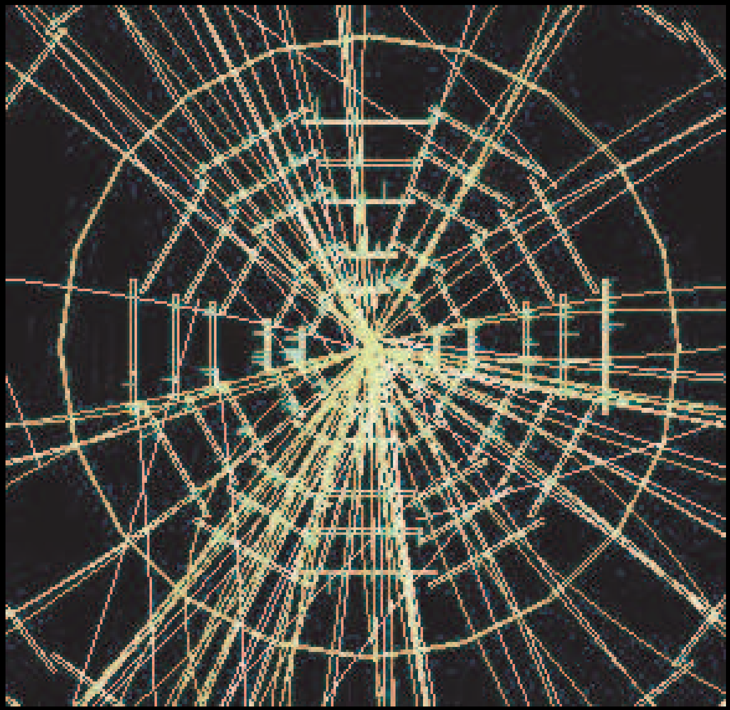
From Hits to Four-Vectors



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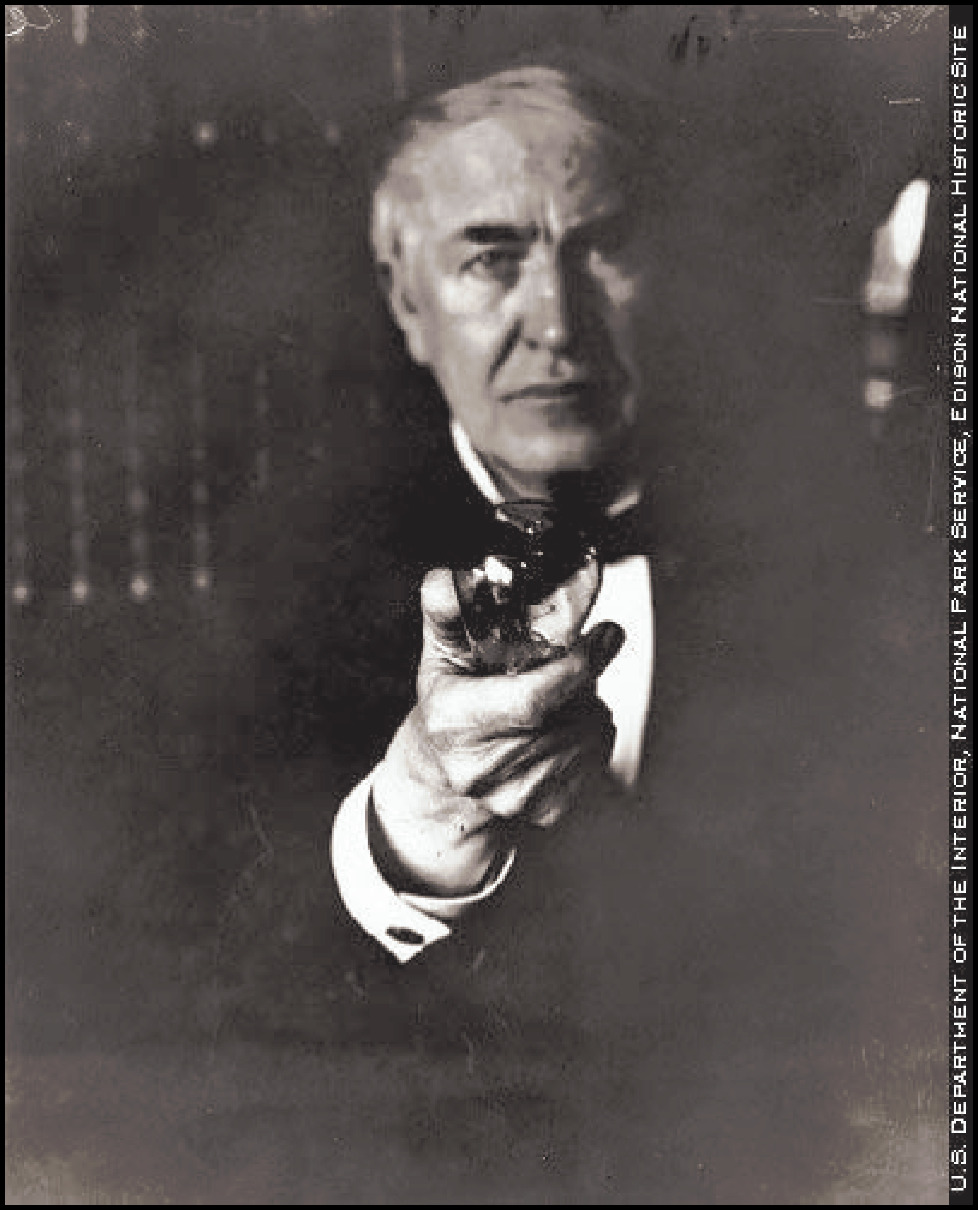

Thomas A. Edison

Outline

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- Detection
- Pattern Recognition
- Parameter Estimation
- Resolution
- Systematics

Learning More

- The CERN Briefbooks
  - http://rkb.home.cern.ch/rkb/titleA.html
  - http://rkb.home.cern.ch/rkb/titleB.html
- Hal Evan's Tracking Bibliography
  - http://www.nevis.columbia.edu/~evans/stt/bibliography.htm
- Silicon Overview
  - Damerell, ASI Lectures 1995
  - Schwarz & Lutz, Ann Rev Nuc Sci 38 (1995)
  - http://www-physics.lbl.gov/~spieler/
- Track pattern recognition, fitting, and systematics
  - Fruehwirth et al, Data Analysis Techniques for High-Energy Physics
  - http://www.phys.ufl.edu/~avery/fitting.html

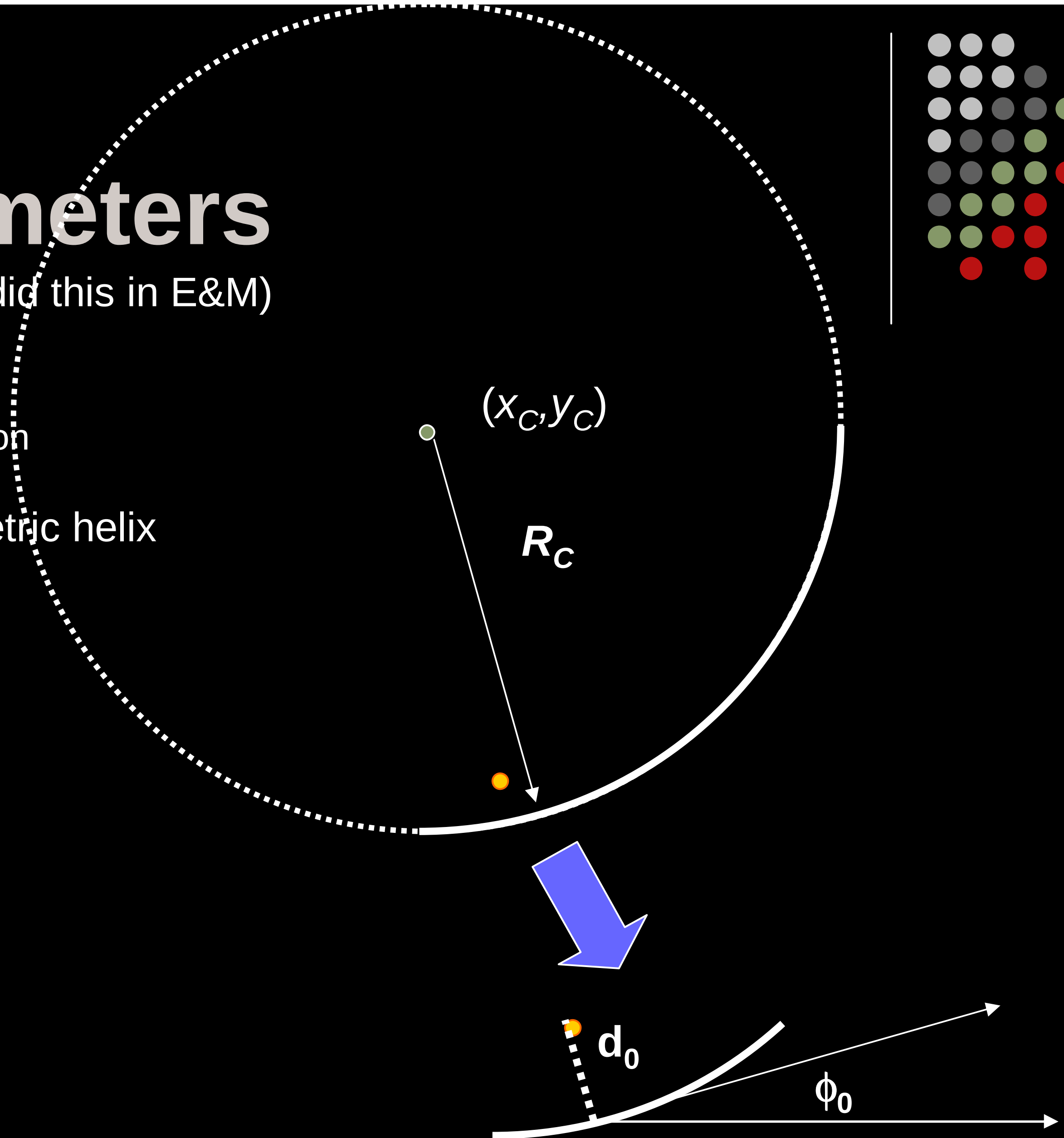
Track Parameters

- Tracks curl in B field (you all did this in E&M)

