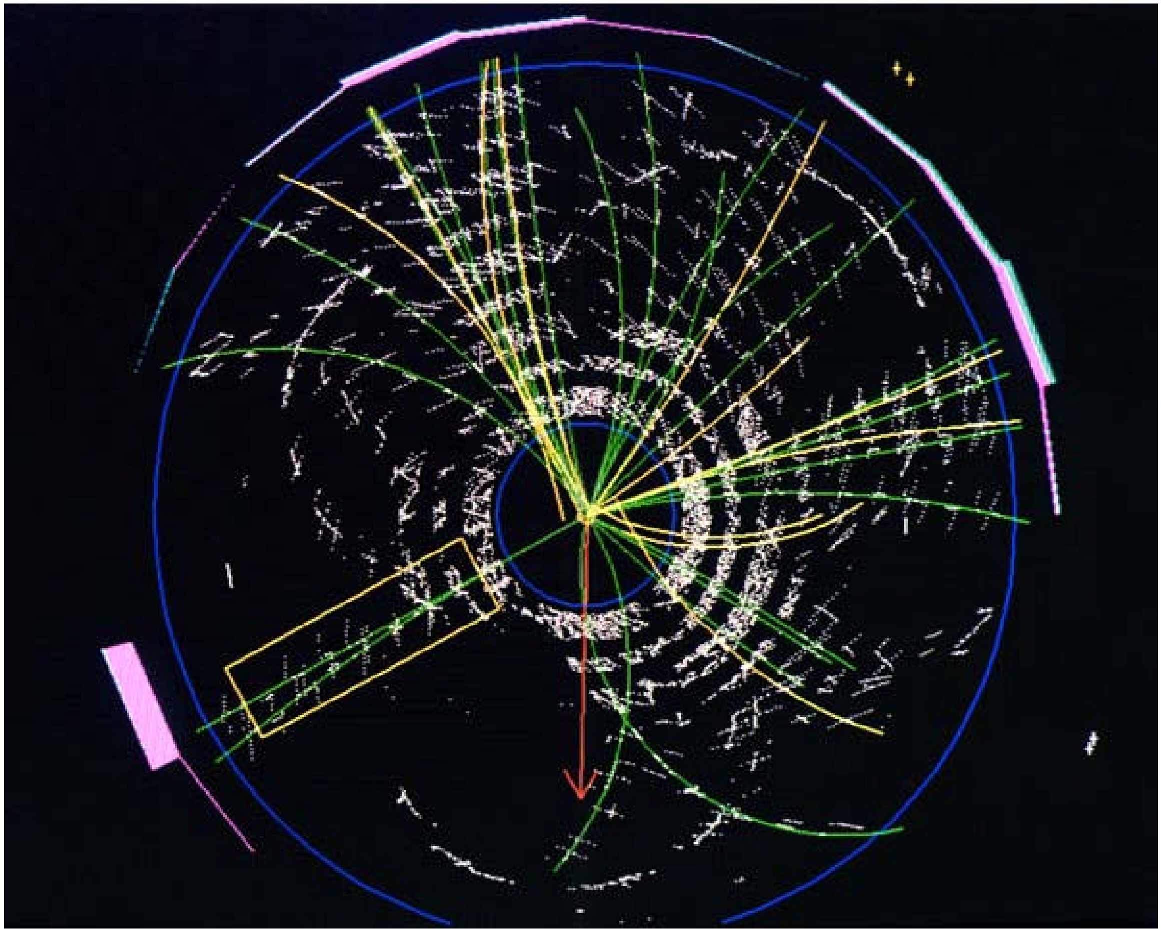
Tracking

Tracking: Find charged particles in event and precisely measure the directions and magnitude of their momenta, that is, $\vec{p}$.



$p\ \bar{p}$ ® $t\ \bar{t}\ X$

from CDF

Different Aspects of Tracking

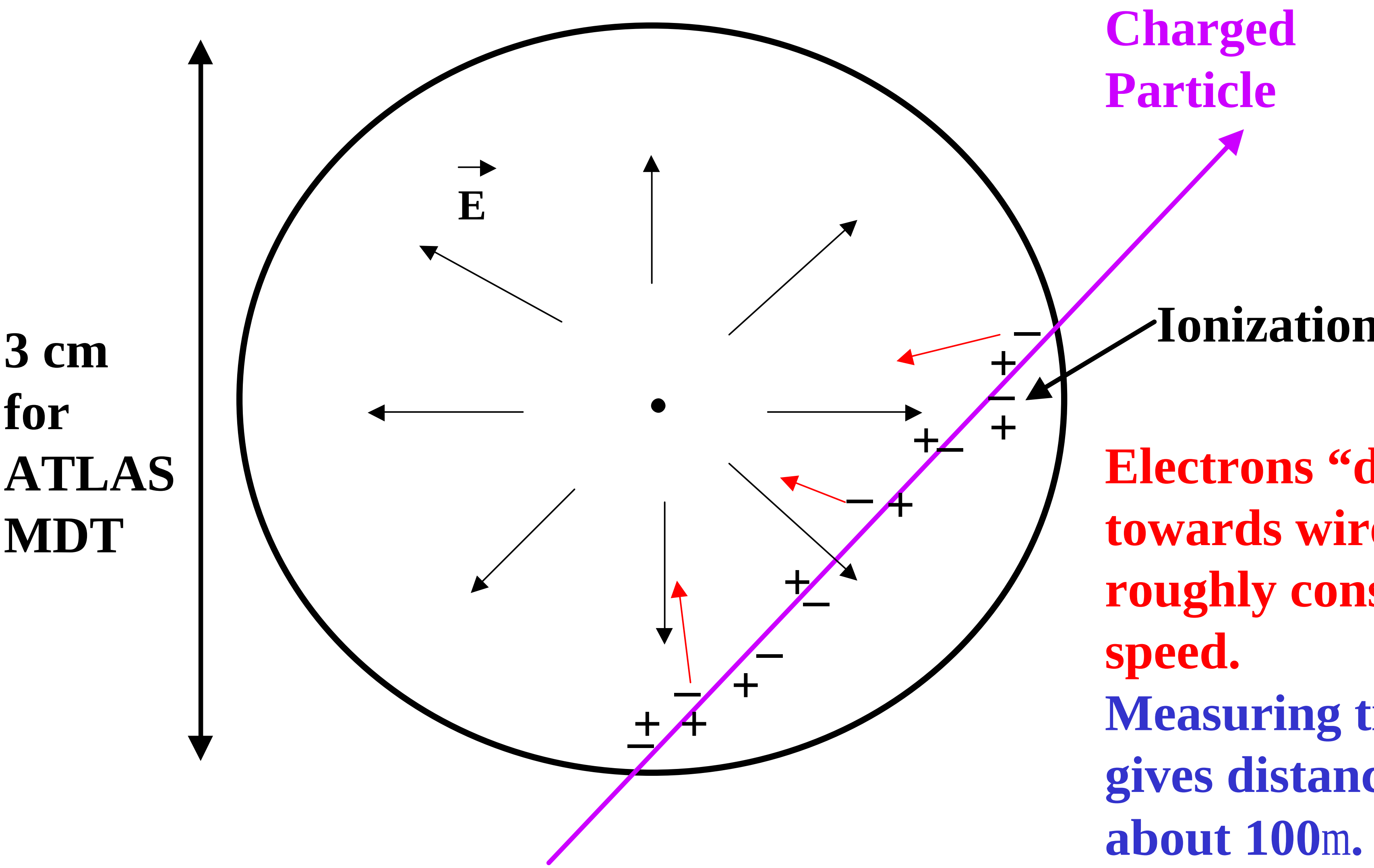
Many different technologies are employed in tracking, such as, multi-wire proportional chambers (MWPC), drift chambers, time projection chambers (TPC), silicon strips, silicon pixels, emulsions, bubble chambers, etc.

Due to time limitations, I will mainly talk about drift chambers.

Due to time (and knowledge) limitations, I will concentrate on the physics of drift chambers (ignoring almost completely pattern recognition and track fitting).

Drift Chambers

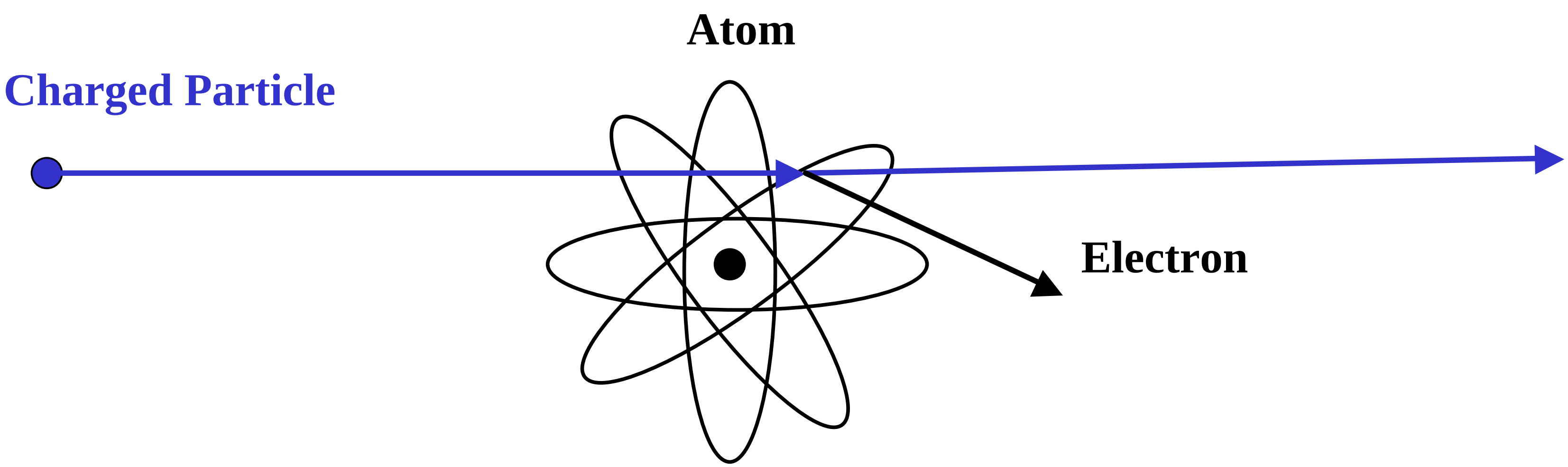
This talk will focus on drift chambers.



Electrons “drift” towards wire at roughly constant speed.

Measuring time gives distance to about 100m.

Ionization



Charged particle can do several things:

1. **Scatter off atom as a whole.**
2. **Excite electron in atom.**
3. **Cause a molecule to rotate or vibrate.**
4. **Disassociate a molecule.**
5. **Knock one or more electron out of atom (ionization).**

Bethe-Bloch Equation

To get the rate of energy loss, we fold the fundamental particle-electron cross section with the properties of the atom (energy levels, number of electrons, etc.).

Bethe, Bloch, and others did this, giving the Bethe-Bloch equation:

$$\frac{dE}{dx} = -4\pi N_0 r_e^2 m_e c^2 \rho \frac{Z}{A} z^2 \frac{1}{\beta^2} \left[\frac{1}{2} \ln\left(\frac{2 m_e c^2 \beta^2 \gamma^2 T_{max}}{I^2} \right) - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

Energy lost per unit length (dE/dx)

Avogadro (N_0)

Classical electron radius (r_e)

$4\pi N_0 r_e^2 m_e c^2$ = 0.3071 Mev/g cm^{-2}

Atomic number (Z)

Atomic mass (A)

Max K.E. of e (T_{max})

Mean excitation energy (I) (188 eV for Argon)

Density Correction ($\delta(\beta\gamma)$)

Note that dE/dx depends on fundamental constants, properties of the gas, and β.

