Introduction to Collider Physics

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The Very Big Picture

History of the Universe

Accelerators

Inflation

Big Bang

Key:
- $w, z$ bosons
- $\gamma$ photon
- $q$ quark
- $g$ gluon
- $e$ electron
- $\mu$ muon
- $\tau$ tau
- $n$ neutrino

Particle Data Group, LBNL, © 2000. Supported by DOE and NSF
Figure of Merit 1: Accelerator energy
===> energy frontier of discovery

The Energy Frontier
(Discoveries)

Hadron colliders

Constituent Center of Mass Energy (GeV)

Year of First Physics

“Livingston plot”
Physics figure of Merit 2: Number of events

\[ \text{Events} = \text{Cross-section} \times \langle \text{Collision Rate} \rangle \times \text{Time} \]

Beam energy: sets scale of physics accessible

\[
\text{Luminosity} = \frac{N_1 \times N_2 \times \text{frequency}}{\text{Overlap Area}} = \frac{N_1 \times N_2 \times f}{4\pi\sigma_x\sigma_y} \times \text{Disruption} \times \alpha \text{ correction}
\]

We want large charge/bunch, high collision frequency & small spot size

Note: L changes in time as collisions deplete the bunches

===> Luminosity lifetime
Discovery space for future accelerators

Luminosity = \frac{\text{Energy} \times \text{Current}}{\text{Focal depth} \times \text{Beam quality}}
FoM 3: Resolution (Energy/\Delta E)

* Intertwined with detector & experiment design
  ➞ In hadron colliders: production change, parton energy distribution
  ➞ In lepton colliders: energy spread of beams (synchrotron radiation)
Characteristics of Beams that Matter to Particle Physics
**Beams: Bunches of particles with a directed velocity**

- Ions - either missing electrons (+) or with extra electrons (-)
- Electrons or positrons
- Plasma - ions plus electrons

- Beams particles have random (thermal) \( \perp \) motion
- Beam must be confined against thermal expansion during transport
- Beam’s have internal (self-forces)
Beam transverse self-fields

\[ E_{sp} (V/cm) = \frac{60 \cdot I_{beam} (A)}{R_{beam} (cm)} \]

\[ B_{\theta} (gauss) = \frac{I_{beam} (A)}{5 \cdot R_{beam} (cm)} \]

In vacuum:

Beam’s transverse self-force scale as \( 1/\gamma^2 \)

- Space charge repulsion: \( E_{sp,\perp} \sim N_{beam} \)
- Pinch field: \( B_{\theta} \sim I_{beam} \sim v_z N_{beam} \sim v_z E_{sp} \)

\[ F_{sp,\perp} = q \left( E_{sp,\perp} + v_z \times B_{\theta} \right) \sim (1-v^2) N_{beam} \sim N_{beam}/\gamma^2 \]

Beams in collision are *not* in vacuum (beam-beam effect)
Envelop Equation: Evolution of beam during transport

Write equation of motion for each particle
\[ \mathbf{F}_i = m \mathbf{a}_i = e (\mathbf{E} + \mathbf{v}_i \times \mathbf{B}) \]

Energy equation for each particle
\[ E_i = \gamma_i mc^2 \]

Assume \( v_\perp \ll v_z \)

Take averages over the distribution of particles

Apply Virial Theorem: K.E = -P.E./2

\[ R'' + \frac{1}{\gamma} \frac{\mathbf{U}_{self}}{R} + \frac{\langle k_\beta^2 \mathbf{r}^2 \rangle}{R^3} - \frac{\varepsilon^2}{R^3} = \frac{1}{R^3} (\varepsilon^2)' \]

\[ \varepsilon^2 = R^2(V^2 - (R')^2)/c^2 \]
Emittance describes the area in phase space of the ensemble of beam particles.