- Describe geometric helix
  - Axis of helix=B-field~z direction

- 5 parameters describe geometric helix
  - Referred to r-ϕ PCAO
  - d_0
  - ϕ_0
  - $C=1/2R_c$ or $1/R_c$
    - *Check in your exp't*
  - Z_0
  - Cotθ

- Bits are cheap
  - x_0, y_0
  - c_x, c_y, c_z
  - x_C, y_C, Rc

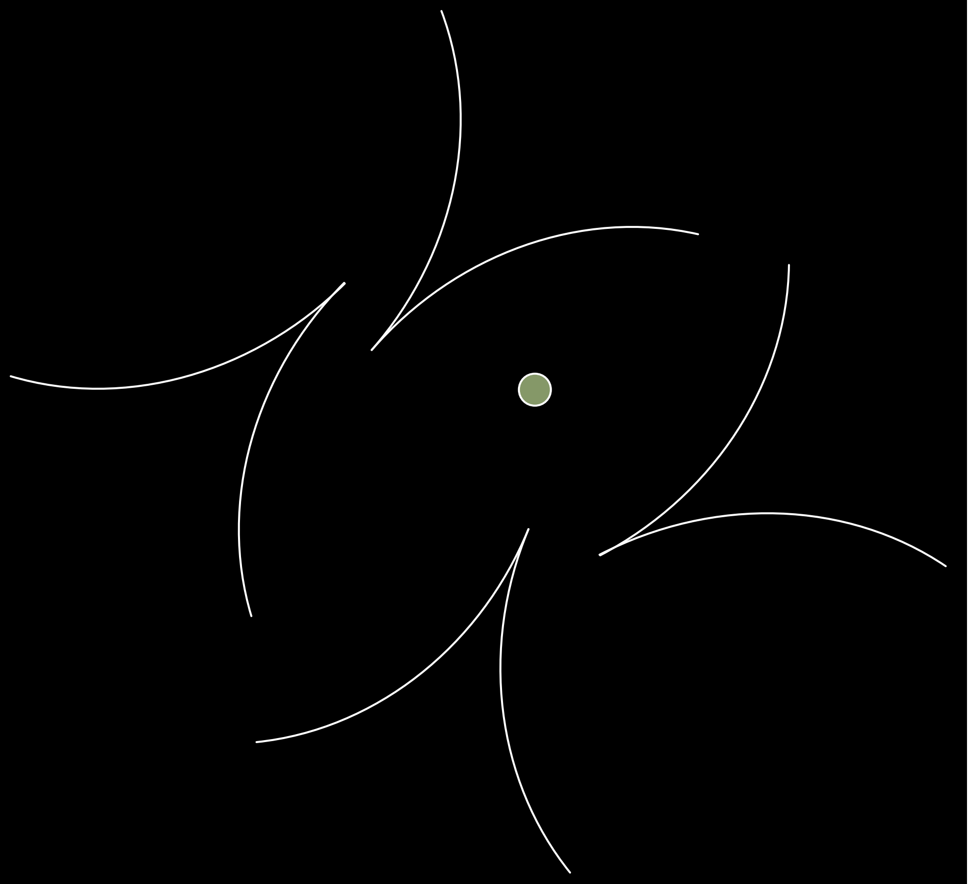


Track Parameters, z view

- s-z view gives straight lines
- For small C, s~r
  - r-z usually used for recognition
    - Look for lines
    - Line fitters fast, fewer hits -> brute force
- Z0
  - Z position at PCAO in r-f plane
    - May NOT be 3-D PCAO!!!
- Cotq (aka l)
  - Slope of helix pitch

Sign Convention

- d_0 sign convention varies experiment to experiment
- USUALLY it is defined as follows, up to a possible overall sign:
  - Sign of d_0 is same as sign of charge, if the coordinate origin is outside the helix circle; opposite otherwise
  - Equivalent definition: rotate track to go outwards along x axis; sign of d_0 is sign of y intercept



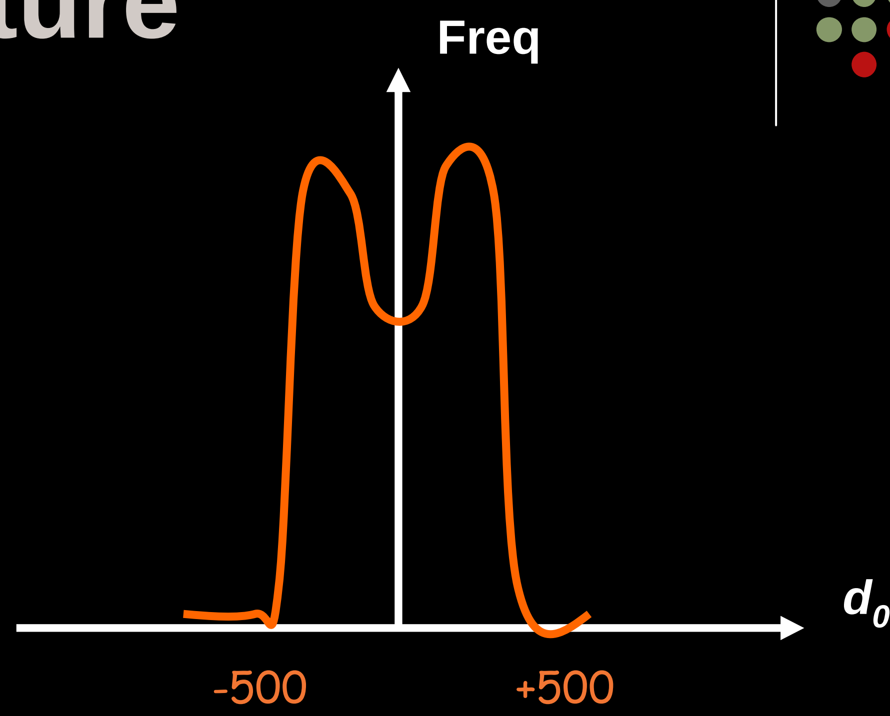
Physics Parameters

- Related to helix parameters
- Best to keep separate
  - Esp. in nonhomogenous fields

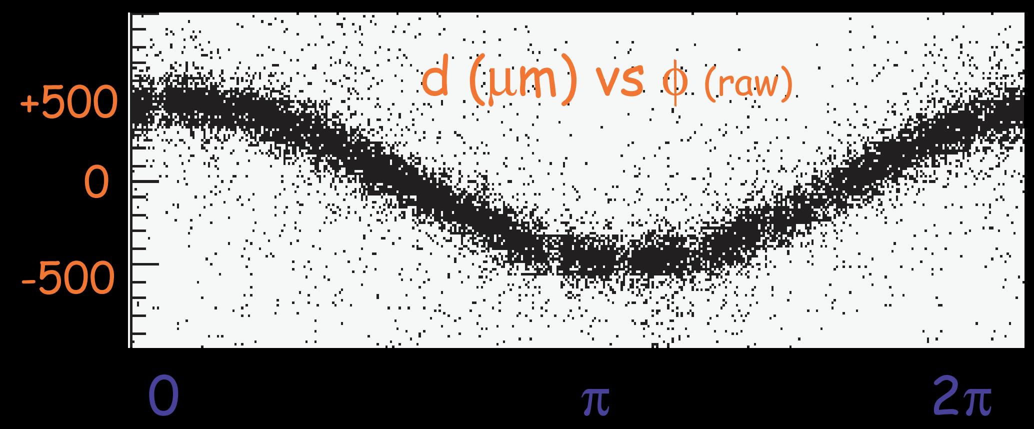
- Cartesian
  - x_0, y_0, z_0
  - p_x, p_y, p_z
- "Theory"
  - p, p_T, η, $\cos\theta$
  - 4 vectors
    - $$\left(\sqrt{\left(\frac{cB}{C}\right)^2 + m^2}, \left(\frac{cB}{C}\right)\cos\phi_0, \left(\frac{cB}{C}\right)\sin\phi_0, \left(\frac{cB}{C}\right)(1 + \cot^2\lambda)\right)$$
    - $$(0, -d_0\sin\phi_0, d_0\cos\phi_0, z_0)$$