Energy Loss

$$\frac{1}{\rho}\frac{dE}{dx}\left(\mathrm{Mev/g\ cm^{-2}}\right)$$

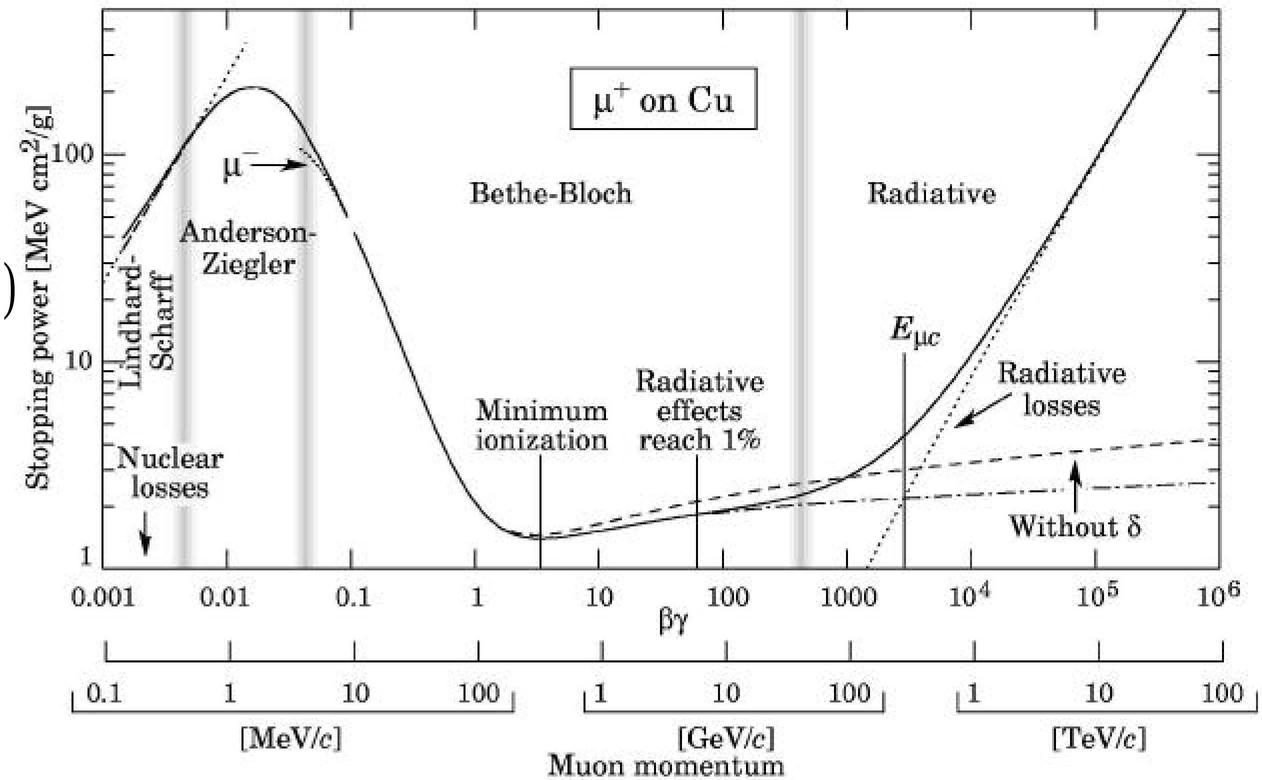


Fig. 27.1: Stopping power ($= \langle -dE/dx \rangle$) for positive muons in copper as a function of $\beta\gamma = p/Mc$ over nine orders of magnitude in momentum (12 orders of magnitude in kinetic energy). Solid curves indicate the total stopping power. Data below the break at $\beta\gamma \approx 0.1$ are taken from ICRU 49 [2], and data at higher energies are from Ref. 1. Vertical bands indicate boundaries between different approximations discussed in the text. The short dotted lines labeled "μ^-" illustrate the "Barkas effect," the dependence of stopping power on projectile charge at very low energies [3].

Energy Loss II

$$\frac{1}{r}\frac{dE}{dx}\left(\mathrm{Mev}/\mathrm{g\ cm}^{-2}\right)$$

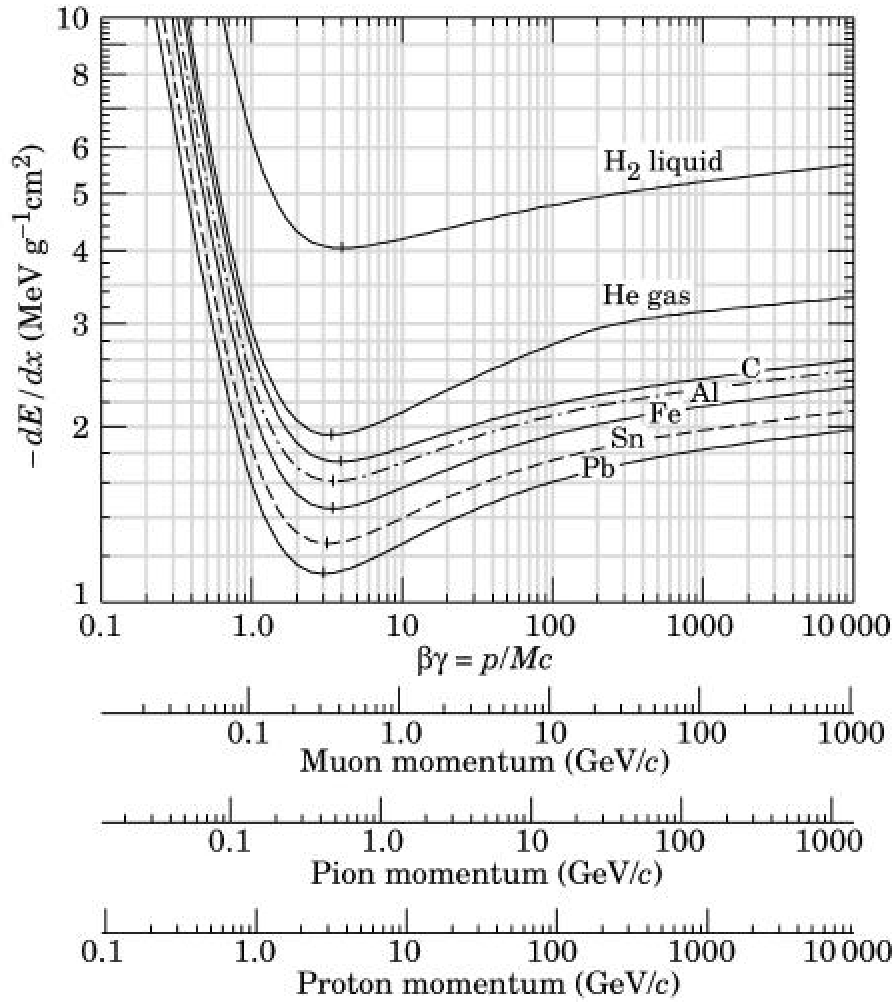


Figure 27.3: Mean energy loss rate in liquid (bubble chamber) hydrogen, gaseous helium, carbon, aluminum, iron, tin, and lead. Radiative effects, relevant for muons and pions, are not included. These become significant for muons in iron for $\beta\gamma \gtrsim 1000$, and at lower momenta for muons in higher-Z absorbers. See Fig. 27.21.

Ionization per Unit Length

In argon, it takes on average W = 26 eV to produce an ion-electron pair. Thus, the number of electrons created by a particle crossing 1 cm of argon gas at STP is

$$N_e = \frac{dE}{dx}\ell\frac{1}{W} = \left(1.52\ \mathrm{Mev\ cm^2/g}\right)\left(1.78\mathrm{x}10^{-3}\ \mathrm{g/cm^3}\right)(1\ \mathrm{cm})\frac{1}{26\,\mathrm{eV}} \text{ » } 100$$

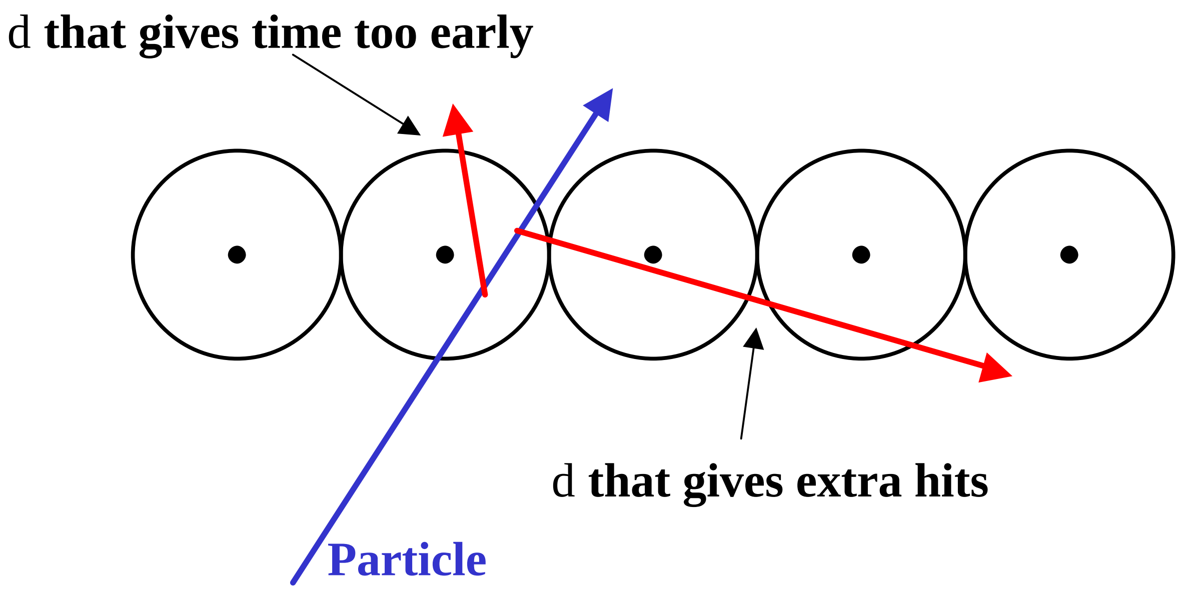
Clustering: Often multiple electrons are produced either because the charged particle knocks 2 (or more) electrons out of an atom or because an electron knocked out has sufficient energy to ionize one or more nearby atoms. The average cluster size for Argon is about 2, although it has a very long high side tail.

Delta Rays

Sometimes a particle knocks an electron out of the atom with sufficient energy to travel several centimeters or more.

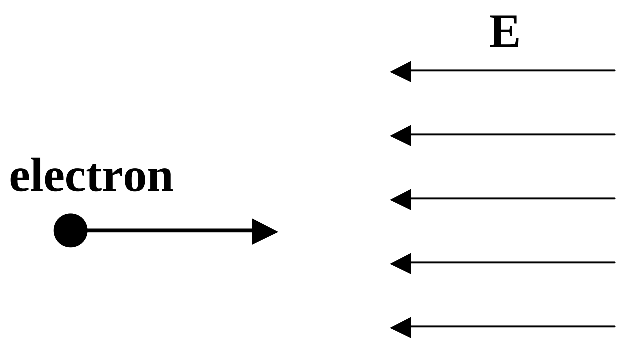
These electrons are known as delta rays.

They can further ionize the gas - often in unwanted ways.



Drift Velocity

First, consider a free electron in a constant electric field.



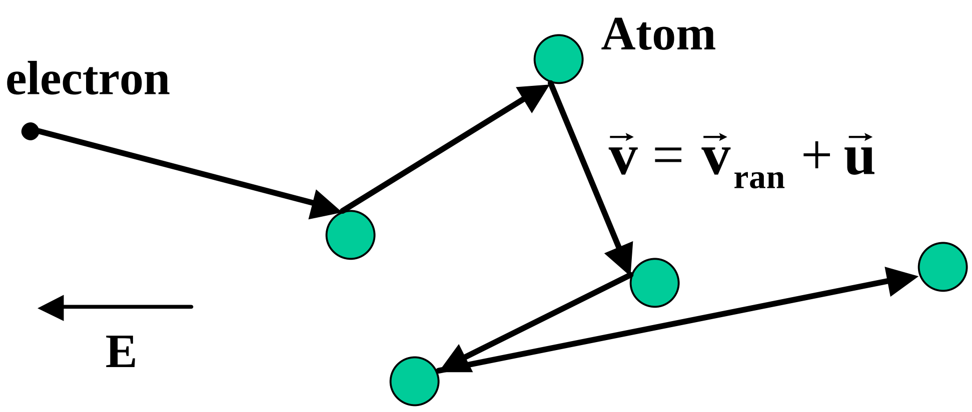
If there is no gas, electron accelerates with $a = \frac{eE}{m}$

If there is gas, electron accelerates until it hits an atom, where it looses some energy and changes direction. Since there are multiple energy loss mechanisms, this is a very complicated process.

