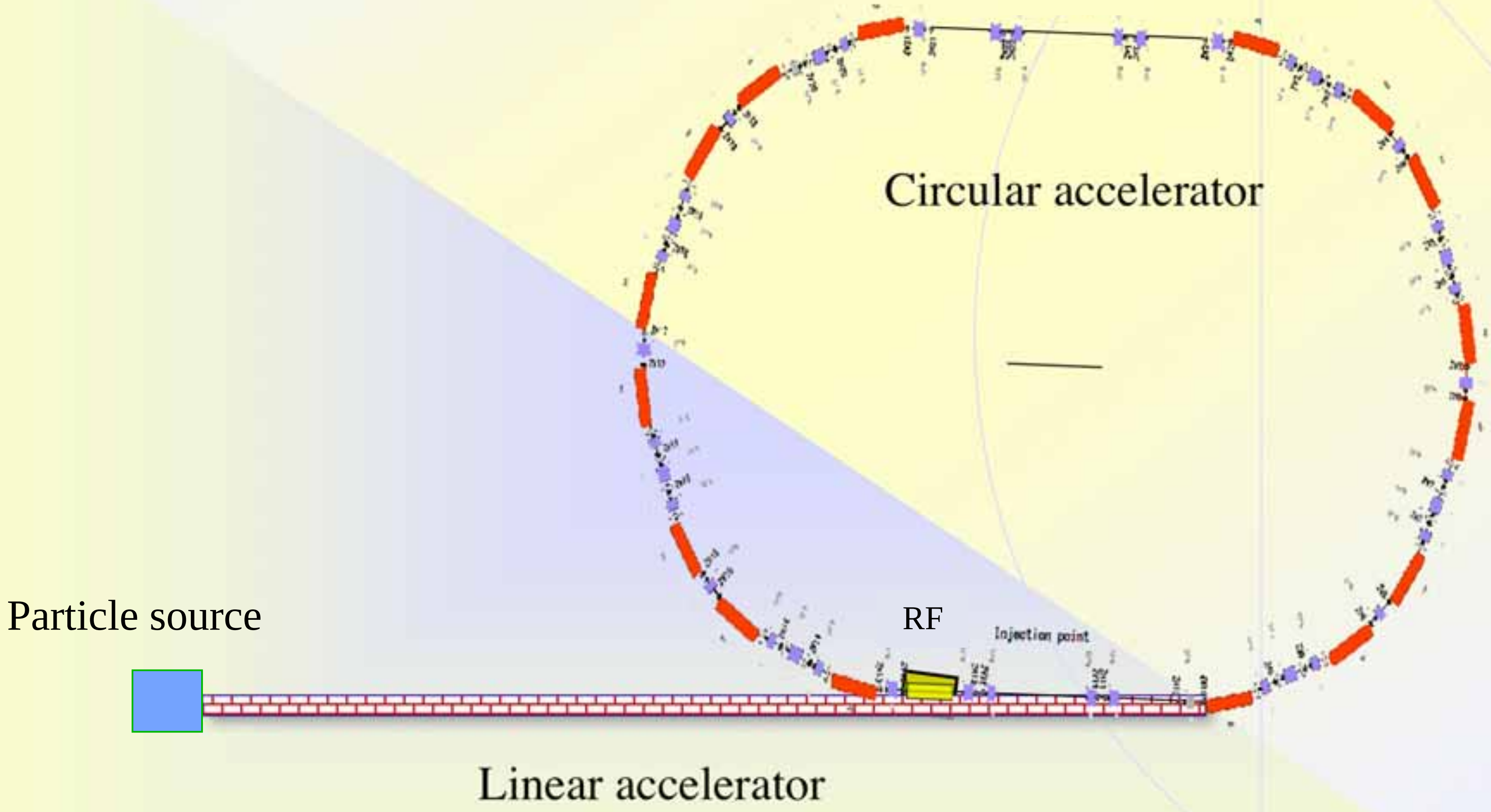
$$\delta_E = \left\langle -\frac{\Delta E}{E}\right\rangle \approx 0.209\frac{r_e^3 N^2 \gamma}{\sigma_z}\left(\frac{2}{\sigma_x+\sigma_y}\right)^2 U_1(\Upsilon_{av}) \approx 1.20\left[\frac{\alpha\sigma_z\Upsilon_{av}}{\lambda_e\gamma}\right]\Upsilon_{av}U_1(\Upsilon_{av})$$

Figure 12: Functions U_0, U_1 and U_γ. The crosses are the approximate formulas in Eqs.(3.71) and (3.73).