A Common Feature

- If your d_0 distribution looks like this

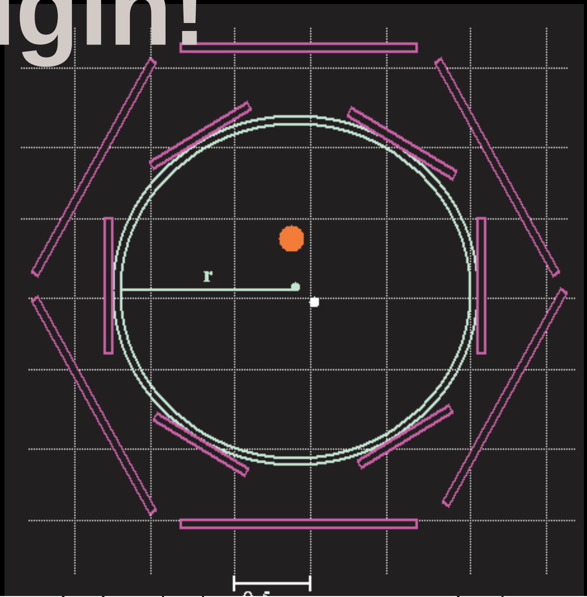


- Plot d_0 vs. ϕ_0 to see this

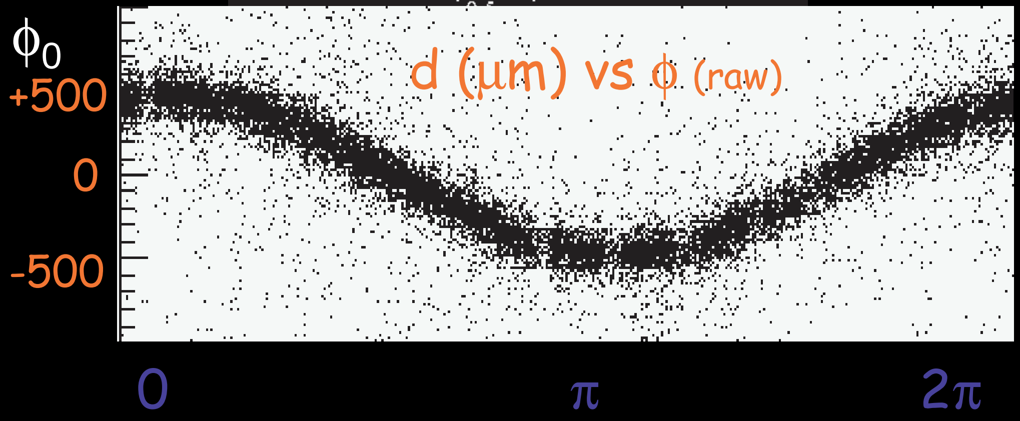


Tracks not from Origin!

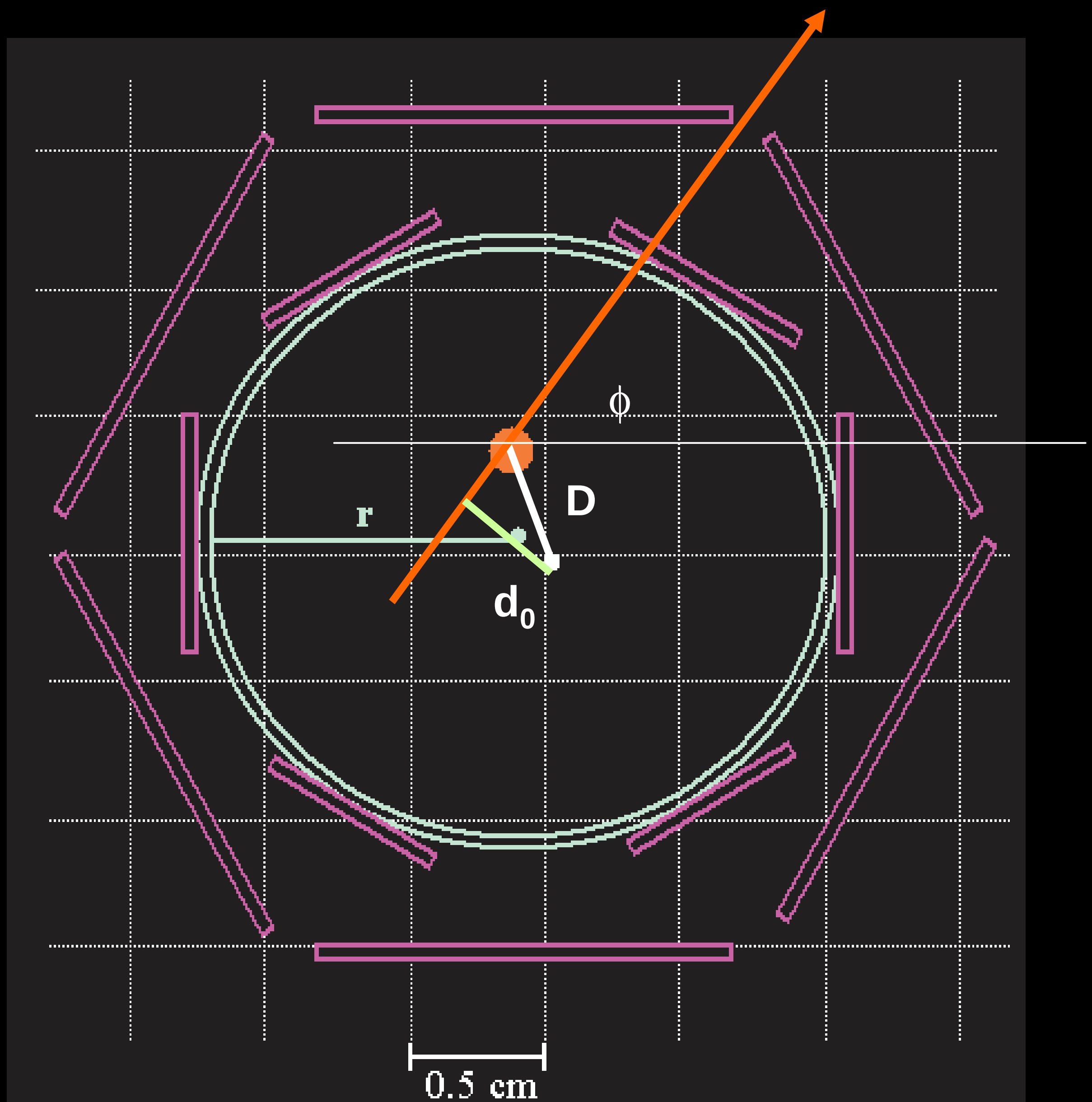
- Offset Beamspot



- Magnitude, Phase of d_0 vs ϕ_0 meaningful
  - $(x_b, y_b) = (M\cos\beta, M\sin\beta)$



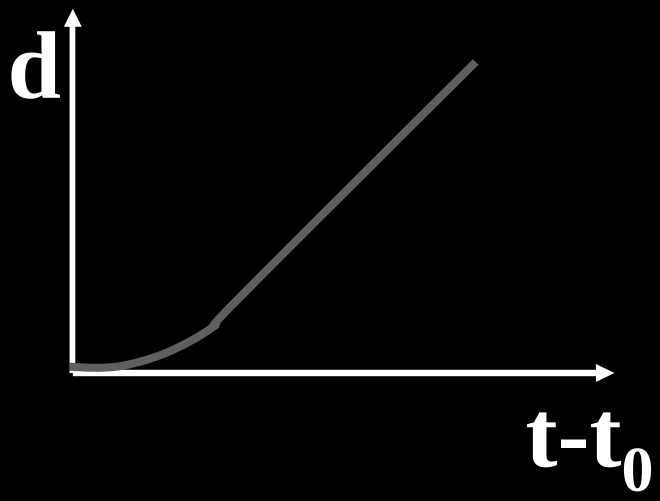
- Coordinate origin usually given by wire chamber