Emittance - Phase space volume of beam

\[ \varepsilon^2 = R^2 \left( V^2 - (R')^2 \right) / c^2 \]

RMS emittance

Phase space of an harmonic oscillator

\[ k_p(x) \text{ - frequency of rotation of a phase volume} \]
Why is emittance an important concept

1) Liouville: Under conservative forces phase space evolves like an incompressible fluid

\[
Z = \frac{\lambda}{12}
\]

\[
Z = \frac{\lambda}{8}
\]

\[
Z = \frac{\lambda}{4}
\]

\[
Z = 0
\]

2) Under linear forces macroscopic emittance is preserved

3) Under acceleration $\gamma \varepsilon = \varepsilon_n$ is an adiabatic invariant
Nonlinear applied & space-charge fields lead to filamentation of phase space

Data from CERN PS

Macroscopic (rms) emittance is not conserved
Damping of coherent beam displacement by frequency spread in non-linear elements

![Graph showing damping of coherent beam displacement by frequency spread in non-linear elements](image)

- Relative force vs. displacement for different orders of non-linear elements:
  - Dipole
  - Quadrupole
  - Sextupole
  - Octupole

Graphical representation of the relationship between the relative force \( R/R_0 \) and the displacement \( x \) for different orders of non-linear elements. The lines indicate how the force increases with displacement for each order, with higher orders showing more significant increases.
What sets spot size?

※ Strength (depth of focus) of lens at interaction point, $\beta^*$
※ Distribution of positions & transverse momenta of beam particles (emittance), $\epsilon$

$$\sigma_{x,y} = \sqrt{\epsilon_{x,y} \beta^*}$$

Luminosity $$= \frac{N_1 \times N_2 \times f}{4\pi \sqrt{\beta_x^* \beta_y^* \epsilon_x \epsilon_y}} \times \text{Pinch effect} \times \text{angle correction}$$

※ For simplicity say $\epsilon$ and $\beta^*$ are equal for $x$ and $y$

Luminosity $$= \frac{N_1 \times N_2 \times f}{4\pi \beta_x^* \epsilon_x} \times H_D \times \text{angle correction}$$
How can we maximize luminosity?

- If $\beta^* < \text{bunch length, } \sigma_z$, bunch has hour glass shape
  - Correction factor lowers luminosity

- Raise N: instability limits
- Raise f: Detector issues
- Can we change the emittance?
  - $\Rightarrow$ Non-conservative forces acting on beams
  - $\Rightarrow$ Beam cooling
Emittance increase from non-conservative forces (scattering) depends on beam size.

Oops, we got the sign wrong! We wanted to **cool** the beam.

*So, remove energy from particles with largest $v_{\perp}$*
Synchrotron radiation (for electrons)

\[ U_o = \frac{4\pi r_p m_p c^2}{3} \frac{\gamma^4}{\rho} = 6.03 \times 10^{-18} \frac{\gamma^4}{\rho} \text{ GeV} \]

\[ N_\gamma \sim 4\pi\alpha \text{ per turn} \]

\[ P_{sr} = 26.5 \text{ kW} \quad E_{GeV}^3 I_A B_T \text{ (for electrons)} \]

\[ E_{crit}(keV) = 0.66 E_{GeV}^2 B_T \]

Implications:
- Power deposition on vacuum chamber
- Beam-beam focusing at IP
- Damping of coherent beam properties
- Energy damping - \( U \sim \gamma^4 \)

Equilibrium energy spread in ring

\[ \Delta E \approx \sqrt{E_{beam} E_{crit}} \]

Radiation damping time \( \sim \frac{E_{beam}}{U_o} \)
Effects of synchrotron radiation: Cooling the particles in rings

- Particles radiate (loose energy) along instantaneous velocity.
- Energy is added along beam axis.
- E-folding emittance requires replacing beam energy
  \[
  \frac{d \epsilon}{\epsilon} = \frac{d p_\perp}{p_\perp} = \frac{d p_\parallel}{p_\parallel}
  \]
- Rate is independent of number of particles per bunch
- Radiation cooling of emittance limited by quantum nature of radiation
- Quantum spread of radiation energy increases longitudinal emittance

BUT, what if radiation is negligible?
Van der Meer’s demon

- Stochastic cooling: Measures fluctuations in beam distribution

\[
\frac{1}{\tau_{x,\text{max}}} = \frac{f}{2N_s}
\]

- The smaller the number of particles in the sample, & the higher the revolution frequency, the higher the cooling rate.
- For example, 1 GHz of bandwidth cools \(10^9\) particles at 1Hz.

