Dynamics of Voter Models on Heterogeneous Networks

Sid Redner (physics.bu.edu/~redner)

Complex Network Program SAMSI Aug. 29-Sept. 1, 2010

T. Antal (Edinburgh), V. Sood (NBI)

NSF DMR0535503 & DMR0906504

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The classic voter model

3 basic results

Voting on complex networks

T. Antal, V. Sood

new conservation law & fixation probabilities
two time-scale route to consensus
short consensus time

Partisan voting

N. Gibert (Paris)
N. Masuda (Tokyo)

can truth be reached?

Classic Voter Model

Clifford & Sudbury (1973)
Holley & Liggett (1975)

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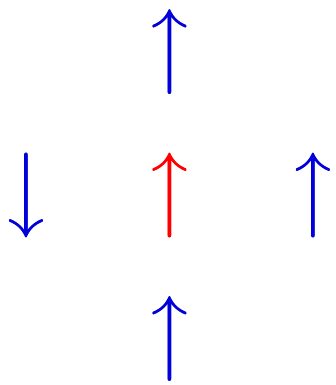

0. Binary voter variable at each site i

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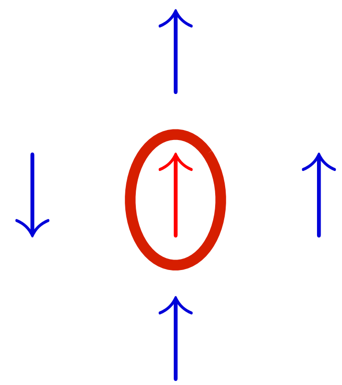

0. Binary voter variable at each site i
1. Pick a random voter

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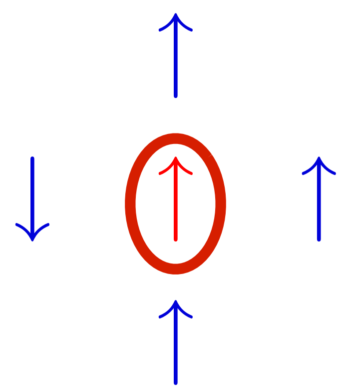

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1. Pick a random voter
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   individual has no self-confidence & adopts neighbor's state

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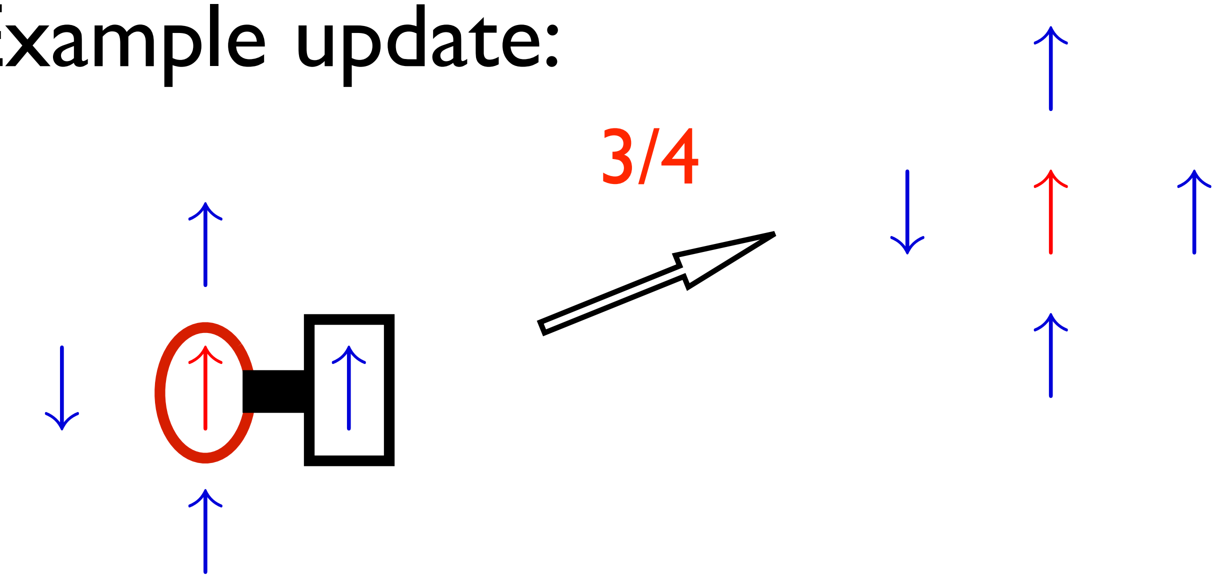
Example update:



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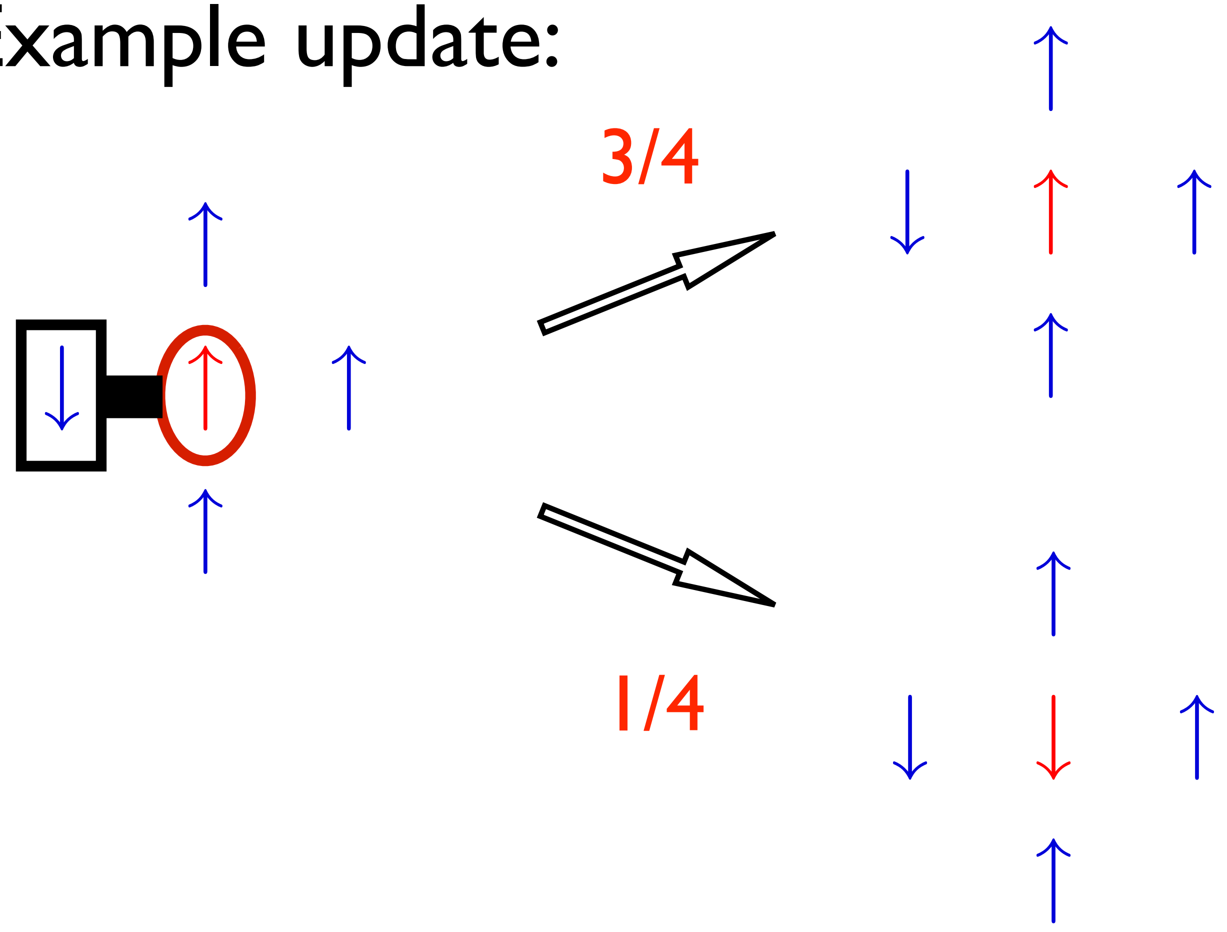

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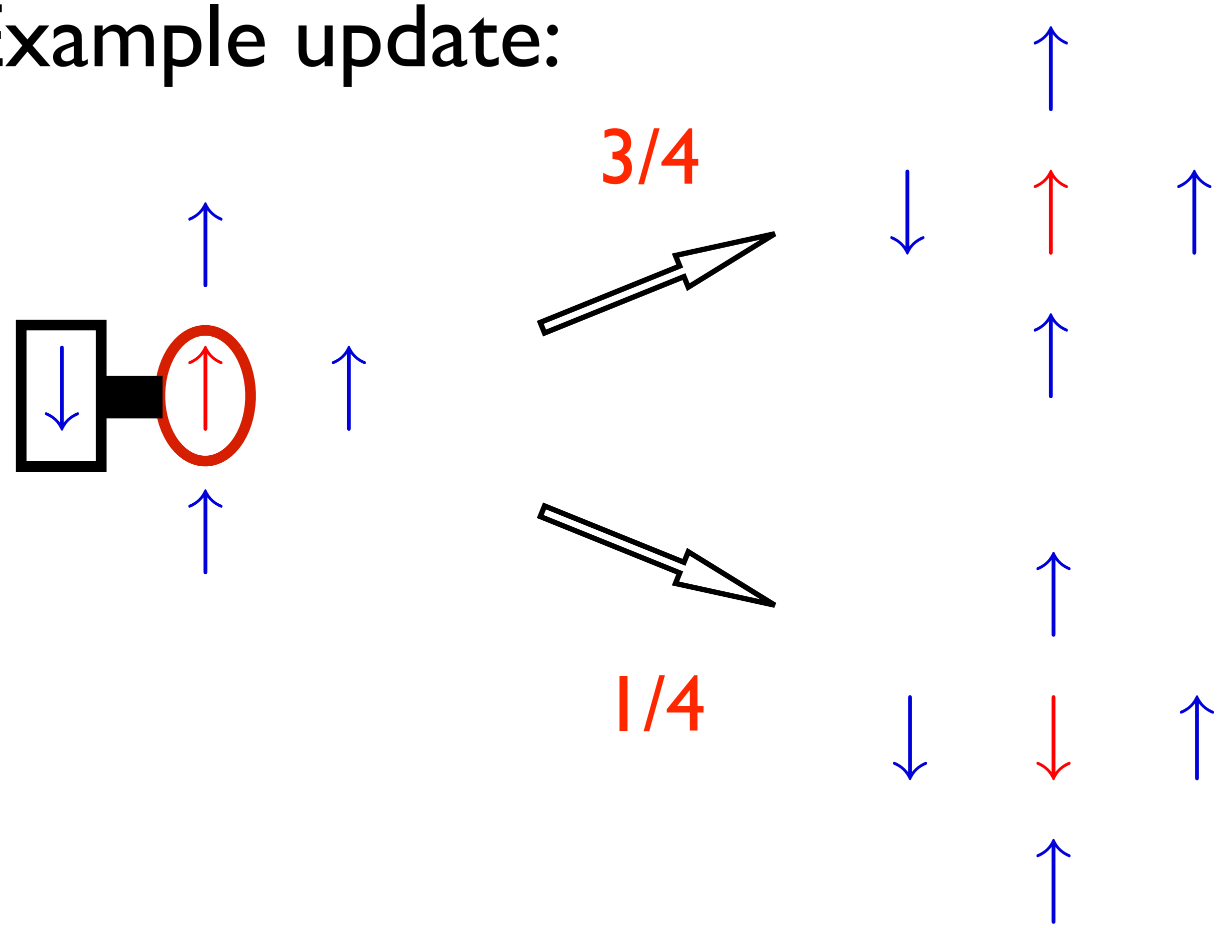

