

Cracking the Unitarity Triangle

— Latest Results from BABAR —



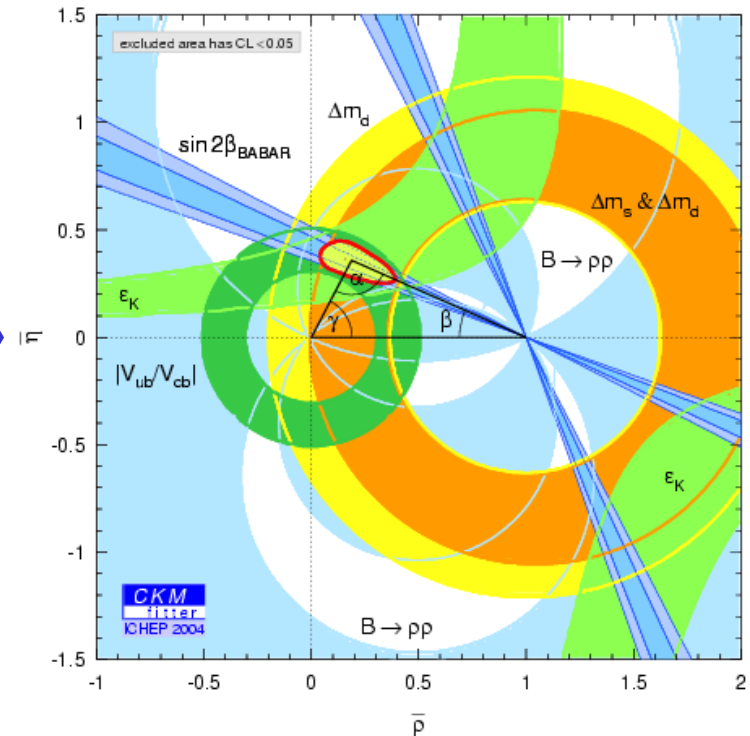
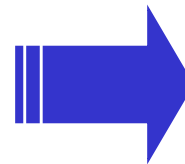
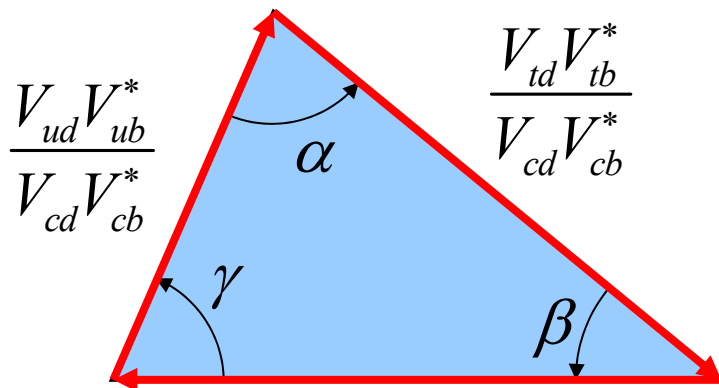
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NEPPSR 2004, Craigville, MA

Unitarity Triangle

- Already seen it a few times this week

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$



- How is this triangle “measured” experimentally?
 - Using CP violation, of course, ... right?

How to Violate CP

- Complex coupling constant (= weak phase)
 - CKM matrix in the SM has 1 such phase
- Interference between ≥ 2 diagrams with different phases
 - e.g., tree and penguin diagrams for $B^0 \rightarrow K^+ \pi^-$ (Carlo's talk)
- With 2 diagrams, the size of the CPV is

$$A_{CP} = \frac{2|A_1||A_2|\sin\Delta\phi\sin\Delta\delta}{|A_1|^2 + |A_2|^2}$$

weak phase difference

strong phase difference

- CKM information in the weak phase $\Delta\phi$
- $|A_1|/|A_2|$ ratio, $\Delta\delta$ usually not calculable

Observing CPV doesn't necessarily teach us much about its origin

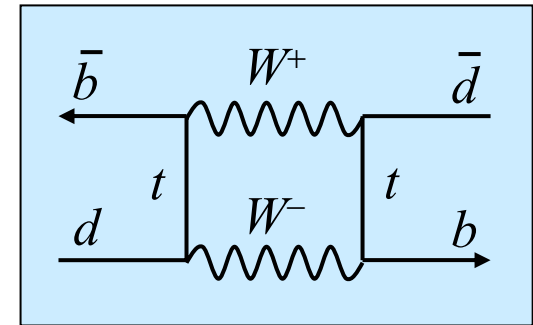
B^0 Mixing to the Rescue

- B^0 and \bar{B}^0 mix, i.e., they turn into each other

- Weak phase = $\arg(V_{tb}^2 V_{td}^{*2}) = 2\beta$

- Mass eigenstates are linear combinations

$$\begin{aligned} |B_L\rangle &= p|B^0\rangle + q|\bar{B}^0\rangle \\ |B_H\rangle &= p|B^0\rangle - q|\bar{B}^0\rangle \end{aligned} \quad p^2 + q^2 = 1$$



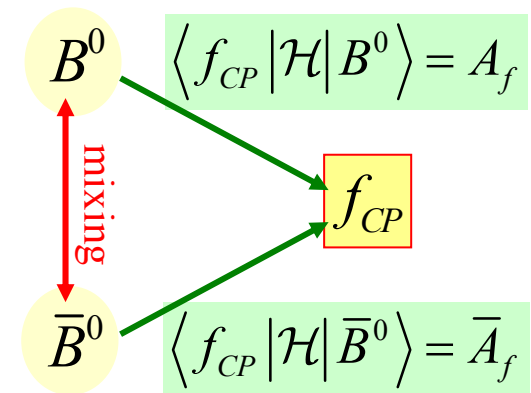
- B_H and B_L differ in mass (Δm), but not in lifetime ($\Delta\Gamma = 0$)
- A little exercise in QM gives us

Pure B^0 state after time t \Rightarrow $|B^0(t)\rangle = e^{-im_B t} e^{-\Gamma t} \left\{ \cos\frac{\Delta m t}{2} |B^0\rangle - i\frac{q}{p} \sin\frac{\Delta m t}{2} |\bar{B}^0\rangle \right\}$

Pure \bar{B}^0 state after time t \Rightarrow $|\bar{B}^0(t)\rangle = e^{-im_B t} e^{-\Gamma t} \left\{ -i\frac{p}{q} \sin\frac{\Delta m t}{2} |B^0\rangle + \cos\frac{\Delta m t}{2} |\bar{B}^0\rangle \right\}$

Mixing and Interference

- Pure B^0 state, after time t , decays into a CP eigenstate f_{CP}
 - Both B^0 or \bar{B}^0 can decay into f_{CP}
 - Interference between “mixed” and “unmixed” paths
- Calculate the CP asymmetry
 - Neat problem for undergrad. QM



$$\mathcal{A}_{CP}(t) = \frac{N(\bar{B}^0(t) \rightarrow f_{CP}) - N(B^0(t) \rightarrow f_{CP})}{N(\bar{B}^0(t) \rightarrow f_{CP}) + N(B^0(t) \rightarrow f_{CP})} = C_f \cos(\Delta mt) + S_f \sin(\Delta mt)$$

$$C_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}$$

$$S_f = \frac{-2\text{Im}\lambda_f}{1 + |\lambda_f|^2}$$

$$\lambda_f = \eta_f \frac{q}{p} \cdot \frac{\bar{A}_f}{A_f}$$

decay term

CP eigenvalue

mixing term

Golden Modes

- Suppose the decay $B^0 \rightarrow f_{CP}$ goes through a single diagram

- A_{CP} and \bar{A}_{CP} differ only by the sign of the weak phase $\frac{\bar{A}_{CP}}{A_{CP}} = e^{2i\phi}$

- Mixing term q/p is known to be almost pure phase

- Weak phase is 2β $\rightarrow \lambda_f = \eta_f \frac{q}{p} \frac{A_{CP}}{\bar{A}_{CP}} = \eta_f e^{2i\beta} e^{2i\phi}$

- Things get nice and simple

$$C_f = 0, \quad S_f = -\eta_f \sin(2\beta + 2\phi)$$

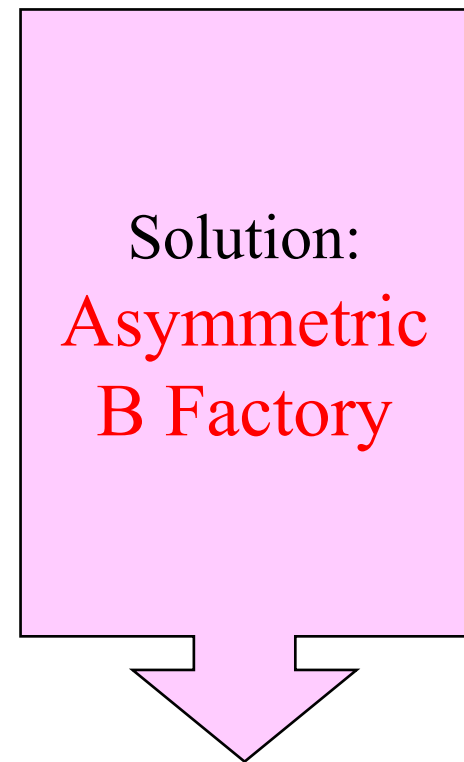
$$\mathcal{A}_{CP}(t) = -\eta_f \sin(2\beta + 2\phi) \sin(\Delta mt)$$

For “golden” decays, the amplitude S_f of the time-dependent CP asymmetry measures the weak phase

Measuring CP Violation

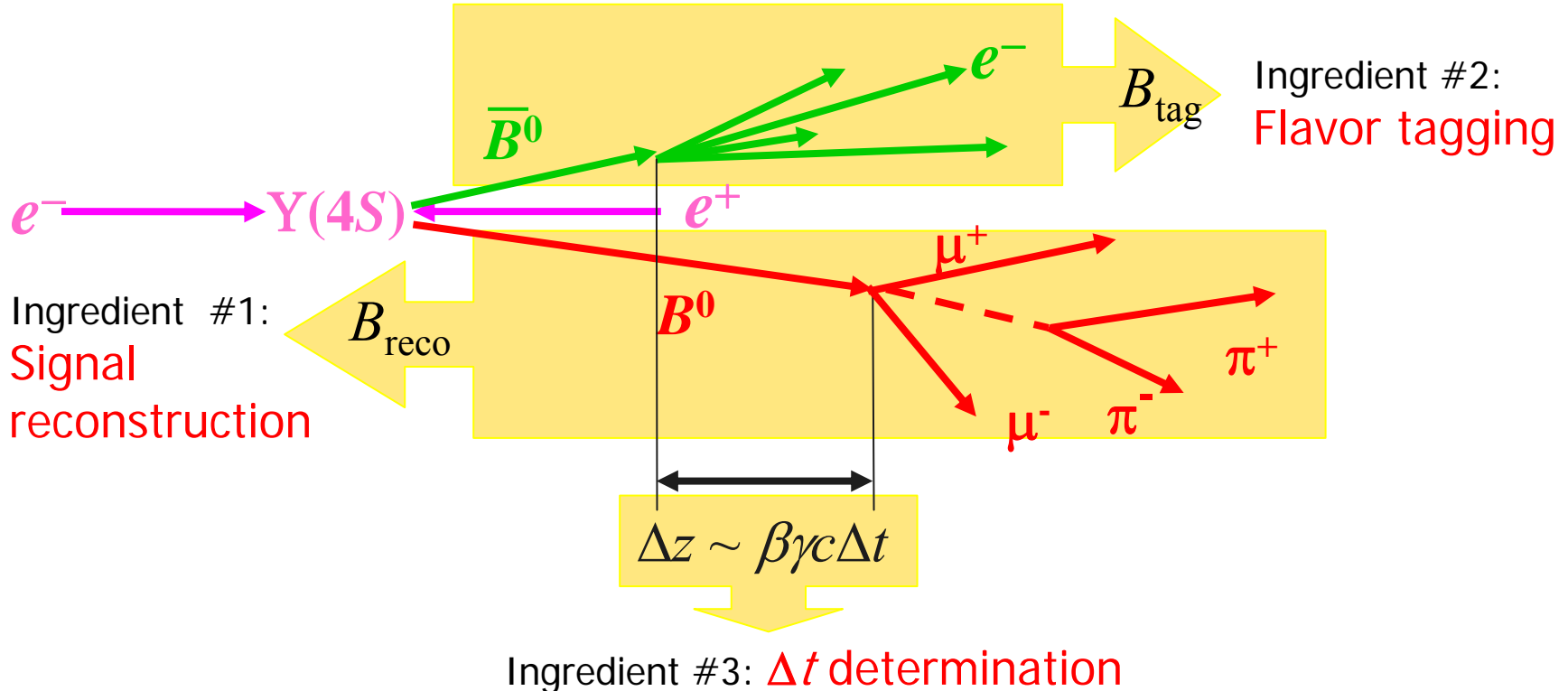
$$\mathcal{A}_{CP}(t) = \frac{N(\bar{B}^0(t) \rightarrow f_{CP}) - N(B^0(t) \rightarrow f_{CP})}{N(\bar{B}^0(t) \rightarrow f_{CP}) + N(B^0(t) \rightarrow f_{CP})} = C_f \cos(\Delta mt) + S_f \sin(\Delta mt)$$