*Good for anti-protons; too slow for muons*
Basic components: Ionization cooling of muons

- Radiation cooling & ionization cooling are analogous
- Strong lens to focus the beam
- Ionization medium in which the particles lose both transverse & longitudinal momentum,
- Accelerating structure to restore longitudinal momentum
- Rate is independent of number of particles per bunch
- Ionization cooling of transverse emittance limited by beam heating due to multiple Coulomb scattering
- Straggling increases longitudinal emittance $\frac{dE}{E} \approx 1.35 \times 10^{-3} \sqrt{E} \text{ (MeV)}$
Beamstrahlung lowers energy resolution in linear collider

At IP space charge cancels; currents add

$$\Rightarrow$$ strong beam-beam focus

- Luminosity enhancement
- Strong synchrotron radiation

Consider 250 GeV beams with 1 kA focused to 100 nm

$$B_{\text{peak}} \sim 40 \text{ Mgauss} \Rightarrow E_{\text{crit}} \sim 166 \text{ GeV}$$

Quantum effects & radiation reaction must suppress synchrotron radiation

A correct calculation yields

$$\delta E/E \sim 4\%$$
For details see: Yakoya & Chen
Beam-beam phenomena in linear colliders (1995)

\[ \gamma = \frac{2 \hbar \omega_c}{3 E} = \frac{\lambda_e \gamma^2}{\rho} = \frac{B}{B_c} \]

\[ \gamma_{av} \approx \frac{5}{6} \frac{N r_e^2 \gamma}{\alpha \sigma_z (\sigma_x + \sigma_y)} \]

\[ \delta_E = \left\langle \frac{-\Delta E}{E} \right\rangle \approx 0.209 \frac{r_e^2 N^2 \gamma}{\sigma_z} \left( \frac{2}{\sigma_x + \sigma_y} \right)^2 U_1(\gamma_{av}) \approx 1.20 \left[ \frac{\alpha \sigma_z \gamma_{av}}{\lambda_e \gamma} \right] \gamma_{av} U_1(\gamma_{av}) \]

Figure 12: Functions \( U_0 \), \( U_1 \) and \( U_f \). The crosses are the approximate formulas in Eqs.(3.71) and (3.73).
Accelerator Components
Generic accelerator facility

Circular accelerator

Linear accelerator

Particle source

RF
Anatomy of an ion source

- Gas in
- Electrons in
- Container for plasma
- Plasma
- Extraction electrodes
- Filter
- Beam out
Accelerator components: Optics

- Optics (lattice): distribution of magnets to guide & focus beam

- Lattice design depends upon the goal & type of accelerator
  - Linac or synchrotron
  - High brightness: small spot size & small divergence
  - Physical constraints (building or tunnel)
Motion of each charged particle is determined by E & B forces that it encounters as it orbits the ring:

→ Lorentz Force

\[ \mathbf{F} = m \mathbf{a} = e (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \]

Lattice design problems:

→ 1. Given an existing lattice, determine the beam properties
→ 2. For a desired set of beam properties, design the lattice.

Problem 2 is not straight-forward – a bit of an art.
Types of magnets in accelerators:

Dipoles:
Used for steering
\[ B_x = 0 \]
\[ B_y = B_0 \]

Quadrupoles:
Used for focusing
\[ B_x = K_y \]
\[ B_y = K_x \]

Sextupoles:
Used for chromatic correction
\[ B_x = 2S_{xy} \]
\[ B_y = S(x^2 - y^2) \]
Function of magnets in a ring