proportional rule

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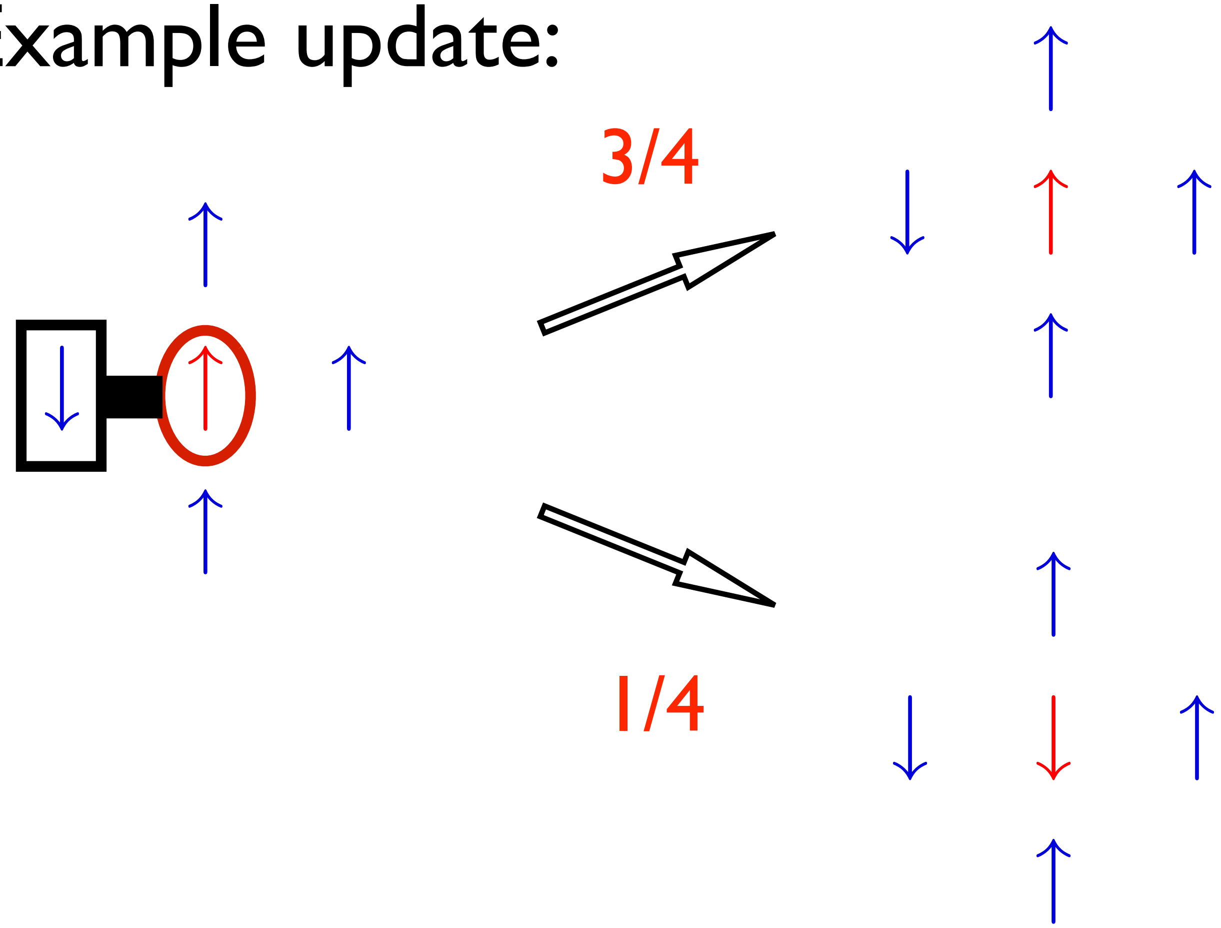


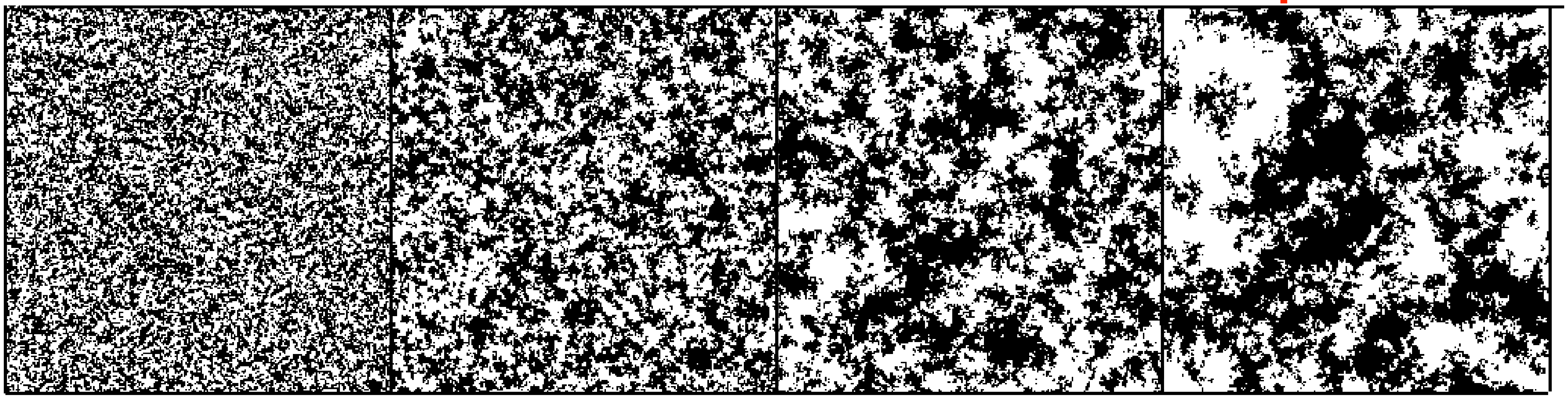
proportional rule

0. Binary voter variable at each site i
1. Pick a random voter
2. Assume state of randomly-selected neighbor
   individual has no self-confidence & adopts neighbor's state
3. Repeat 1 & 2 until consensus *necessarily* occurs in a finite system

Voter Model Evolution

Dornic et al. (2001)

random initial condition, 256 x 256 square:



t=4 t=16 t=64 t=256

Voter Model Evolution

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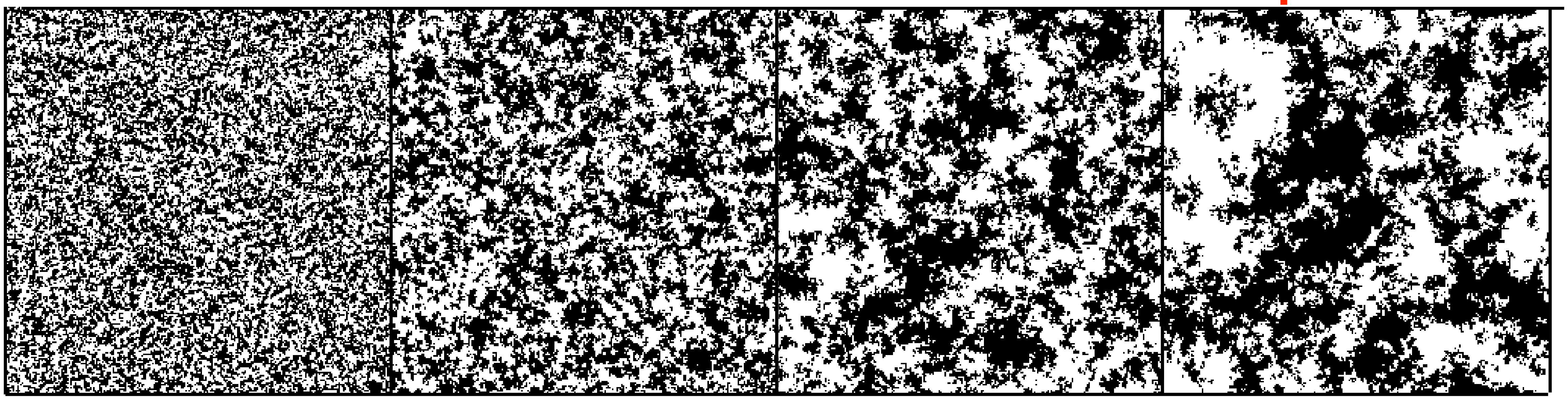
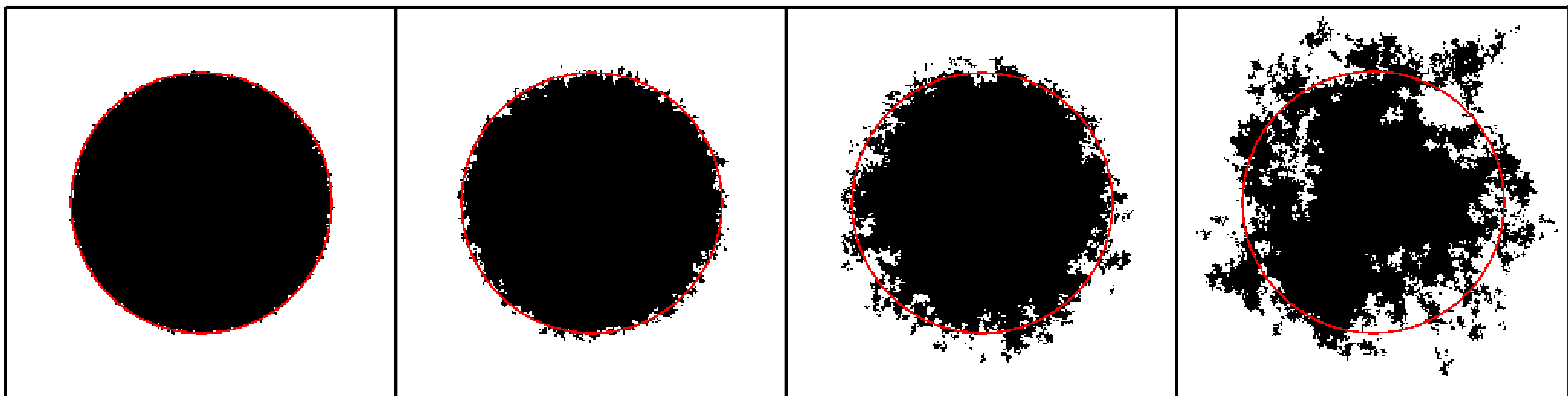


t=4 t=16 t=64 t=256

droplet initial condition:



no surface tension

Voter Model & Cousins

lemming

Voter Model: Tell me how to vote

Voter Model & Cousins

lemming

Voter Model: Tell me how to vote

persuasive "friend"

Invasion Process: I tell you how to vote

Voter Model & Cousins

lemming

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Link Dynamics: Pick two disagreeing agents and change one at random

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Link Dynamics: Pick two disagreeing agents and change one at random

identical on regular lattices, *distinct* on random graphs

Suchecki, Eguiluz & San Miguel (2005), Castellano (2005), Sood & SR (2005)

Lattice Voter Model: 3 Basic Properties

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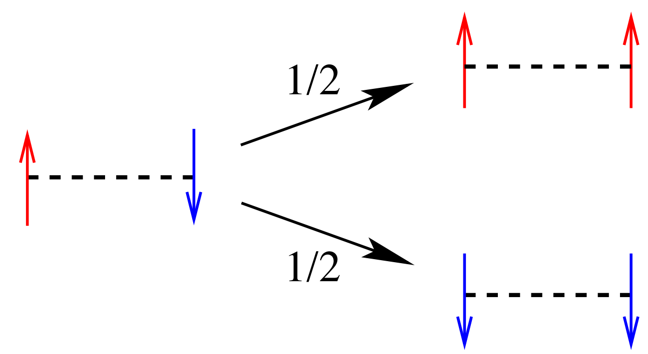
1. Final State (Exit) Probability $\mathcal{E}(\rho_0)$

Evolution of a single active link:



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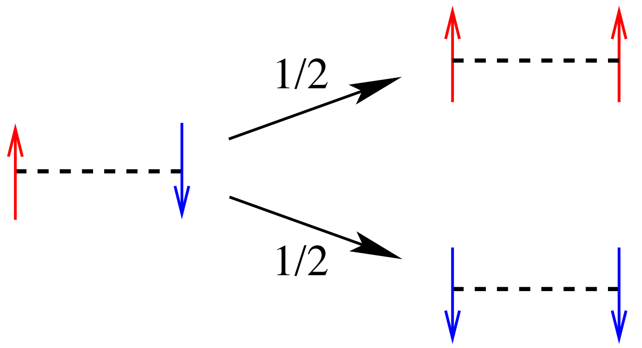

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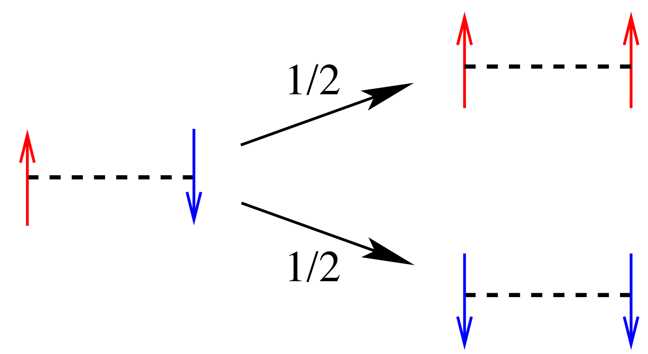