We will model it by assuming

1. Change of direction is isotropic.
2. Fraction of energy lost in each collision is f.
3. In equilibrium, energy lost is balanced by energy gained from the electric field.

Drift Velocity Model



v_{ran} is random velocity due to scattering.
u is the non-random component due to electric field.

t = mean time between collisions
n = number density of atoms
s = cross section
x = mean distance in drift direction between collisions
f = fraction of energy lost per collission
M = mass per molecule
r = density

$$\mathbf{u} = \frac{\mathbf{eE}}{\mathbf{m}} t \ (\text{constant acceleration})$$

$$E_{lost} = f\left(\tfrac{1}{2} m v^2\right)$$

$$x = u t$$

$$E_{gain} = xeE = u t eE$$

Setting the energy lost equal to the energy gained, gives

$$u^2 = \frac{eE}{mn s}\sqrt{\frac{f}{2}} = \frac{eEM}{m r s}\sqrt{\frac{f}{2}}$$

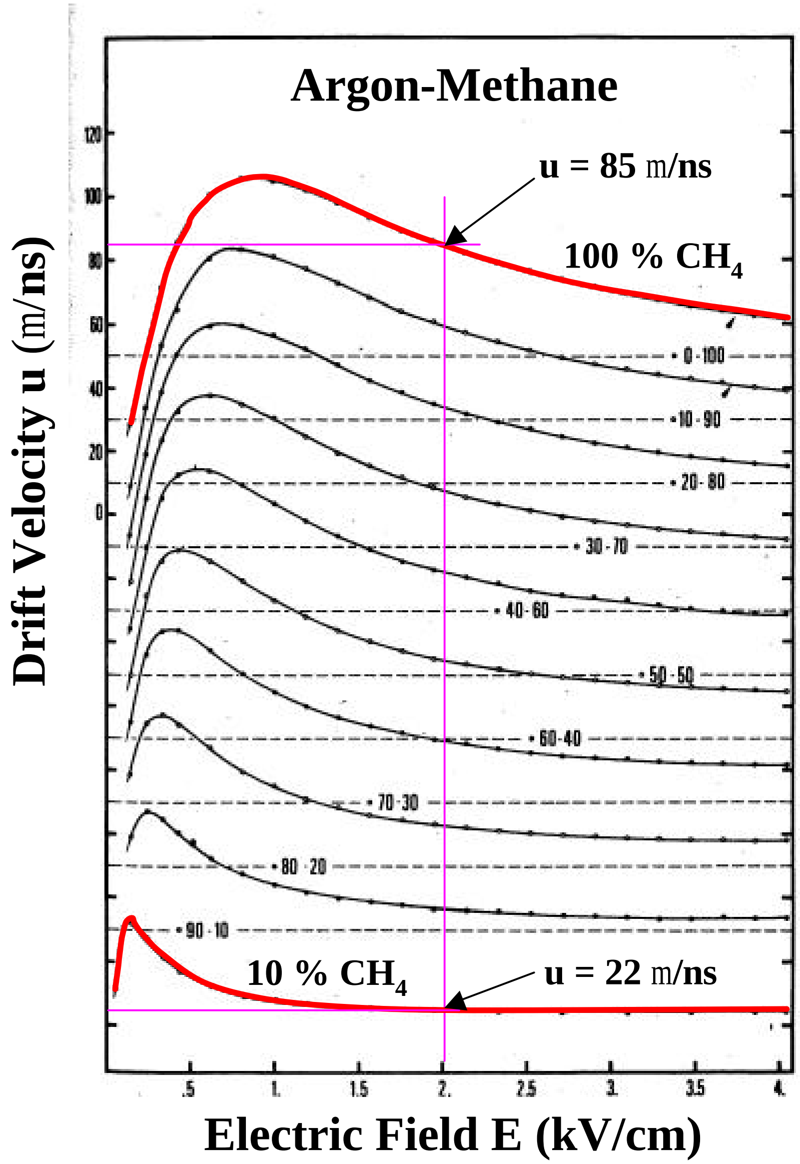
Drift Velocity Example

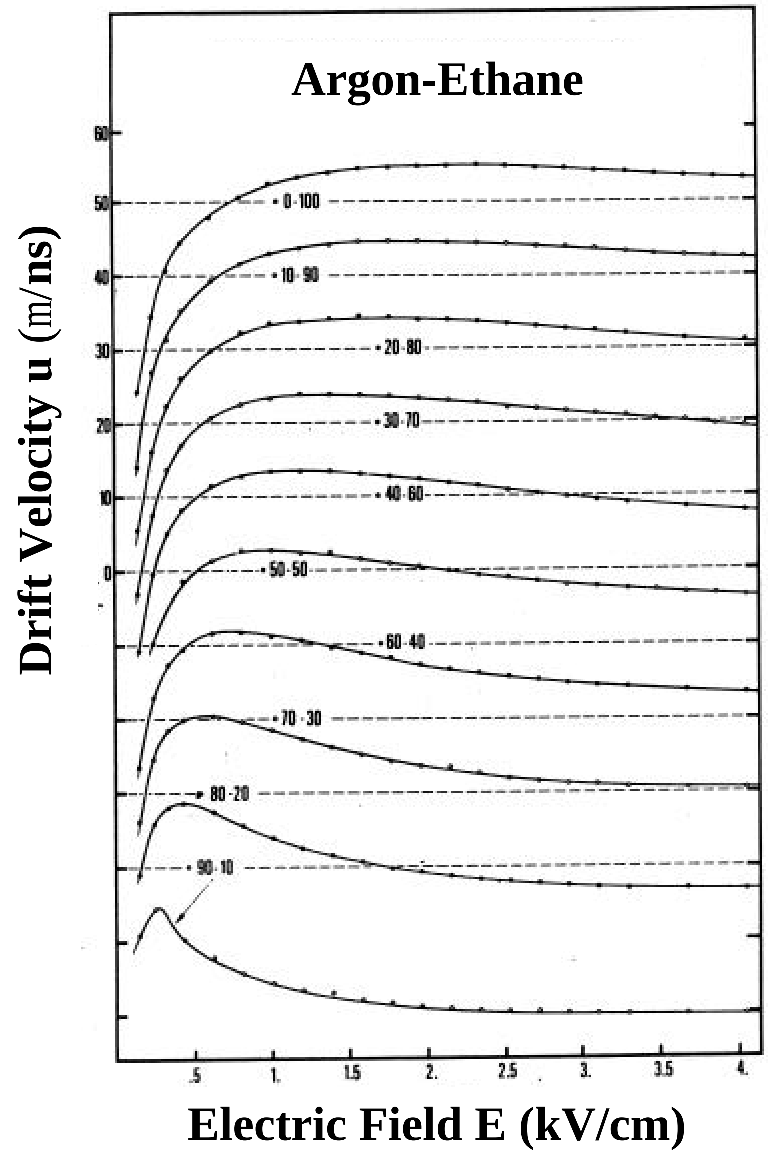
Typical numbers are E = 2 kV/cm and $\sigma = 5x10^{-20}$ m^2. We will take standard temperature and pressure for the gas.

For Argon, electrons don't have enough energy to excite atoms, so electrons "bounce" off entire atom, giving $f \approx 10^{-4}$. Plugging into the formula gives u = 10 μ/ns.

**For Methane, electrons in the relevant energy range can excite the molecules (primarily rotation) giving $f \approx 10^{-2}$.
Plugging into the formula gives u = 100 μ/ns.**

Measured Drift Velocities

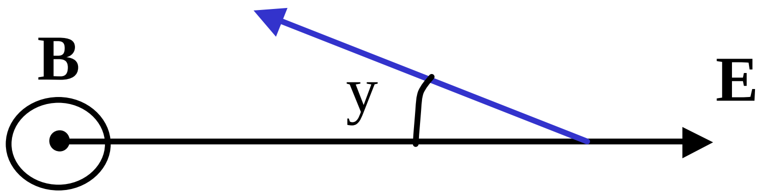




Note: dotted line for each mixture is 50 m/ns.

Lorentz Angle

If there are both E and B fields, the electrons don't "drift" along the E field. They drift an an angle y (known as the Lorentz angle).



An effective equation for the drift velocity is

$$\vec{F} = e\vec{E} + e\vec{u} \times \vec{B} - K\vec{u}$$

where K represents the "friction" term.

In equilibrium, the total force is zero, giving $\tan y = \frac{u_0 B}{E}$, **where u_0 is the drift speed when B = 0.**

Drift Distance vs Time (r-t) Relation

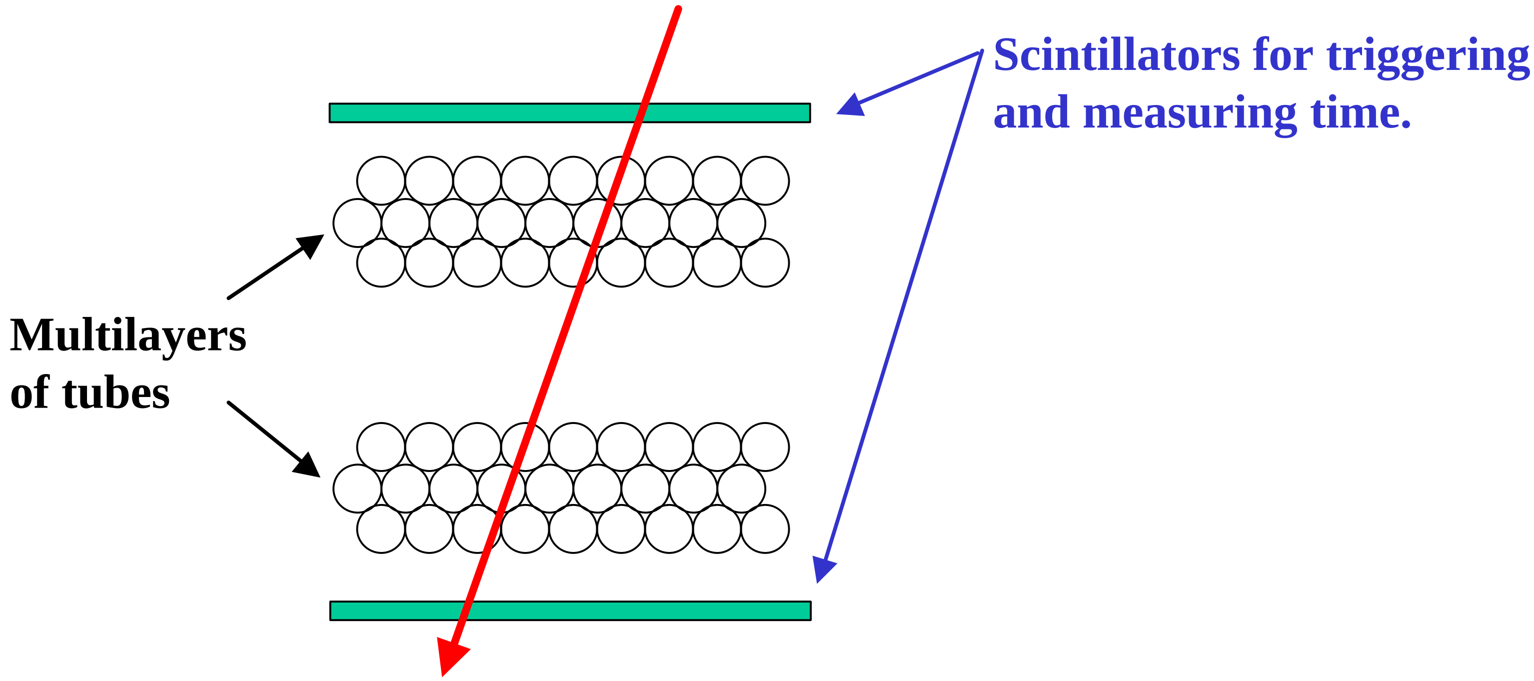
For real chambers, the electric field is seldom constant. Consider the drift tubes in the ATLAS muon system - they are tubes 3 cm in diameter, up to several meters long, with a 50 μm diameter gold-plated tungsten-rhenium wire down the center. The high voltage is 2.35 kV.