- Experiment must do 3 things:
 - Produce and detect $B \rightarrow f_{CP}$ events
 - Typical branching fraction: $10^{-4} - 10^{-5}$
 - Need a lot of lot of lot of lot of B 's
 - Separate B^0 from \bar{B}^0 = “Flavor tagging”
 - Use $Y_{4S} \rightarrow B^0 \bar{B}^0$ and tag one B
 - Measure the decay time
 - Measure the flight length $\beta\gamma ct$
 - But B 's are almost at rest in Y_{4S} decays



Three Ingredients

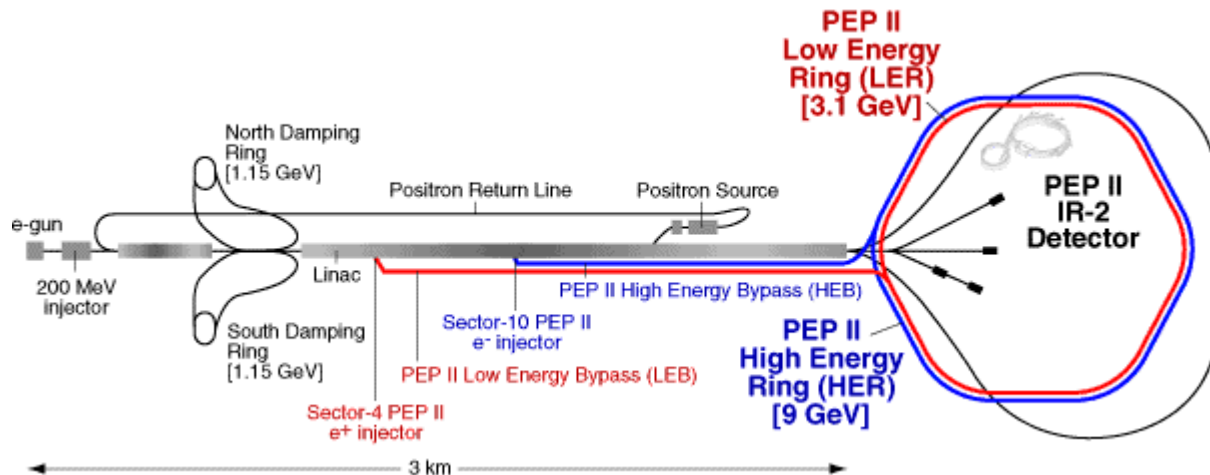
$$A_{CP}(t) = \frac{N(\bar{B}^0(t) \rightarrow f_{CP}) - N(B^0(t) \rightarrow f_{CP})}{N(\bar{B}^0(t) \rightarrow f_{CP}) + N(B^0(t) \rightarrow f_{CP})} = C_f \cos(\Delta mt) + S_f \sin(\Delta mt)$$



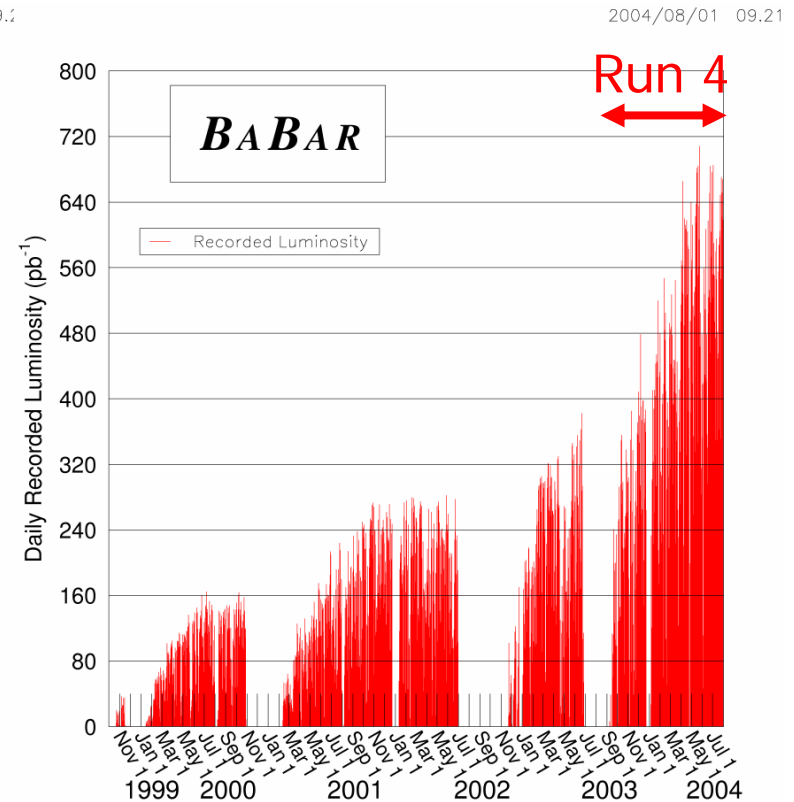
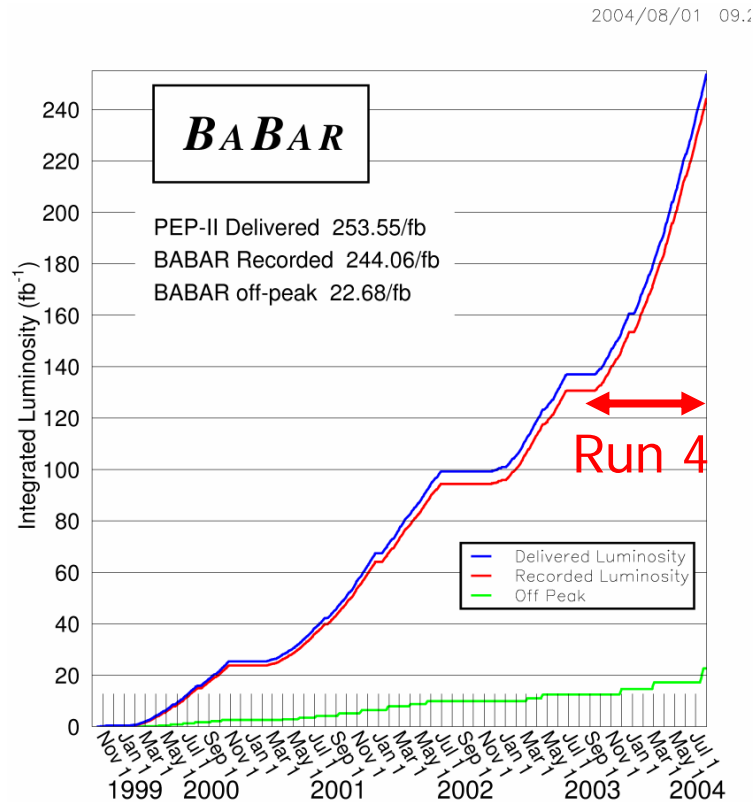
Asymmetric B Factory

- Collides e^+e^- at $E_{CM} = m(Y_{4S})$ but with $E(e^+) \neq E(e^-)$
 - PEP-II: 9 GeV e^- vs. 3.1 GeV e^+ $\rightarrow \beta\gamma = 0.56$
 - The boost allows measurement of Δt
- Collides lots of them: $2.4A(e^+) \times 1.6A(e^-)$
 - PEP-II luminosity $9.2 \times 10^{33}/\text{cm}^2/\text{s} = 9.2 \text{ Hz/nb}$
 - That's $>3x$ the design