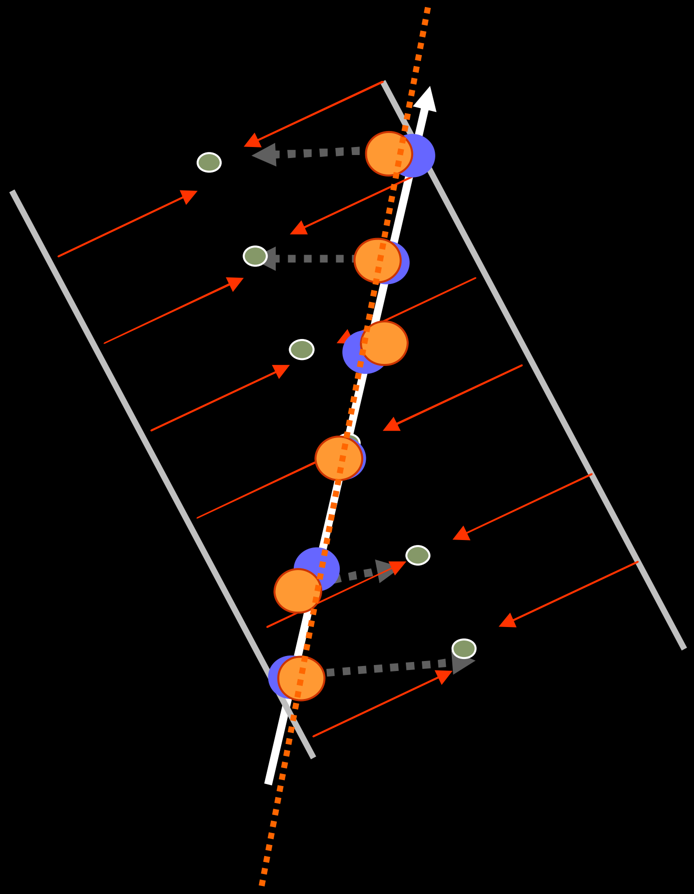


ϕ
D
r
d_0
0.5 cm

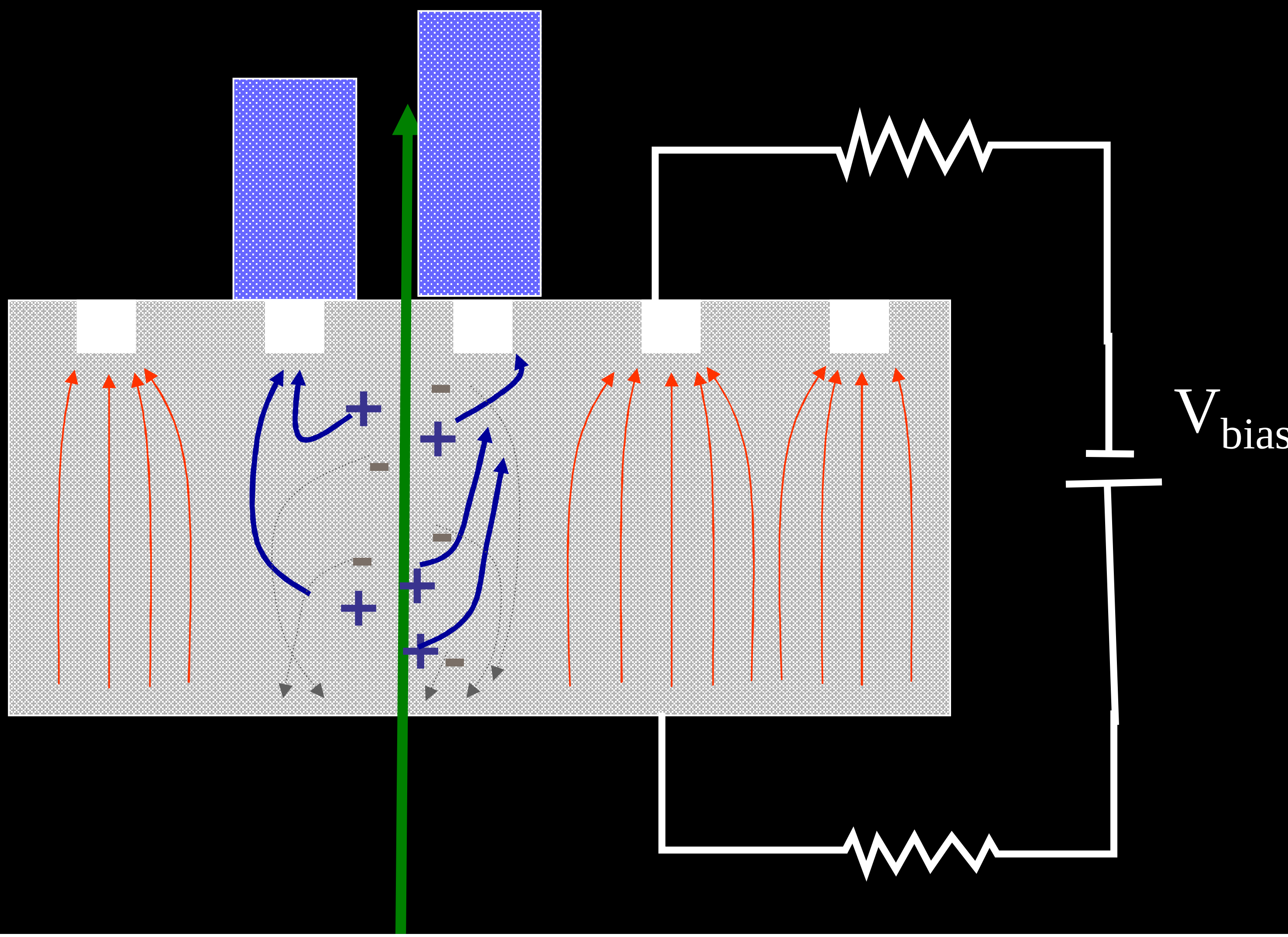
Drift Chambers

- Charged particle ionizes gas as it passes through chamber
  - Lorentz effect (solenoid)
- Large electric fields to drift charge to wires
- Time measurement gives distance from track to wire





Silicon



Fibers

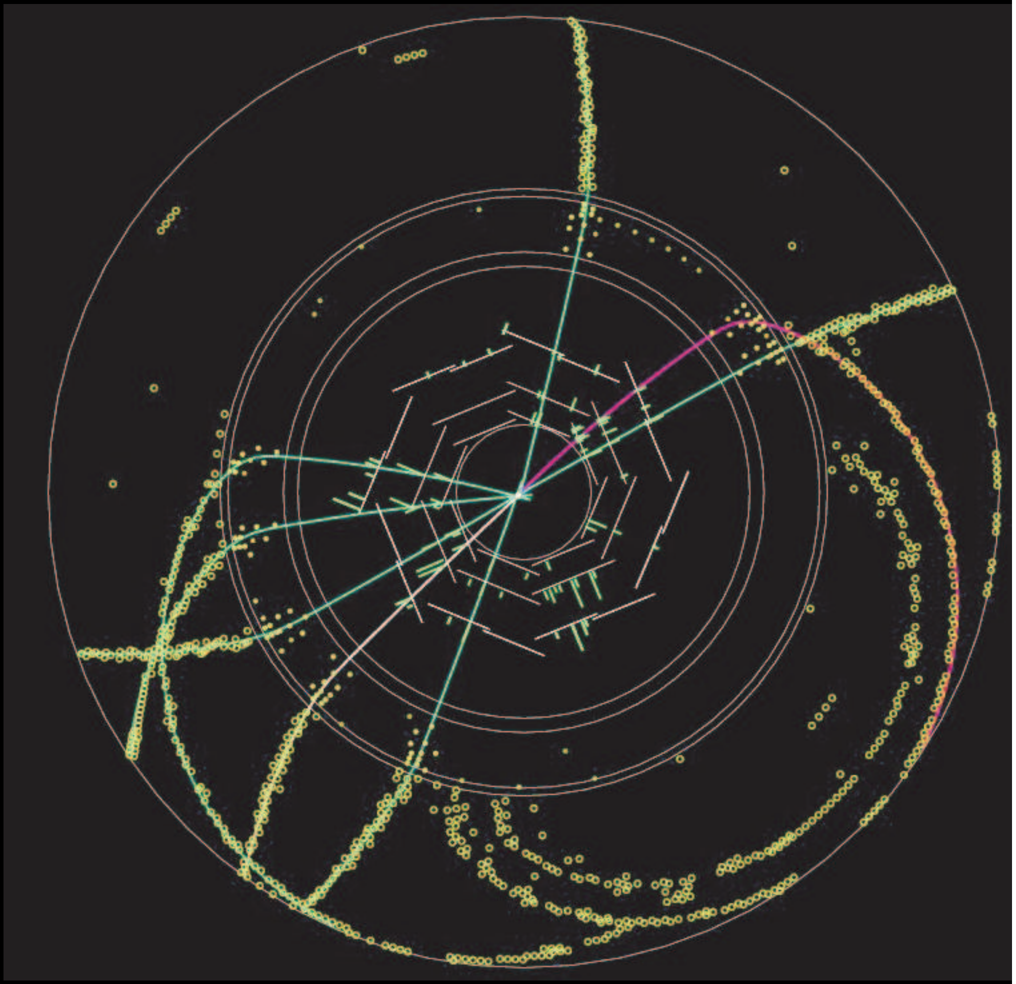
- I know essentially nothing about fibers you couldn’t find yourself on Google

Pattern Recognition

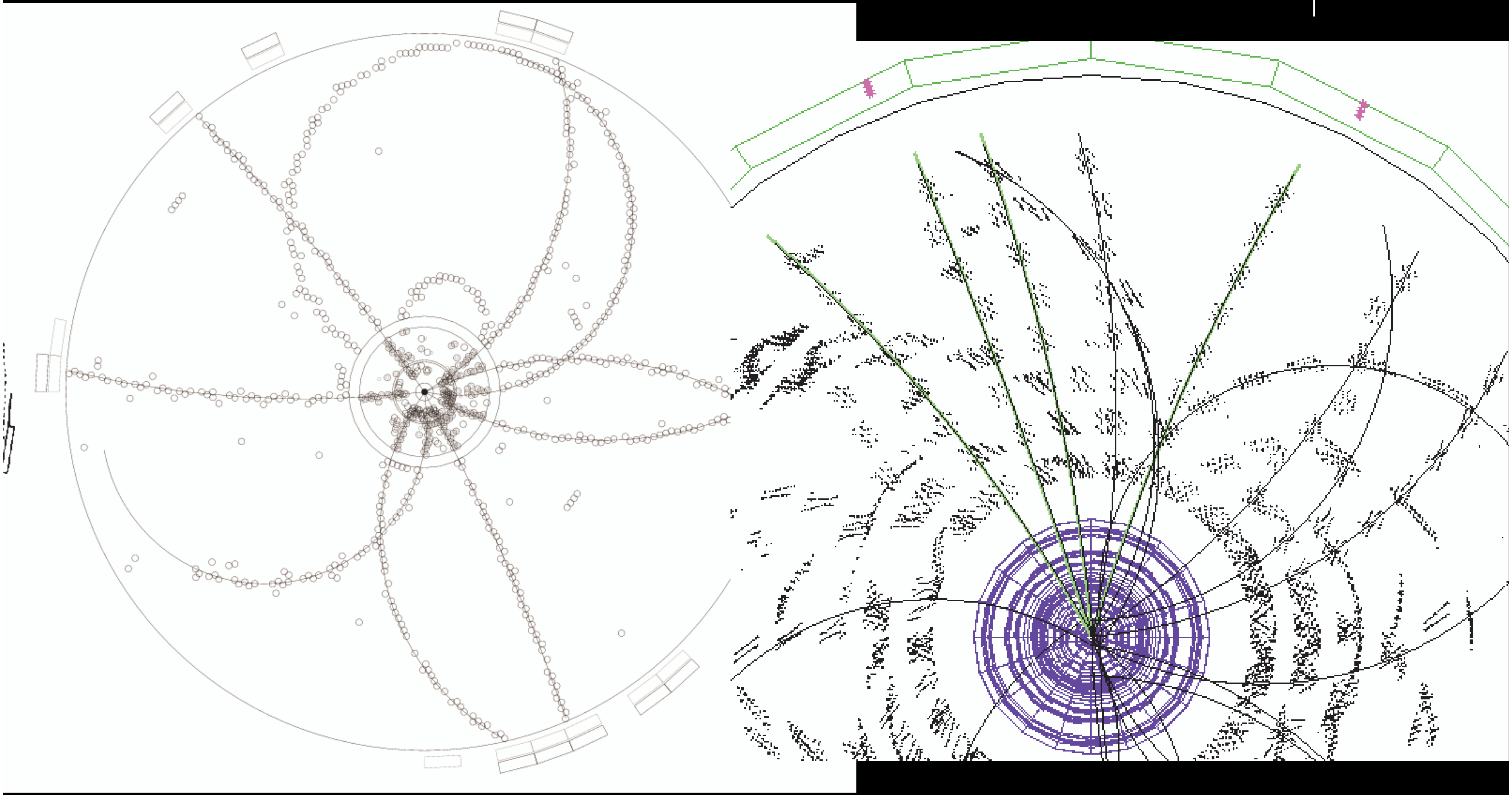
- Event Displays
- Seeding
- Hough and Histograms
- Sharing, Lock-in / Lock-out
  - Tracking and dE/dx
- Curlers