In cases like this (and more complicated ones), the relationship between the drift distance and the drift time is normally determined experimentally.

Must be able to measure time that test particle passes through and the position of the test particle in the drift tube.

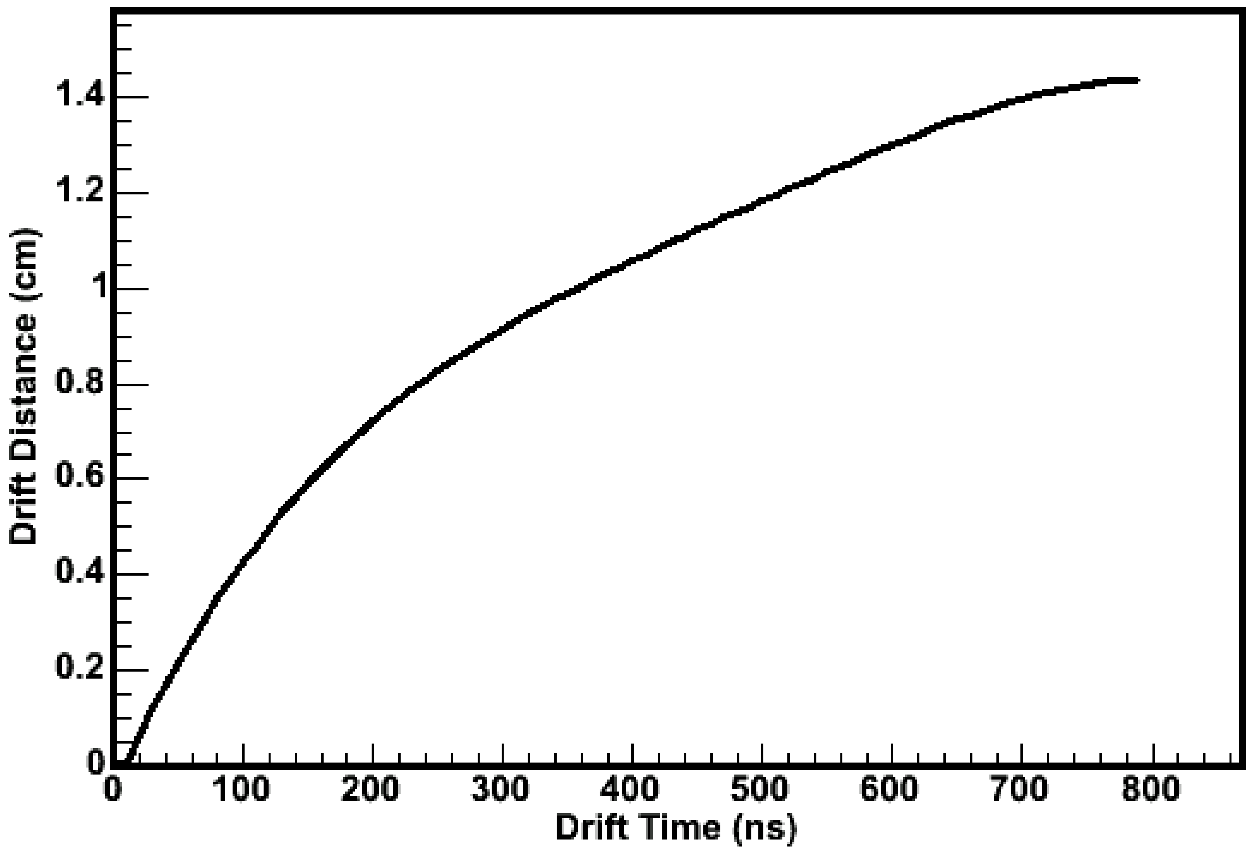
This can be done with test beam, cosmic rays, or data from the experiment.

Measuring (r-t) Relation



For each tube hit, fit track to other hits (to avoid bias) and determine position in that tube. Get time from scintillators.

ATLAS MDT r-t Relation



Diffusion

Collisions are a random process. Thus, if we start with an electron at a given position and let it drift, the distance covered in a given time will have fluctuations.

Consider a simple 1-d model in the drift direction where the mean free path is λ and the mean drift speed is u. Suppose in time t, the electron makes N collisions.

The probability P(*l*) of going a distance *l* between collisions is

$$P(\ell) = \frac{1}{\ell} e^{-\frac{\ell}{\lambda}}$$

The variation in the distance traveled in N collisions is found by

$$L = \sum_{i=1}^{N} \ell_i \Rightarrow \overline{L} = \sum_{i=1}^{N} \overline{\ell}_i = N\lambda$$

$$L^2 = \sum_i \ell_i \sum_j \ell_j = \sum_{i \neq j} \ell_i \ell_j + \sum_i \ell_i^2$$

$$\Rightarrow \overline{L^2} = \sum_{i \neq j} \overline{\ell_i}\,\overline{\ell_j} + \sum_i \overline{\ell_i^2} = \left(N^2 - N\right)\lambda^2 + N\left(2\lambda^2\right) = \left(N^2 + N\right)\lambda^2$$

$$\sigma_L^2 = \overline{L^2} - \overline{L}^2 = N\lambda^2$$

Diffusion II

This is sometimes written in terms of a diffusion constant D as

$$\sigma_L^2 = N\lambda^2 \equiv 2Dt \Rightarrow D = \frac{N\lambda^2}{2t} = \frac{L\lambda}{2t} = \frac{\upsilon\lambda}{2}$$

Thus, we have an uncertainty σ_L on the drift distance that grows as the square root of time.

We can also estimate this uncertainty for a typical drift distance L of 1 cm as

$$\sigma_L = \sqrt{2Dt} = \sqrt{N\lambda^2} = \sqrt{L\lambda} \approx \sqrt{(1\text{ cm})(1\ \mu\text{m})} = 100\ \mu\text{m}$$

Note that $\lambda \sim 1/(\rho\sigma)$, so that the uncertainty on the drift disance decreases with greater density.

Avalanche

When drifting electrons get close to the wire, the electric field is high and the electrons gain sufficient energy between collisions to ionize atoms/molecules. These ionization electrons can in turn further ionize the gas. This gives an exponentially increasing number of electrons.

Simple model: Avalanche starts when energy gained by drifting electron between collision is eΔV = I_0 (the ionization potential of the atom). Each time the electrons pass through a potential change of ΔV, the number of electron doubles.

Gas Gain $G = 2^{\frac{V(r_0) - V(a)}{\Delta V}}$

where r_0 is the radius at which avalanche starts. Let λ be the mean free path.

Gas Gain

$$I_0 = e\Delta V = e\mathbf{E}\lambda = \frac{eV\lambda}{r\ln\left(\frac{b}{a}\right)} = \frac{eVM}{r\rho\sigma\ln\left(\frac{b}{a}\right)}$$

$$r_0 = \frac{eVM}{I_0\rho\sigma\ln\left(\frac{b}{a}\right)}$$

For argon at STP with V = 2kV, I_0 = 18 eV, and $\sigma = 5 \times 10^{-21}\ m^2$, r = 60 μm = 2.3 a.

$$\ln G = \ln 2\,\frac{eV(r_0)}{I_0} = \frac{eV\ln 2}{I_0\ln(b/a)}\ln\left[\frac{eVM}{I_0\rho\sigma\ln(b/a)}\right]$$

Under the conditions we are assuming, $G = 5 \times 10^5$.

Note that G is roughly exponential in V and that $\frac{\Delta G}{G} = \frac{eV\ln 2}{I_0\ln(b/a)}\frac{\Delta\rho}{\rho} \approx 12\frac{\Delta\rho}{\rho}$

Avalanche Simulation

Simulation of an avalanche. The Note that it begins a few diameters from the wire and develops somewhat unevenly, but most of the avalanche is on one side. Also, the avalanche spreads a few diameters along the wire.

Matoba, *et al.*, IEEE Trans. Nucl. Sci., NS-**32**, 541 (1985)

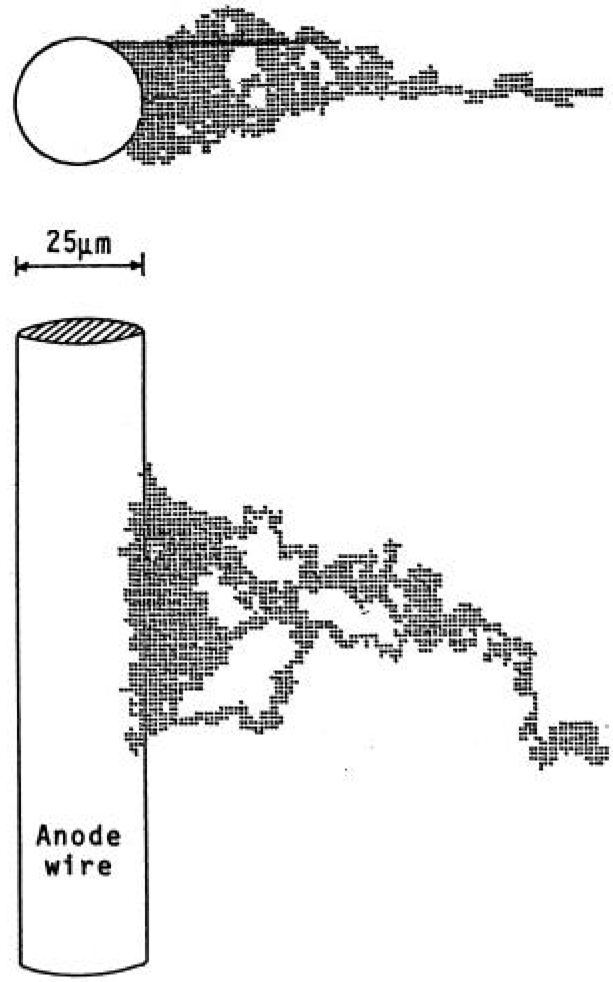
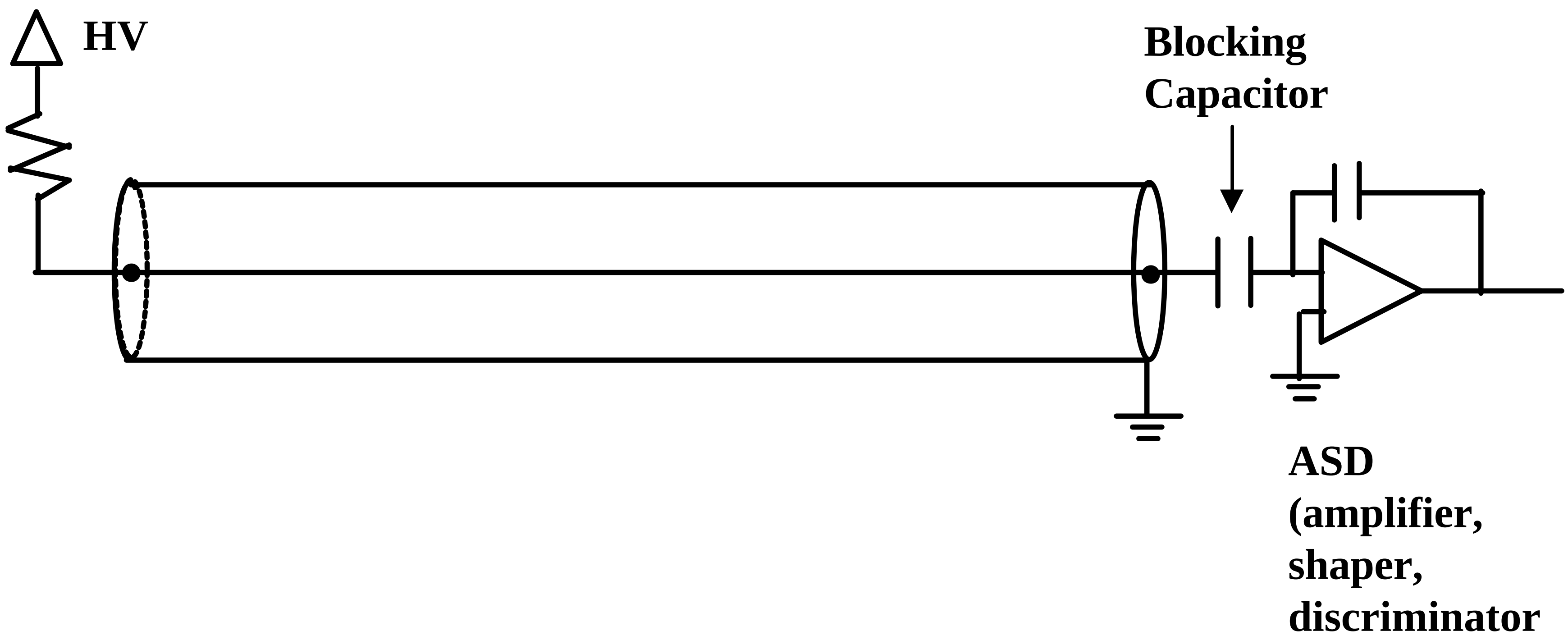


Fig. 7 Two dimensional displays of a simulated electron avalanche. Shading shows the density of electrons in the avalanche.