$$\sigma(e^+e^- \rightarrow b\bar{b}) \sim 1 \text{ nb}$$

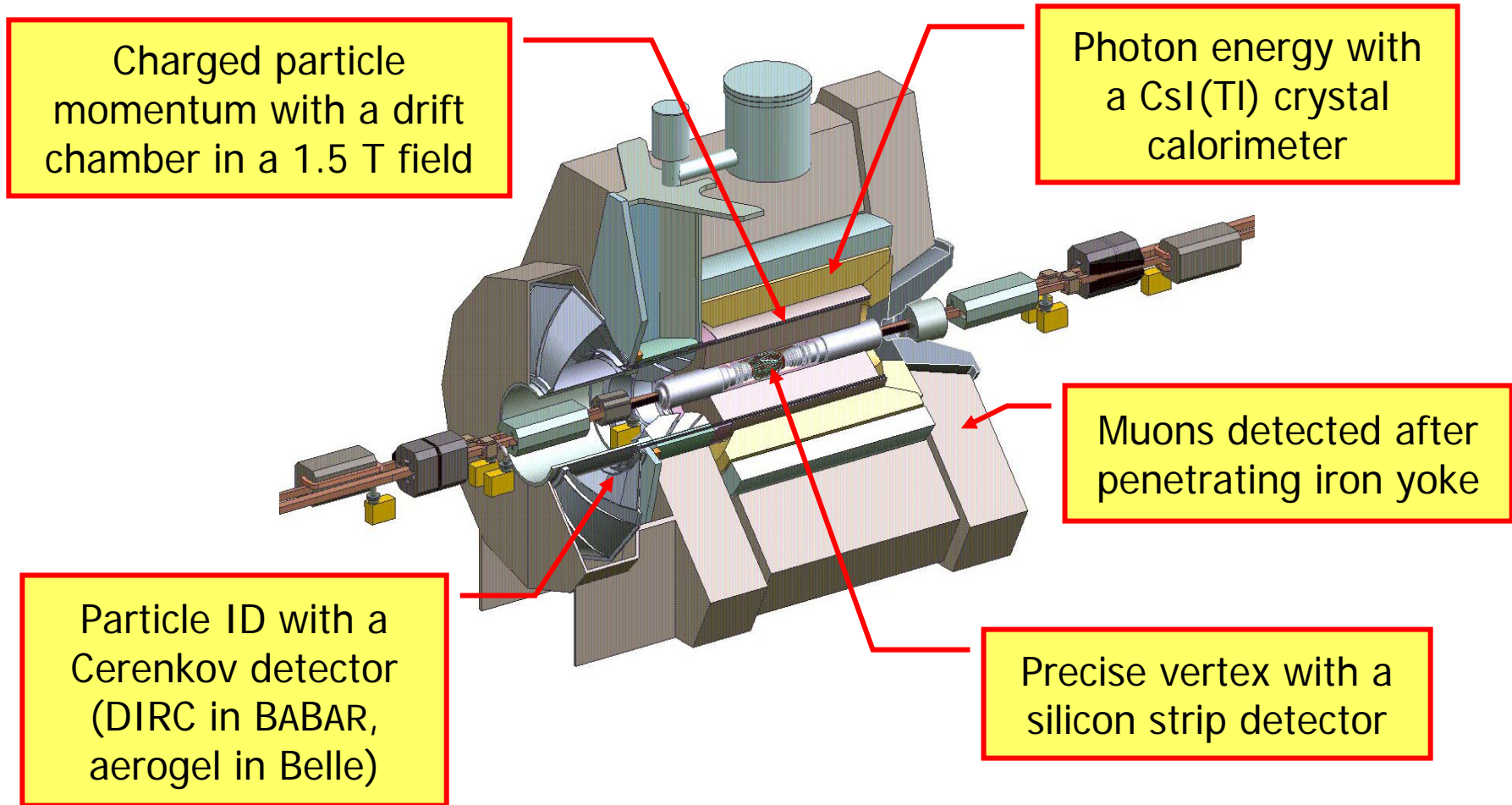


PEP-II Luminosity



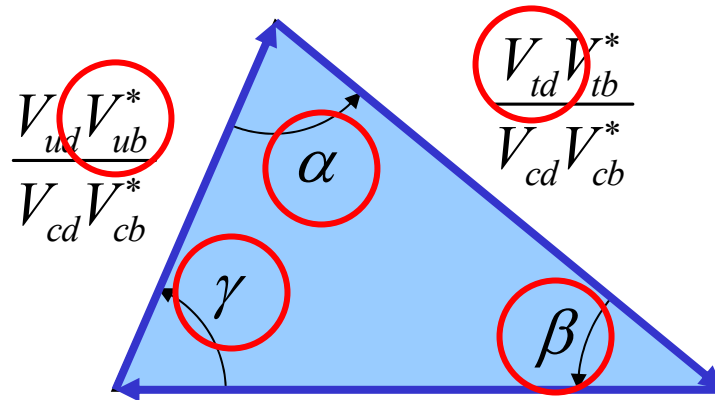
- BABAR has accumulated 244 fb^{-1} of data
- Run 4 (Sep'03-Jul'04) was a phenomenal success

Detector: BABAR



Physics Results

- Will walk through the 3 angles and 2 sides



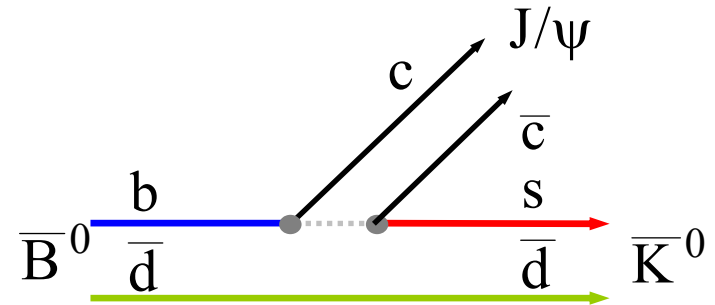
- Almost everything is **PRELIMINARY**
- All ICHEP 2004 results from BABAR/Belle are found at
 - <http://www.slac.stanford.edu/BFROOT/www/Public/ichep2004/>
 - <http://belle.kek.jp/conferences/ICHEP2004/>
- BABAR results use data samples between 80 to 227M $B\bar{B}$ events

Angle β from $b \rightarrow c\bar{c}s$

- “Golden mode” of CP violation

- Tree diagram dominates \rightarrow
- No weak phase in decay

$$\mathcal{A}_{CP}(t) = -\eta_f \sin(2\beta) \sin(\Delta mt)$$



Clean measurement of angle β

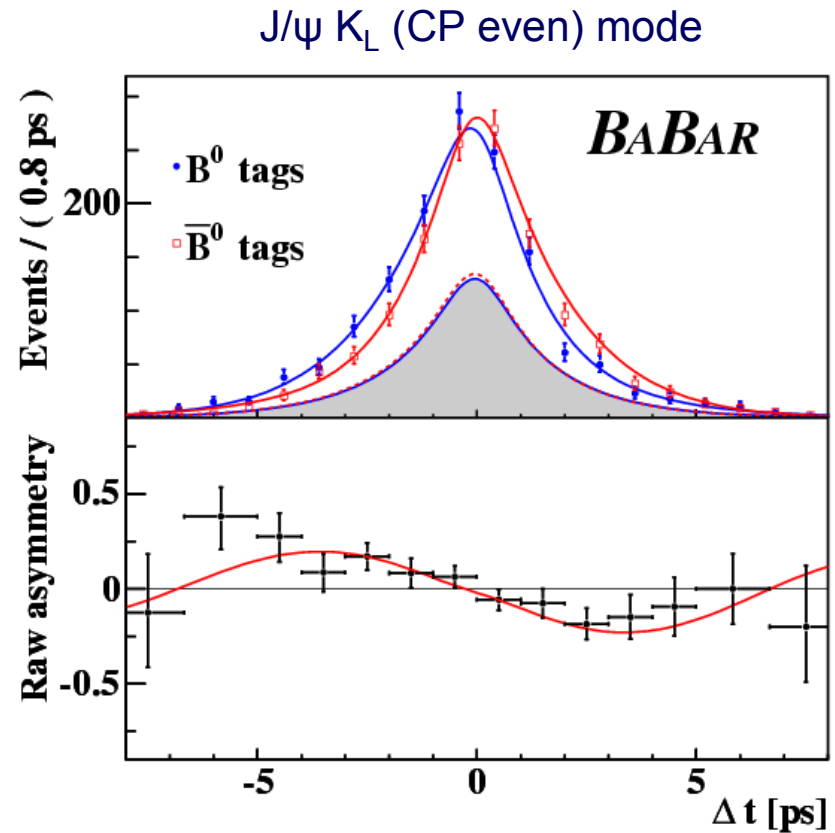
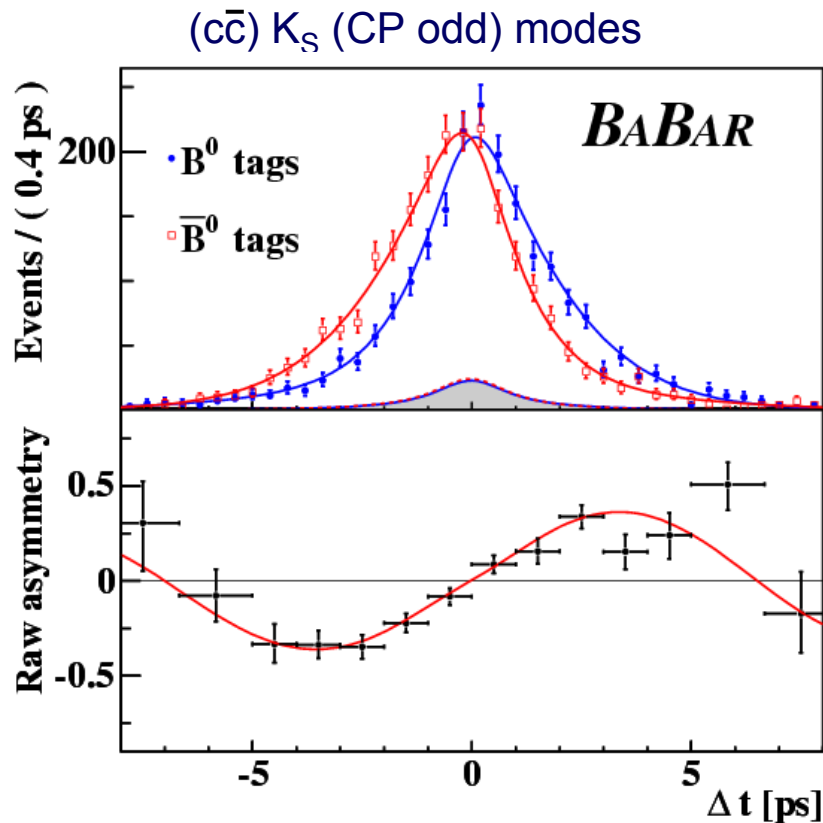
- $c\bar{c}$ pair can be J/ψ , ψ_{2S} , χ_{c1} , or η_c

- They are all $CP = -1$

- $s\bar{d}$ pair can be K_S ($CP = +1$), K_L (-1), or K^{*0} (mixed)