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2. Two-Spin Correlations

$$\frac{\partial c_2(\mathbf{r},t)}{\partial t} = \nabla^2 c_2(\mathbf{r},t)$$

$$c_2(r=0,t) = 1$$
$$c_2(r>0,t=0) = 0$$

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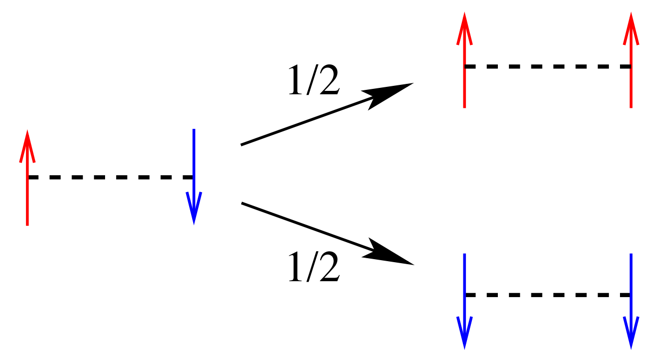

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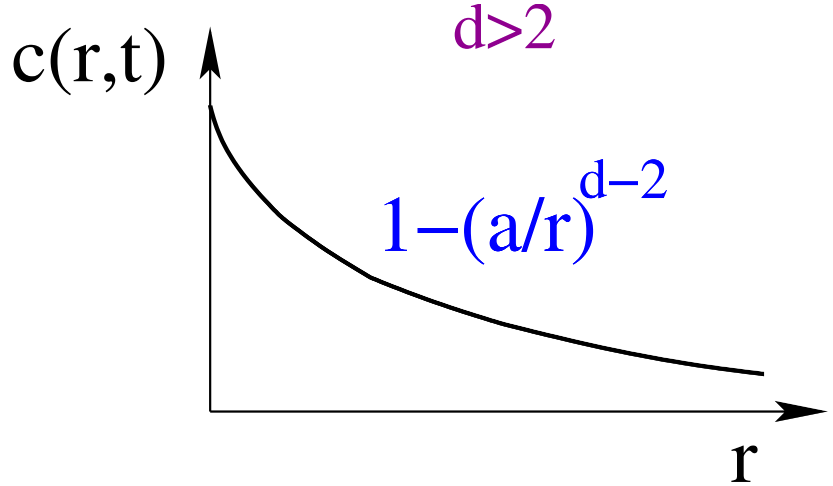

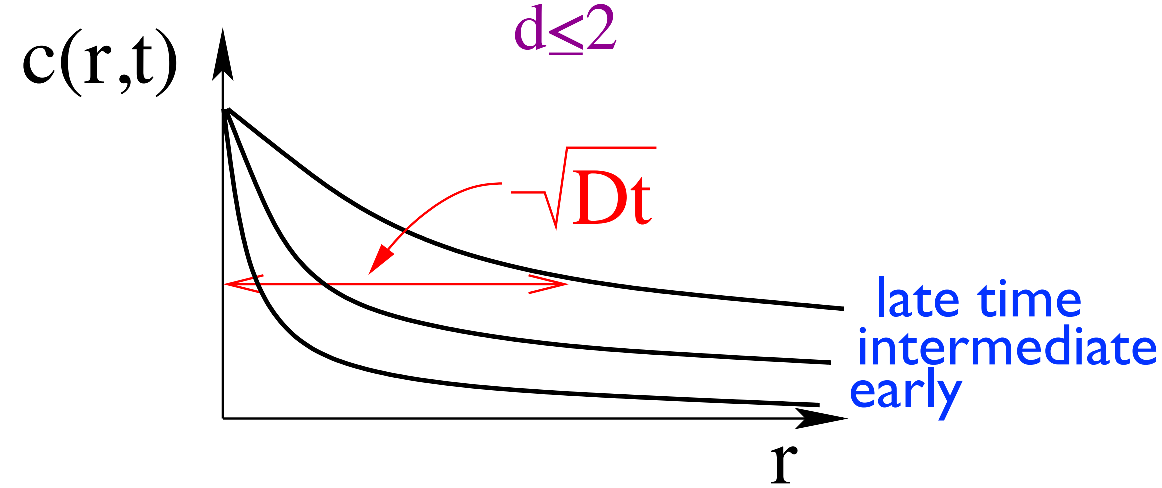

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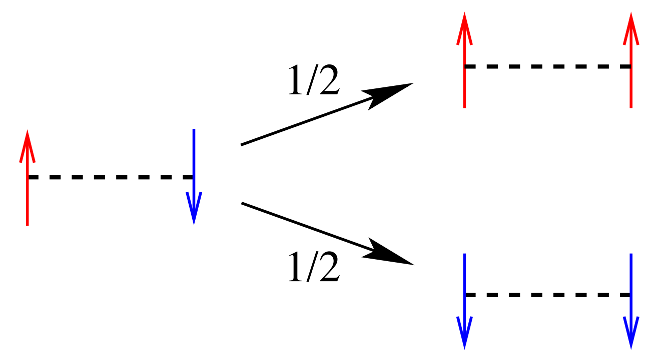

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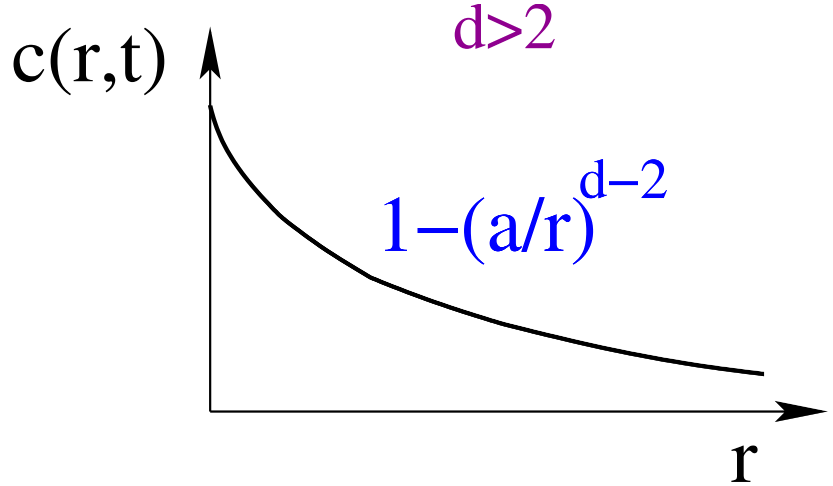

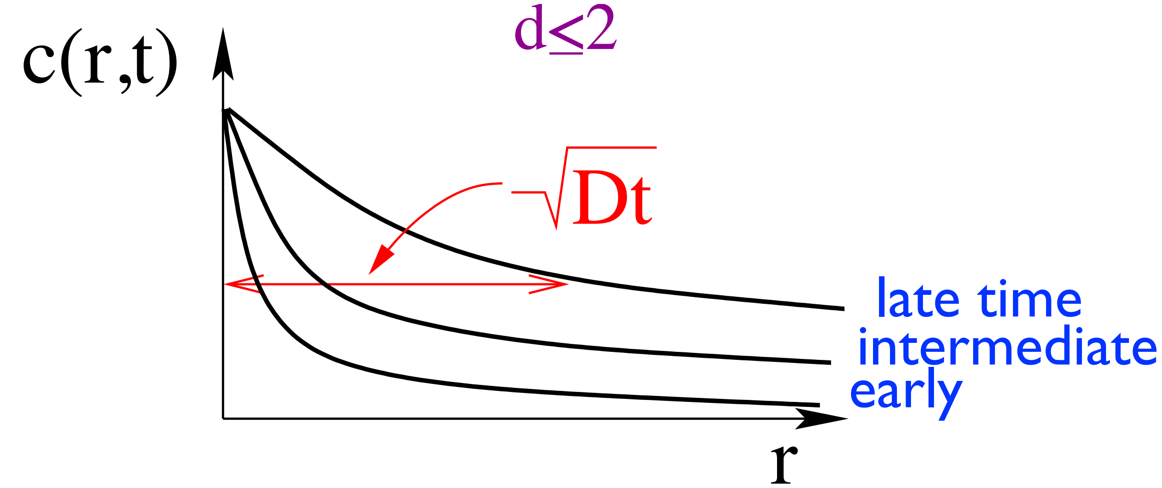

3. Consensus Time

$$\int^{\sqrt{Dt}} c(r,t)\, r^{d-1}\, dr = N$$

dimension	consensus time
1	N^2
2	N ln N
>2	N

Voter Model on Complex Networks

C. Castellano, D. Vilon, A. Vespignani, EPL **63**, 153 (2003)
K. Suchecki, V. M. Eguiluz, M. San Miguel, EPL **69**, 228 (2005)
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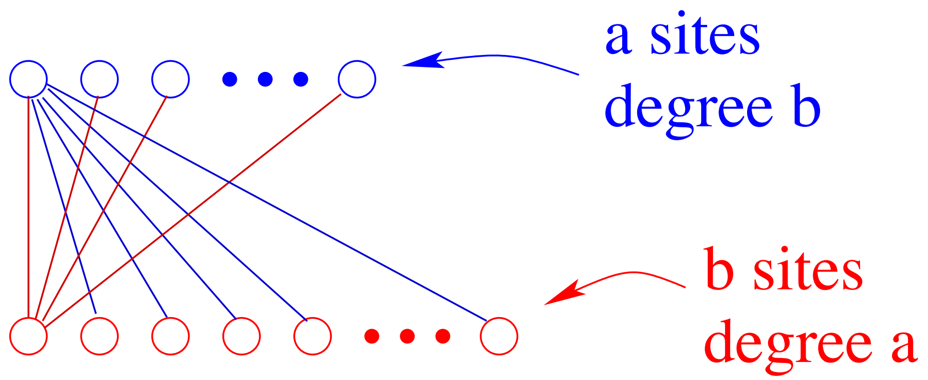
illustrative example:
complete bipartite graph



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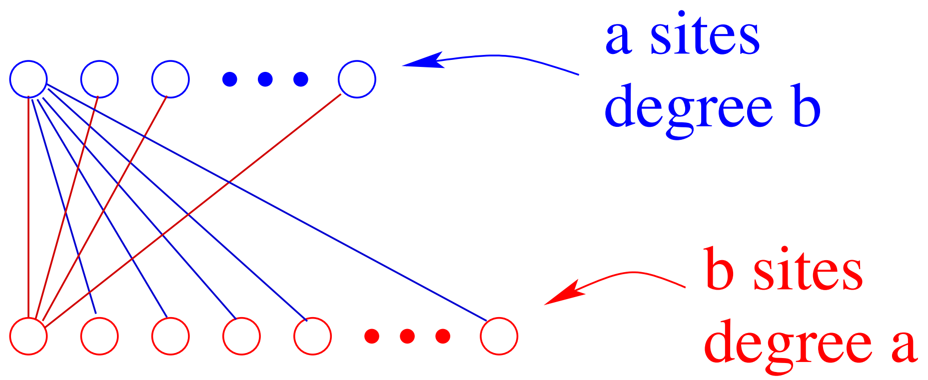

rate equation

$$dN_{\uparrow,a} = \frac{N_{\downarrow,a}N_{\uparrow,b} - N_{\uparrow,a}N_{\downarrow,b}}{(a+b)b}$$

$$dN_{\uparrow,b} = \frac{N_{\downarrow,b}N_{\uparrow,a} - N_{\uparrow,b}N_{\downarrow,a}}{(a+b)a}$$

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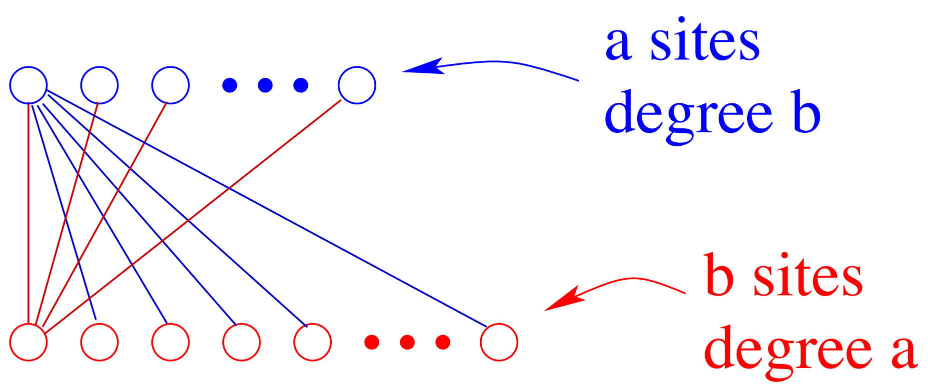