Signal Extraction

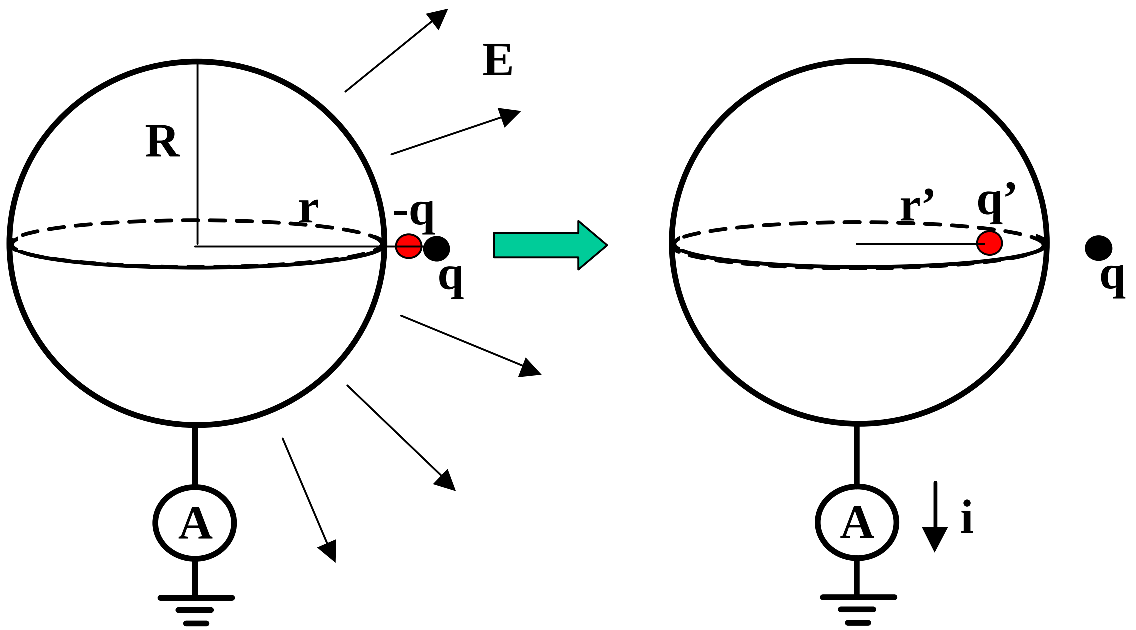
A rough schematics of a common signal electronics connection is shown.



Current due to image charge

Electrons from avalanche do not leave sense wire immediately, even for ideal electronics.

To see this, consider an avalanche near a grounded conducting sphere (for which I can do the calculations).



$$q' = -q\frac{R}{r} = \begin{cases} -q & \text{for } r = R \\ 0 & \text{for } r = \infty \end{cases}$$

$$i = -\frac{dq'}{dt} = -\frac{qR}{r^2}\frac{dr}{dt} = -\frac{qR}{r^2}u_{ion} = -\frac{qR}{r^2}\mu E(r)$$

Note: signal depends on movement of positive ions.

Current Due To Image Charge II

A more careful treatment of the cylindrical case (based on energy and work methods) gives

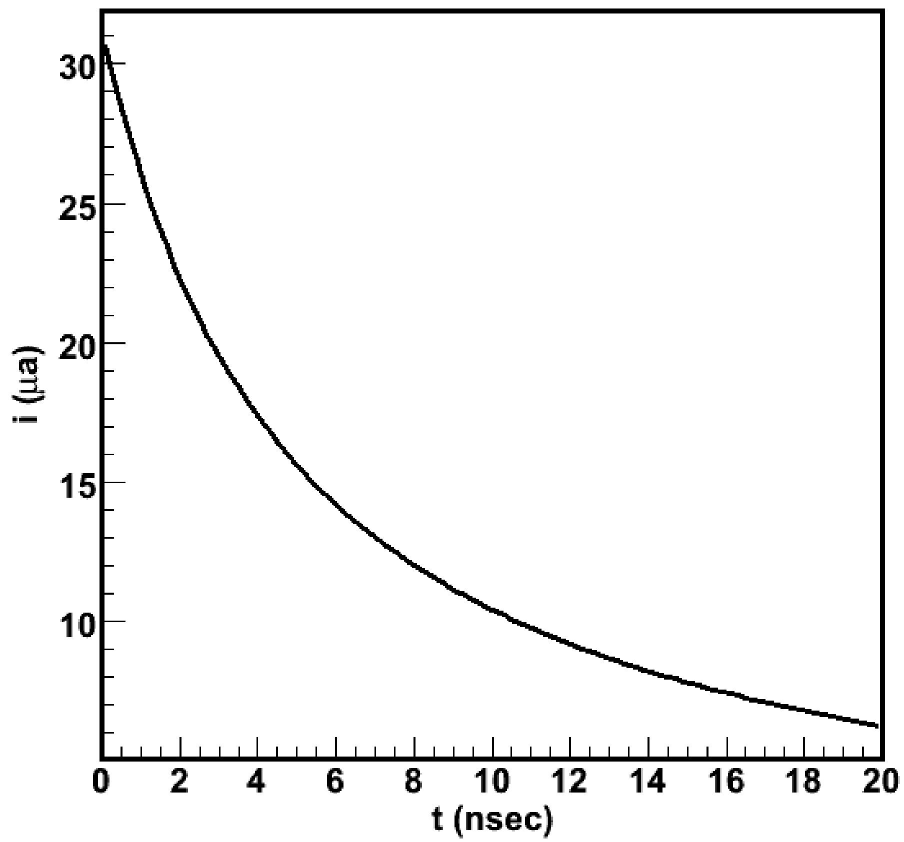
$$\mathbf{i(t)} = \frac{\mathbf{q}}{\ln\left(\frac{\mathbf{b}}{\mathbf{a}}\right)} \frac{\mathbf{1}}{\mathbf{t} + \mathbf{t_0}} \quad \textbf{where } \mathbf{t_0} = \frac{\mathbf{a}}{\mathbf{2\mu E(a)}}$$

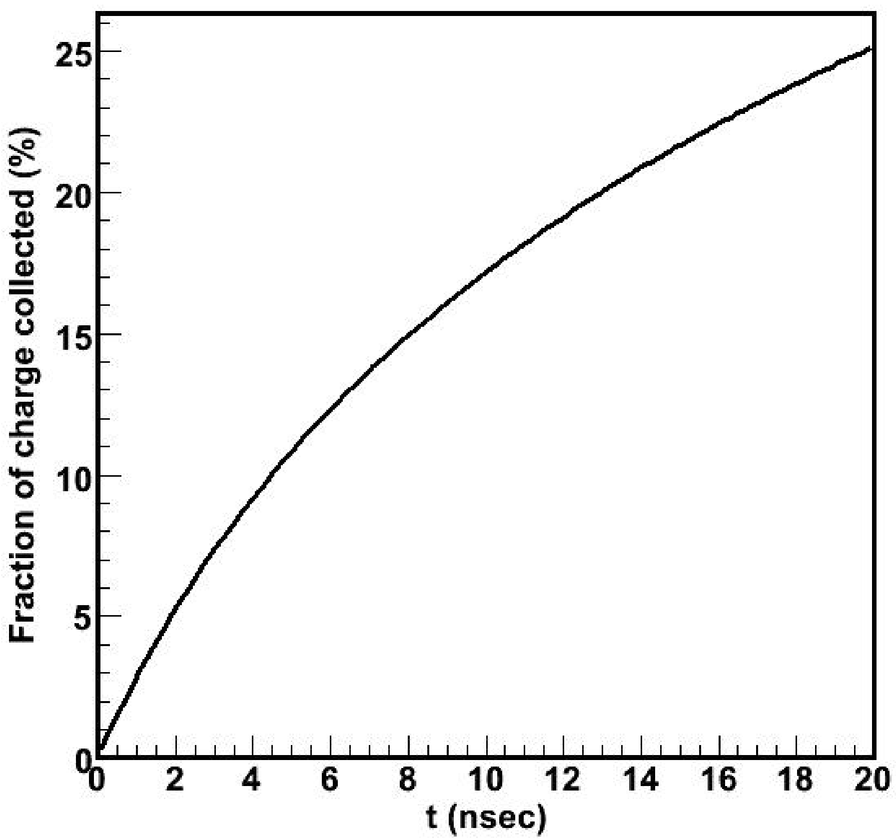
Note that the signal still depends on the positive ion motion.

For V = 2 kV, a = 25 μm, b = 1.5 cm, and typical = 1.5 $cm^2\ V^{-1}\ s^{-1}$, t_0 = 6.6 ns.

Signal Shape and Duration

Look at i(t) and fraction of charge collected for q = 1 pC and t_0 = 5 ns.





Space Charge

The number of positive ions in an avalanche is comparable to the surface charge on the sense wire for a length of a few diameters along the wire.

Thus, the local distortions of the electric field are significant.

Because of their low drift speeds and the long distances they must drift to cathode, positive ions remain in the drift volume for long times.

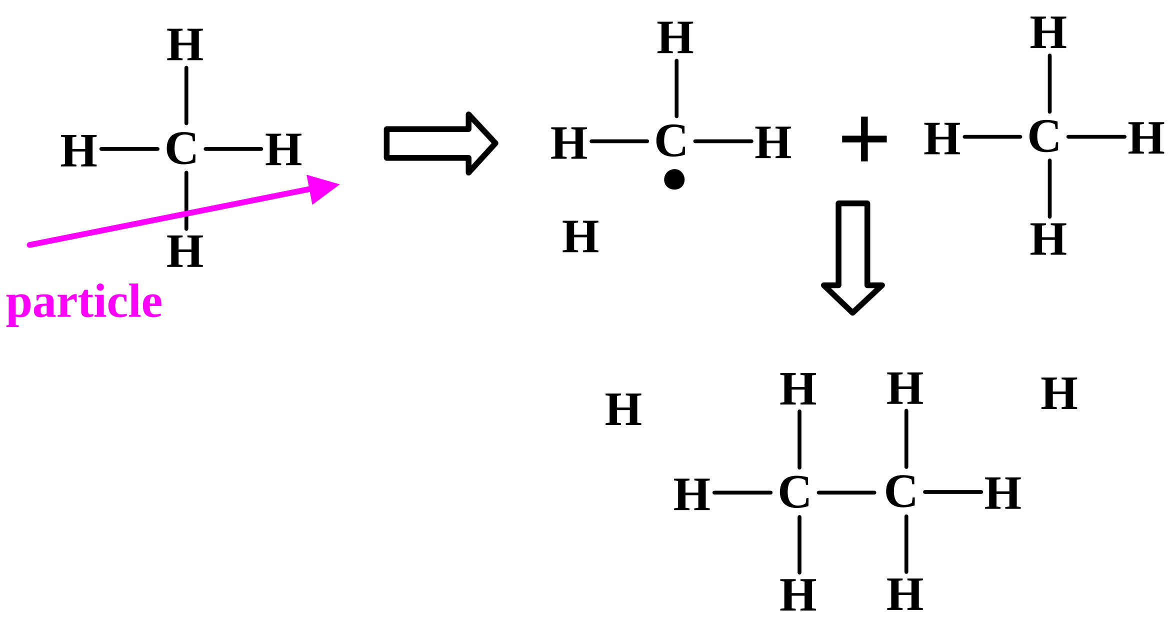
If the rate of particle passing through the drift cell is high enough, the cloud of positive ions that haven't cleared (known as space charge) have significant detrimental effects on the performance.

