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Hanbury-Brown-Twiss Effect: Correlation Between Photons in Two Coherent Beams of Light

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In radio astronomy, to effectively measure the angular diameter of radio stars using an Hanbury-Brown-Twiss (HBT) interferometer, it is essential to show that the time of arrival of photons in coherent beams of light is correlated. This paper discusses an experiment aiming to measure this correlation between photons using an HBT interferometer in a laboratory setting. The experiment, despite significant progresses, did not present a positive correlation between photons as predicted in theory. This deviation from the original HBT experiment result might either be caused by detections of too much reflected light from the light source or due to the usage of a coincidence instead of a correlator to merge the signals from the two detectors.

I. INTRODUCTION

In 1950s, R. Hanbury Brown and R. Q. Twiss conducted a series of experiments based on a new type of interferometer they developed, which later earned its name, Hanbury-Brown-Twiss (HBT) interferometer. As shown in Fig. 1, the HBT interferometer is consisted of two independent radio receivers tuned to the same frequency with identical band-pass characteristics and they are separated by a distance called baseline [1]. Through a series of signal conversions, the outputs from these two receivers are eventually fed to three recorders. Two recorders independently record signals from each of the two receivers while the third recorder records the signal fed from a correlator that combines the outputs from both receivers. Unlike the famous Michelson interferometer which makes detections after the two radio signals are combined and thereby preserving their relative phase, an HBT interferometer makes signal combinations after detections and losses measurements on the phase. In other words, only the correlation in their intensity fluctuation is measured in an HBT interferometer. In practices, on the other hand, this radio intensity interferometer has its advantages. It is theoretically predicted and experimentally proved that when an HBT interferometer is used to measure the angular diameter of radio stars, it is substantially unaffected by atmospheric scintillations and ionospheric scintillations [2,3,4]. Due to these advantages, Hanbury Brown and Twiss shows that one can replace the two radio receivers by two photomultipliers (PMTs) and the two aerials by two mirrors in the HBT interferometer, as shown in Fig. 2, and this setup is more frequently used in optical experiment settings [5].

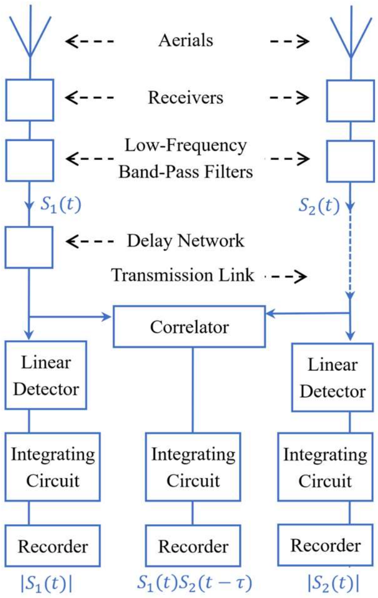

FIG. 1. Block diagram of an HBT interferometer.

When using such optical HBT interferometer to take astronomical measurements, the angular diameter of radio stars can be deduced by measuring, as a function of the distance between the two mirrors, the correlation between the signals from the two PMTs [5]. To ensure that an HBT interferometer is effective in making such measurement, it is necessary to show that, if the light beams incident on the two mirrors are coherent, then the time of arrivals of photons at the two PMTs is correlated [5].

This correlation between photons in coherent light beams had never been directly observed until Hanbury Brown and Twiss carried out a laboratory experiment and established such correlation [5]. We mimicked their experiment by setting up an HBT interferometer and measuring the correlation between photons, as discussed in this paper.

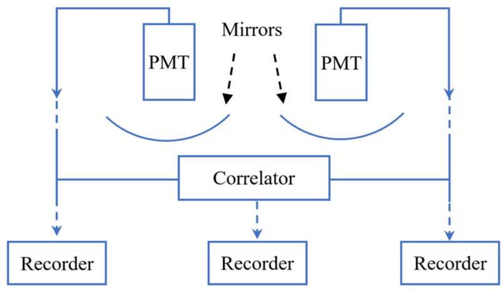

FIG. 2. Block diagram of a HBT interferometer modified for an optical laboratory setting.