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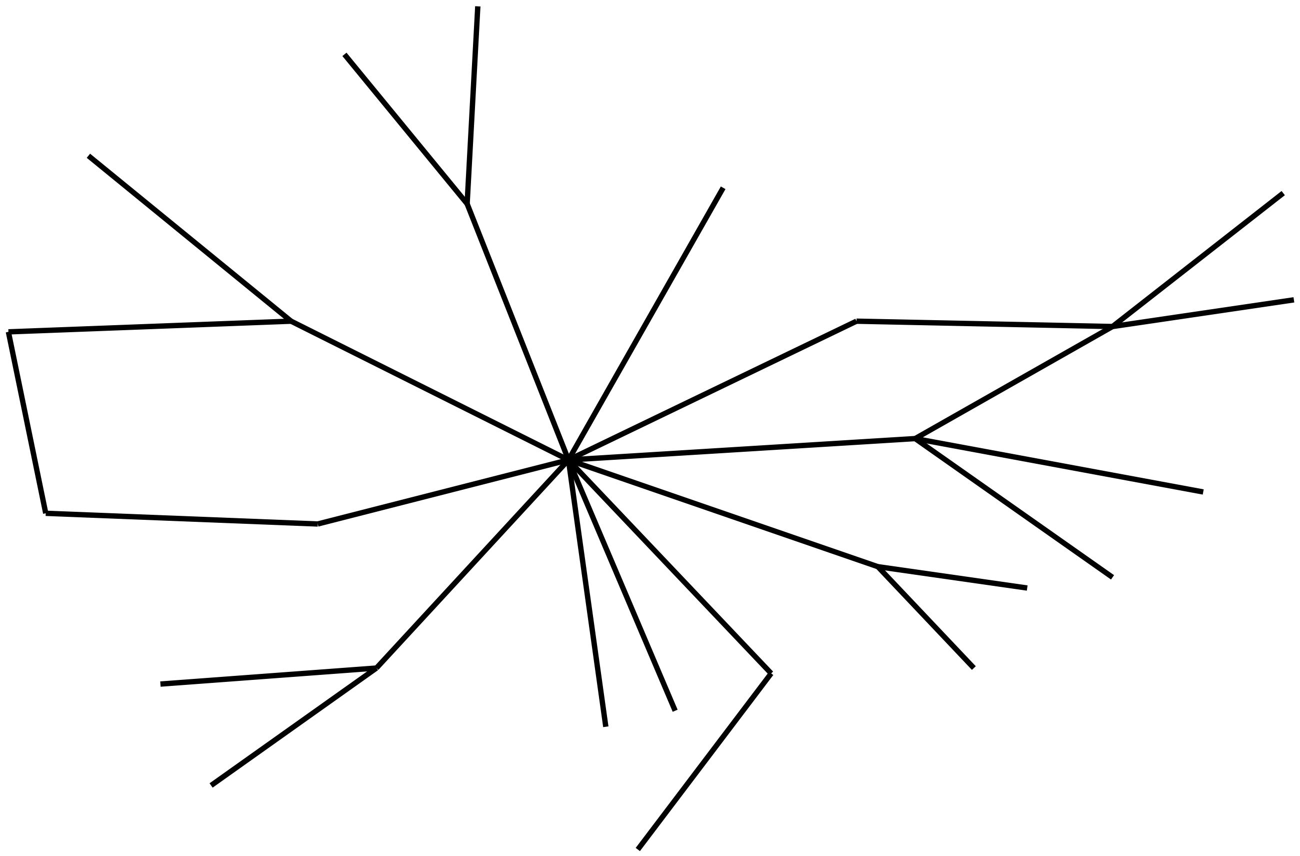
Subgraph densities: $\rho_a = N_{\uparrow,a}/a,\ \rho_b = N_{\uparrow,b}/b \quad dt = 1/(a+b)$

$$\rho_{a,b}(t) = \frac{1}{2}[\rho_{a,b}(0) - \rho_{b,a}(0)]\,e^{-2t} + \frac{1}{2}[\rho_a(0) + \rho_b(0)]$$

$$\to \frac{1}{2}[\rho_a(0) + \rho_b(0)]$$

magnetization ***not*** conserved

Voter Model on Complex Networks



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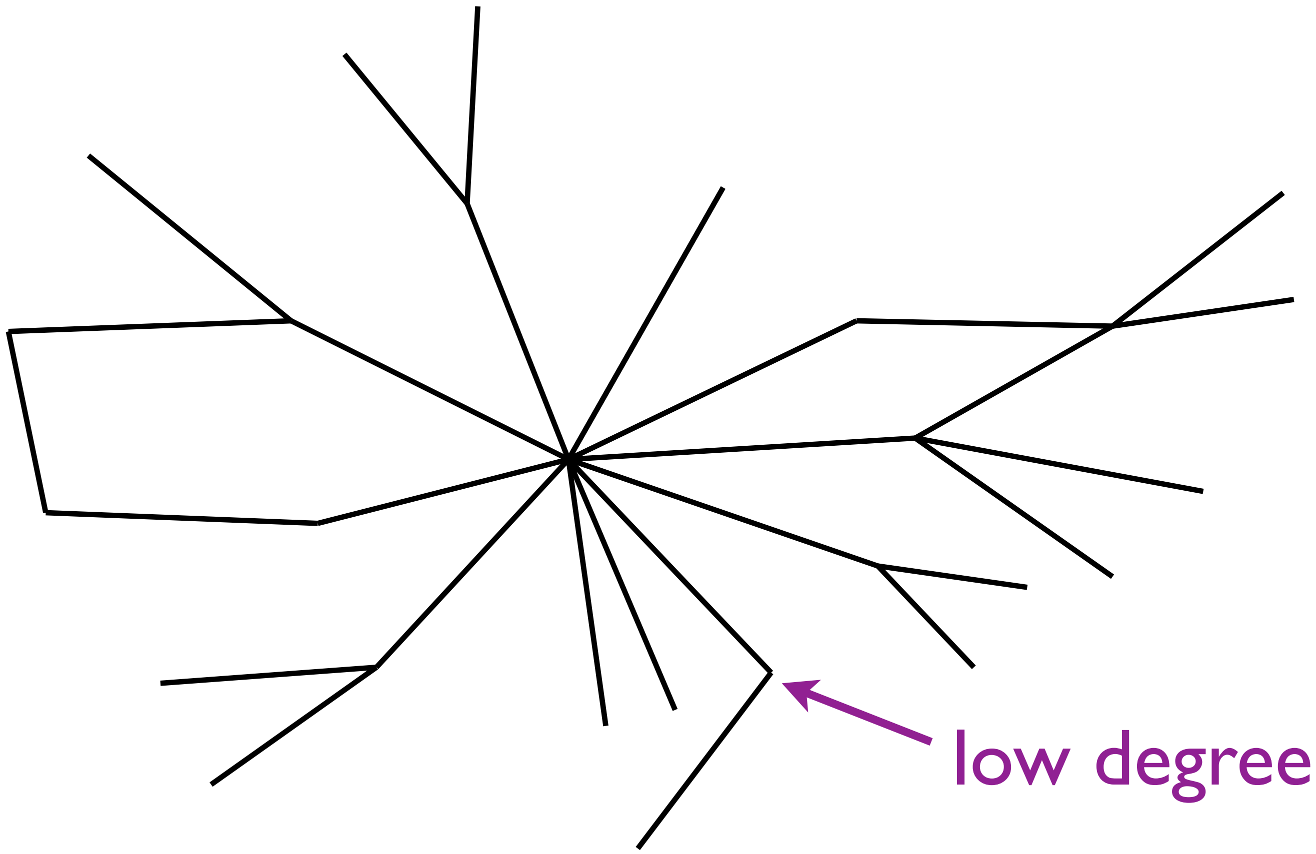
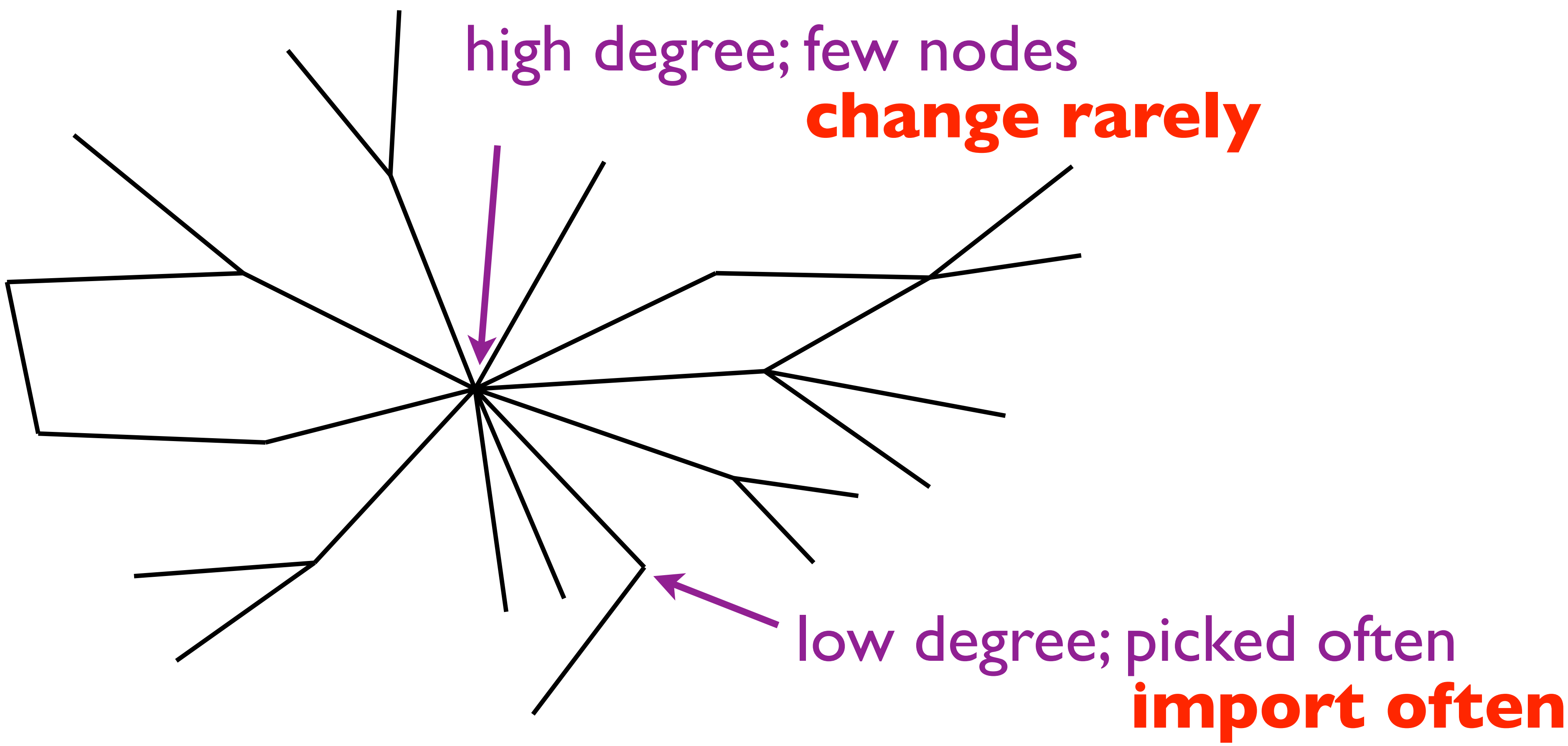