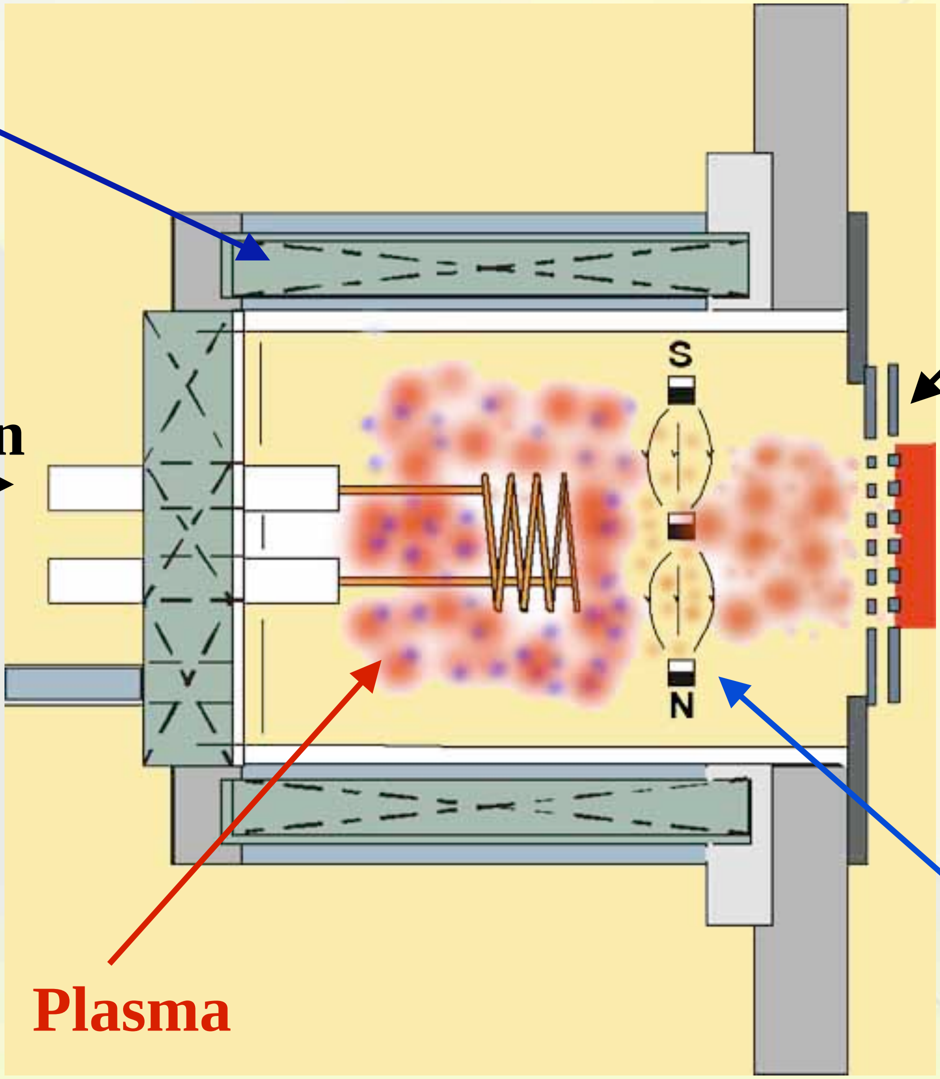


Accelerator Components

Generic accelerator facility



Anatomy of an ion source



Container for plasma

Extraction electrodes

Electrons in

Beam out

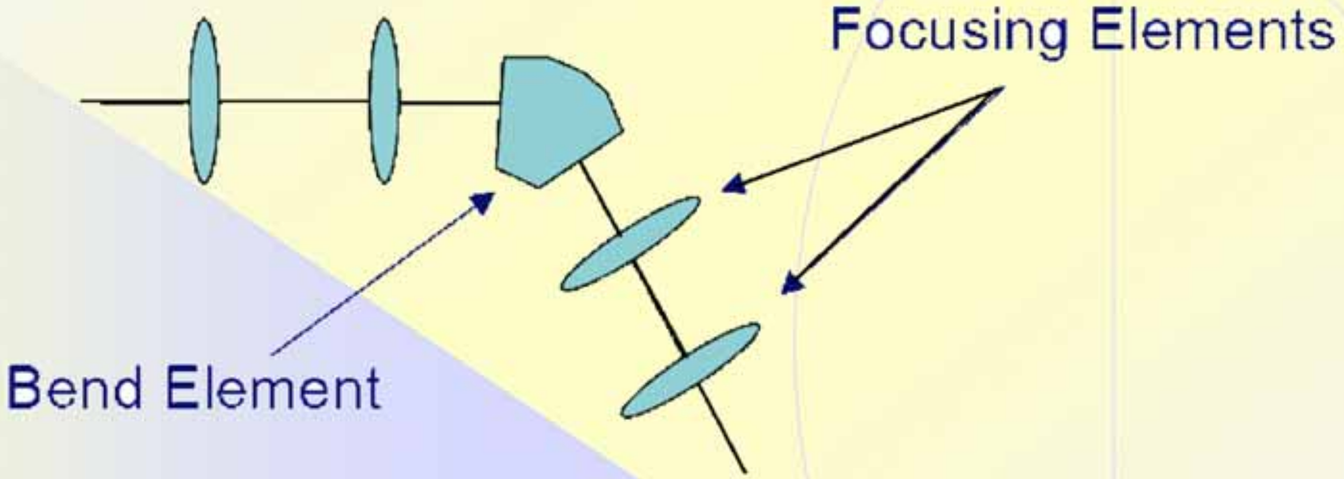
Gas in

Filter

Plasma

Accelerator components: Optics

- Optics (lattice): distribution of magnets to guide & focus beam



- Lattice design depends upon the goal & type of accelerator
  - Linac or synchrotron
  - High brightness: small spot size & small divergence
  - Physical constraints (building or tunnel)

Particle trajectories (orbits)

- Motion of each charged particle is determined by E & B forces that it encounters as it orbits the ring:
  - ➞Lorentz Force

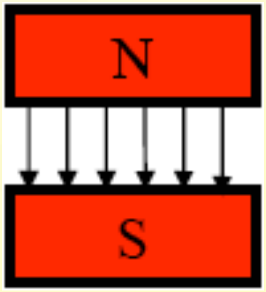
$$\mathbf{F} = m\mathbf{a} = e\,(\mathbf{E} + \mathbf{v}\times\mathbf{B}\,)$$

- Lattice design problems:
  - ➞ 1. Given an existing lattice, determine the beam properties
  - ➞ 2. For a desired set of beam properties, design the lattice.
- Problem 2 is not straight-forward – a bit of an art.

Types of magnets in accelerators:

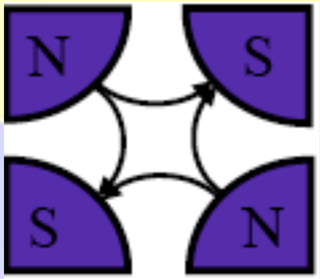
Dipoles:
Used for steering

$$B_x = 0$$
$$B_y = B_o$$



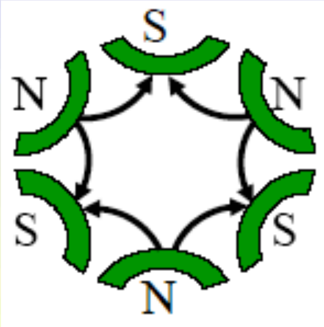
Quadrupoles:
Used for focusing

$$B_x = Ky$$
$$B_y = Kx$$

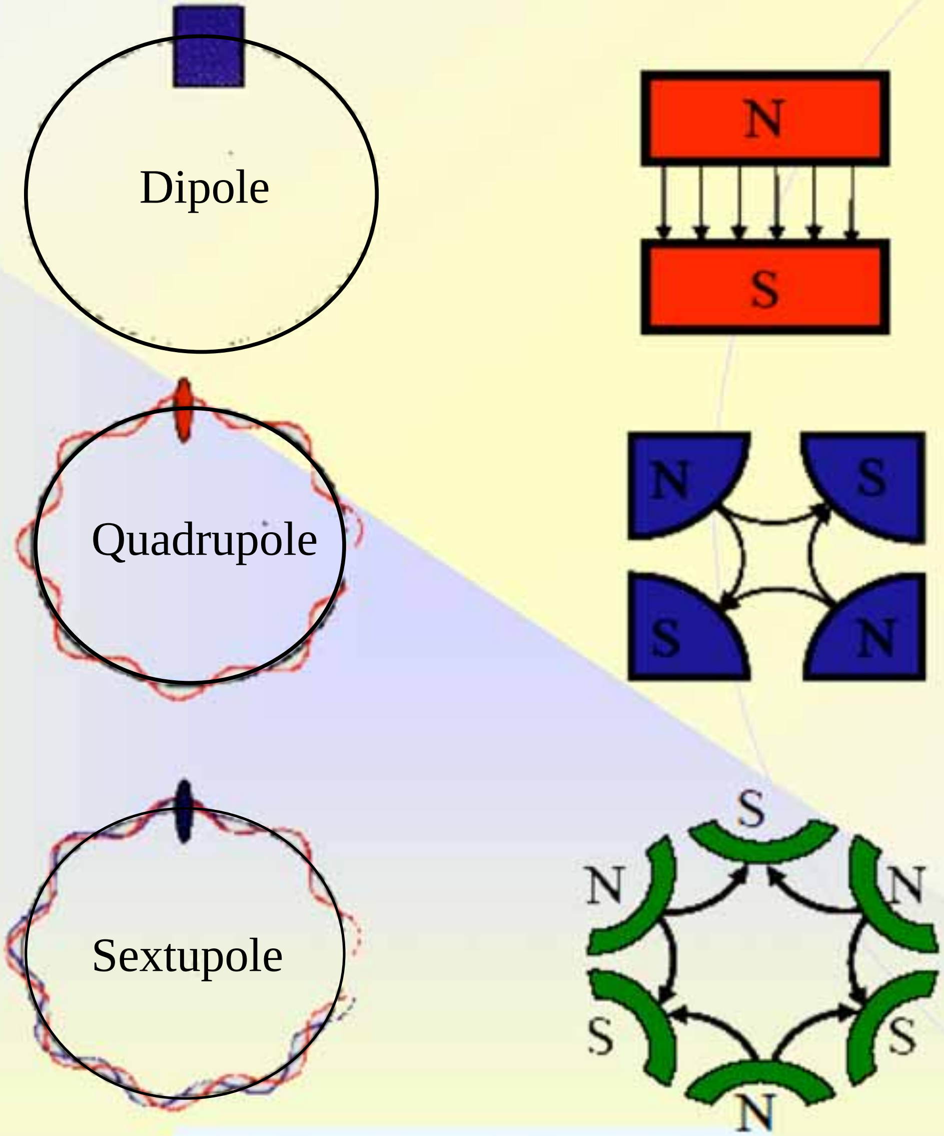


Sextupoles:
Used for chromatic correction

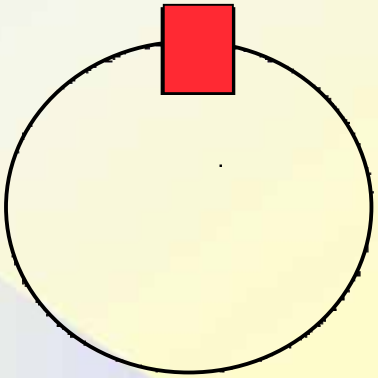
$$B_x = 2Sxy$$
$$B_y = S(x^2 - y^2)$$

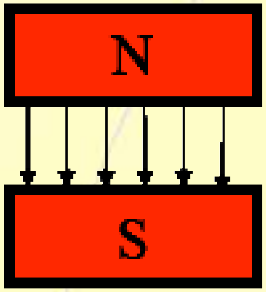


Function of magnets in a ring



Average dipole strength drives ring size & cost