- Total 7730 candidates (78% purity) found in 227 M $B\bar{B}$ events

Time-Dependent A_{CP} Fit

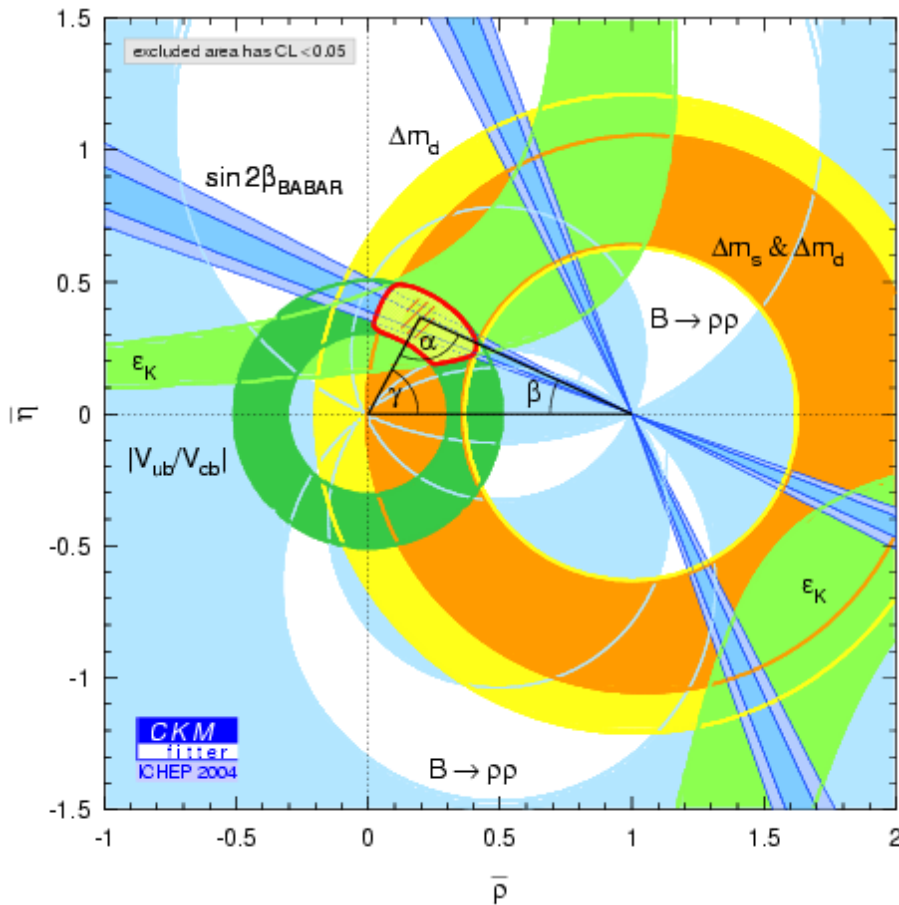


$$\sin 2\beta = 0.722 \pm 0.040 \text{ (stat)} \pm 0.023 \text{ (sys)}$$

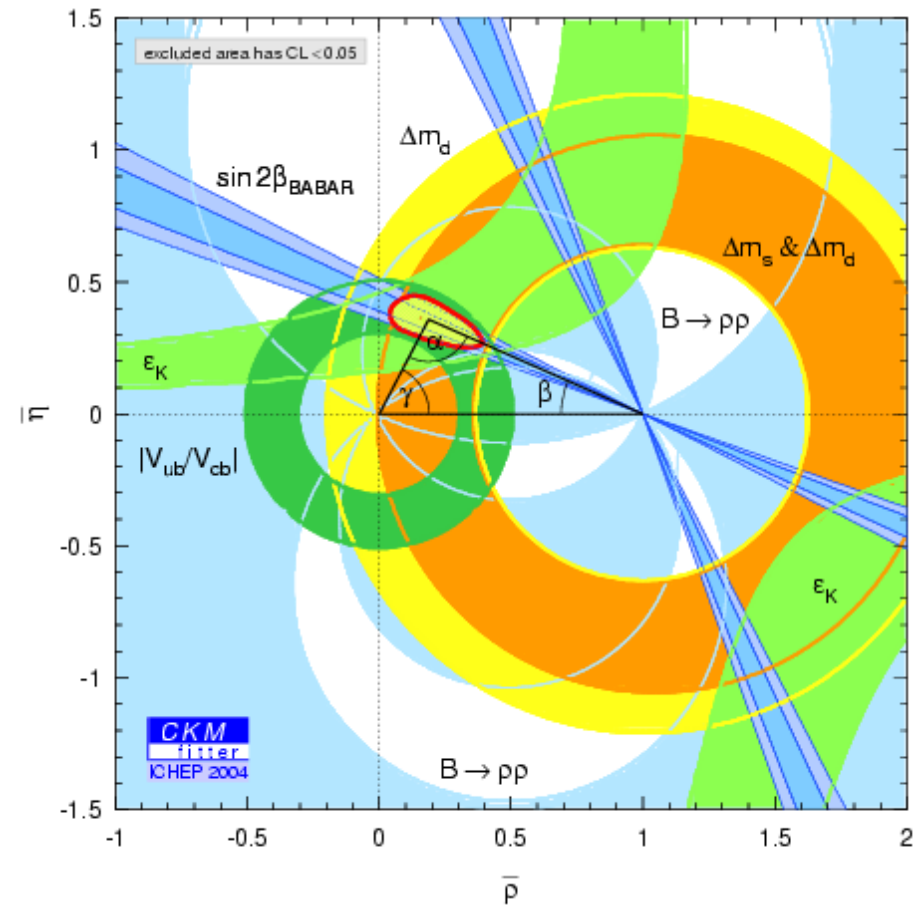
205 fb^{-1}

Unitarity Triangle

CKM fit without $\sin(2\beta)$ measurement



CKM fit with $\sin(2\beta)$ measurement



■ Precise measurement of $\sin 2\beta$ agree with the SM expectation

Angle β from $b \rightarrow s\bar{s}s$

- $b \rightarrow s\bar{s}s$ decay dominated by the “penguin” diagram

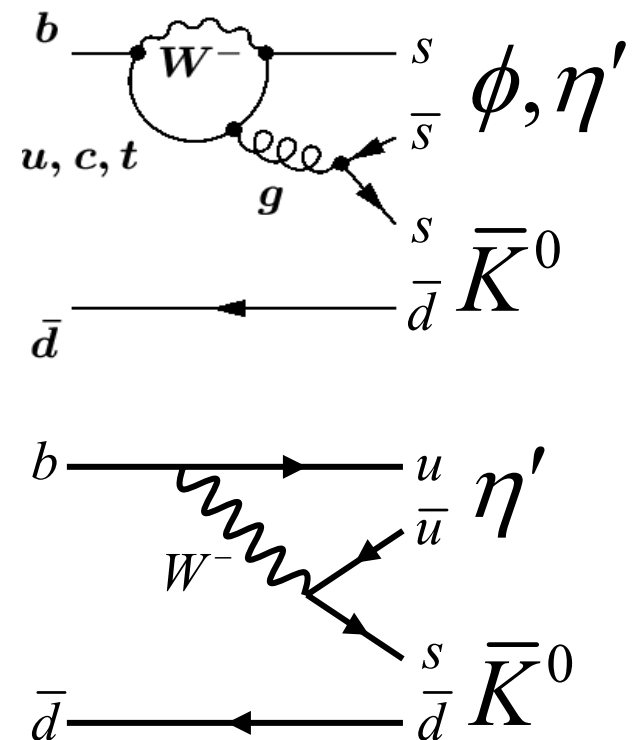
- In the SM, $\mathcal{A}_{CP}^{b \rightarrow s\bar{s}s} = \mathcal{A}_{CP}^{b \rightarrow c\bar{c}s} = \sin 2\beta$
- New Physics may enter the loop $\rightarrow \mathcal{A}_{CP}$ may not agree!

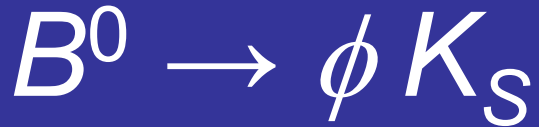
- ϕK_S is pure-penguin

- Small BF: 7.6×10^{-6}

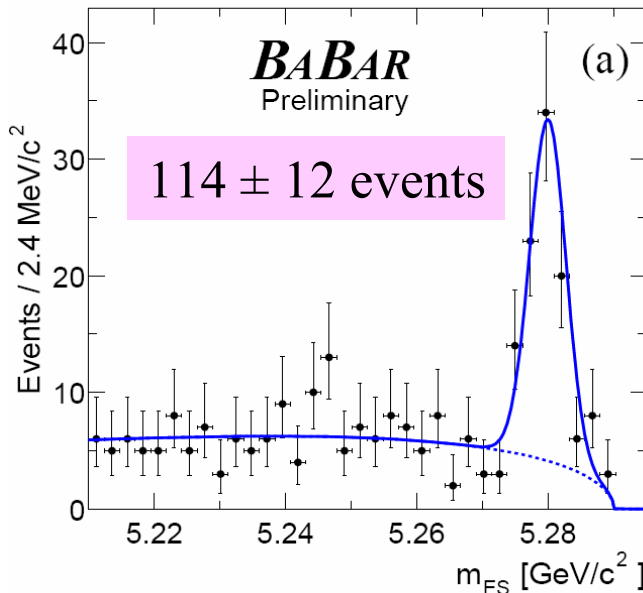
- $\eta' K_S$ has tree diagrams too

- Suppressed by small V_{ub}
- \mathcal{A}_{CP} affected by 0.01 to 0.1
- Larger BF: 5.5×10^{-5}





$B^0 \rightarrow \phi K_S$ candidates

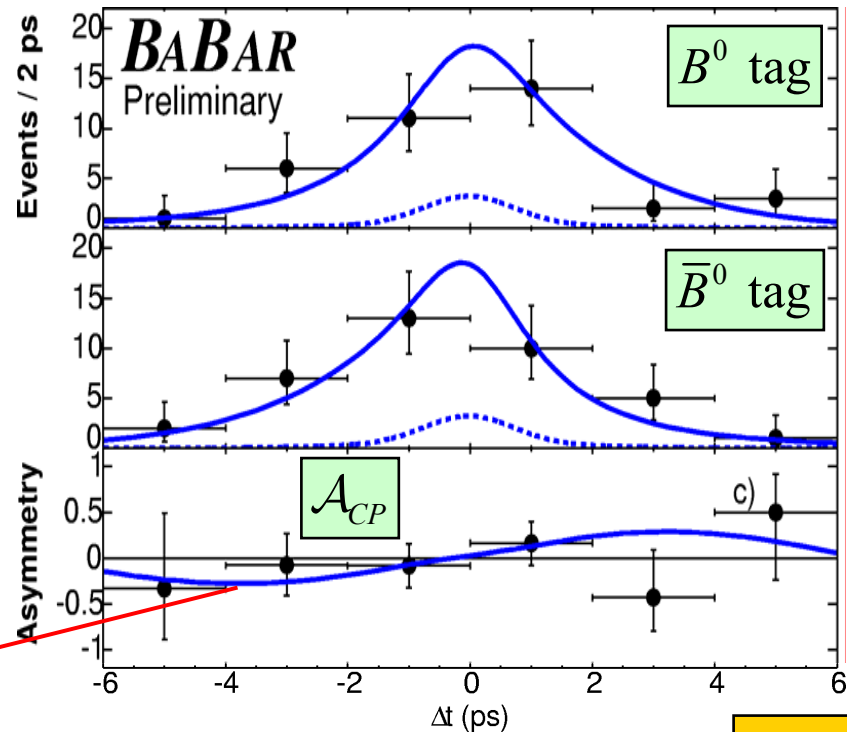


$$S_{\phi K_S} = 0.29 \pm 0.31(\text{stat})$$

■ Combining ϕK_S and ϕK_L together, we get

$$S_{\phi K^0} = 0.50 \pm 0.25(\text{stat})_{-0.04}^{+0.07}(\text{syst}), \quad C_{\phi K^0} = 0.00 \pm 0.23(\text{stat}) \pm 0.05(\text{syst})$$