II. THEORY

As shown in Fig. 3, consider a wave packet arriving from $\vec{R}_1$ at a and b within the coherence time of the photon and another wave packet arriving from $\vec{R}_2$ at a and b within the same coherence time interval:

$$A^a = A_1\cos(\vec{k}_1\cdot\vec{r}_a - \omega t + \varphi_1) + A_2\cos(\vec{k}_2\cdot\vec{r}_a - \omega t + \varphi_2)$$
$$A^b = A_1\cos(\vec{k}_1\cdot\vec{r}_b - \omega t + \varphi_1) + A_2\cos(\vec{k}_2\cdot\vec{r}_b - \omega t + \varphi_2)$$

The intensity of signals at a and at b, measured by PMTs, are:

$$I_a = (A^a)^2$$
$$I_a = (A^a)^2$$

Since the PMTs does not preserve optical frequency in their detections of photons, we take out ωt from the equations and obtain:

$$I_a = \frac{1}{2}(A_1^2 + A_2^2) + A_1A_2\cos((\vec{k}_1 - \vec{k}_2)\cdot\vec{r}_a + \varphi_1 - \varphi_2)$$
$$I_b = \frac{1}{2}(A_1^2 + A_2^2) + A_1A_2\cos((\vec{k}_1 - \vec{k}_2)\cdot\vec{r}_b + \varphi_1 - \varphi_2)$$

A statistical average of I_a and I_b over many coherence time intervals, gives:

$$\langle I_a\rangle = \langle I_b\rangle = \frac{1}{2}(A_1^2 + A_2^2)$$

Similarly, statistical average of product of I_a and I_b is:

$$\langle I_aI_b\rangle = \frac{1}{4}(A_1^2 + A_2^2)^2 + {A_1}^2{A_2}^2\cos((\vec{k}_1 - \vec{k}_2)\cdot(\vec{r}_a - \vec{r}_b))$$

Thus, the correlation between intensity of signals at a and at b is:

$$Cr \propto \frac{\langle I_aI_b\rangle}{\langle I_a\rangle\langle I_a\rangle} = 1 + \frac{2\,{A_1}^2{A_2}^2}{(A_1^2 + A_2^2)^2}\cos((\vec{k}_1 - \vec{k}_2)\cdot(\vec{r}_a - \vec{r}_b))$$

The mathematical deduction written above are for two light sources. Using a similar manner, one can deduce that, as a function of the distance between the detectors B, the correlation for a light source in the shape of a circular disk with radius R is:

$$Cr(B) \propto \frac{\langle I_aI_b\rangle}{\langle I_a\rangle\langle I_a\rangle} = 1 + \left(\frac{2\,J_1\left(\frac{kRB}{L}\right)}{\frac{kRB}{L}}\right)^2$$

where k is the wave number of the incident light, L is the distance from the light source and the detectors, and $J_1(x)$ is the first order Bessel function. Since the signal intensity correlation coefficient needs to be in the range of 0 to 1, we adjust this formula and obtain:

$$Cr(B) = \frac{\langle I_aI_b\rangle}{\langle I_a\rangle\langle I_a\rangle} - 1 = 4\times\left(\frac{J_1\left(2\pi\frac{RB}{\lambda L}\right)}{2\pi\frac{RB}{\lambda L}}\right)^2$$

where λ is the wave length of the incident light.

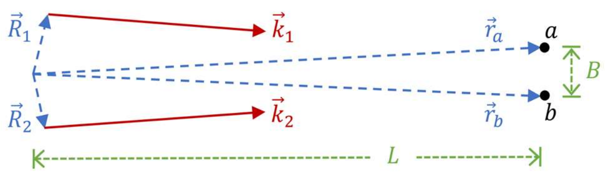

FIG. 3. An illustration of two wave packets arriving at a and b.

III. APPARATUS

We used the Dolan-Jenner MH-100 Metal Halide Fiber Optic Illuminator as our light source, which has a spectral output ranging from 360 nm to 630 nm [6]. We guided the light into an optical case using a Dolan-Jenner single, straight, flexible light guide, which has 7.9 mm in diameter [7]. We used a 546 nm light color filter to filter the light, a neutral density filer to reduce the light intensity, and a circular aperture to reduce the light source diameter down to 200 µm.