In a ring for particles with energy E with N dipoles of length l, bend angle is

$$\theta = \frac{2\pi}{N}$$

The bending radius is $\rho = \frac{l}{\theta}$

The integrated dipole strength will be $Bl = \frac{2\pi}{N}\frac{\beta E}{e}$

The on-energy particle defines the central orbit: y = 0

Focusing the beam for its trip through the accelerator

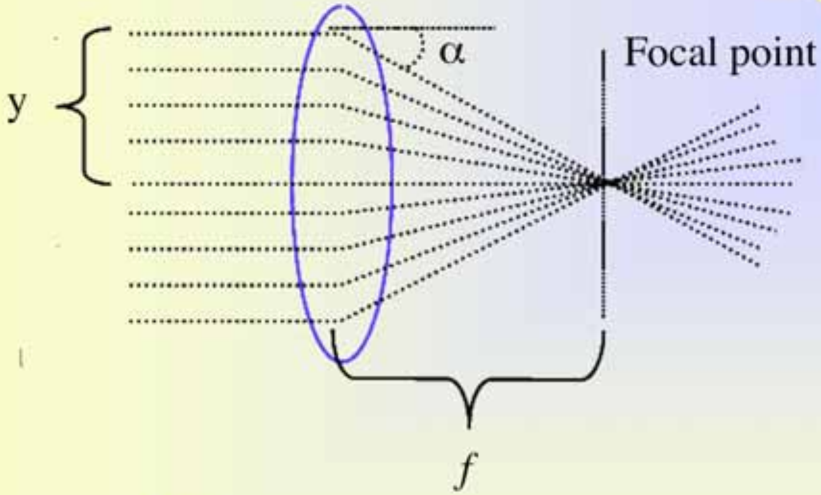
For a lens with focal length *f*, the deflection angle, $\alpha = -y/f$