- Dipole
- Quadrupole
- Sextupole
In a ring for particles with energy $E$ with $N$ dipoles of length $l$, bend angle is

$$\theta = \frac{2\pi}{N}$$

The bending radius is

$$\rho = \frac{l}{\theta}$$

The integrated dipole strength will be

$$Bl = \frac{2\pi}{N} \frac{\beta E}{e}$$

The on-energy particle defines the central orbit: $y = 0$
Focusing the beam for its trip through the accelerator

For a lens with focal length \( f \), the deflection angle, \( \alpha = -\frac{y}{f} \)

Then,

Quadupole with length \( l \) and with a gradient \( g \) ==> \( B_x = gy \)

\[ \alpha = -\frac{l}{f} = -\frac{e}{\beta E} B_x l = -\frac{e}{\beta E} g yl \]

BUT, quadrupoles are focusing in one plane & defocusing in the other

# oscillations = tune
Alternate gradient focusing

From optics we know that a combination of two lenses, with focal lengths $f_1$ and $f_2$ separated by a distance $d$, has

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

If $f_1 = -f_2$, the net effect is focusing!

N.B. This is only valid in thin lens approximation
Motion of particles in a storage ring

Use local cartesian coordinates

\[ x, \quad x' = \frac{dx}{ds}, \quad y, \quad y' = \frac{dy}{ds}, \quad \delta = \frac{\Delta p}{p_0}, \tau = \frac{\Delta L}{L} \]

Hill’s equations: Harmonic oscillator with time dependent frequency

\[
x'' - \left( k(s) - \frac{1}{\rho(s)^2} \right) x = \frac{1}{\rho(s)} \frac{\Delta P}{P} \\
y''' + k(s) y = 0
\]

Hill’s equations: Harmonic oscillator with time dependent frequency

\[ k_\beta = \frac{2\pi}{\lambda_\beta} = \frac{1}{\beta(s)} \]

The term \( 1/\rho^2 \) corresponds to the dipole weak focusing

The term \( \Delta P/(P\rho) \) is present for off-momentum particles

\[ \text{Tune} = \# \text{ of betatron oscillations in one trip around the ring} \]
Space-charge forces at IP give impulsive kicks to beams

- Tune shift
  \[ \xi = \frac{r_i N \beta^*}{4\pi \gamma \sigma^2} = \frac{r_i N}{4\pi \gamma \epsilon} \]

- For gaussian bunches, space charge forces are non-linear
  - Tune spread

- Tunes where
  \[ m v_x + n v_y = M \]
  resonantly kick the beams
  - Unstable particle motion

\[ \Rightarrow \text{Limits beam intensity} \]
Acceleration with rf-fields requires mode with non-zero $E_z$ on axis

Lowest order rf-mode in pillbox cavity

Rf-loop coupler

Beam excitation

Higher order modes have non-zero $E_r$ on axis
Conditions on rf-cavities

* In ring: $V_{rf} > U_o$ (energy loss/turn);
  → rf must be synchronized to beam revolution frequency
  → Provides longitudinal focusing (confinement)

* In linac: Match $v_{ph}$ of rf field to $v_z$ of beam

* Standing wave structures:

* Traveling wave structures:
Beam structure interaction links machine characteristics with beam parameters

- Transverse wakefields (Volts/m/pC)
  - Exponentially growing deflecting modes (scales as a^{-3})
  - Driven by beam current (∼I_{av}I_{peak})
    - ==> strong focusing, cavity tailoring
  - Bunch-to-bunch coupling ==> multi-bunch instabilities

- Longitudinal wakefields & beam-loading
  - Beam removes energy stored in cavity ➔ Energy spread
  - Chromatic effects in focusing lattice

- Fourier transform of the wakefield = impedance Z(ω)
  ➔ Limits I_{beam} & grows emittance ➔ Limits Luminosity
RF-bucket: range of phases with bounded energy orbits

Remember: for $v_z \sim c$, the greater the energy, the greater the revolution time

$\Rightarrow$ particles in bucket oscillate in phase & energy

Size of bucket depends on chromatic properties (energy acceptance) of the lattice
For more details: Attend the USPAS

http://uspas.fnal.gov/

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