We used a half-silvered mirror to split the light beam, and used two Hamamatsu R329-02 Photomultiplier Tubes (PMTs) as our detectors, which are powered by a High Voltage (HV) Supply. At an HV supply of 2.7 kV, the PMT has a gain of $10^{7.5}$ [8].

We used a Tektronix TDS 380 Digital Real-Time Oscilloscope to monitor the signal waveform, and used a Hewlett Packard 8011A Pulse Generator to generate testing signals.

For signal processors and recorders, we used a LeCroy Octal Discriminator, a LeCroy Quad Coincidence, and a CAEN Mod. N1145 Quad Scaler and Preset Counter/Timer. Multiple Lemo cables and RG-58 cables were also used to connect the apparatus.

IV. EXPERIMENTAL PROCEDURE

We setup the apparatus in the optical case in the way shown in Fig. 4. We first setup the half-silvered mirror and two PMTs. We called the PMT that detected transmitted light "PMT-T", and the PMT that detected reflected light "PMT-R". PMT-T was fixed on the case while PMT-R was mounted on a horizontal slide. Then we connected cables and power cords onto the PMTs. We also covered the PMTs with circular apertures, whose diameters were measured. Then we adjust the height of the PMTs and mirror to ensure that they were on the same horizontal level. Then we fixed the light guide, the color filter, and the light source onto an optical stand and also ensure that the light source aperture was of the same height as the two PMT apertures. We measured the distance between the light source aperture and the mirror and also the distance between the mirror and the PMT-T, and then added up these two measurements to obtain L. Finally, we adjusted the position of the PMT-R to a place such that its image in the mirror superimposed with that of PMT-T, and record this PMT-R position as x_0.

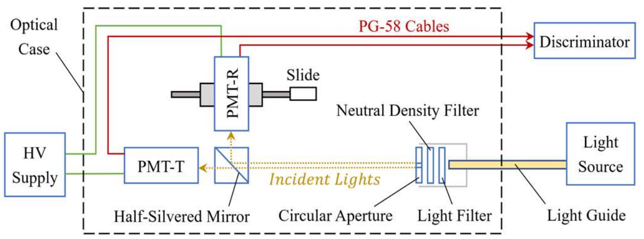

FIG. 4. Experimental setup in the optical case. Notice that the image of PMT-R overlaps with that of PMT-T in the mirror, and effectively resulting in a very small distance (B) between the two detecting points. We changed this effective distance using a slide.

We then used Lemo cables to connect the discriminator, the coincidence, and the counter together. Using the pulse generator, we sent testing signals to these signal processors to ensure that they function properly. We used the oscilloscope to monitor the signal waveform sent by the discriminator and the coincidence, and adjusted the width of these square wave signal to be 50 ns.

Then we covered the optical case with a lid and a black cloak. Then we turned on the high voltage supply, fed the signals from the PMTs to the discriminator using RG58 cables, and started monitoring the waveform on the oscilloscope. We then adjusted the voltage level on the high voltage and subsequently found that we could achieve high efficiency of the PMT when we set the voltage at 2.7 kV. The oscilloscope showed that at this level basically all of photon signal has a height no less than 30 mV. Thus, we set the threshold of the discriminator at 30 mV.

We then turned off the room light in the lab and performed a light test on the optical case to find potential light leaks. I used a torch to shine on different areas of the case while my collaborator stared on the oscilloscope screen without knowing where on the optical case I was shining the light. He would call out "light leak" whenever he saw active signals on the screen, and I would label the corresponding area on the case and sealed it with black tapes. We performed several rounds of light test until no more light leaks were found.