Ageing

Hydrocarbon molecules struck by charged particles or avalanche electrons can ionize or disassociate.

They makes them chemically active and they can form longer chains.

This gives long chain hydrocarbons that can “freeze out” onto the sense wires, adversely affecting performance.



Ageing Examples

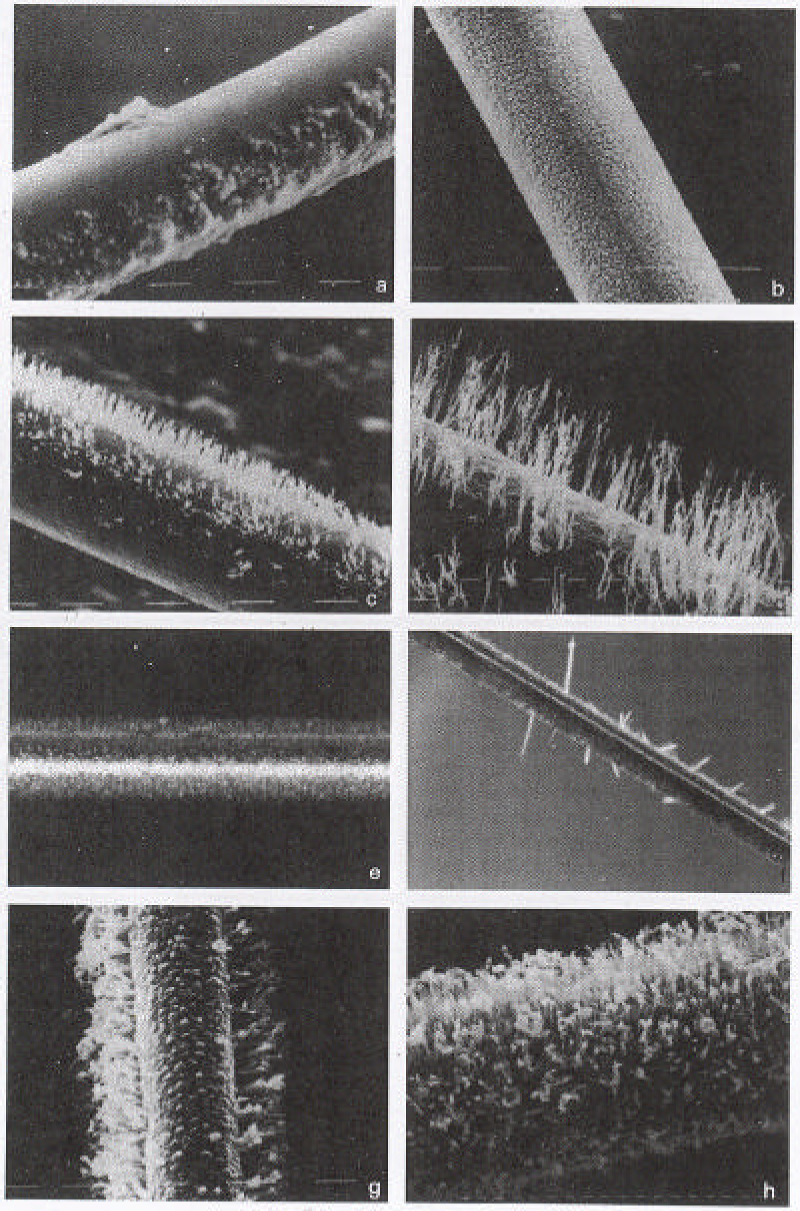


Fig. 13. Examples of deposits on anode wires [79].

J. Kadyk, *et al.*, IEEE Trans. Nucl. Sci. NS-**37**, 478 (1990)

Gas selection

Selection of gas is a compromise among many things:

1. **Low electron affinity (noble gas)**
2. **Large ionization (heavy noble gas)**
3. **Cost (usually means people choose argon)**
4. **Must have quenching (methane, ethane, alcohol, carbon dioxide, etc., etc., etc.)**
5. **Drift velocity**
6. **Resolution (diffusion)**
7. **Lorentz angle**
8. **Gas Gain**
9. **Positive ion mobility**
10. **Multiple scattering**
11. **Ageing**

Momentum Measurement

We want the direction and magnitude of the momentum.

The uncertainty on direction is usually less important for the physics (except for secondary vertex detection).

The uncertainty on the magnitude is determined by the hit resolution (usually dominated by diffusion) and the relationship between the geometry and the momentum.

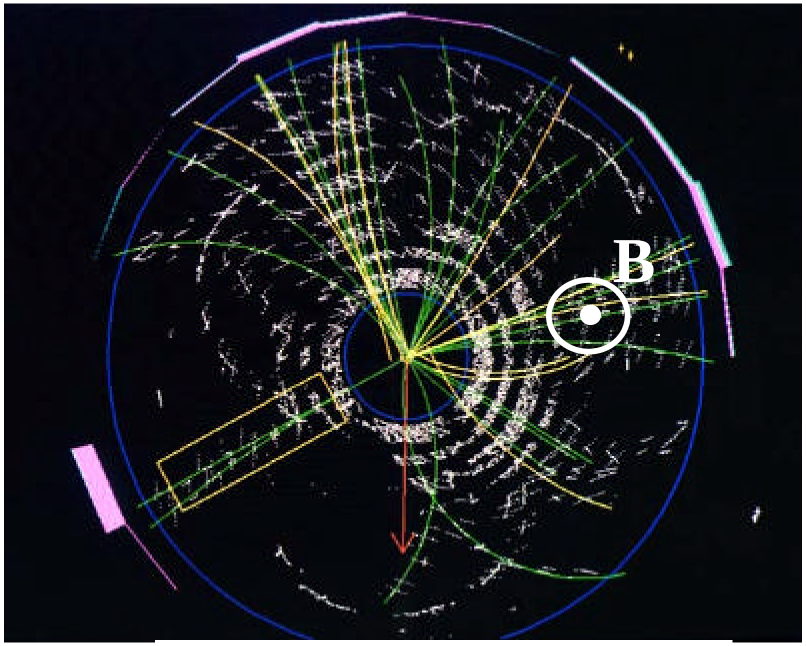
Drift Direction

The best momentum resolution is obtained if

1. **the drift direction is perpendicular to the track direction and**
2. **the drift direction is perpendicular to the B field.**

Remember that the actual drift direction is given by the Lorentz angle in addition to the E field.

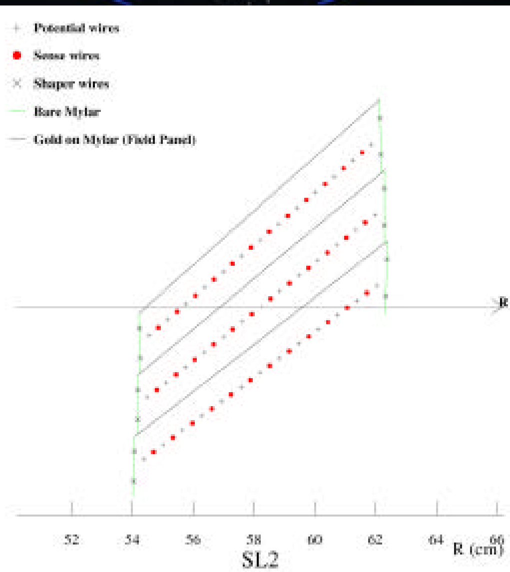
Drift Chamber in Solenoidal Field

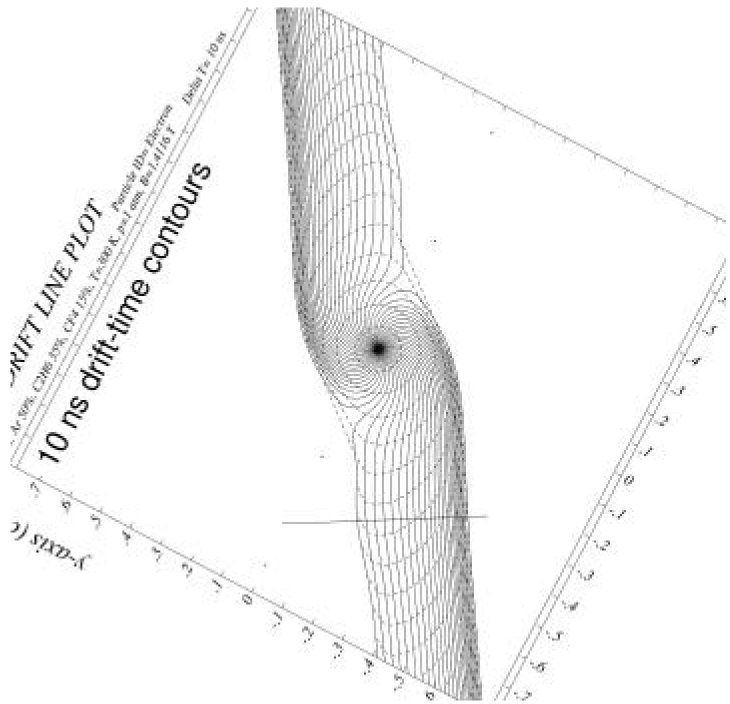


B is along beam direction (z).

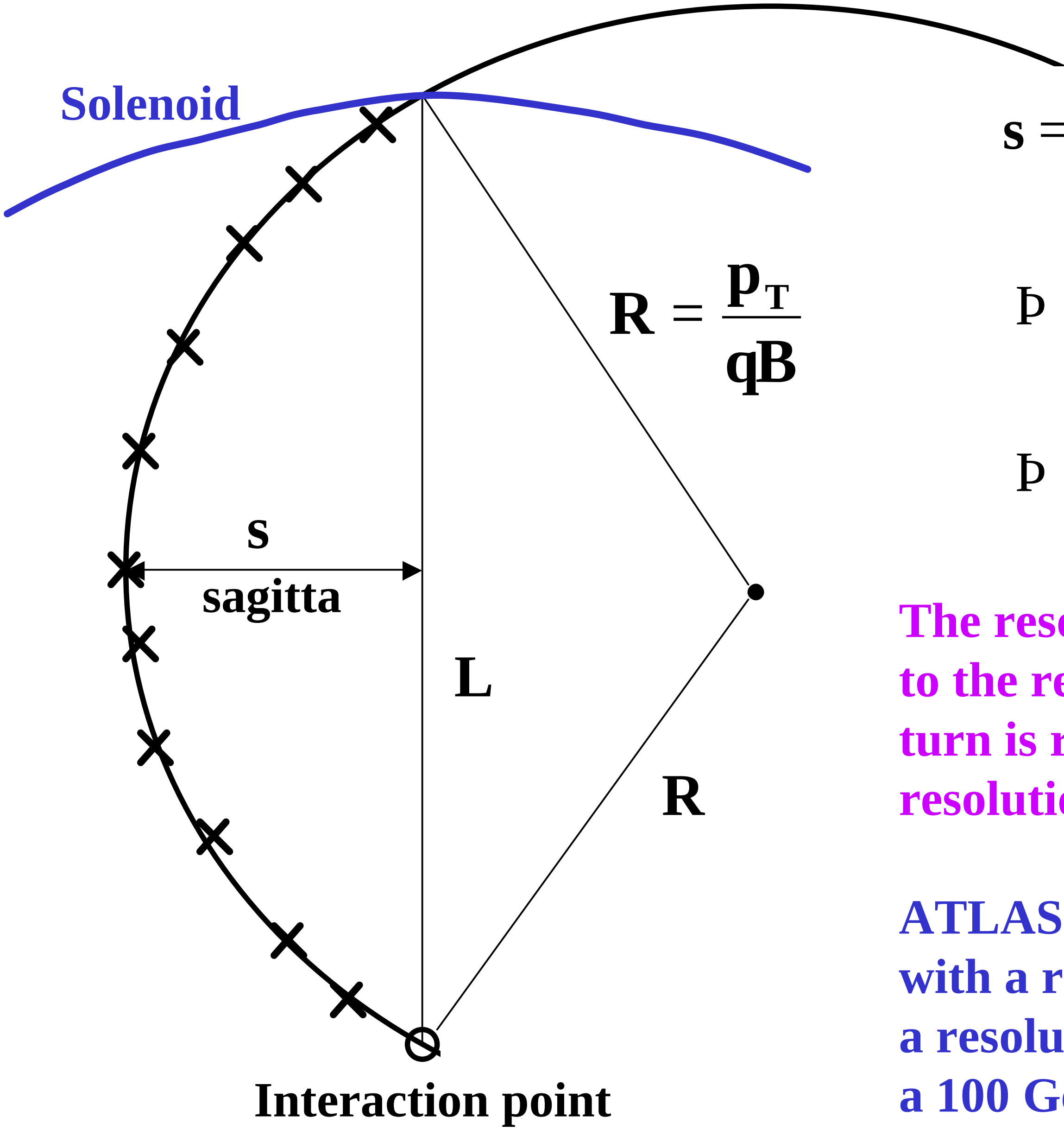
Tracks are roughly radial.