Event Displays

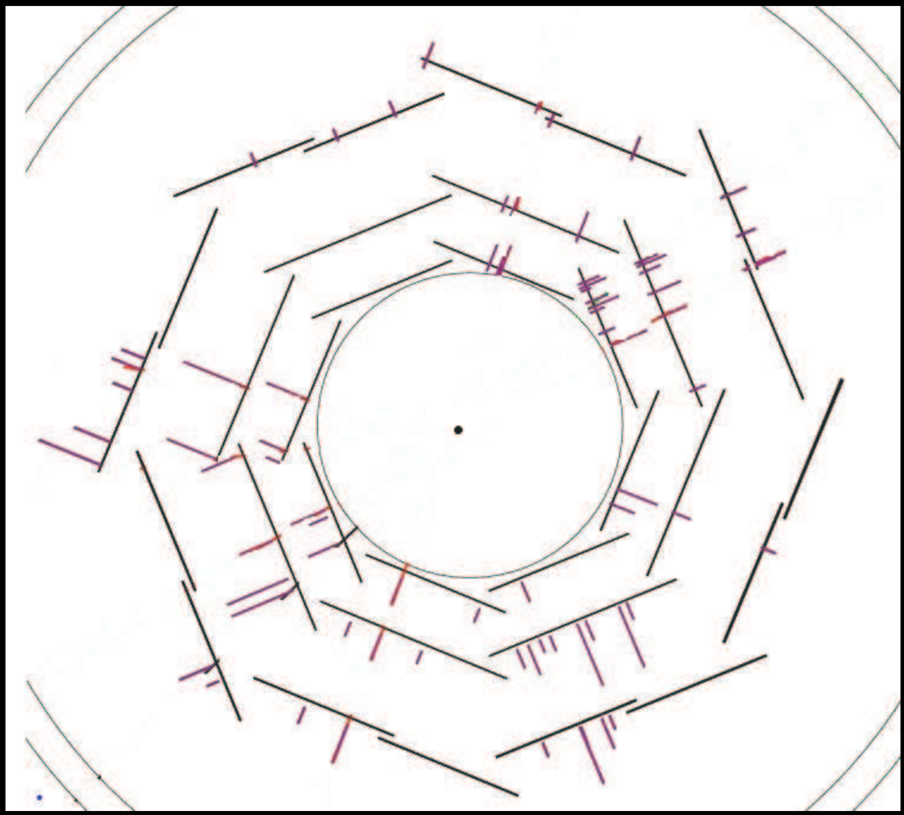
- Service to collaborators
  - Require good event display
    - Refuse to work without one
    - Stable
    - Accurate
    - Ability to display hit information
      - Esp. t_0
- Fisheye view often useful

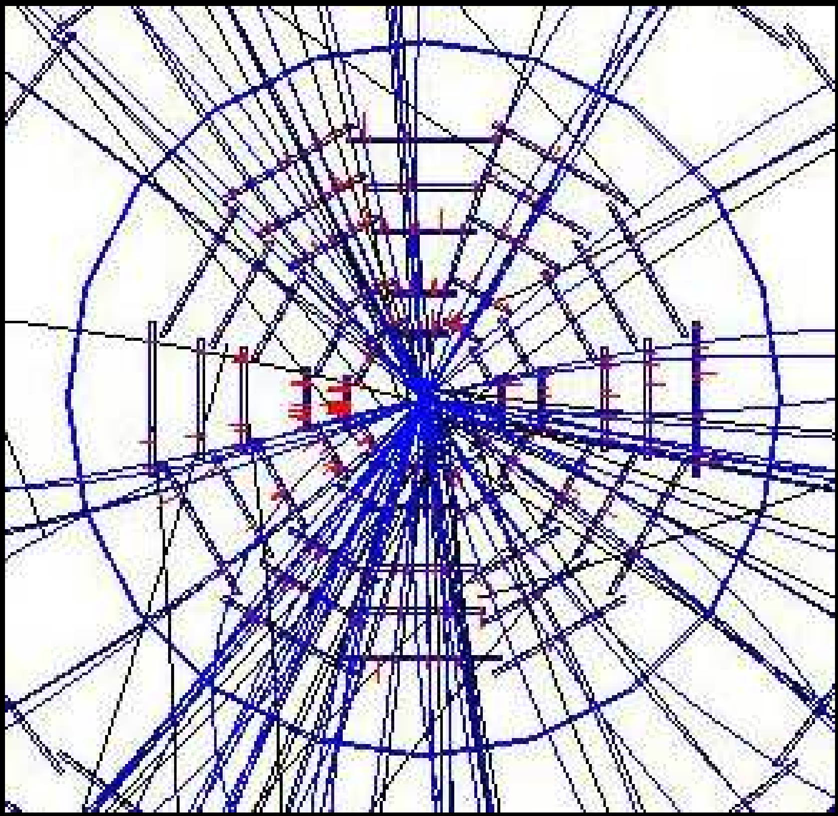


Wire Chambers



Silicon Detectors

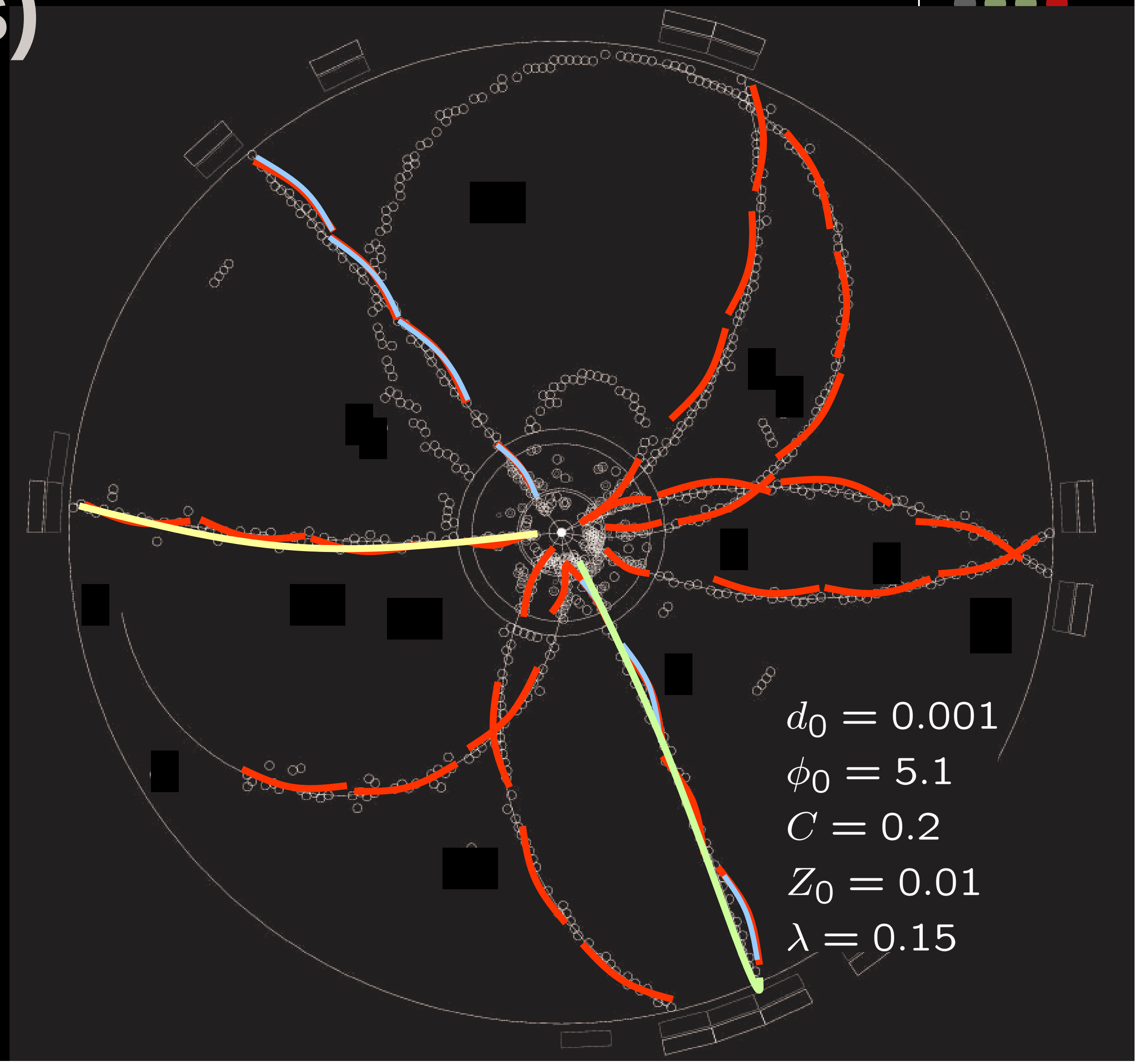




Usual Procedure (Wire Chambers)