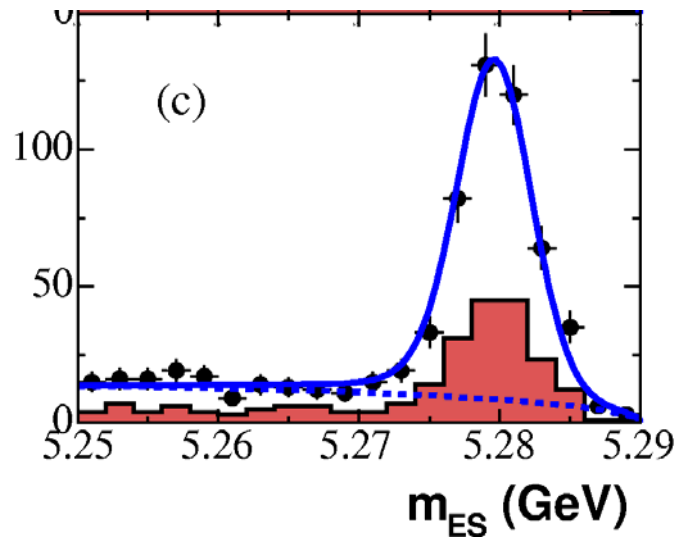
Time-dependent CP asymmetry



205 fb⁻¹

$B^0 \rightarrow \eta' K_S$

$B^0 \rightarrow \phi K_S$ candidates

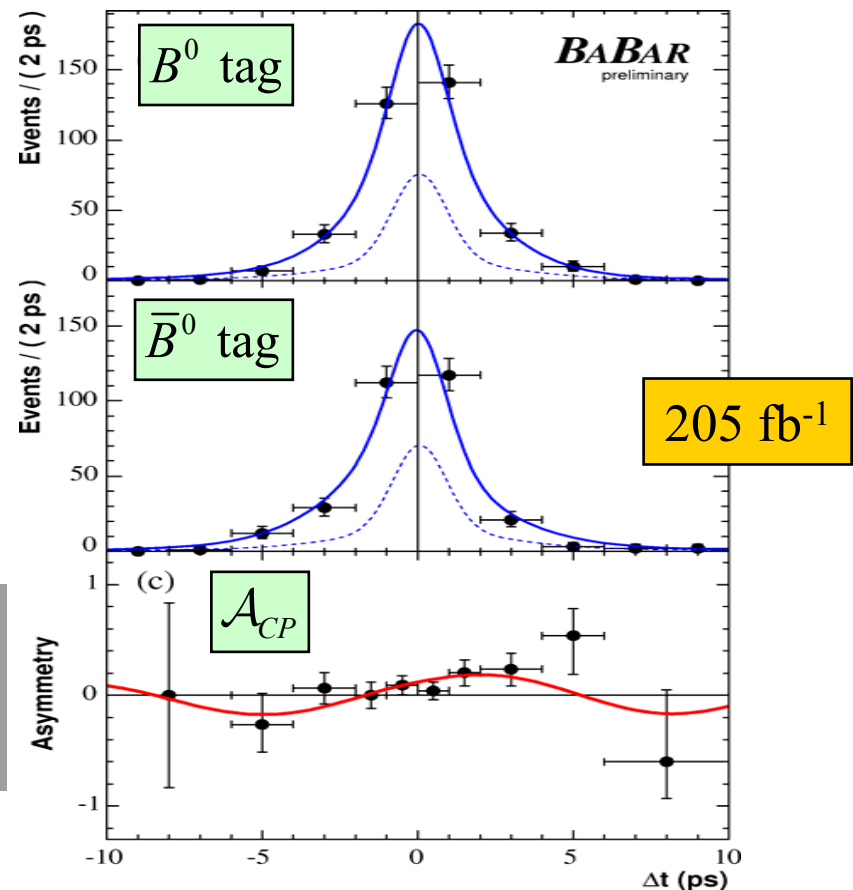


$$S_{\eta' K^0} = 0.27 \pm 0.14(\text{stat}) \pm 0.03(\text{syst})$$

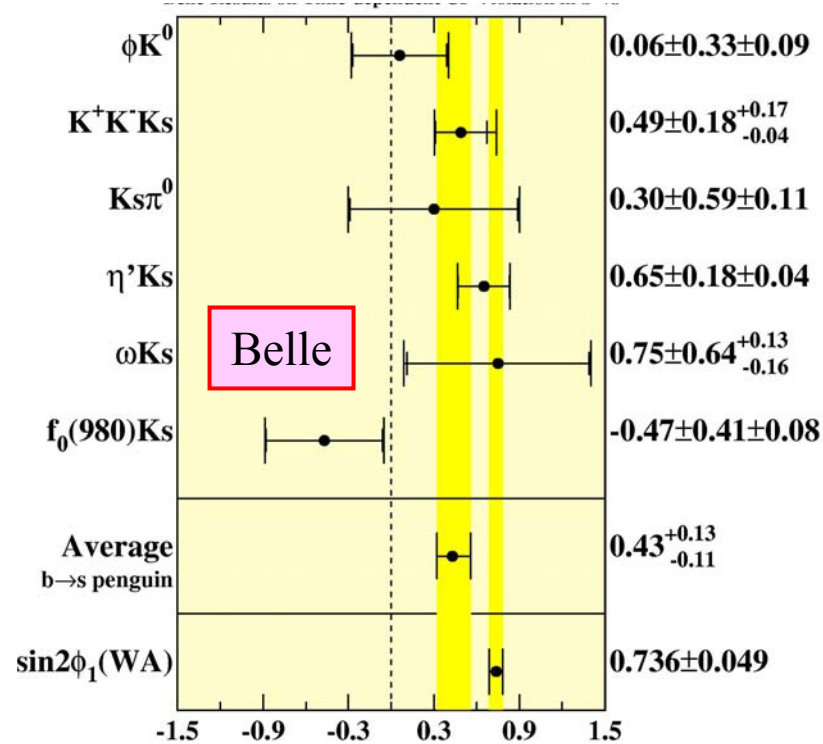
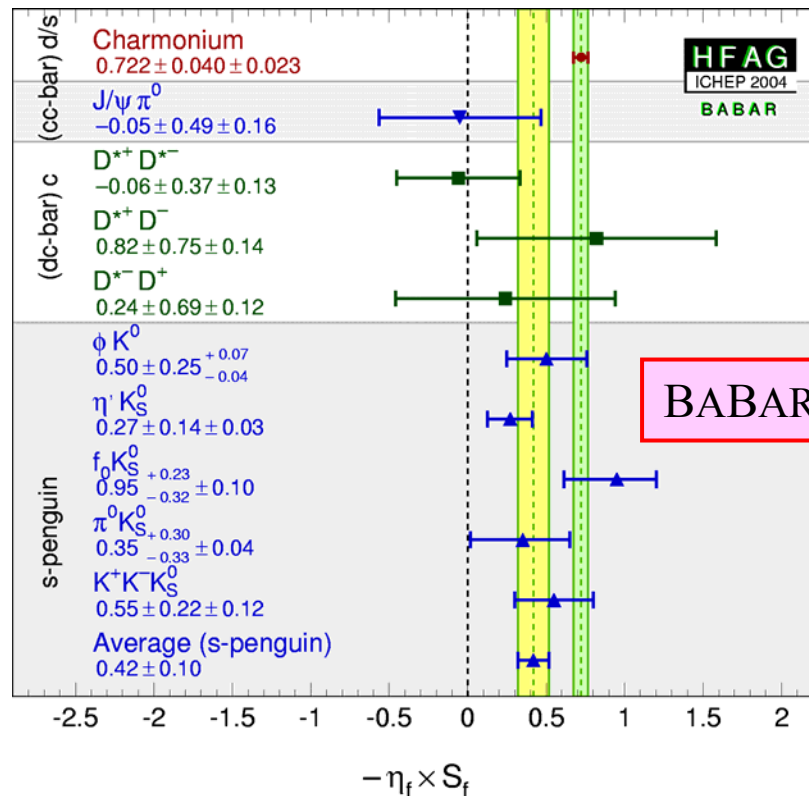
$$C_{\eta' K^0} = -0.21 \pm 0.10(\text{stat}) \pm 0.03(\text{syst})$$

S is 3.0σ from $\sin 2\beta$ from $c\bar{c}s$

Time-dependent CP asymmetry



Status of Angle β



- Measurements of CPV in s -penguin channels improving rapidly
- Average $S < \sin 2\beta$ by 2.7σ (BABAR), 2.4σ (Belle)

Angle α

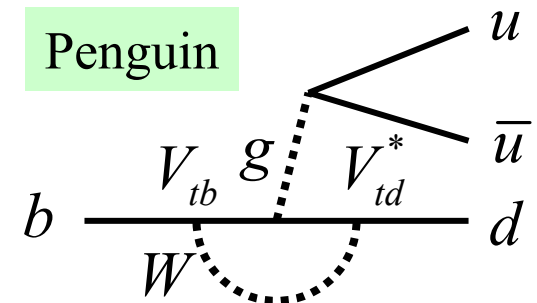
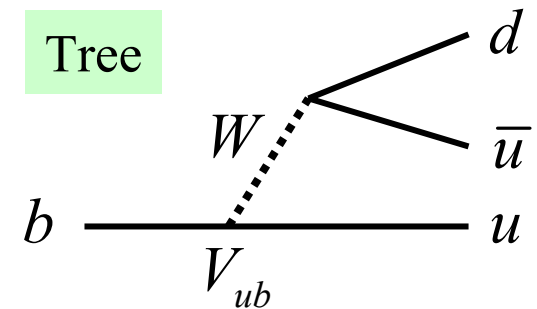
- Consider $b \rightarrow u\bar{u}d$ decay

- Example: $B^0 \rightarrow \pi^+ \pi^-$
- Decay involves $V_{ub} \rightarrow$ Weak phase γ
- CP asymmetry should measure

$$\sin(2\beta + 2\gamma) = -\sin 2\alpha$$

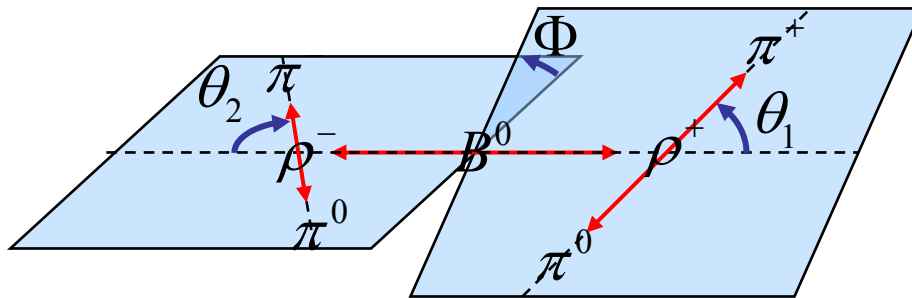
- Not so fast – There are penguins

- We can measure $\sin 2\alpha$ only if the penguins are much smaller than the tree
- It's about 1/3 in $B^0 \rightarrow \pi^+ \pi^-$
 - Not so good...
- Is there a better way?



$$B^0 \rightarrow \rho^+ \rho^-$$

- $B^0 \rightarrow \rho^+ \rho^-$ has much smaller penguin
 - Known from small $\mathcal{B}(B^0 \rightarrow \rho^0 \rho^0) < 1.1 \times 10^{-6}$ (90% C.L.) 205 fb⁻¹
 - $\mathcal{B}(B^0 \rightarrow \rho^+ \rho^-) = (30 \pm 4 \pm 5) \times 10^{-6}$
- ρ is vector meson $\rightarrow \eta_{CP}$ depends on the polarization
 - Determine the polarization from decay-angle distribution

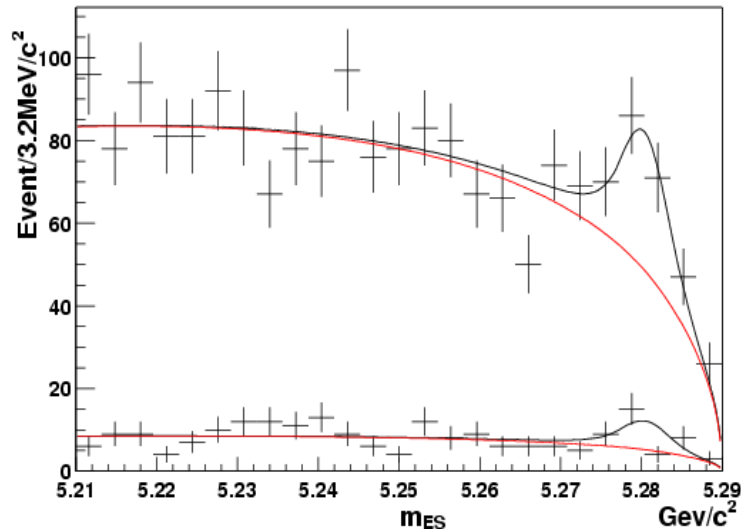


Longitudinal $\propto \cos^2 \theta_1 \cos^2 \theta_2$
 Transverse $\propto \sin^2 \theta_1 \sin^2 \theta_2$

- Longitudinal fraction $f_L = 0.99 \pm 0.03^{+0.04}_{-0.03}$
- Almost pure CP eigenstate



