Measuring the Verdet constant through Faraday rotation

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***Abstract*—The objective of this lab is to use Faraday rotation of a laser in a solenoid wrapped around a sample of SF-59 glass to calculate the Verdet constant. We used the extinction method and the Malus' Law method and our best result was 25.4 $\pm$ 0.19 rad/Tm, compared to the referenced value of 23 rad/Tm [1].**

I. Introduction

As polarized light goes through a magnetic field, the angle of polarization will rotate. Michael Faraday discovered this when he applied a magnetic field in the same direction as the path of light. The rotation depends on the length of the material, the B field, and the Verdet constant. We measured the Verdet constant in two ways, by measuring the change in polarization angle directly and by measuring the change in voltage to obtain the change in angle. For the extinction method, we measured the initial angle of the polarizers with no current through the solenoid, then measured the change in angle for different applied currents. With this data we used the equation

$$v = \frac{\theta}{Bl} \tag{1}$$

where

$$B = (11.1 * 10^{-3} mT/A)I \tag{2}$$

and theta is the change in polarization angle, and l is the length of the glass. For the Malus' Law method, we measured the initial voltage with no current, and the voltage for different applied currents. We then used the equation

$$v = \frac{45(\frac{\pi}{180}) - cos^{-1}(\sqrt{\frac{V}{2V_o}})}{Bl} \tag{3}$$

where the numerator is equal to the change in polarization angle by the relationship

$$I = I_o cos^2(\theta) \tag{4}$$

assuming a linear relationship between intensity and voltage.

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II. Experimental Set-up

We used a 642 nm laser and bounced the light off 3 mirrors. The light then goes through a chopper at 76 Hz to reduce the amount of light coming in. The light goes through a polarizer, then an SF-59 glass wrapped in a 15 cm solenoid. The light then goes through an analyzer and photodiode, which is connected to an oscilloscope. The solenoid is connected to a DC power source. The photodiode is connected to a pre-amplifier to increase the signal and a lock in amplifier to reduce the noise.

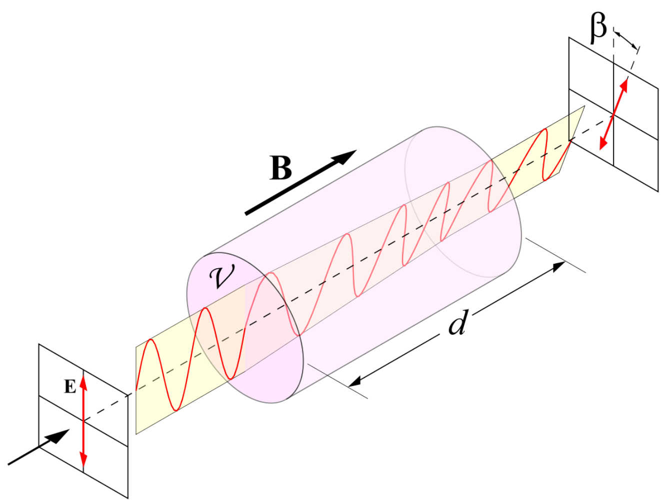

Fig. 1. Schematic representation of Faraday rotation [4]

For the extinction method, we set the two polarizers orthogonal to each other to allow for maximum extinction. For the Malus' Law method, we could not measure the initial intensity directly since it overloaded the lock-in amplifier, so we set the polarizers at 45° with respect to one another and adjusted our equation to include the 45° offset.

III. Data and Error Analysis

For the extinction method we plotted a weighted average of our points for 4 trials on a graph of Current vs Change in Angle (Fig. 2) and used the numpy library in Python to estimate a best fit straight line:

$$y = mx + b \tag{5}$$

where

$$m = 1.49 \pm 0.16 \quad (6)$$

$$b = 0.01 \pm 0.28 \quad (7)$$

and where the slope, when converted to radians and divided by the magnetic field and the length of the solenoid as in eq (1) gives the Verdet constant:

$$\bar{v} = 16.5 \pm 0.16 \frac{radians}{teslameter} \quad (8)$$

We used the following formula to obtain the weighted error for the Verdet constant [3]:

$$\sigma_{\bar{x}} = \sqrt{1/\sum \sigma^{-2}} \quad (9)$$

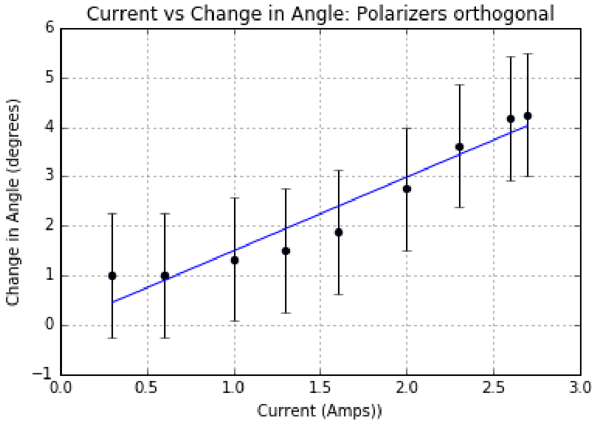

Fig. 2. Plot of Current vs Change in Angle using extinction method when polarizers are set to 90° difference

After the experiment we did a mathematical least squares fit. We calculated chi square for the extinction method and got a value of:

$$\mathcal{X}^2 = 6 \pm 4.0 \quad (10)$$

For the Malus' Law method, we used the following formula to calculate change in polarization angle:

$$\theta = \frac{45(\frac{\pi}{180}) - cos^{-1}(\sqrt{\frac{V}{2V_o}})}{Bl} \quad (11)$$

We then plotted our values for Current vs Change in Angle (Fig. 3).

From this graph we calculated the line of best fit:

$$y = mx + b \quad (12)$$

where

$$m = 2.71 \pm 0.03 \quad (13)$$

$$b = 0.11 \pm 0.05 \quad (14)$$

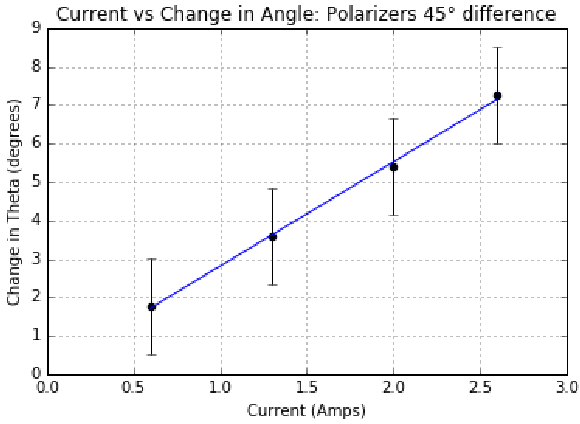

Fig. 3. Plot of Current vs Change in Angle using Malus'Law when polarizers are set to 45° difference

The slope, when converted to radians and plugged into eq (1) yields a Verdet constant of

$$v = 25.4 \pm 0.19 \frac{radians}{teslameter} \quad (15)$$

The chi square for the plot using Malus' Law is

$$\mathcal{X}^2 = 3.0 \pm 2.0 \quad (16)$$

IV. Discussion

We compared our results to the referenced value for a sample of SF-59 glass which is 23 rad/Tm [1]. The percent error for the Malus' Law method is 10 percent. Our Malus' Law value of 25.4 rad/Tm is closer to this value than the extinction method value of 16.5 rad/Tm. This could be due to the fact that when the polarizers are set to 45 relative to each other, the intensity readings are larger and less susceptible to noise. One source of error for the extinction method was the difficulty in reading the correct angle, which yielded a large uncertainty of 1.25 degrees. Another source of error could be that the chopper was not operating at the exact frequency we set it to. There was a systematic error of the oscilloscope reading fluctuating by 0.04 Volts when set to Volts and 20 mV when set to mV. The reading of the current also fluctuated by about 0.02 Amps. The error of the oscilloscope reading and the current source are on the order of 1 magnitude smaller than the uncertainty in the calculated Verdet constant. We propagated these errors through the calculations while deeming the uncertainty in the measurement of the length of the solenoid negligible.

V. Conclusion

This experiment attempted to calculate the Verdet constant by applying current to a solenoid and by measuring change in polarization angle using the extinction method and the Malus' Law method. Our result of 25.4 rad/Tm was within 10 percent of the referenced value.

REFERENCES

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[4] Faraday Effect. https://en.wikipedia.org/wiki/Faraday_effect.

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