- r-ϕ first
- Local noise removal
  - Nearest-neighbor
- Local fitting
  - Often triplets or tracklets
  - Fast fitter needed
- Coalescing
  - Crawling
  - Histogramming
- Pickup

- Final Fitting (e.g. Kalman)



Usual Procedure (Silicon)

- Extrapolate wire chamber tracks into detector
- Add nearby hits
  - Fast extrapolator
  - Generally with large tree of possible hits; best tree chosen

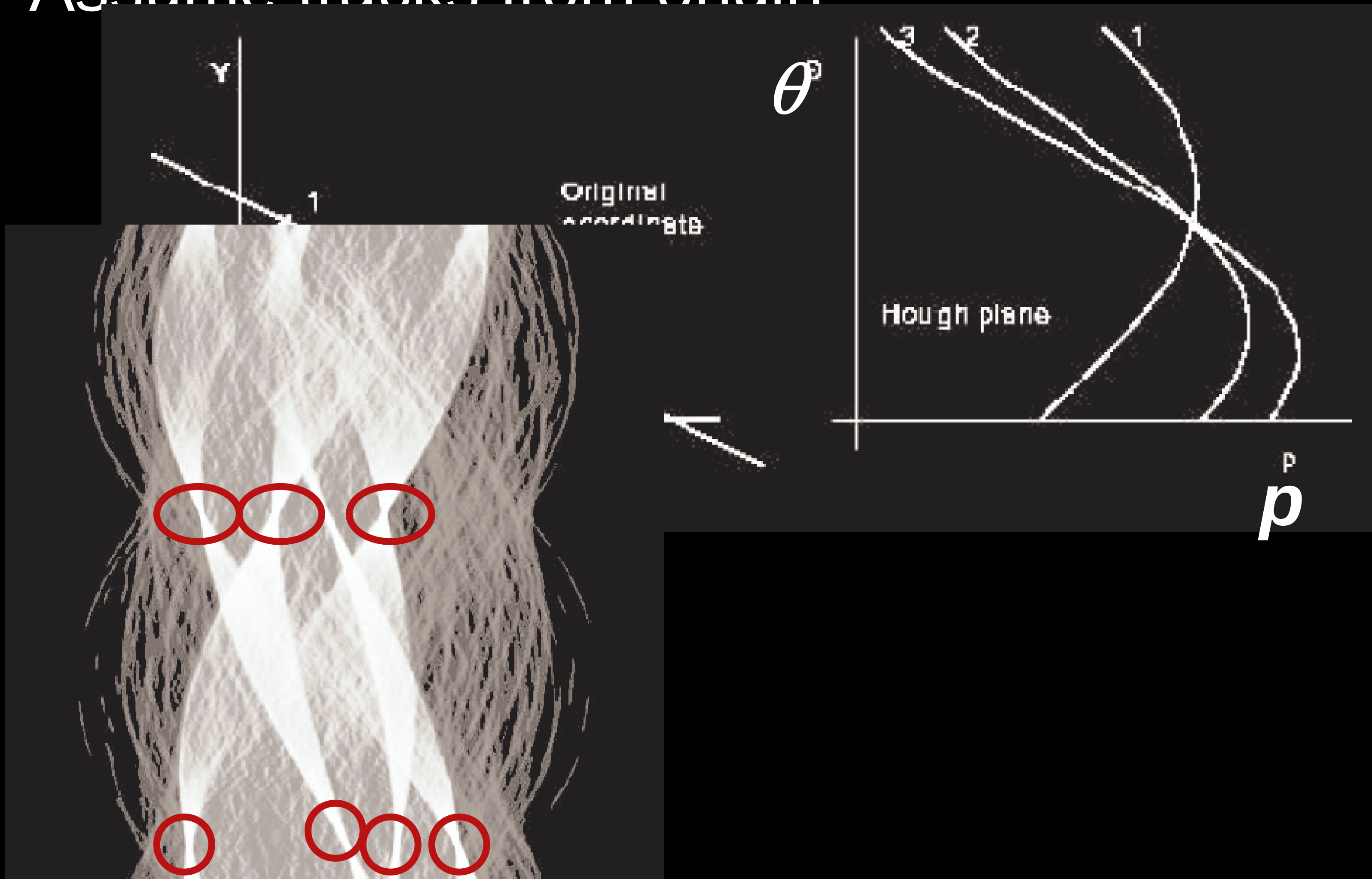
OR

- Find standalone tracks
  - High redundancy required

- Z-information always hard
  - Always
  - Even if your advisor tells you otherwise

Hough and Histograms

- Assume tracks from origin



High Multiplicity Environments

- Where tracks cross
  - Shared hits
- Multihit electronics
  - May be able to partition correctly
- Best to leave until unambiguous hits already assigned
- Decision to add or not add could depend
  - Tracking or dE/dx

Frontiers for You

- Low-multiplicity, highly redundant tracking "well advanced"

- Many high-multiplicity are just n x low mult

- New techniques?
  - Global hit partitioning
  - Multi-vertex tracking
  - High-occupancy tricks
    - Even CDF chamber is less than 5%

Tracking Pathologies

- "Ghosts"
  - Two tracks formed from one particle
- "Kinks"
  - Either
    - Hard-scatter
    - Decay in-flight
  - Usually made into two tracks
  - Bad things happen when fit into 2 tracks
- "Curlers"
  - Not simple to know head from tail
    - Get momentum, charge , initial angle backwards

Resolution

- Point resolution
  - r-ϕ view
    - Typically O(100 μm) wire
    - O(10 μm) silicon
  - s-z view
    - Typically O(mm) wire
    - O(100 μm) silicon
- Parameter resolution
  - Obviously related to point resolution
  - Estimating resolution on line parameters

Likelihood Fit to a Line

- Set of points
  - Known to be a line $$\hat{y}_i = mx_i + b$$
  - Known x_i
  - Measured y_i

$$(y_i - \hat{y}_i) = (y_i - (mx_i + b))$$

  - χ^2: sum of normalized deviations squared

$$\left(\frac{y_i - \hat{y}_i}{\sigma_i}\right)^2 = \left(\frac{y_i - (mx_i + b)}{\sigma_i}\right)^2$$

$$\chi^2 = \sum_i \left(\frac{y_i - \hat{y}_i}{\sigma_i}\right)^2 = \sum_i \left(\frac{y_i - (mx_i + b)}{\sigma_i}\right)^2$$

Reminder: why χ?

- If measured y_i is gaussian about true

$$P(y_i)dy_i = C_i e^{-\left(\frac{(y_i-\hat{y}_i)^2}{2\sigma_i}\right)} dy_i$$

- Likelihood to measure the complete set

$$\mathcal{L} = \Pi_i C_i e^{-\left(\frac{(y_i-\hat{y}_i)^2}{2\sigma_i}\right)}$$

- Maximize $\mathcal{L}$ = Maximize ln $\mathcal{L}$ = Minimize -2 ln $\mathcal{L}$

$$-2\ln\mathcal{L} = \sum_i \left(\frac{(y_i-\hat{y}_i)^2}{\sigma_i^2}\right) \equiv \chi^2$$

Minimizing χ^2

$$\chi^2 = \sum_i \left(\frac{y_i - \widehat{y}_i}{\sigma_i}\right)^2 = \sum_i \left(\frac{y_i - (mx_i + b)}{\sigma_i}\right)^2$$

- Find parameters m, b that minimize χ^2

$$\frac{\partial \chi^2}{\partial m} = 0$$

$$\frac{\partial \chi^2}{\partial b} = 0$$

$$\sum_i x_i \left(\frac{y_i - (mx_i + b)}{\sigma_i^2} \right) = 0$$

$$\sum_i \left(\frac{y_i - (mx_i + b)}{\sigma_i^2} \right) = 0$$

$$\sum_i \frac{x_i^2}{\sigma_i^2} m + \sum_i \frac{x_i}{\sigma_i^2} b = \sum_i \frac{x_i y_i}{\sigma_i^2}$$

$$\sum_i \frac{x_i}{\sigma_i^2} m + \sum_i \frac{1}{\sigma_i^2} b = \sum_i \frac{y_i}{\sigma_i^2}$$

Tedious algebra

$$b = \frac{\sum_i \frac{x_i y_i}{\sigma_i^2} - \frac{\sum_i \frac{x_i^2}{\sigma_i^2}}{\sum_i \frac{x_i}{\sigma_i^2}} \sum_i \frac{y_i}{\sigma_i^2}}{\sum_i \frac{x_i}{\sigma_i^2} - \frac{\sum_i \frac{x_i^2}{\sigma_i^2}}{\sum_i \frac{x_i}{\sigma_i^2}} \sum_i \frac{1}{\sigma_i^2}}$$

$$m = \frac{\sum_i \frac{x_i y_i}{\sigma_i^2} - \frac{\sum_i \frac{x_i}{\sigma_i^2}}{\sum_i \frac{1}{\sigma_i^2}} \sum_i \frac{y_i}{\sigma_i^2}}{\sum_i \frac{x_i^2}{\sigma_i^2} - \frac{\sum_i \frac{x_i}{\sigma_i^2}}{\sum_i \frac{1}{\sigma_i^2}} \sum_i \frac{x_i}{\sigma_i^2}}$$