low degree; picked often

import often

Voter Model on Complex Networks



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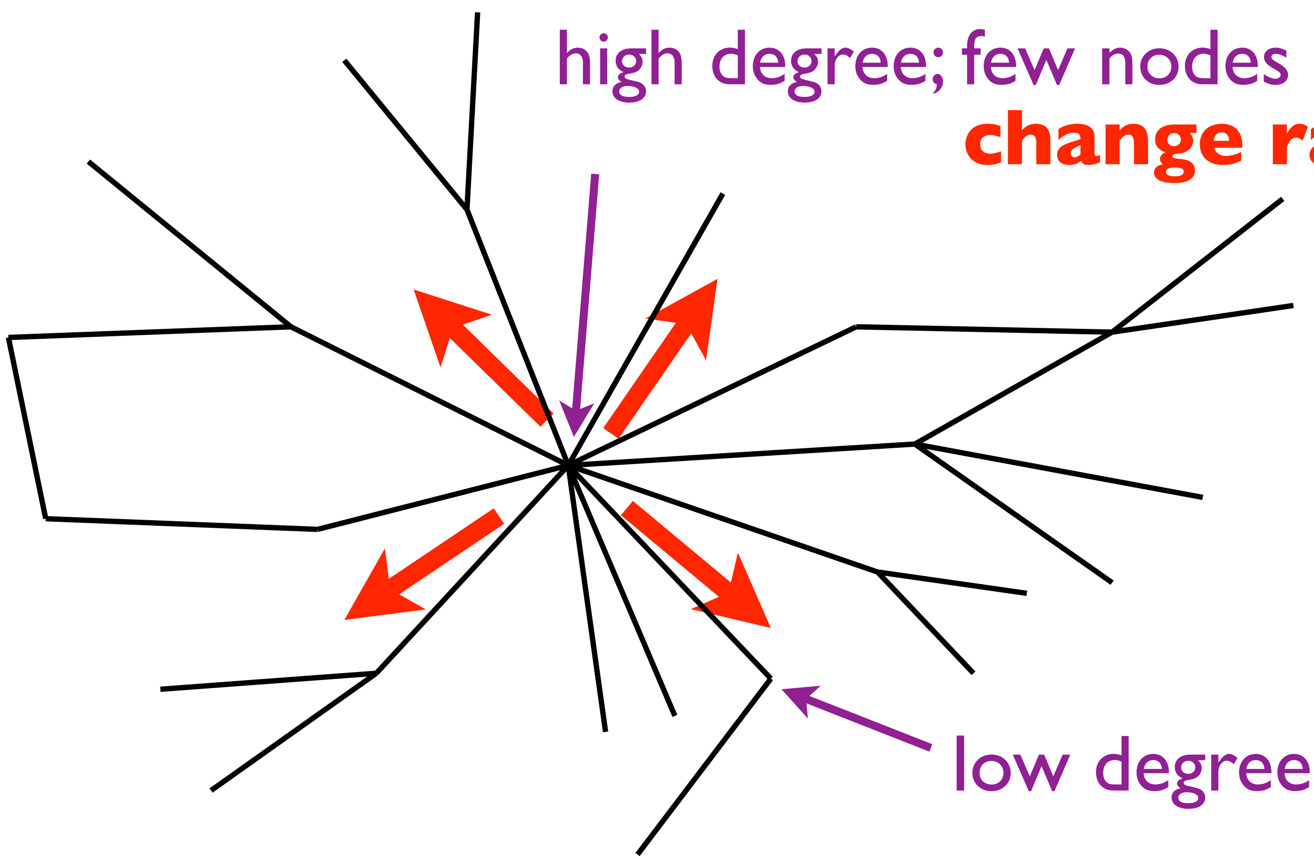
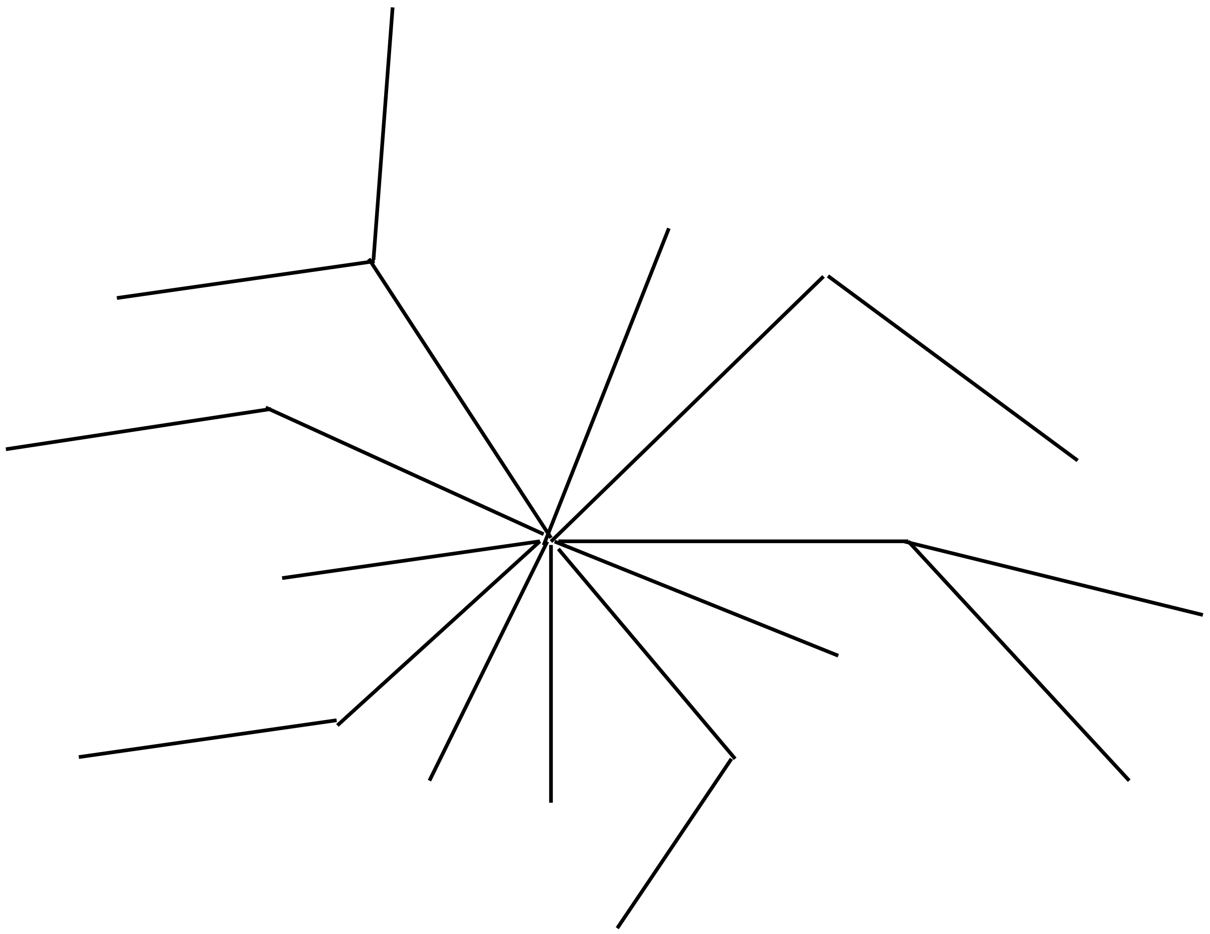


*"flow" from **high** degree to **low** degree*

Invasion Process on Complex Networks

Castellano, AIP Conf Proc **779,** 114 (2005)



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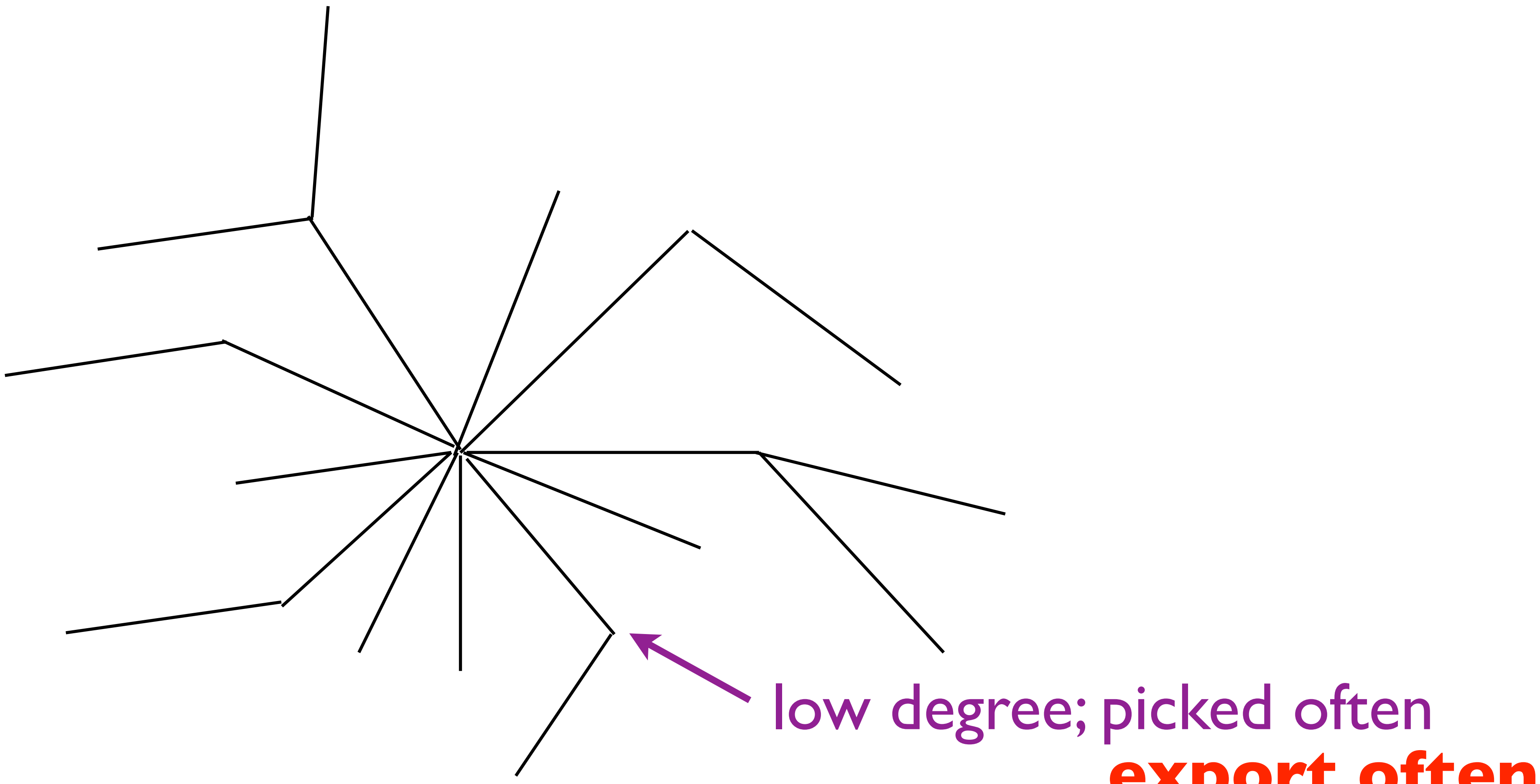

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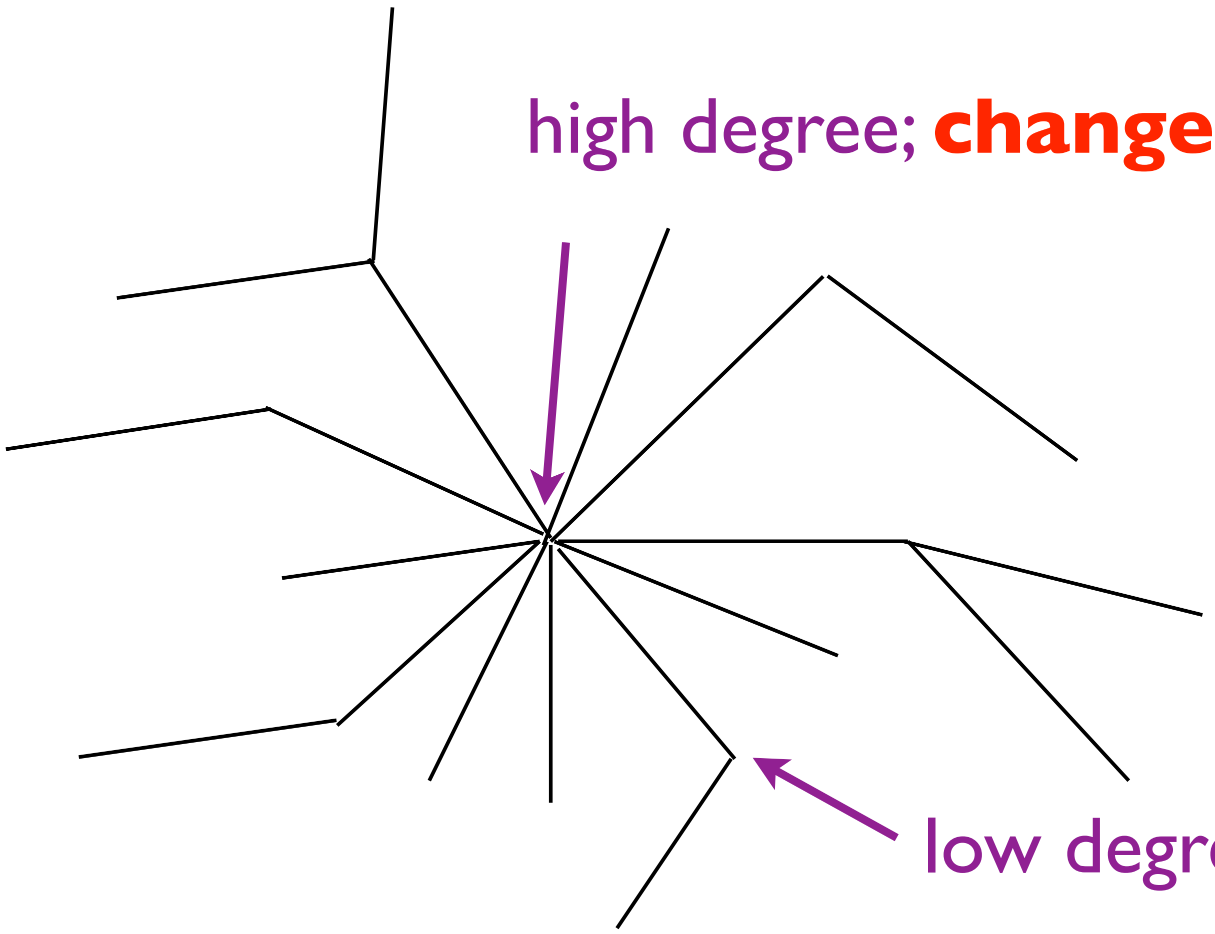