Then we connected the cables in the manner we intended for this experiment (Fig. 5). For each photoelectric signal from the two PMTs fed to the discriminator, the discriminator generated a square-wave signal that had a width of 50 ns. The discriminator generates two sets of signals, one for PMT-T and one for PMT-R, which were then fed to the coincidence. Based on these two sets of input signals, the coincidence generated four sets of signals, which were PMT-T signals, PMT-R signals, AND signals, and OR signals. Specifically, AND signals were generated in the presence of both input PMT-T and PMT-R signals, while OR signals were generated in the presence of either input PMT-T signals or input PMT-R signals. These four sets of output signals were then sent to the counters, which counted the number of square waves during a preset interval of time. From the readings of these counters, we recorded down corresponding counting data under the names, "T", "R", "AND", and "OR".

With all these properly set up, we were ready to take measurements. With the light source off and the room light off, we first counted the background signal during a 5-minute interval, and got 6-digit counting results. Then, we turned on the light source and took the counts. As the counts quickly jumped to 8 digits within 10 s, it turned out that the background signal was insignificant.

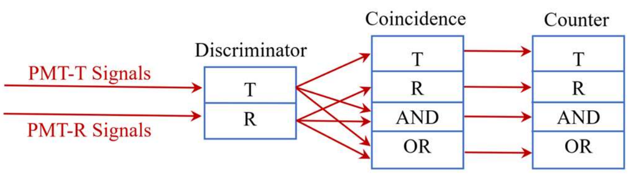

FIG. 5. Scheme of Lemo cable connections and data processing.

We recorded multiple sets of counts as we adjusted the horizontal position of PMT-R using the slide. To do this, each time we first turned down the high voltage supply down to 0, opened the optical case, adjusted and recorded the position of PMT-R, closed the case, turned up voltage supply back to 2.7 kV, and then did the counting. Each time we changed the position of the PMT-R by 0.025 inch. We repeated this process until we have 30 sets of data, each corresponding to a certain PMT-R position.

Based on these results, we performed a rough data analysis, which is detailed in the next section of this paper, and we subsequently realized that the intensity of the light source was too high. Thus, we used a neutral density filter to reduce the intensity of the light source. We then repeat the measuring process and obtained another 11 sets of data.

V. DATA ANALYSIS

For each of the first 30 sets of data, we have about 63 million counts for each PMT within 10 s, which correspond to a signal frequency of 6.3 MHz or a period of about 160 ns, assuming that the pulses in the signal are evenly distributed. Since the width of the discriminator signal is 50 ns, even if the signals from the two PMTs are not correlated, the predicted probability of incurring a coincidence count between PMT-T and PMT-R, as measured by the AND counter, is as high as $2 \times 50/160 = 0.625$. This predicted probability roughly matches our AND counts in experiment, which is about 33 million counts within 10 s. In other words, at a signal frequency this high, we could not determine whether the photons are correlated or not, so we lowered the intensity of the light source. The following data analysis is performed using the new 11 sets of data collected after we reduced the light intensity.

The background signal was insignificant because our counts in background signal was no more than 7400 per 10 s, which is less than 0.85% of any PMT-R counts and PMT-T counts.

From the data, we aim to derive the photon correlation as a function of PMT-R position, x. The theory predicts that the correlation

$$Cr(x) = \frac{\langle I_a I_b \rangle}{\langle I_a \rangle \langle I_a \rangle} - 1 = 4 \times \left(\frac{J_1\left(2\pi \frac{R}{\lambda L}(x - x_0)\right)}{2\pi \frac{R}{\lambda L}(x - x_0)} \right)^2$$

where x_0 is the PMT-R position such that the images of the two PMTs precisely superimpose with each other in the mirror and the correlation reaches its peak at 1. In the experiment, we had

$$\lambda = 546 \text{ nm},$$
$$R = 100 \text{ µm},$$

and measured

$$L = 575.5 \pm 0.5 \text{ mm},$$
$$x_0 = 1.00 \pm 0.01 \text{ inch},$$

with their systematic errors. Using these parameters, we have a curve of predicted correlation $Cr(x)$ as a function of PMT-R position x, as shown in Fig. 6 (a).