$B^0 \rightarrow \rho^+ \rho^-$ candidates

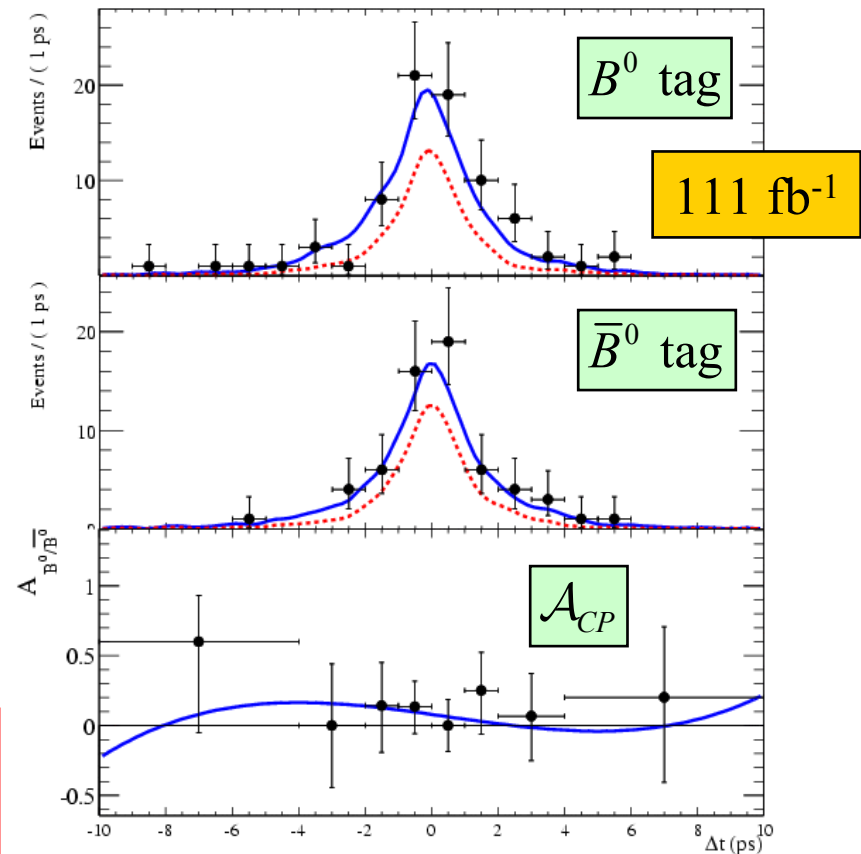


- Simultaneously fit for f_L and S , C of the long. component

$$S_{long} = -0.42 \pm 0.42(stat) \pm 0.14(syst)$$

$$C_{long} = -0.17 \pm 0.27(stat) \pm 0.14(syst)$$

Time-dependent CP asymmetry

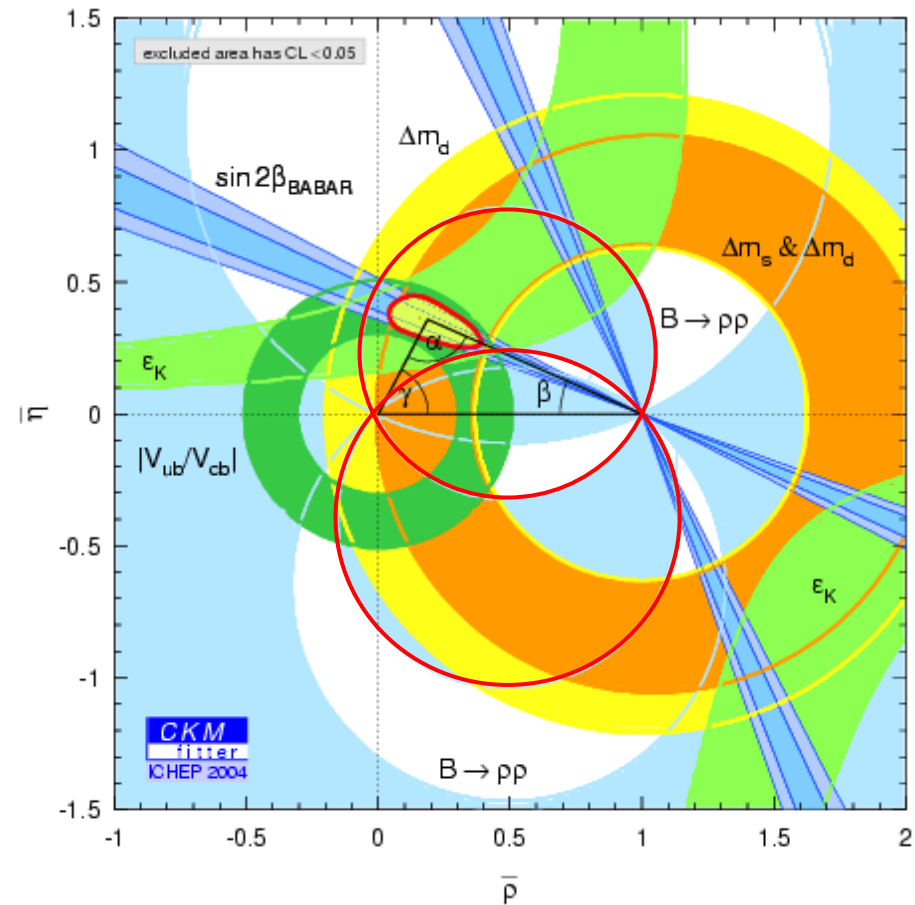


Status of Angle α

- A_{CP} in $B^0 \rightarrow \rho^+\rho^-$ combined with BF's for $B^0 \rightarrow \rho^+\rho^-$, $B^+ \rightarrow \rho^+\rho^0$, and $B^0 \rightarrow \rho^0\rho^0$

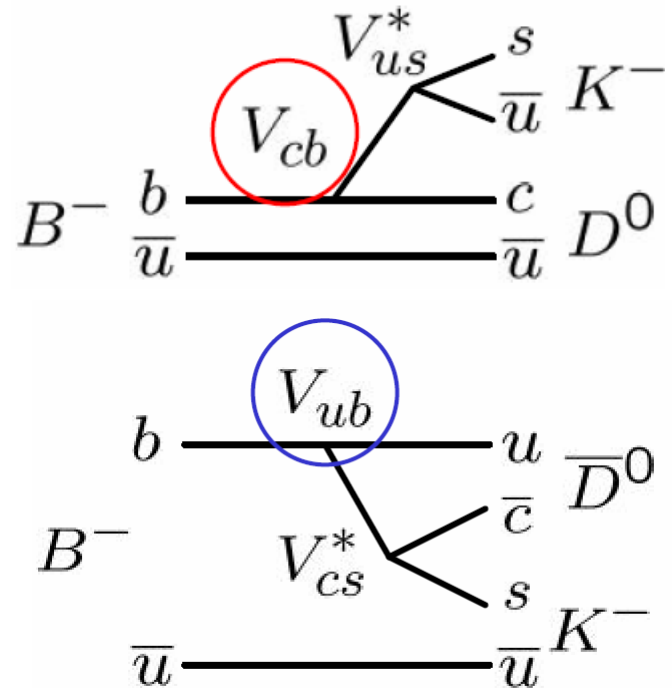
$$\alpha = \left[\begin{array}{l} 96 \pm 10(\text{stat}) \pm 4(\text{syst}) \\ \pm 11(\text{penguin}) \end{array} \right] \text{deg}$$

- A promising method for measuring α
- Agrees with the SM with a large error



Angle γ

- Hard to find a good channel that measure γ
 - A lot of not-so-good techniques are being studied
- Example: $B^- \rightarrow D^0 K^-$, $D^0 \rightarrow K_S \pi^+ \pi^-$
 - 2 decay diagrams \rightarrow
 - Weak phases differ by γ
 - If D^0 decays into a CP eigenstate, interference violates CP
- Back to the old question
 - How do we know the relative amplitudes?
 - How do we know the strong phase?



$$B^- \rightarrow D^0 K^-, \quad D^0 \rightarrow K_S \pi^+ \pi^-$$

- Suppose we know the amplitude for $D^0 \rightarrow K_S \pi^+ \pi^-$ as a function of $m_+ = m(K_S \pi^+)$ and $m_- = m(K_S \pi^-)$

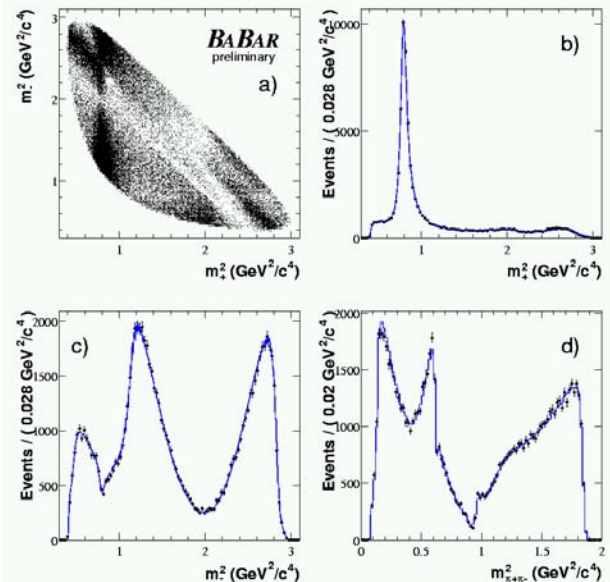
- Total decay amplitude should be

$$M_-(m_-, m_+) = f(m_-, m_+) + r_B e^{i(\delta-\gamma)} f(m_+, m_-) \quad \text{for } B^- \rightarrow K_S \pi^+ \pi^- K^-$$

$$M_+(m_-, m_+) = f(m_+, m_-) + r_B e^{i(\delta+\gamma)} f(m_-, m_+) \quad \text{for } B^+ \rightarrow K_S \pi^+ \pi^- K^+$$

- Experiments must

- Determine $f(m_-, m_+)$ from fit to the $D^0 \rightarrow K_S \pi^+ \pi^-$ Dalitz plot \rightarrow
- Fit the decay rates for B^+ and B^- and extract r_B , δ and γ



Status of Angle γ

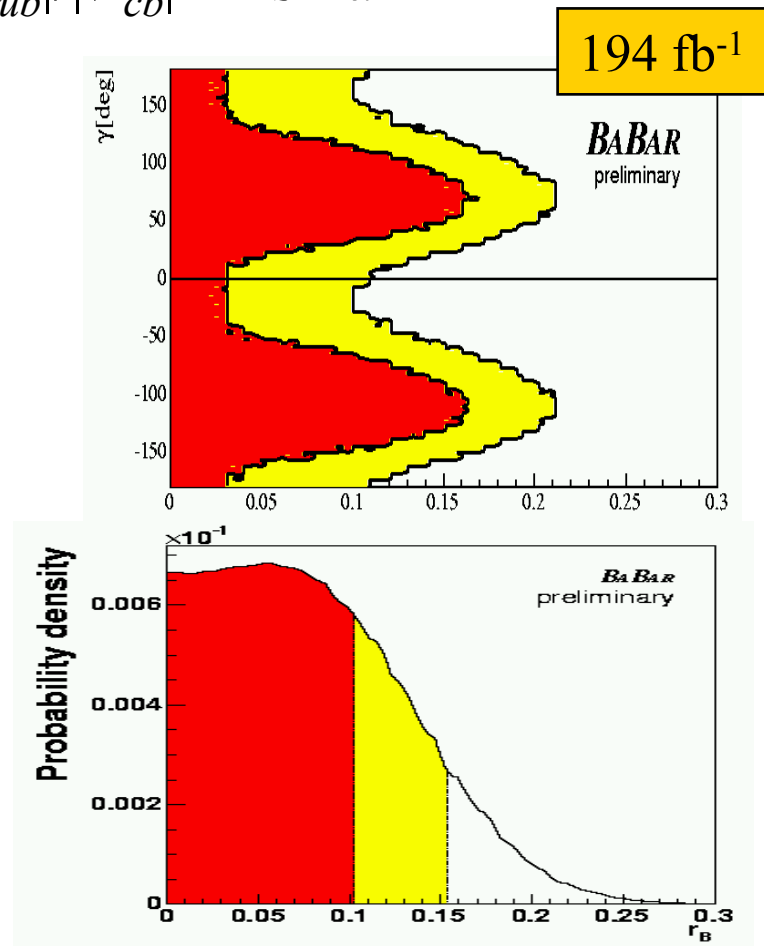
- Sensitivity is proportional to $r_B \sim |V_{ub}|/|V_{cb}| \leftarrow$ small

- No sensitivity for $r_B < 0.1$ with current statistics
- Bayesian probability with flat prior gives $r_B < 0.18$ (90% C.L.)