We want direction in azimuthal direciton.





Solenoidal Momentum Resolution



$$s = \frac{L^2}{8R} = \frac{qBL^2}{8P_T}$$

$$\Rightarrow P_T = \frac{qBL^2}{s}$$

$$\Rightarrow \sigma_{P_T} = \frac{P_T^2 \sigma_s}{qBL^2}\sqrt{\frac{720}{N+4}}$$

Stat. factor

The resolution on P_T is related to the resolution on s, which in turn is related to the hit resolution (~100 microns).

ATLAS has a 2 Tesla solenoid with a radius of 1.25 m, giving a resolution of about 4% for a 100 GeV/c track.

Alignment

In order to take advantage of resolution, we must know the location (at least relative) of all the wires to better than the resolution.

This is done with

1. **Precision construction**
2. **Surveying**
3. **Optical alignment (lasers, cameras, lenses)**
4. **Lasers simulating tracks**
5. **Tracks**

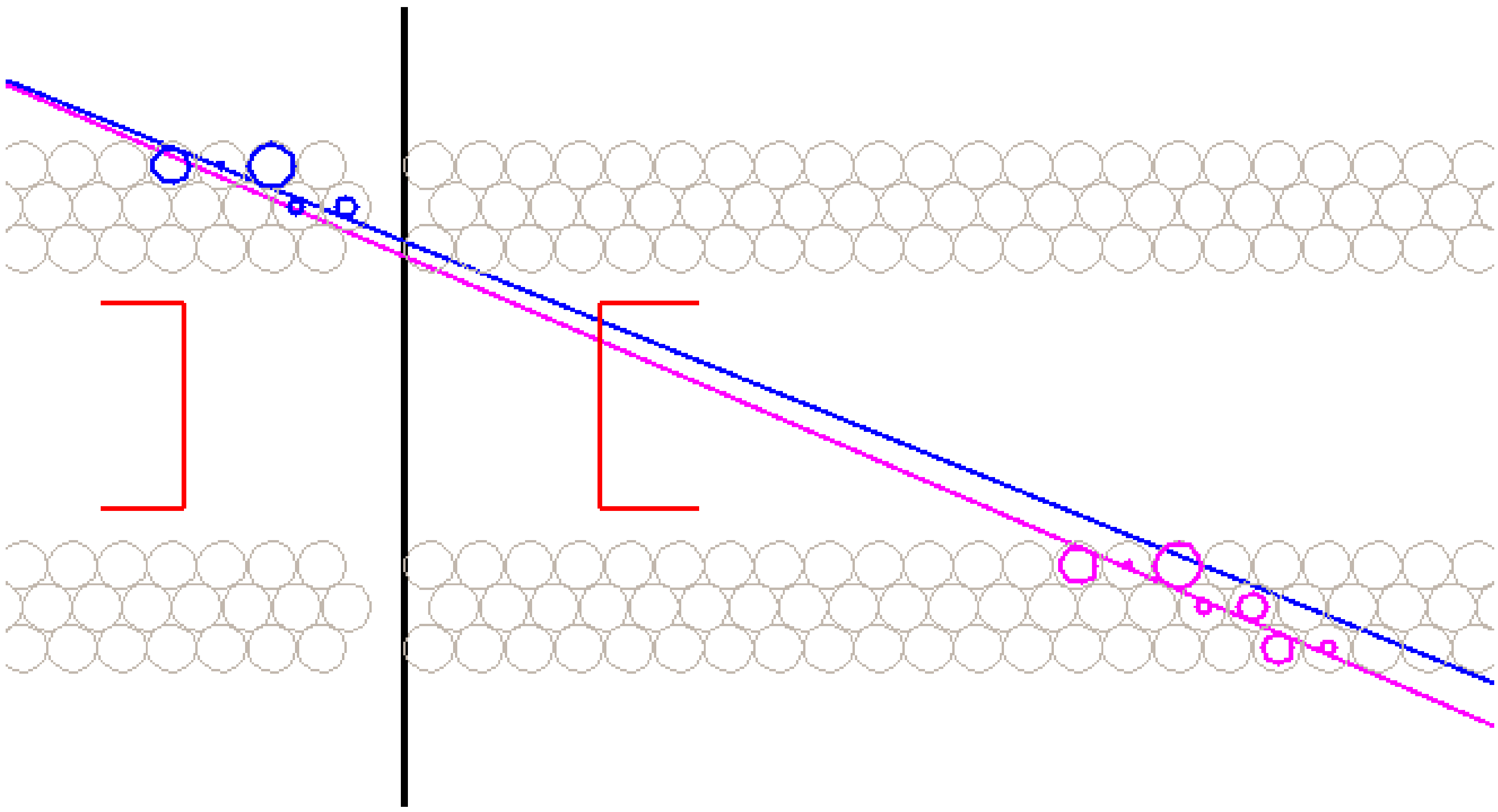
Pattern Recognition and Track Fitting

Pattern is recognition is the process of deciding which hits go on which tracks. It is often the most CPU intensive step in data reconstruction.

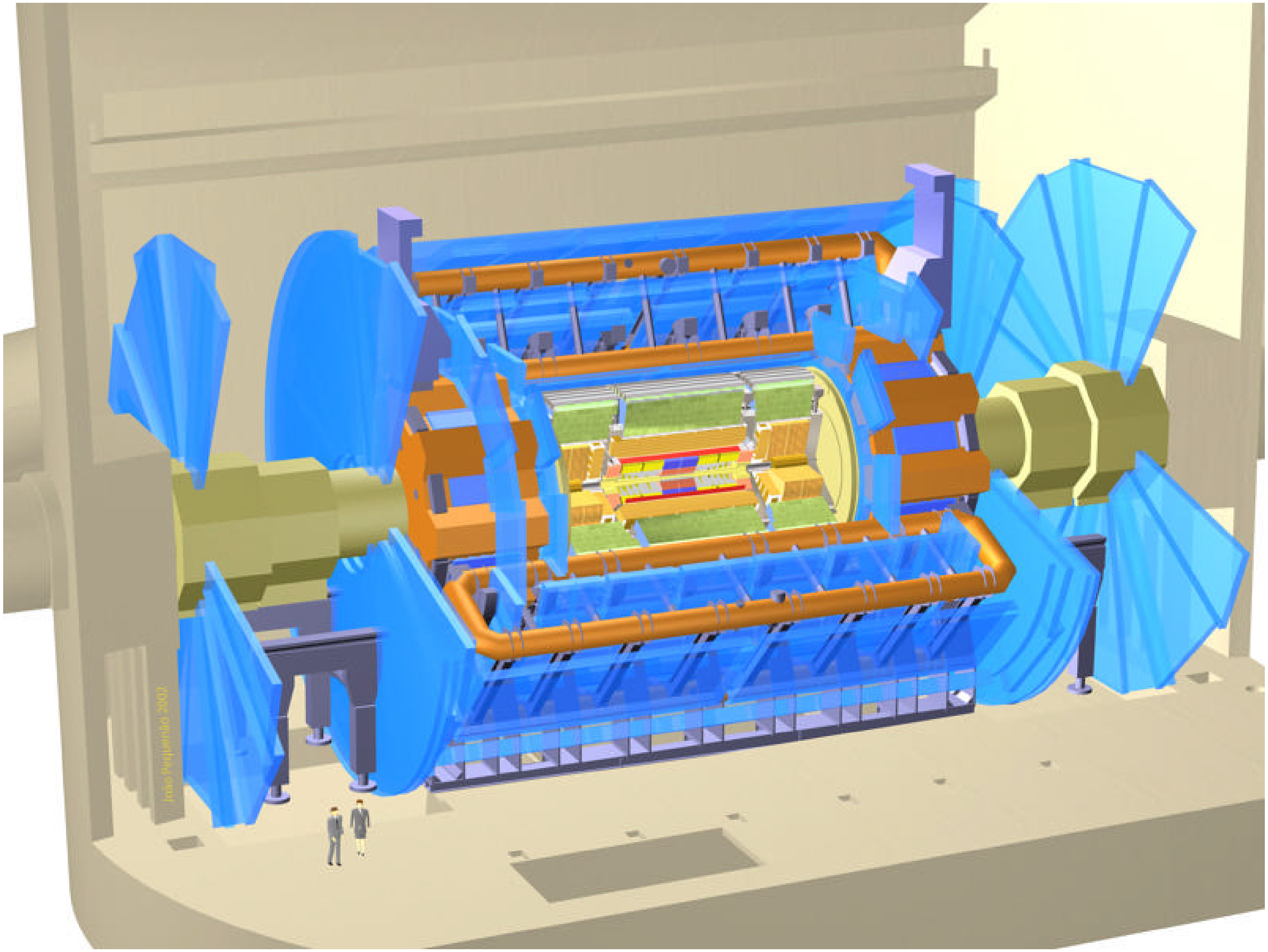
Track fitting is the process of determining the best estimate of the track parameters (for example, helix parameters in a solenoidal chamber) from a set of hits.

These are two very important and interesting topics, about which I will say nothing further, except that many of the tools used here are becoming usesful in physics analyses, such as, Kalman filters, Hough transforms, and neural nets.