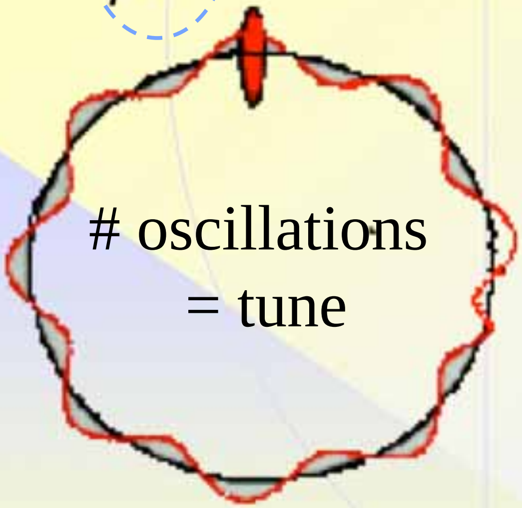
Then,

Quadupole with length l and with a gradient g ==> $B_x = gy$

$\therefore$

$$\alpha = -\frac{l}{f} = -\frac{e}{\beta E} B_x l = -\underbrace{\frac{e}{\beta E}}_{k} g\, yl$$





oscillations = tune

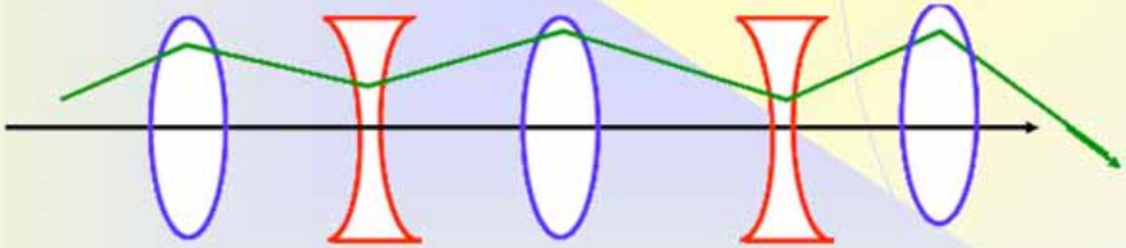
BUT, quadrupoles are focusing in one plane & defocusing in the other

Alternate gradient focusing

From optics we know that a combination of two lenses, with focal lengths f_1 and f_2 separated by a distance d, has

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

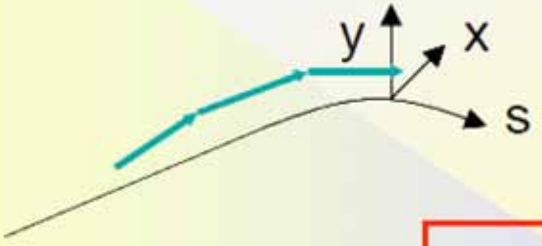
If $f_1 = -f_2$, the net effect is focusing!



N.B. This is only valid in thin lens approximation

Motion of particles in a storage ring

Use local cartesian coordinates



$$x, \quad x' = \frac{dx}{ds}, \quad y, \quad y' = \frac{dy}{ds}, \quad \delta = \frac{\Delta p}{p_0}, \tau = \frac{\Delta L}{L}$$

$$\boxed{\begin{aligned} x'' - \left(k(s) - \frac{1}{\rho(s)^2}\right) x &= \frac{1}{\rho(s)} \frac{\Delta P}{P} \\ y'' + k(s)\, y &= 0 \end{aligned}}$$

Hill's equations: Harmonic oscillator with time dependent frequency

$$k_\beta = 2\pi/\lambda_\beta = 1/\beta(s)$$

The term $1/\varrho^2$ corresponds to the dipole weak focusing

The term $\Delta P/(P\varrho)$ is present for off-momentum particles

Tune = # of betatron oscillations in one trip around the ring

Tune-shifts of operating point limits luminosity

- Space-charge forces at IP give impulsive kicks to beams
  - Tune shift

$$\xi = \frac{r_i}{4\pi}\frac{N\beta^*}{\gamma\sigma^2} = \frac{r_i}{4\pi}\frac{N}{\gamma\varepsilon}$$