Need $\sum_i \frac{1}{\sigma_i^2}, \sum_i \frac{x_i}{\sigma_i^2}, \sum_i \frac{y_i}{\sigma_i^2}, \sum_i \frac{x_i^2}{\sigma_i^2}, \sum_i \frac{x_i y_i}{\sigma_i^2}$

Recapitulation

- Line formula gives expectation
  - As function of line parameters

$$\hat{y}_i = m x_i + b$$

- Expectation-Measured gives χ^2

$$\chi^2 = \sum_i \left(\frac{y_i - \hat{y}_i}{\sigma_i}\right)^2 = \sum_i \left(\frac{y_i - (m x_i + b)}{\sigma_i}\right)^2$$

- Minimizing χ^2 = Maximizing $\mathcal{L}$

$$\mathcal{L} = \Pi_i C_i e^{-\left(\frac{(y_i - \hat{y}_i)^2}{2\sigma_i}\right)}$$

- Maximum $\mathcal{L}$ = Best-fit parameters

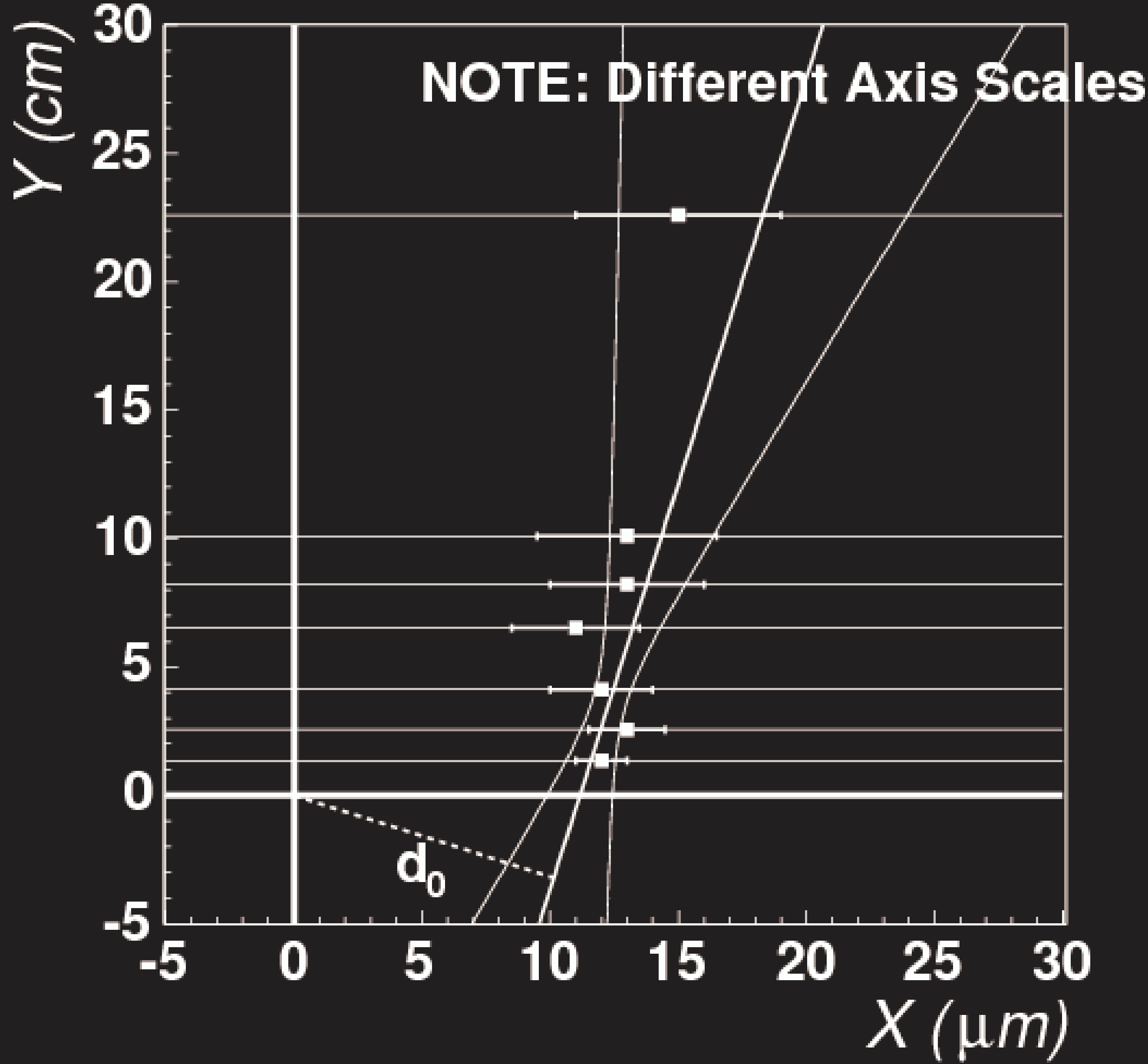
$$\frac{\partial \chi^2}{\partial m} = 0 \qquad \frac{\partial \chi^2}{\partial b} = 0$$

Wobblin’ Goblin

Probably apocryphal

Uncertainties

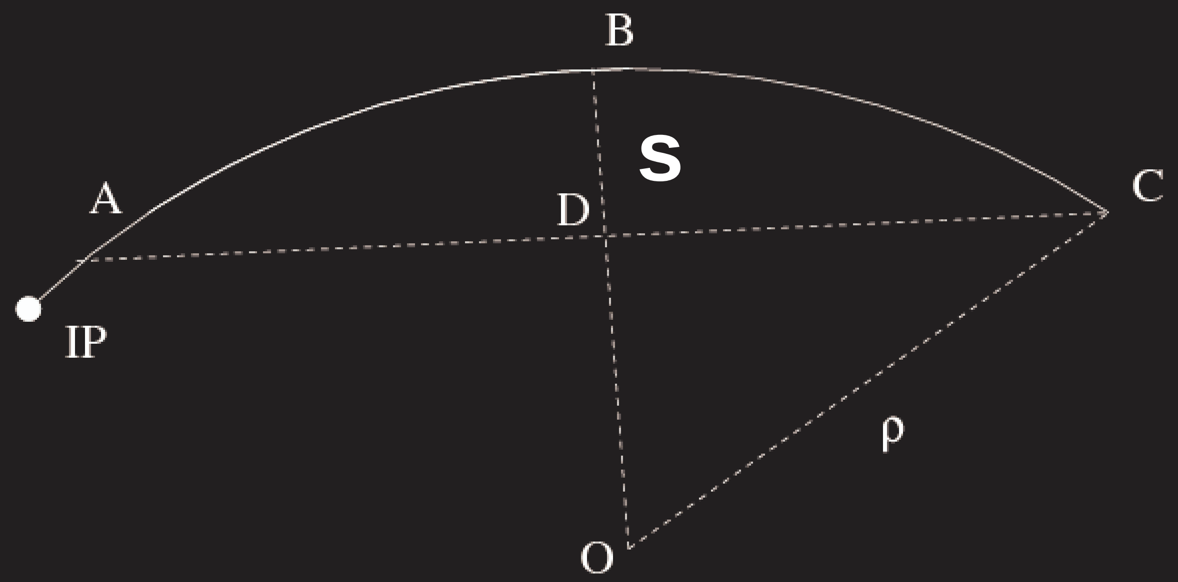
- 1-σ uncertainty on parameter
  - When -2 ln $\mathcal{L}$ changes by one unit
  - Why?
- At first order, -2 ln $\mathcal{L}$ doesn't change as parameters change
  - We set derivatives to 0
- Second derivatives give uncertainties
  - Error on parameter β $\propto \dfrac{1}{\dfrac{\partial^2 \ln \mathcal{L}}{\partial \beta^2}}$



NOTE: Different Axis Scales
Y (cm)
X (μm)
d0
30
25
20
15
10
5
0
-5
-5
0
5
10
15
20
25
30

Introducing Curvature

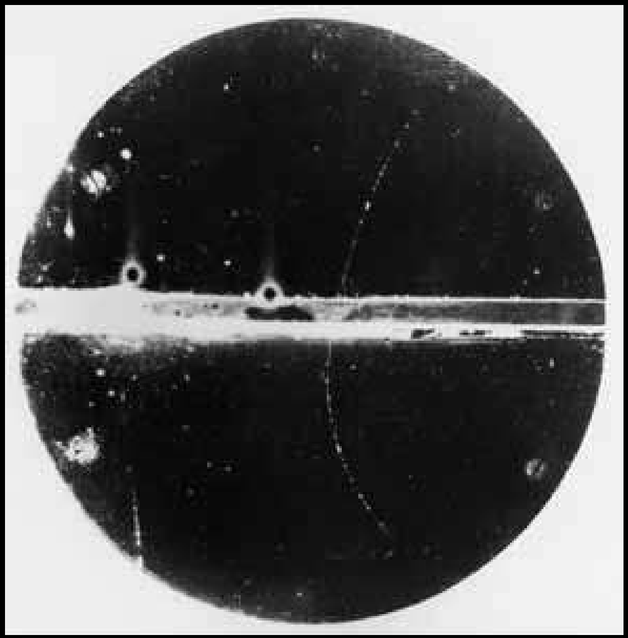
$$s = \rho - \sqrt{\rho^2 - \frac{|AC|^2}{4}}$$



$$\rho = \frac{\frac{|AC|^2}{4} + s^2}{2s}$$

Material

- Material has two separate effects
  - Energy loss
    - Relatively predictable loss of momentum
    - Increase of curvature
    - Mean is non-zero
      - S.D. is called "energy straggling"
  - Multiple scattering
    - Significant angular deflection in material
    - Average deflection is zero



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Multiple Scattering

- nb point resolution not degraded!!!!
  - Points lie less well on line
    - Reflect actual trajectory
      - Not initial trajectory
- Common “cheap” model
  - Increase point resolution
    - Generally p-dependent
  - Just a model