We considered several different ways of interpreting the counter data into the correlation coefficient. The most plausible way we consider is:

$$Cr = \frac{{Counts_{\mathrm{AND}}}^2}{Counts_{\mathrm{R}} \cdot Counts_{\mathrm{T}}}$$

since R and T count the number of photons detected by PMT-R and PMT-T, and AND counts the number of times that both PMTs simultaneously detect a photon. Using this interpretation of correlation, we fit the predicted curve to the data using CERN ROOT, but the curve fits very poorly and obtains an extremely large χ^2, as shown in Fig. 6 (b).

Though indicating almost zero correlation, the data points in Fig. 6 (b) show a pattern, which is in the shape of a "bell" curve. An analysis of the data reveals that this pattern does not necessarily imply a change in correlation and it shows up merely because the R counts have the same pattern, as presented in Fig. 6 (c). A linear regression shows that AND counts increases linearly with the R counts in our experiment (Fig. 7), meaning that, by demanding

$$Counts_{\mathrm{AND}} = \mathrm{C} \cdot Counts_{\mathrm{AND}} \text{ ,}$$

in which C is a constant, we can rewrite:

$$Cr = \frac{\mathrm{C}^2 \cdot {Counts_{\mathrm{R}}}^2}{Counts_{\mathrm{R}} \cdot Counts_{\mathrm{T}}} = \frac{\mathrm{C}^2}{Counts_{\mathrm{T}}} \cdot Counts_{\mathrm{R}} \text{ .}$$

In the experiment, the position of PMT-T did not change, meaning that we should, and indeed did, obtain $Counts_{\mathrm{T}}$ that were roughly constant in all 11 sets of data. As a result, the above equation then yields a linear relation between Cr and $Counts_{\mathrm{R}}$, which explains the similarity in the patterns of Cr and $Counts_{\mathrm{R}}$ as x changes.

Since the measured Cr values are all closed to zero and the variation in $Counts_{\mathrm{R}}$ accounts for most of the variation in measured Cr, we failed to establish a positive correlation between the photons.

There are several possible reasons that may explain the absence of the positive correlation in our experiment. One reason could be that the light was not focused and the PMTs detected not only photons directly coming out of the light source but also many photons that were caused by reflections in the optical case. These many photons "bumped" around in the case before they got detected, and since their timings of arrival is random, the resulting correlation is low.

Another possible reason could be that we used a coincidence counter to correlate the signals from the two PMTs instead of using a correlator like the way Hanbury Brown and Twiss did in their original design [2]. As shown in the original HBT interferometer design (Fig. 1), signals from the two PMTs are merged by the correlator before they enter linear detectors. The correlator effectively produces signals that mimic $I_a I_b$, which are not necessarily square waves. In our experiment, however, signals from the two PMTs were merged by the coincidence after they passed through the discriminator (Fig. 5). The discriminator transformed the signal into square waves, which are then correlated using the coincidence. Since the mechanism is different from the original HBT design, the AND counts from our experiment may not be a good measure of $I_a I_b$.