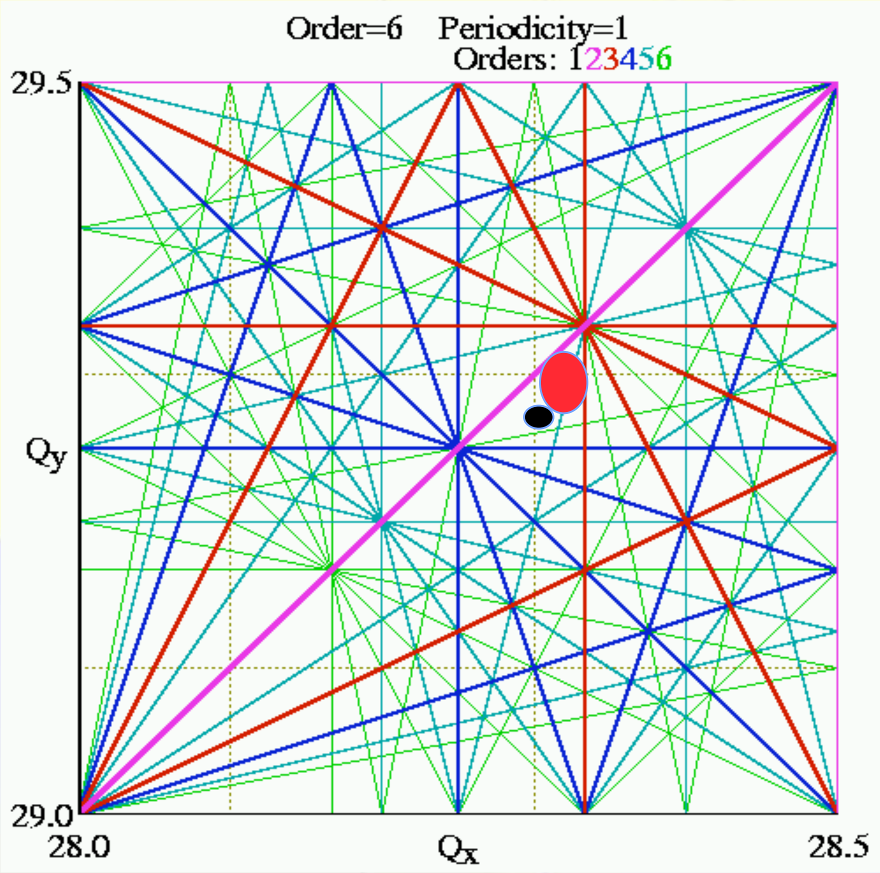
- For gaussian bunches, space charge forces are non-linear
  - Tune spread
- Tunes where