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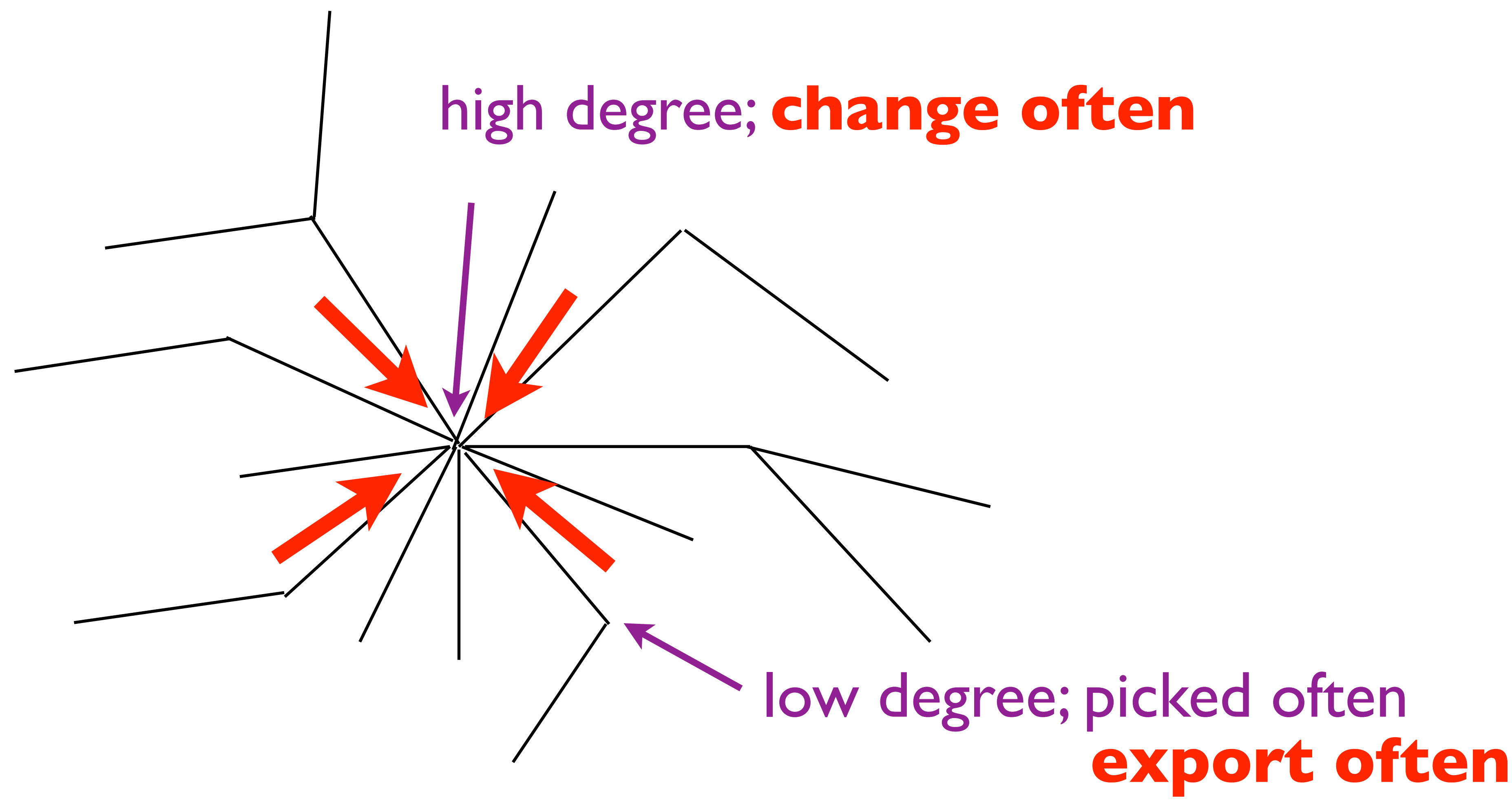

*"flow" from **low** degree to **high** degree*

Formal Approach for Conservation Law

flip rate: $$\mathbf{P}[\eta \to \eta_x] = \sum_y \frac{A_{xy}}{\mathcal{Z}} \left[\Phi(x,y) + \Phi(y,x)\right]$$

$\eta = \{1, 1, 0, 0, \ldots, 1\}$ system state

$\eta_x =$ system state when voter at x flips

$\eta(x) =$ state of voter at x

Formal Approach for Conservation Law

$$\Phi(x,y) \equiv \eta(x)[1-\eta(y)]$$
$$A_{xy} = \text{adjacency matrix}$$

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Formal Approach for Conservation Law

2 connected nodes
in different states

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$$\eta = \{1,1,0,0,\ldots,1\} \quad \text{system state}$$

$$\eta_x = \text{system state when voter at } x \text{ flips}$$

$$\eta(x) = \text{state of voter at } x$$

$$\mathcal{Z} \equiv \begin{cases} Nk_x & \text{VM} \\ Nk_y & \text{IP} \\ N\mu_1 & \text{LD} \end{cases}$$

- VM: choose x, choose neighbor of x with prob. $(Nk_x)^{-1}$
- IP: choose y (neighbor of x), choose of x with prob. $(Nk_y)^{-1}$
- LD: choose link & update x with prob. $(N\mu_1)^{-1}$

Formal Approach for Conservation Law

$$\langle \Delta\eta(x) \rangle = [1 - 2\eta(x)]\, \mathbf{P}[\eta \to \eta_x] = \sum_y \frac{A_{xy}}{\mathcal{Z}} \, [\eta(y) - \eta(x)]$$

Formal Approach for Conservation Law

$$\langle \Delta\eta(x)\rangle = [1-2\eta(x)]\,\mathbf{P}[\eta\to\eta_x] = \sum_y \frac{A_{xy}}{\mathcal{Z}}\,[\eta(y)-\eta(x)]$$

degree-weighted moments $$\langle\omega_m\rangle \equiv \frac{1}{N\mu_m}\sum_x k_x^m\,\eta(x) = \frac{1}{\mu_m}\sum_k k^m\,n_k\,\rho_k$$

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$$\Delta\langle\omega_1\rangle = \sum_{x,y}\frac{A_{xy}}{Nk_x}k_x[\eta(y)-\eta(x)] = 0$$

voter model

$$\Delta\langle\omega_{-1}\rangle = \sum_{x,y}\frac{A_{xy}}{Nk_x k_y}[\eta(y)-\eta(x)] = 0$$

invasion process

$$\langle\Delta\omega_0\rangle = \langle\Delta\rho\rangle = \sum_{x,y}\frac{A_{xy}}{N\mu_1}\,[\eta(y)-\eta(x)] = 0$$

link dynamics

Exit Probability on Complex Networks

Voter model: $\mathcal{E}(\omega) = \omega$

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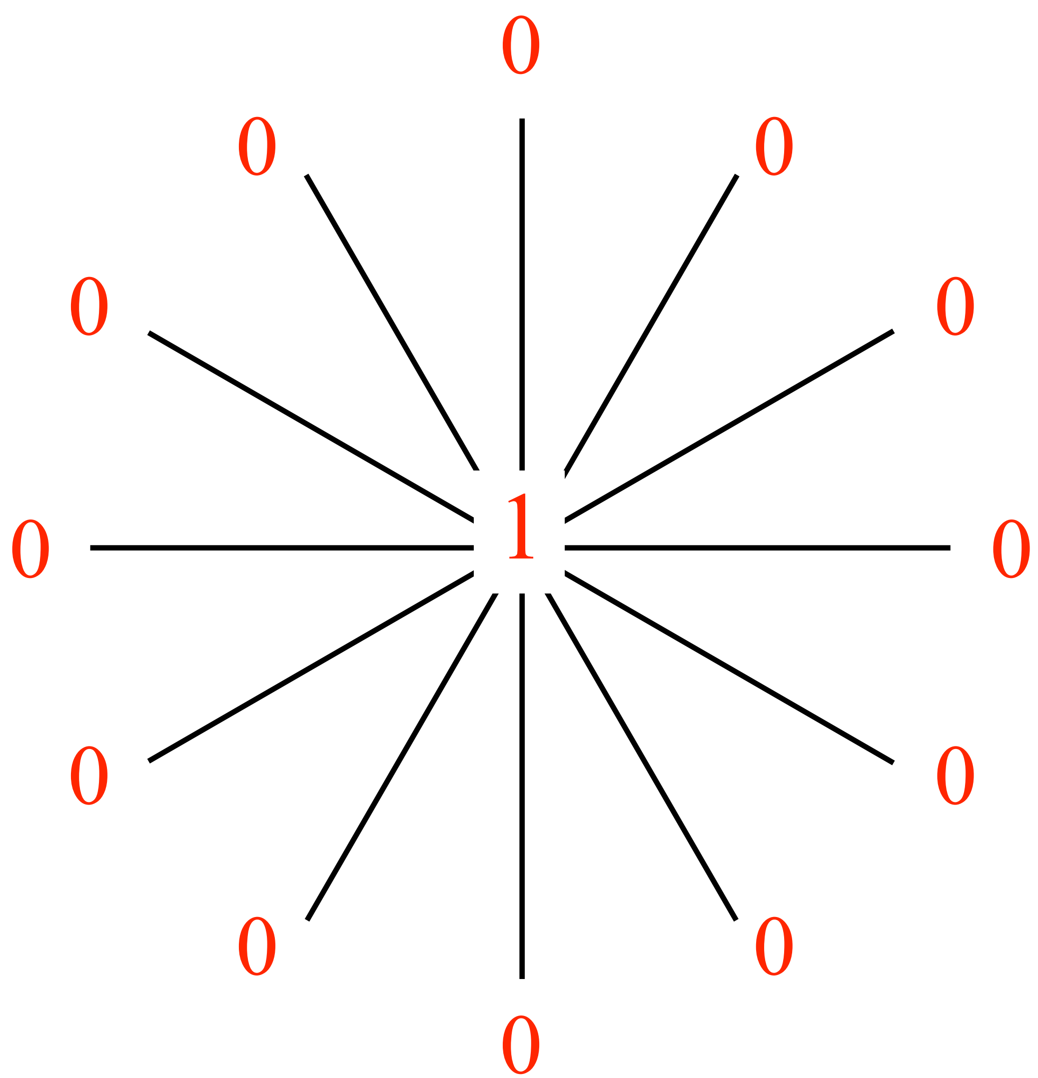
Extreme case: star graph