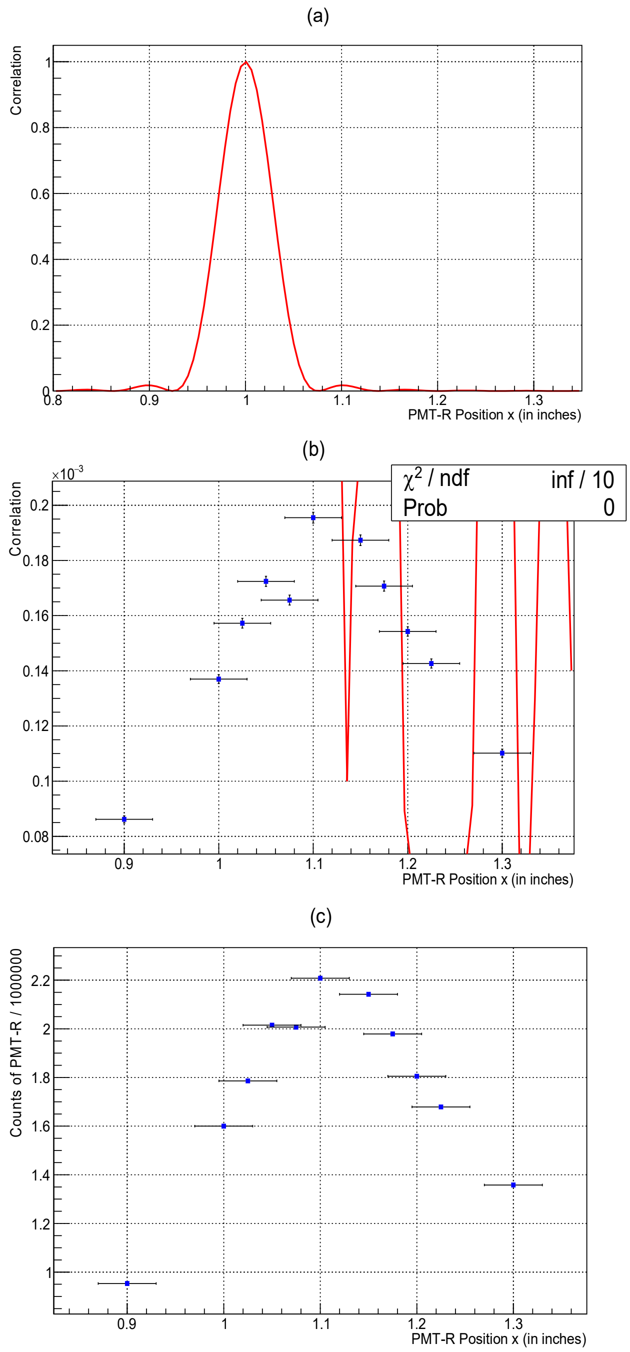

FIG. 6. Plots of predicted and measured values. (a) Predicted value of correlation as a function of PMT-R position, x, using parameters measured in experiment. We made this plot before we took measurements so that we could control the resolution of our measurement and determine the appropriate amount of x we want to increase as we took data to ensure that we did not miss the peak of the correlation pattern. (b) Least-square fit of the expected value to the measured correlation using CERN ROOT. The error bar of the correlation is obtained by propagating the statistical errors of the "T", "R", and "AND" counts, while each counts error is derived by taking square root of the measured counts. The error bars of the independent variable, x, correspond to systematic errors when taking measurements. (c) Plots of "R" counts as a function of x. The error bars are calculated similarly to (b).

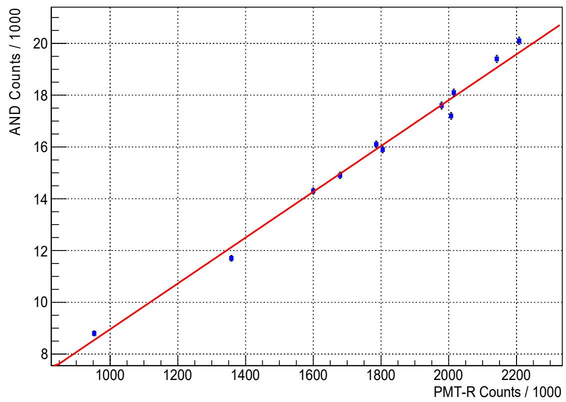

FIG. 7. Plot of "AND" counts as a function of "R" counts reveals a linear relationship between them.

There are also other systematic errors in our experiment, including errors arising from the measurements for L and x_0, the precision of the aperture radius R, the range of the frequency of the light that got filtered out by the light filter, and the difference in the gain between the two PMTs. We also could not completely eliminate the background noise or ensure that the two PMTs were perfectly at the same height. All the systematic errors discussed here, however, have an effect on the measurement greatly smaller than the two factors discussed in the former two paragraphs, and the reductions of these systematic errors are also unlikely to reverse the result of observing a zero photon correlation.

VI. CONCLUSION

We performed an experiment aiming to measure this correlation between photons using an HBT interferometer in a laboratory setting. The experiment, however, failed to establish a positive correlation between photons as predicted in theory. To effectively improve the experiment, one might consider making the light focused and reducing the light reflections in the optical case, or consider using a correlator to merge the PMT signals like Hanbury Brown and Twiss did in their original experiment [2,3].

VII. ACKNOWLEDGEMENTS

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