- Combining D^0K^- and D^0K^{*-}

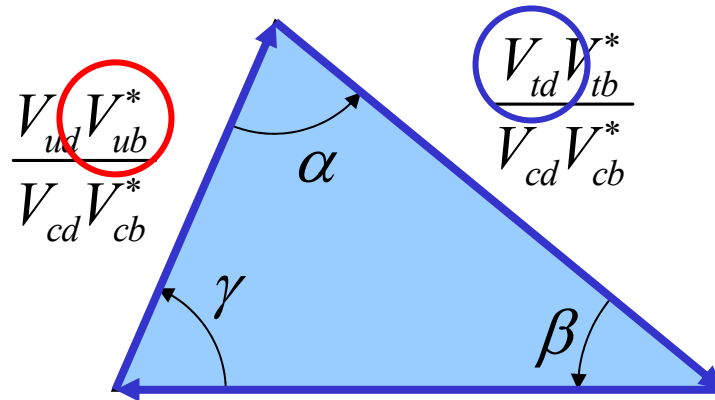
$$\gamma = (88 \pm 41 \pm 19 \pm 10)^\circ$$

- Not a precision measurement...
- Future: combine multiple methods to constrain r_B better



The Sides

- In addition to the angles, we measure the lengths of the sides

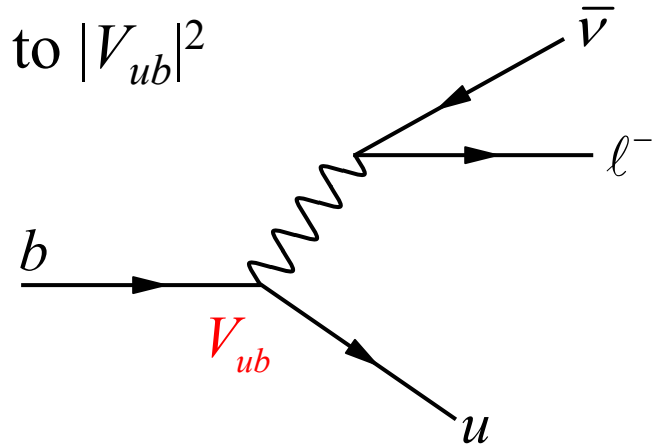


- Uncertainties of the “**left**” and “**right**” sides are dominated by the smallest CKM elements V_{ub} and V_{td}

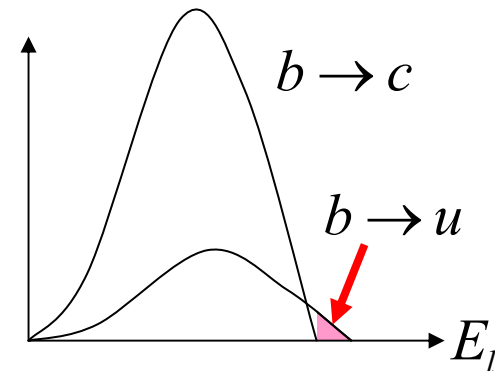
“Left” Side – $|V_{ub}|$

- Rate of $b \rightarrow u\ell\nu$ decay is proportional to $|V_{ub}|^2$
 - Leptonic part is free from strong final-state interaction

$$\frac{\Gamma(b \rightarrow u\ell\bar{\nu})}{\Gamma(b \rightarrow c\ell\bar{\nu})} \approx \frac{|V_{ub}|^2}{|V_{cb}|^2} \approx \frac{1}{50}$$



- Must suppress $50\times$ larger $b \rightarrow c\ell\nu$ decays
- Traditional approach: select events with large lepton energy
 - “Endpoint” above kinematical limit for the $b \rightarrow c\ell\nu$ decay
 - Only $\sim 6\%$ of $b \rightarrow u\ell\nu$ events are accessible



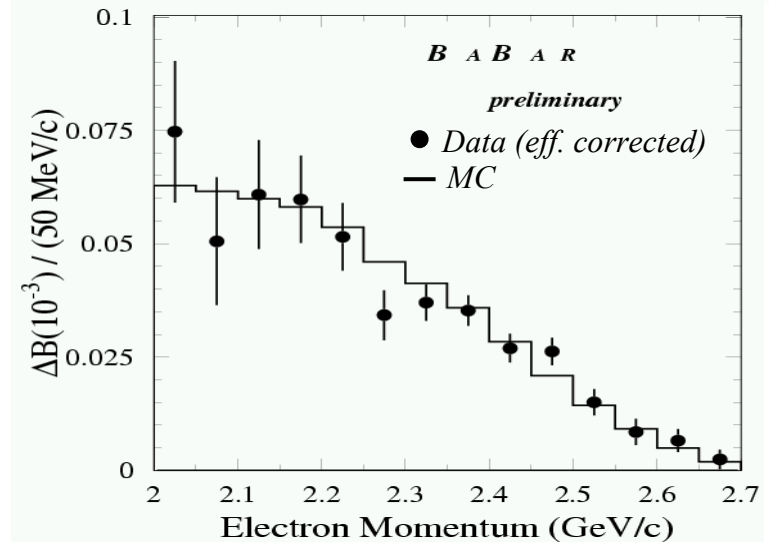
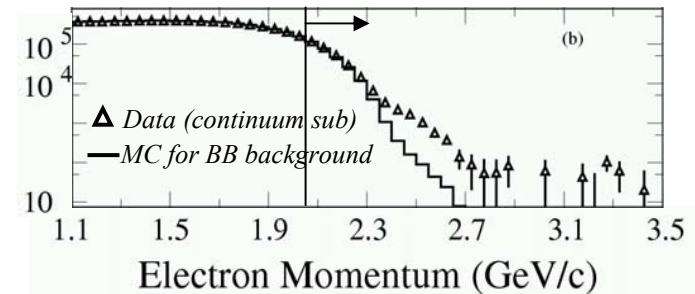
Lepton Endpoint

- Select electrons in $2.0 < E_\ell < 2.6$ GeV
 - Accurate subtraction of background is crucial!
 - Data taken below the Y_{4S} resonance for non- BB background
 - Fit the E_ℓ spectrum with $b \rightarrow u\ell\nu$, $B \rightarrow D\ell\nu$, $B \rightarrow D^*\ell\nu$, $B \rightarrow D^{**}\ell\nu$, etc. to extract

$$\mathcal{B}(B \rightarrow X_u e \nu) = (1.73 \pm 0.22_{\text{exp}} \pm 0.33_{\text{theo}}) \times 10^{-3}$$

- Turn into

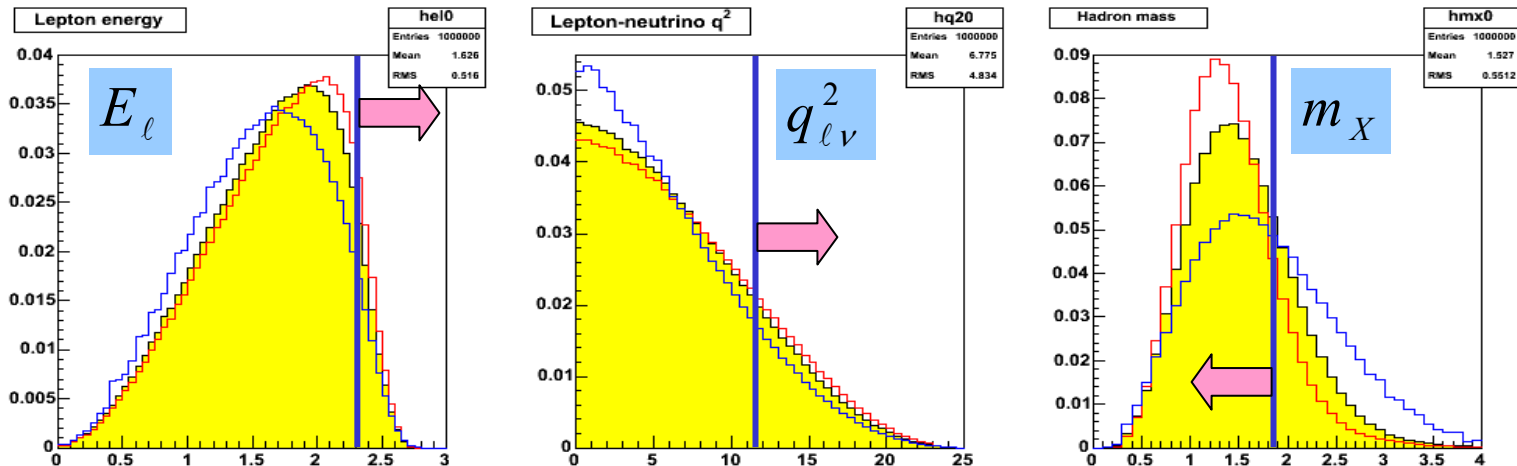
$$|V_{ub}| = (3.94 \pm 0.25_{\text{exp}} \pm 0.37_{\text{theo}} \pm 0.19_{\text{HQET}}) \times 10^{-3}$$



80 fb⁻¹

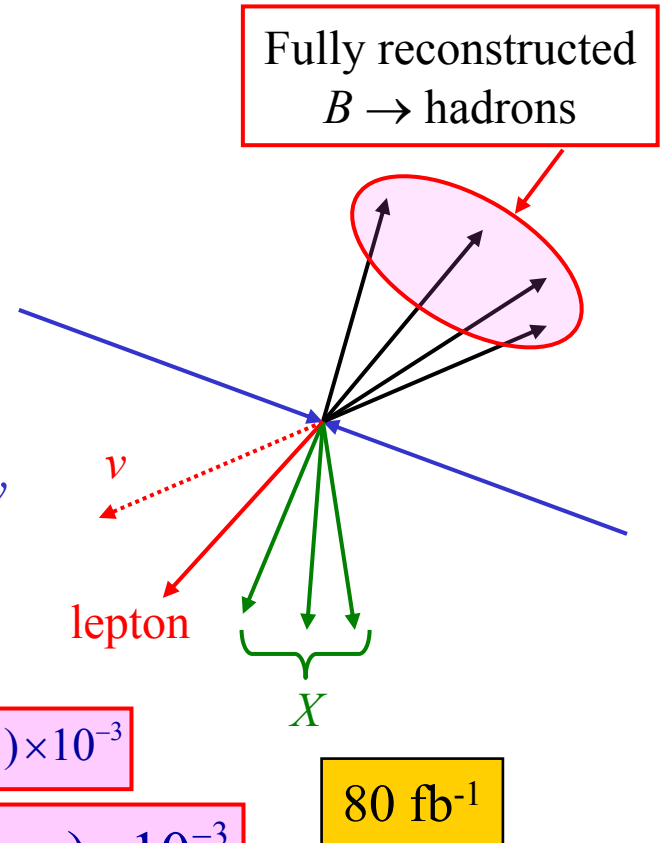
Hadronic Mass and q^2

- E_ℓ is not the only kinematic variable available
 - There are 3 independent variables
 - Consider m_X (hadronic mass) and q^2 (lepton-neutrino mass²)
- $\sim 70\%$ of $b \rightarrow ul\nu$ has $m_X < m_D$
 - High efficiency + smaller extrapolation
- Cut on q^2 is less efficient ($\sim 20\%$) but smaller theoretical errors