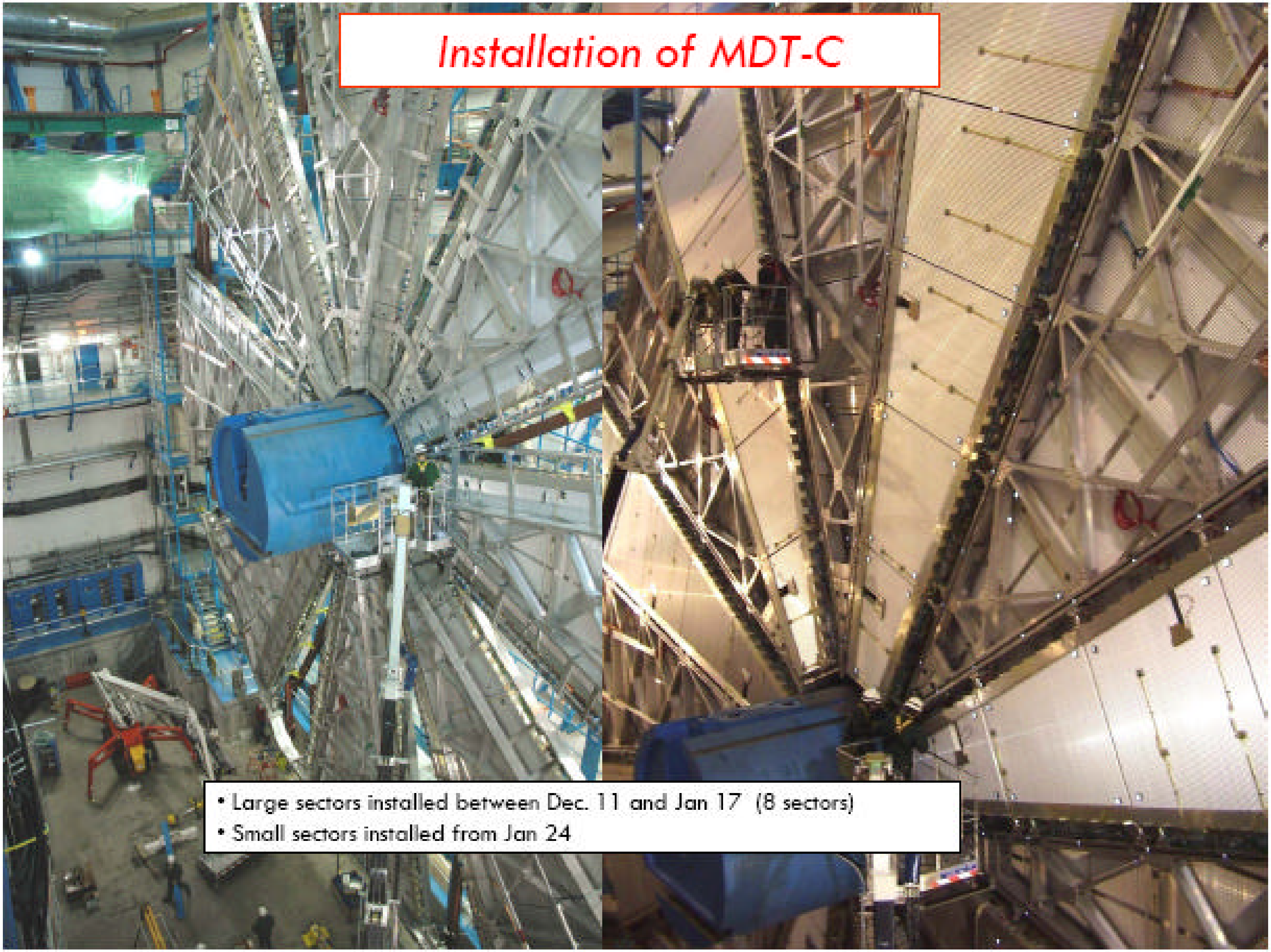
Example Track



ATLAS Detector



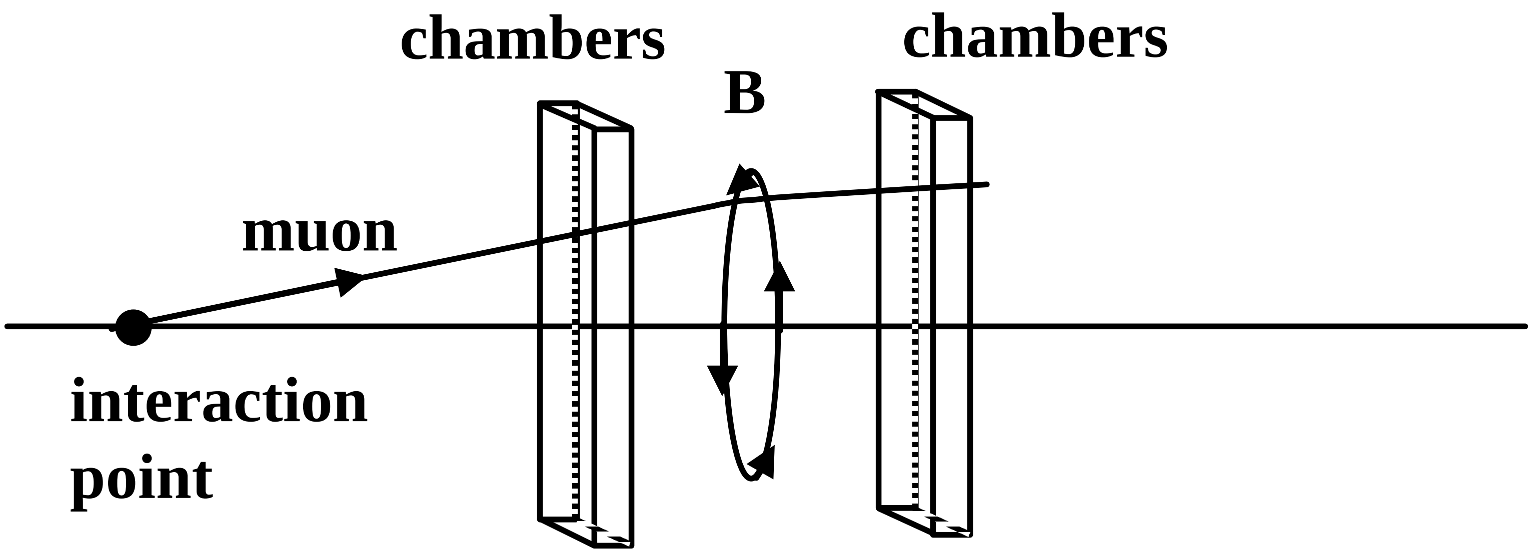
Installation of MDT-C



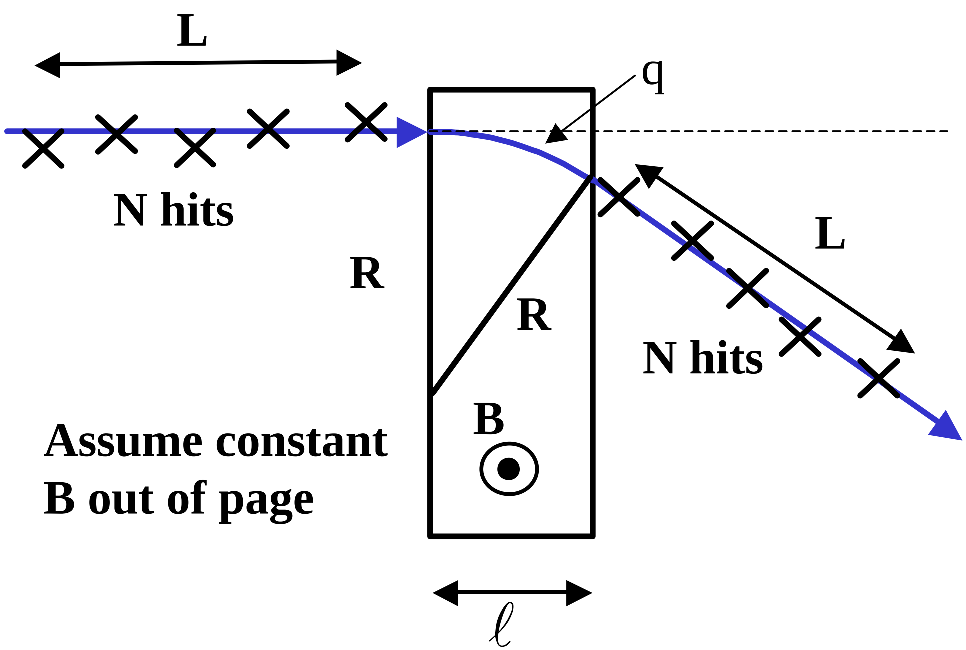
- Large sectors installed between Dec. 11 and Jan 17 (8 sectors)
- Small sectors installed from Jan 24

Chambers With Toroidal Field

In ATLAS endcap muon system B field runs azimuthally and particles travel roughly in z direction, so we want the tubes (wires) to run azimuthally.



Resolution With Toroidal Field



Assume N equal spaced hits before and after B field.

Difference in slopes β of the two lines depends on p (and B and ℓ.

Let δs be the hit resolution.

$$\tan\theta = \frac{\ell}{\mathbf{R}} = \frac{\ell \mathbf{eB}}{\gamma \mathbf{mv}} = \frac{\ell \mathbf{eB}}{\mathbf{p}}$$

$$\tan\theta = \beta_2 - \beta_1 \equiv \Delta\beta \Rightarrow \mathbf{p} = \frac{\ell \mathbf{eB}}{\Delta\beta}$$

$$\delta \mathbf{p} = \frac{\ell \mathbf{eB}}{(\Delta\beta)^2}\,\delta(\Delta\beta) = \frac{\mathbf{p}^2}{\ell \mathbf{eB}}\sqrt{\frac{\mathbf{6}}{\mathbf{N}}}\,\frac{\delta s}{\mathbf{L}}$$

Resolution With Toroidal Field II

In practice, the magnetic field is nonuniform, so be must replace $B\ell$ with $\int_0 B\,d\ell$.

$$\frac{dp}{p} = \frac{p\,ds}{eB\ell L}\sqrt{\frac{6}{N}}$$

is very similar to the solenoidal result of

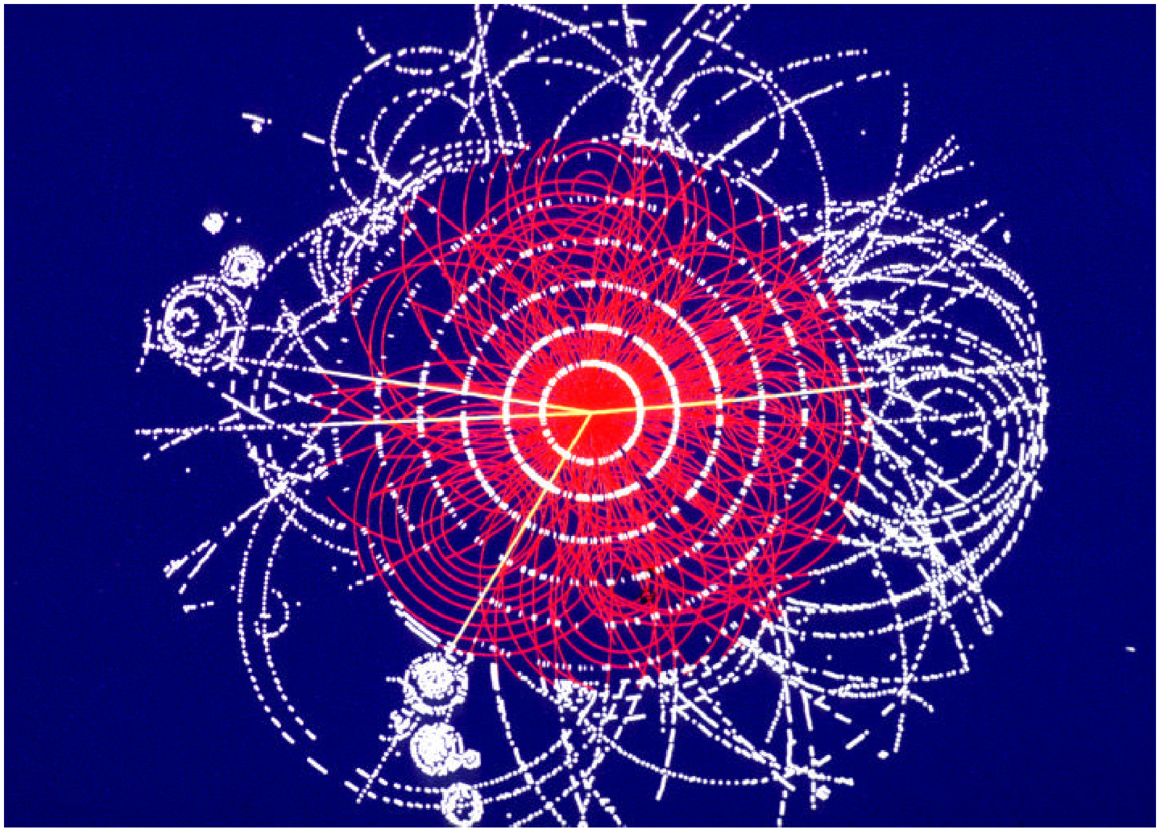
$$\frac{\sigma_{P_T}}{P_T} = \frac{P_T\,\sigma_s}{eBL^2}\sqrt{\frac{720}{N+4}}$$

with the main difference being ℓL versus L^2. You can think of the solenoid case as have one factor of L from the bending in the magnetic field and one factor from the length over which the track is measured.

Conclusion

Drift chambers are important tracking detectors. They are complex but much about them can be understood with a few simple models containing reasonable parameters.

gg ® **H** ® $\mathbf{Z^0Z^0}$ ® $m^+m^-m^+m^-$



Simulation of an event in ATLAS detector. White lines are the four muons. The other tracks are due to particles from quarks in protons.