$$m\nu_x + n\nu_y = M$$

resonantly kick the beams

  - Unstable particle motion

==> **Limits beam intensity**

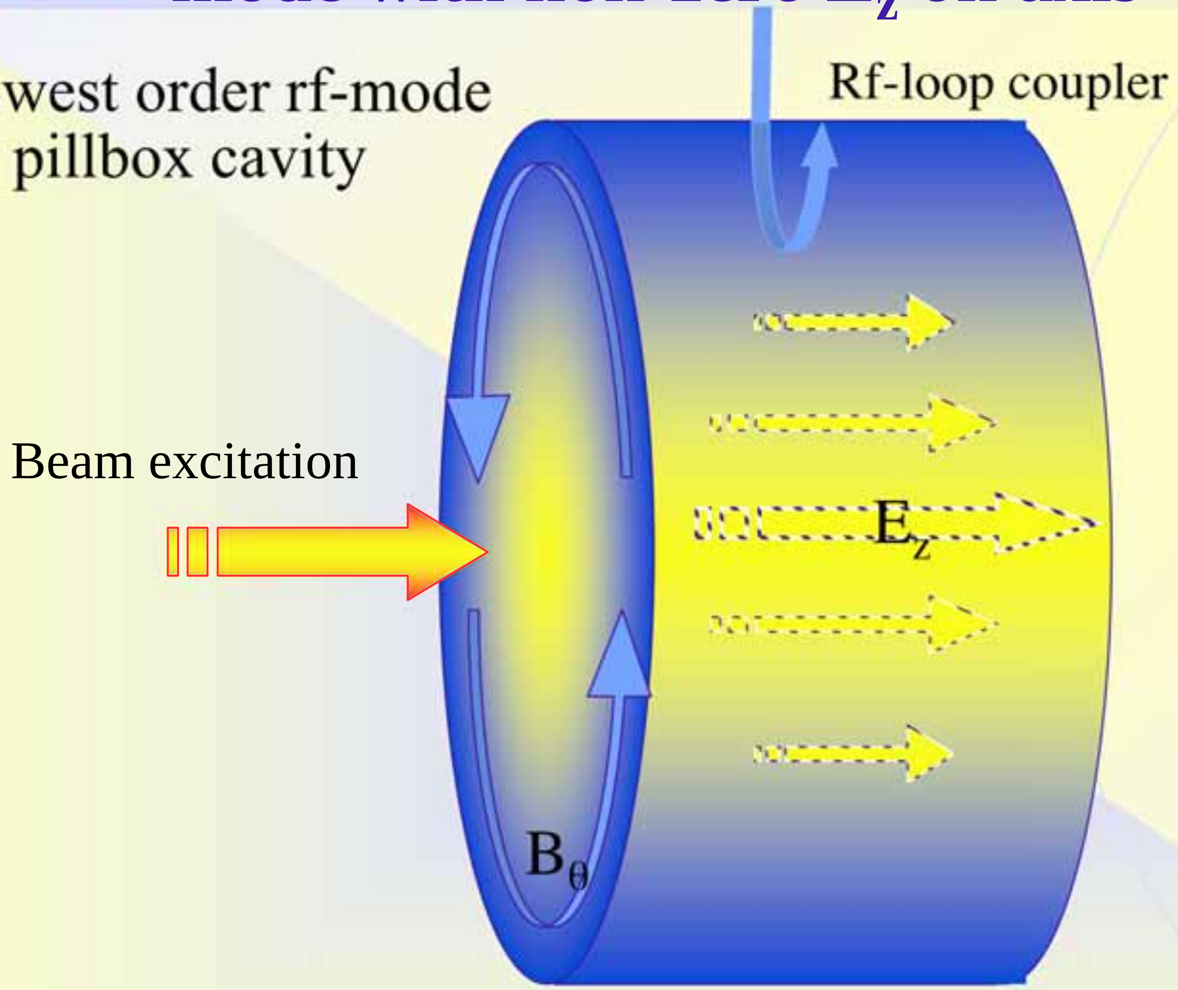


● No collisions

🔴 Beams in collision

Acceleration with rf-fields requires mode with non-zero E_z on axis

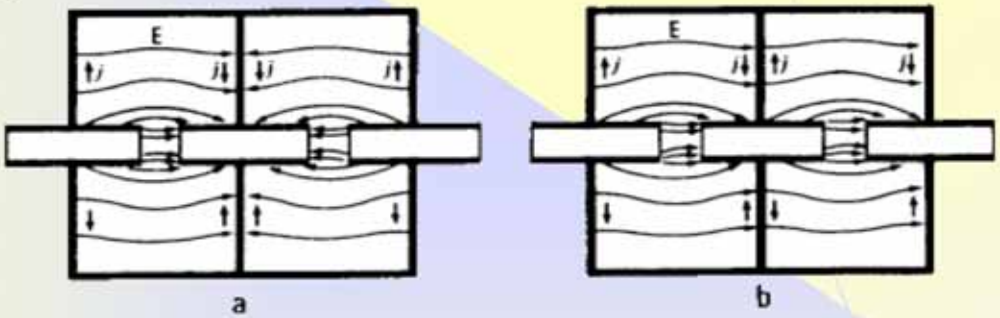
Lowest order rf-mode
in pillbox cavity



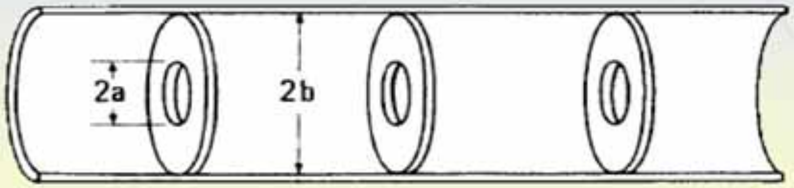
Higher order modes have non-zero E_r on axis

Conditions on rf-cavities

- In ring: $V_{rf} > U_o$ (energy loss/turn);
  - → rf must be synchronized to beam revolution frequency
  - → Provides longitudinal focusing (confinement)
- In linac: Match v_{ph} of rf field to v_z of beam
- Standing wave structures:

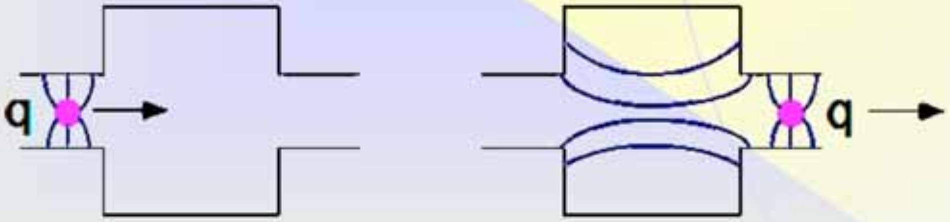


- Traveling wave structures:



Beam structure interaction links machine characteristics with beam parameters

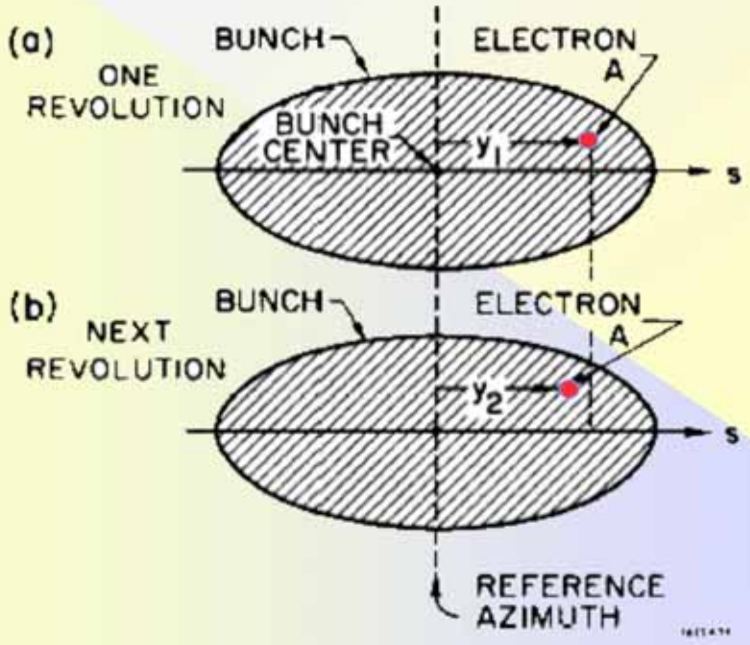
- Transverse wakefields (Volts/m/pC)
  - → Exponentially growing deflecting modes (scales as a^{-3})
  - → Driven by beam current ($\sim I_{av} I_{peak}$)
    ==> strong focusing, cavity tailoring
  - → Bunch-to-bunch coupling ==> multi-bunch instabilities
- Longitudinal wakefields & beam-loading
  - → Beam removes energy stored in cavity → Energy spread
  - → Chromatic effects in focusing lattice

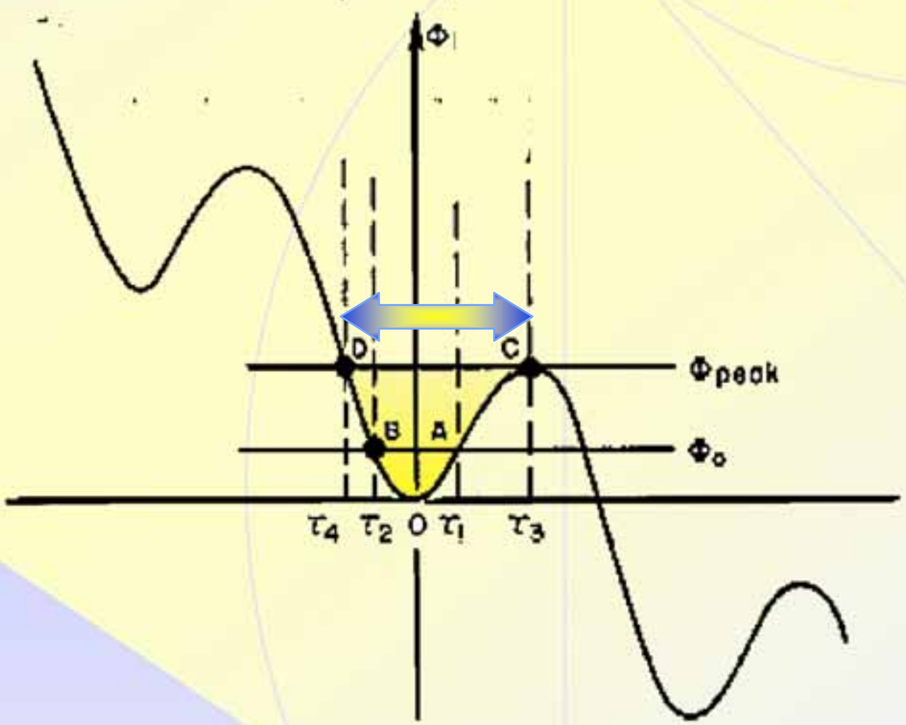


- Fourier transform of the wakefield = impedance $Z(\omega)$

==> Limits I_{beam} & grows emittance ==> ***Limits Luminosity***

RF-bucket: range of phases with bounded energy orbits





Remember: for $v_z \sim c$, the greater the energy, the greater the revolution time ==> particles in bucket oscillate in phase & energy

Size of bucket depends on chromatic properties (energy acceptance) of the lattice

For more details: Attend the USPAS

http://uspas.fnal.gov/

Scholarship support available to for-credit students