Recoil Method

- Must reconstruct all decay products to measure m_X or q^2
 - E_ℓ was much easier
- B mesons produced in pairs
 - Reconstruct one B in any mode
 - Rest of the event contains exactly one **recoil B**
 - Find a lepton in the recoil B
 - Remaining part must be X in $B \rightarrow X\ell\nu$
 - Calculate m_X and q^2
- For $m_X < 1.7 \text{ GeV}$ and $q^2 > 8 \text{ (GeV)}^2$

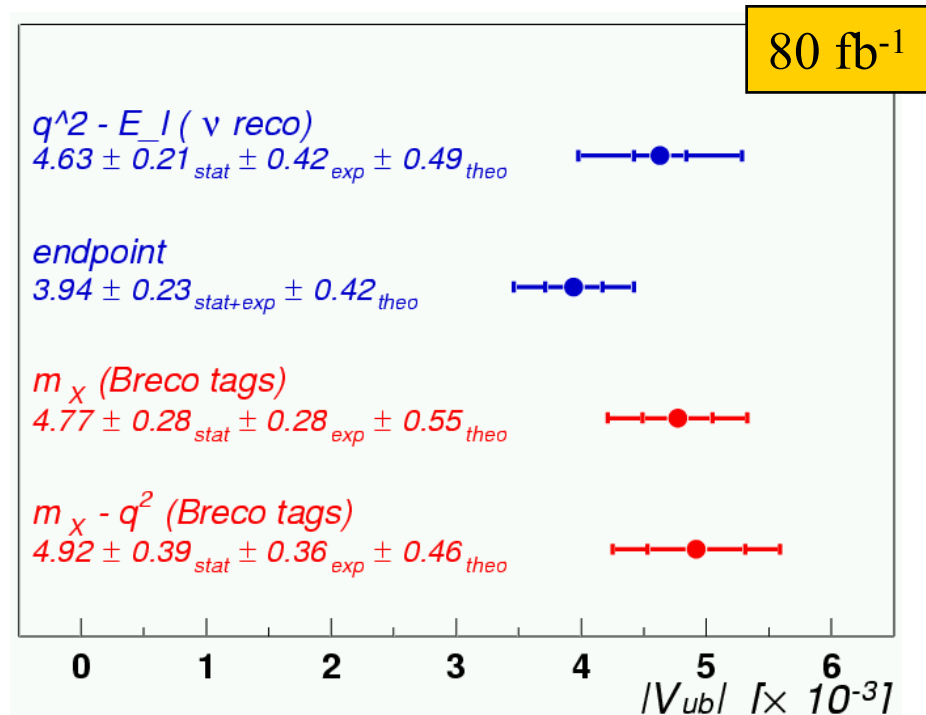


$$\Delta B(B \rightarrow X_u \ell \nu) = (0.88 \pm 0.14_{\text{stat}} \pm 0.13_{\text{exp syst}} \pm 0.02_{(m_b, a) \text{ syst}}) \times 10^{-3}$$

$$|V_{ub}| = (4.92 \pm 0.39_{\text{stat}} \pm 0.36_{\text{exp syst}} \pm 0.46_{\text{theo syst}}) \times 10^{-3}$$

Status of $|V_{ub}|$

- Different approaches for $|V_{ub}|$ pursued to tackle theoretical uncertainty
 - Largest error from the “shape function”
 - Determined by CLEO measurement of photon energy spectrum in $b \rightarrow s\gamma$
- Future improvement from
 - New techniques ← More data helps
 - Theoretical progress
 - Better measurement of $b \rightarrow s\gamma$

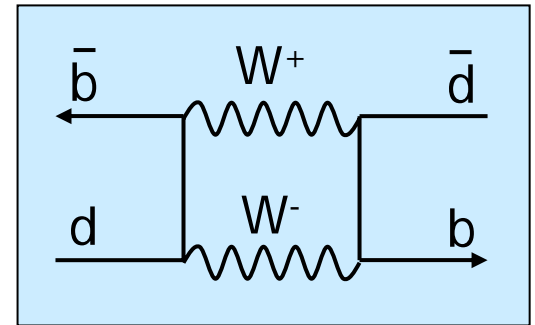


“Right” Side – $|V_{td}|$

- B^0 mixing involves virtual top exchange

- Once B_s mixing rate is measured

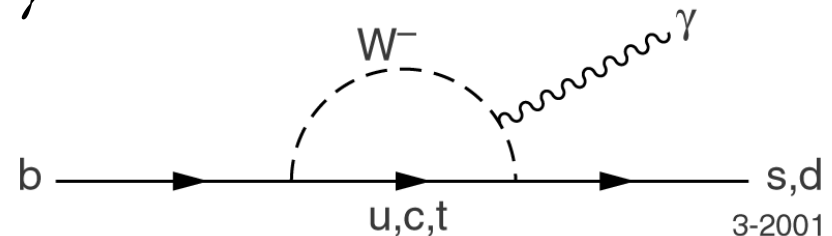
$$\begin{array}{l} B^0 \text{ mixing rate} \\ B_s \text{ mixing rate} \end{array} \Rightarrow \frac{\Delta m_d}{\Delta m_s} \approx \frac{|V_{td}|^2}{|V_{ts}|^2}$$



- What can we do besides waiting for Tevatron?

- Radiative penguin decays $b \rightarrow s/d \gamma$

$$\frac{\Gamma(b \rightarrow d\gamma)}{\Gamma(b \rightarrow s\gamma)} \approx \frac{|V_{td}|^2}{|V_{ts}|^2}$$



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- Expect

$$\mathcal{B}(B^+ \rightarrow \rho^+ \gamma) \approx \mathcal{B}(B^+ \rightarrow K^{*+} \gamma) \frac{|V_{td}|^2}{|V_{ts}|^2} = 1.6 \times 10^{-6}$$

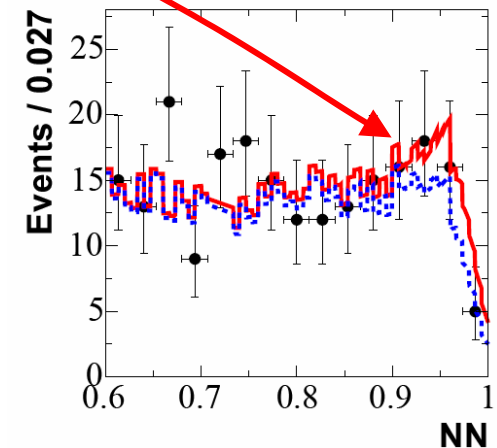
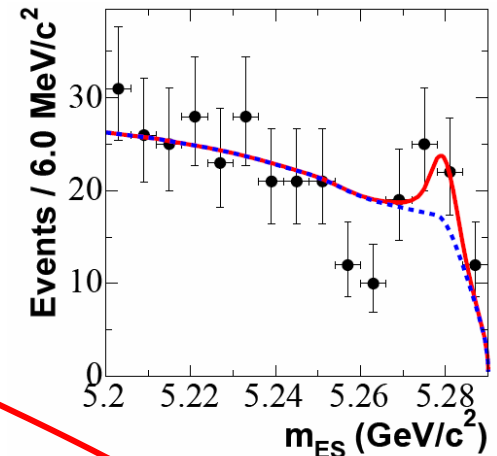
Reachable at
B Factories

$B \rightarrow \rho \gamma$

- Main background from non- BB events
 - Jet-like geometry \rightarrow Use event shape variables \rightarrow Neural Net
- $B \rightarrow K^* \gamma$ is $40\times$ larger
 - Particle ID with the Cherenkov detector
- Combine $\rho^+ \gamma$, $\rho^0 \gamma$ and $\omega \gamma$, assuming
$$\Gamma(B^+ \rightarrow \rho^+ \gamma) = 2\Gamma(B^0 \rightarrow \rho^0 \gamma) = 2\Gamma(B^0 \rightarrow \omega \gamma)$$

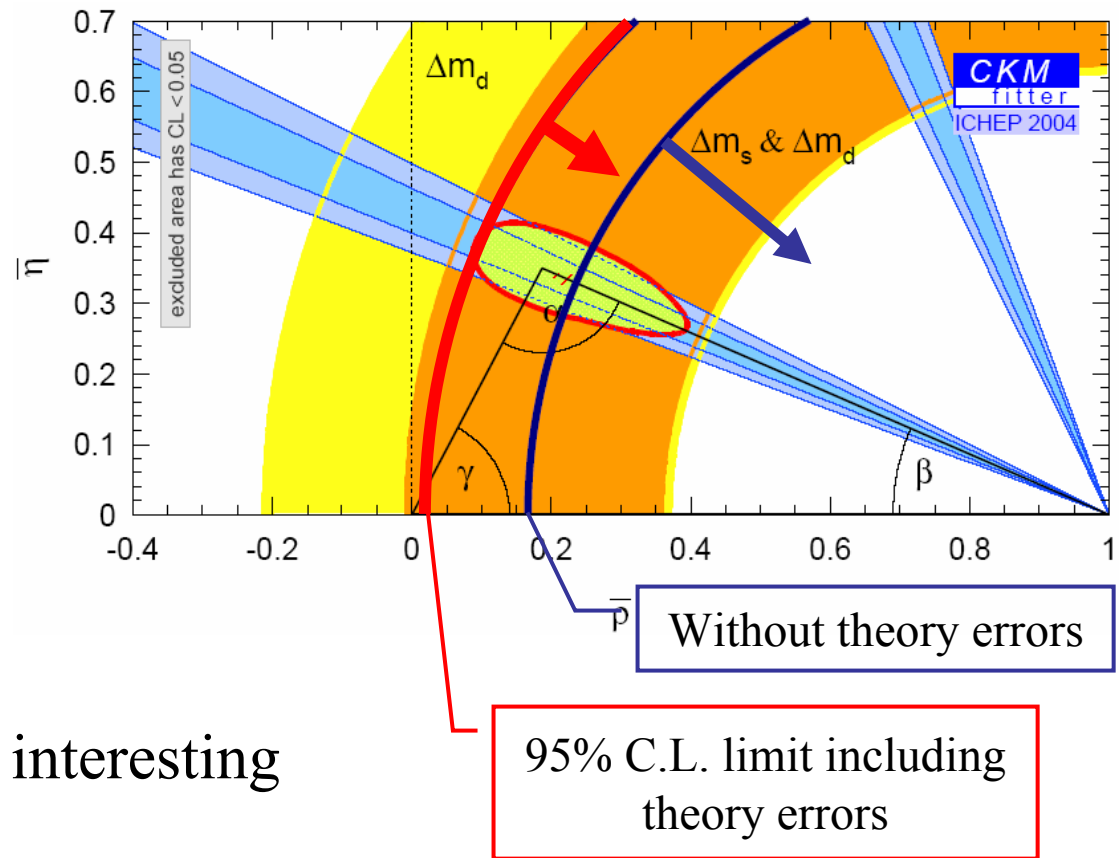
$$\mathcal{B}(B^+ \rightarrow \rho^+ \gamma) = (0.6 \pm 0.3 \pm 0.1) \times 10^{-6}$$
$$< 1.2 \times 10^{-6} \text{ (90\%C.L.)}$$

 - A little smaller than expected by the SM



Status of $|V_{td}|$

- $\mathcal{B}(B \rightarrow \rho\gamma)$ limits the length of the “right” side
 - Bound comparable to that from the mixing
- Theory errors from ρ vs. K^* form factor difference and weak annihilation
- Observing $B \rightarrow \rho\gamma$ with additional statistics will be *very* interesting



Summary

- The unitarity triangle is under attack from all directions
 - Huge data sample allow more and more measurements
- In addition to $\sin 2\beta$, many measurements are reaching interesting precisions
 - CPV in penguin decays, $B \rightarrow \rho\rho$, $B \rightarrow ul\nu$, $B \rightarrow \rho\gamma$, ...
- Will we crack the unitarity triangle?