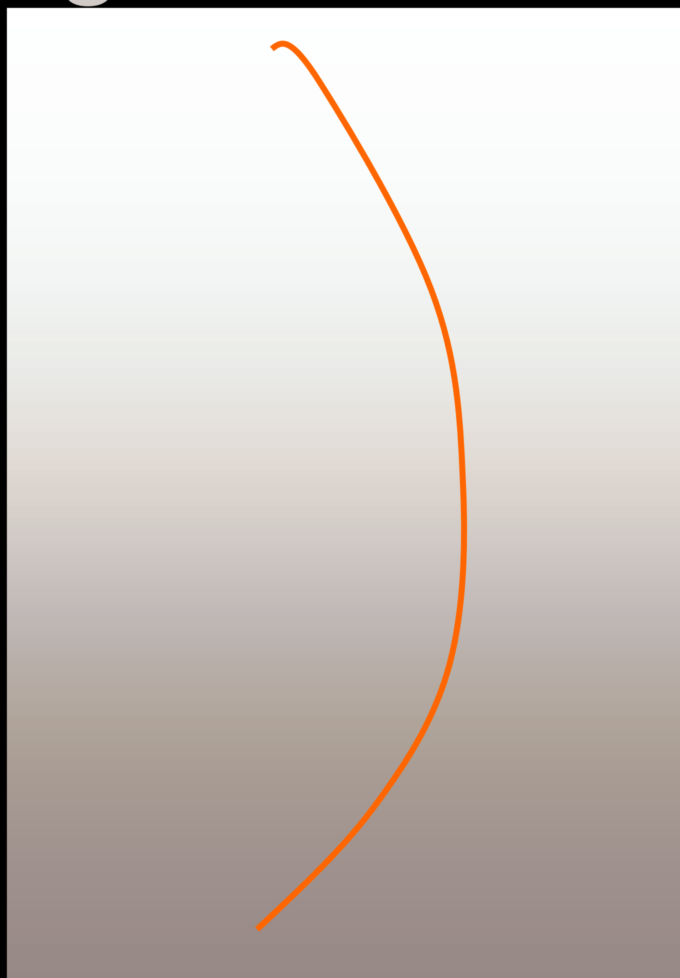
Purpose of Fitting

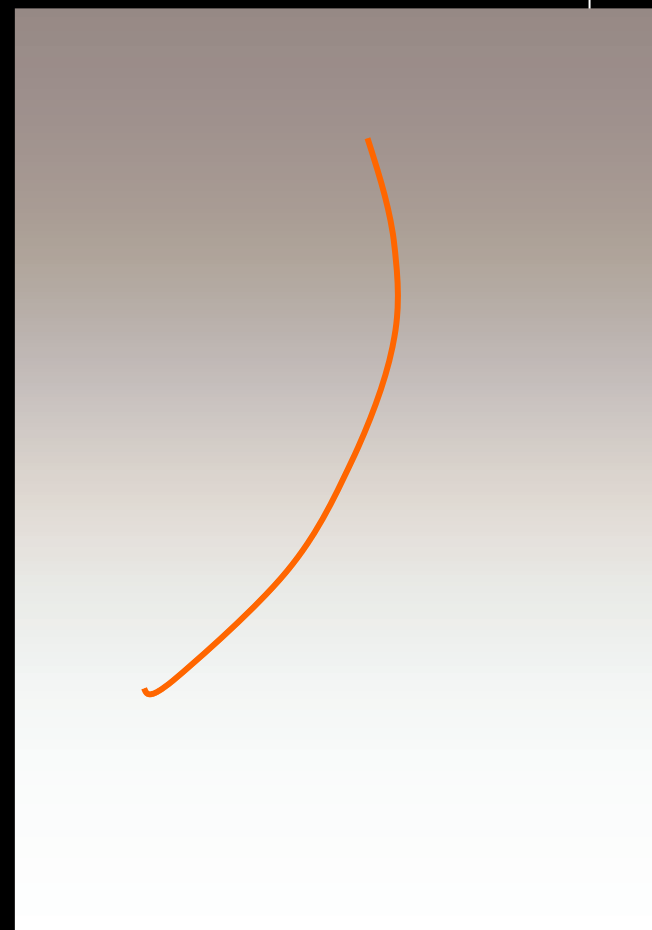
- Usual purpose
  - Best estimate of parameters at origin
  - Reported parameters: tangent helix @ origin
  - Extrapolation does not yield optimal parameters elsewhere

- Occasional purpose
  - Best estimate of parameters at exit
    - Reported: tangent helix @ exit
      - But in terms of its PCAO (possibly confusing)

- Extrapolation to other points does not yield the best possible estimate if it passes through material
  - But it nearly always produces an adequate estimate

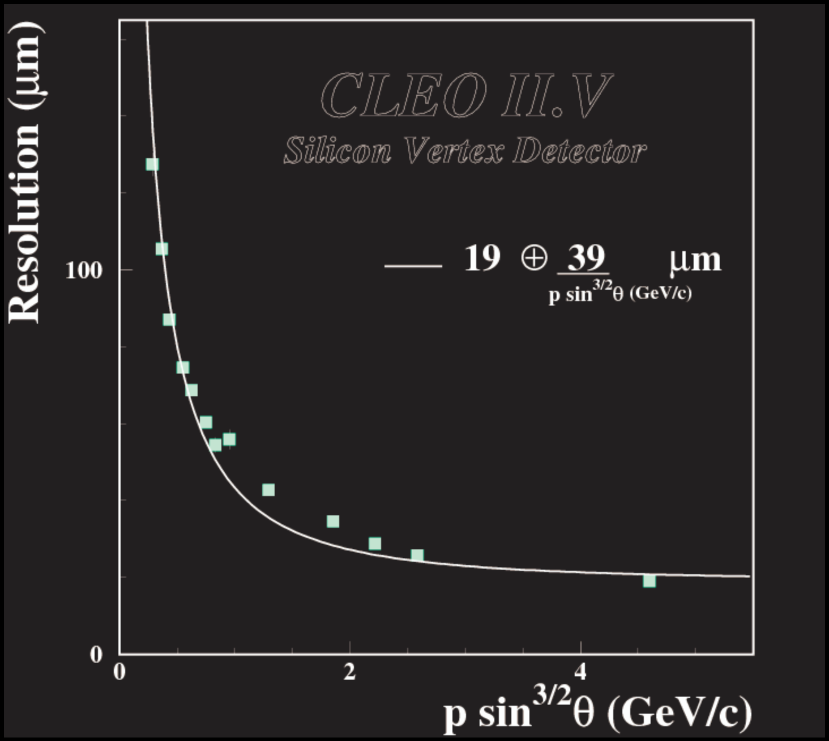
Fog

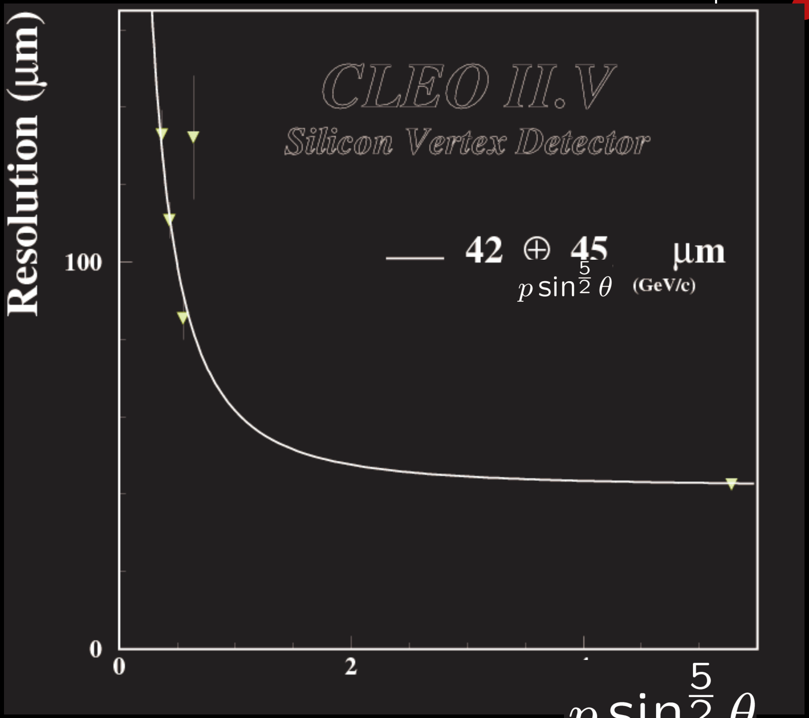




Scatters correlate measurement errors (Kalman)
Different Weightings Depend on Your Goal

Resolution





$p \sin^{\frac{5}{2}} \theta$

$$\sigma_{d_0} = a \oplus \frac{b}{p\beta \cos^{\frac{3}{2}} \theta_{inc}}$$

$$\sigma_{Z_0} = a \oplus \frac{b}{p\beta \cos^{\frac{5}{2}} \theta_{inc}}$$

Depends on material distribution

Depends on point resolution

Useful Symbol

$$x \oplus y \equiv \sqrt{x^2 + y^2}$$

$$3 \oplus 4 = 5$$

$$5 \oplus 12 = 13$$

$$1 \oplus 2 = 2.23$$

- Resolutions often add this way

Systematics

- Material
- Field inhomogeneities
- Overlap / pileup / confusion
- Alignment
- Detector effects
  - Saturation
  - Ion statistics
  - Discriminator timewalk
  - Gravitational wire sag
    - Silicon plane sag
  - Lorentz angle
  - E-field modeling
  - Hall effect
  - Clustering