N nodes: degree 1
1 node: degree N



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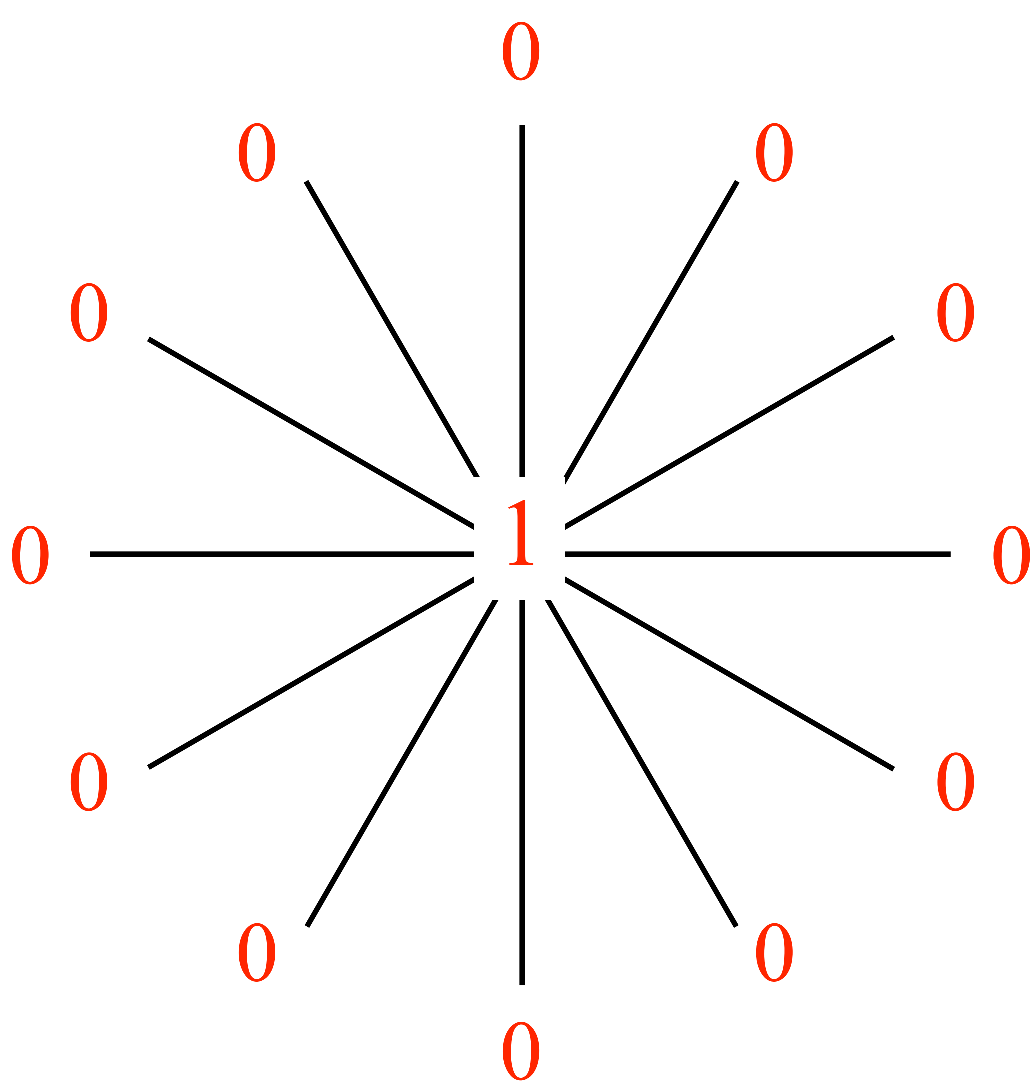

$$\omega = \frac{1}{\mu_1}\sum_k k\, n_k \rho_k = \frac{1}{2}$$

Final state: all 1 with prob. 1/2!

Byproduct: Voter Model Fixation Probability

What is the probability that a single mutant "takes over" a population?

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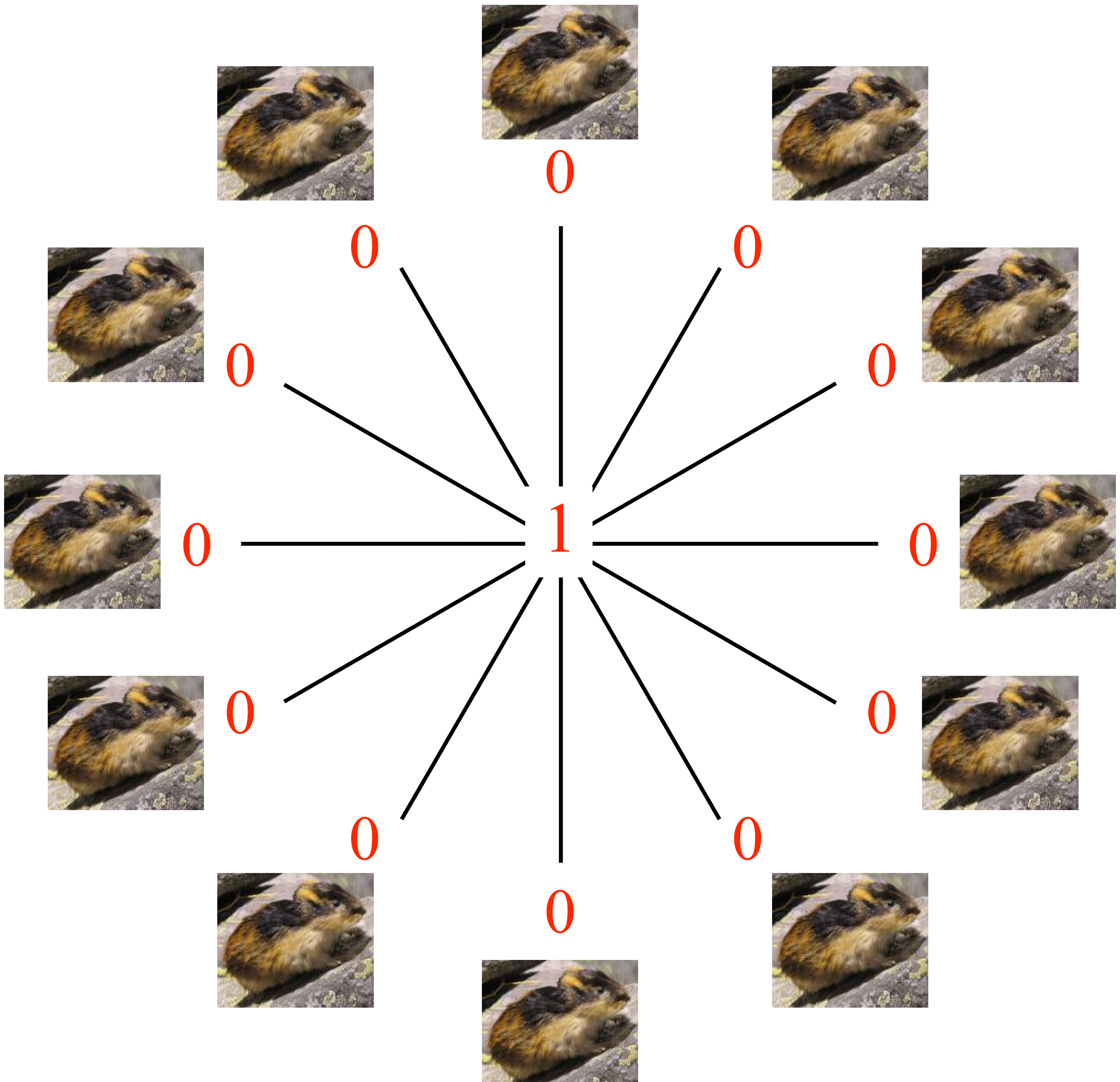
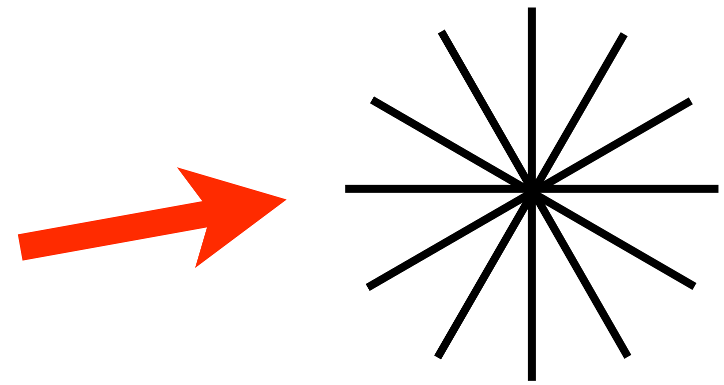
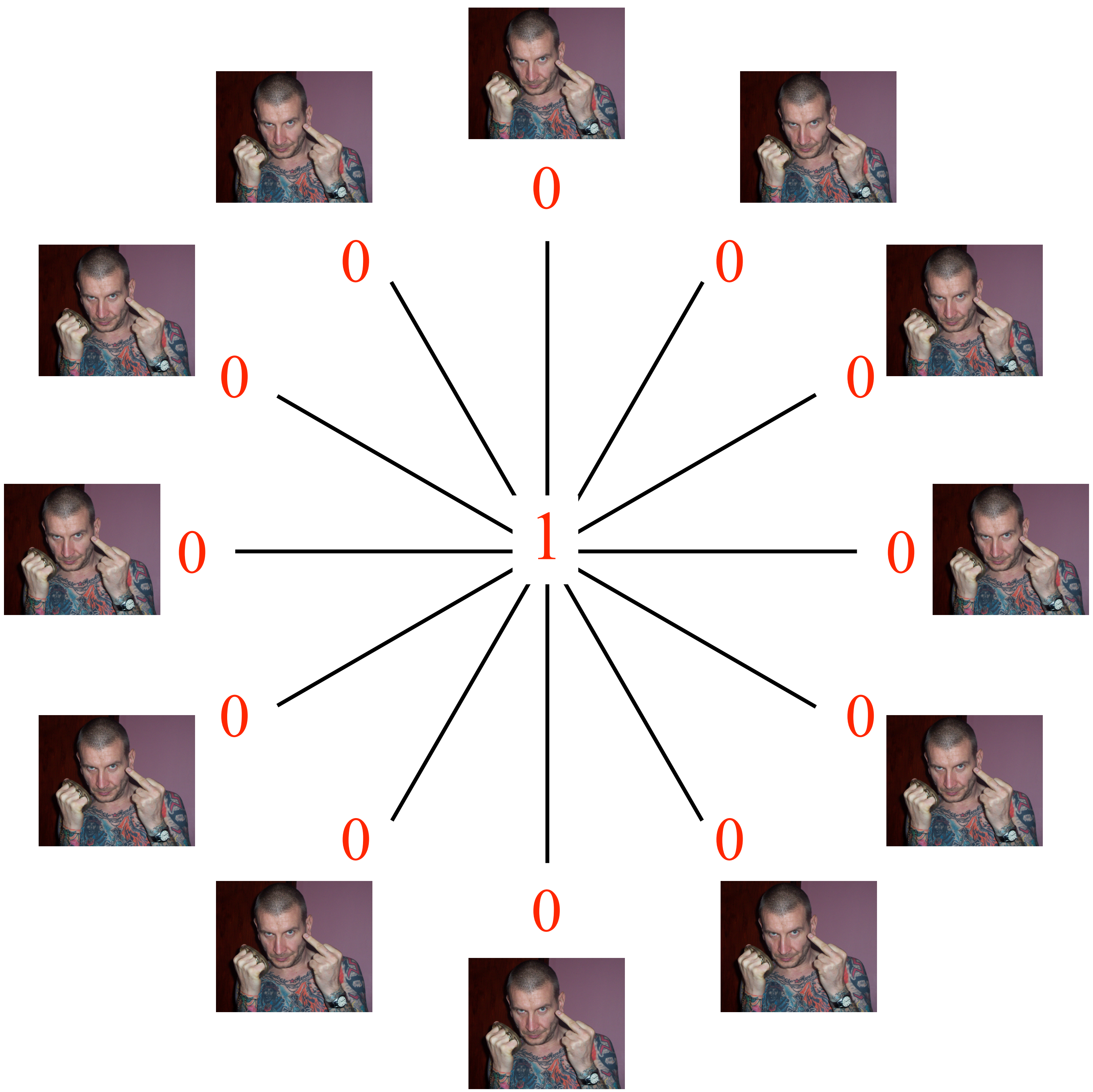


1 consensus

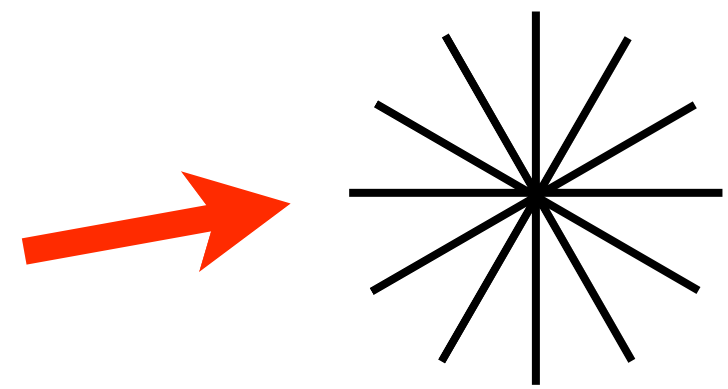


fixation probability $\propto$ node degree

Invasion Process Fixation Probability

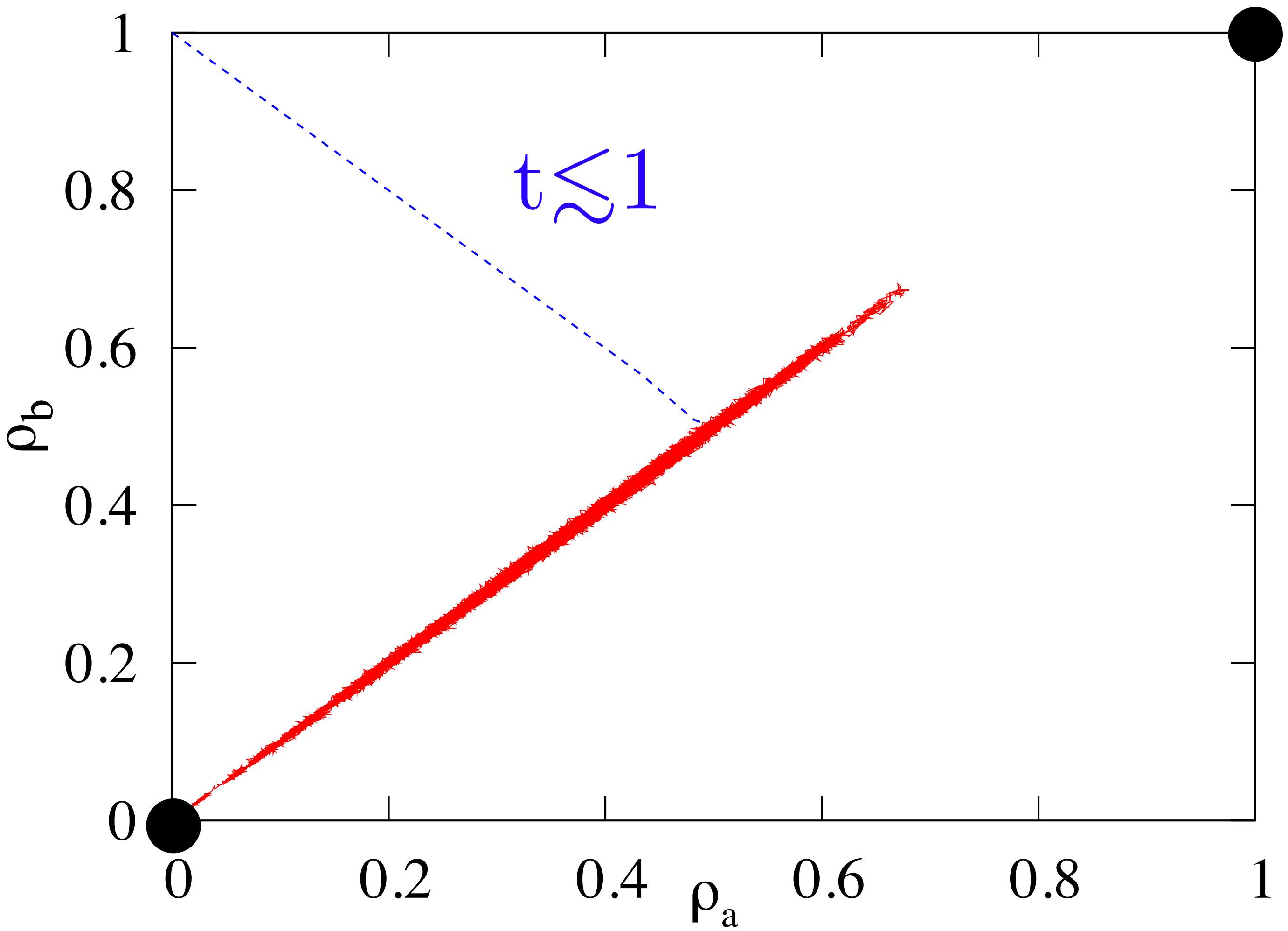


0 consensus

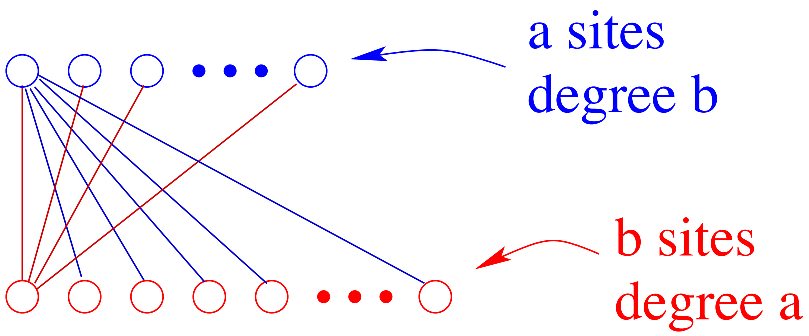


fixation probability
$\propto$ 1/(node degree)

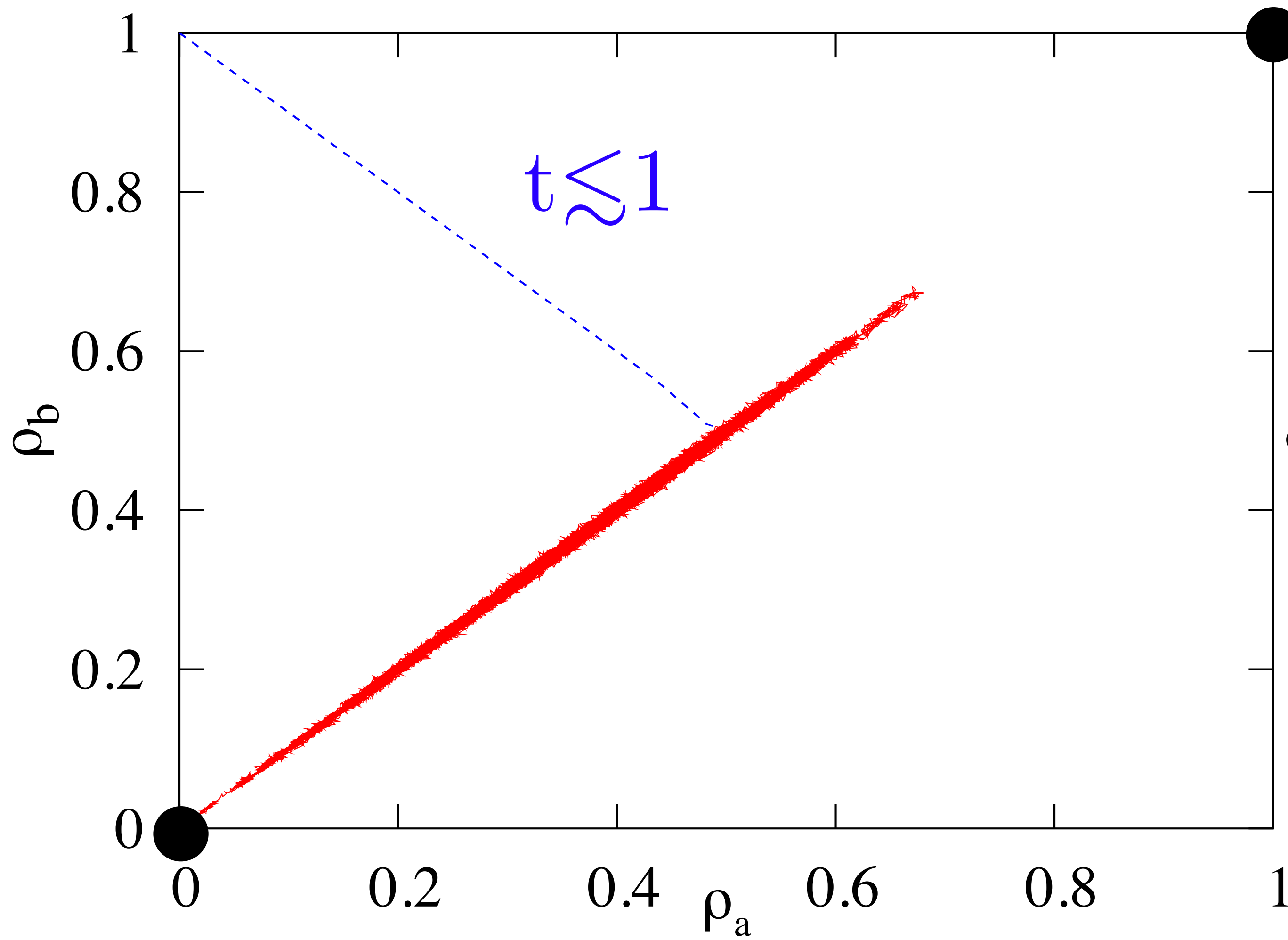
Route to Consensus on Complex Graphs

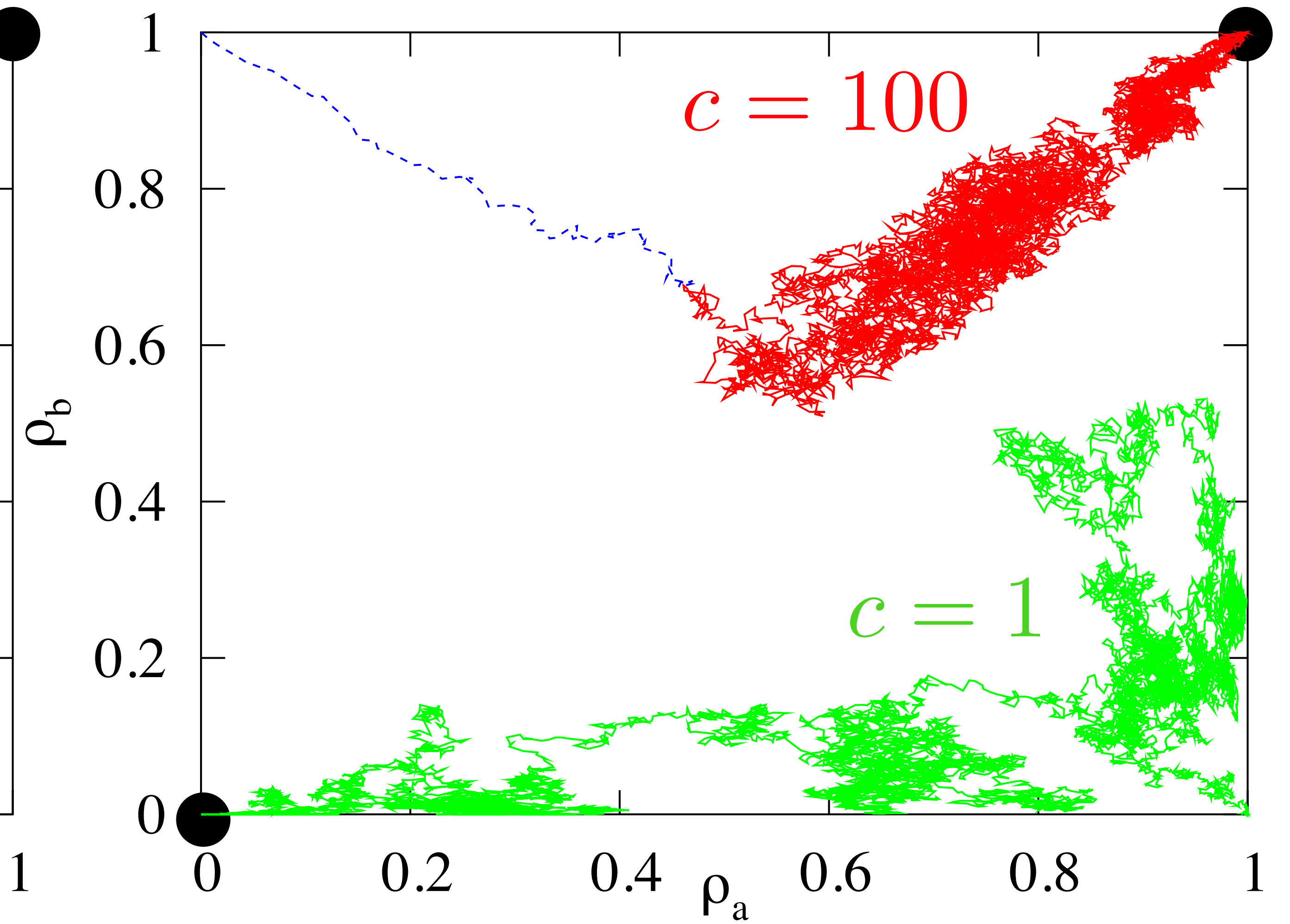


complete bipartite graph

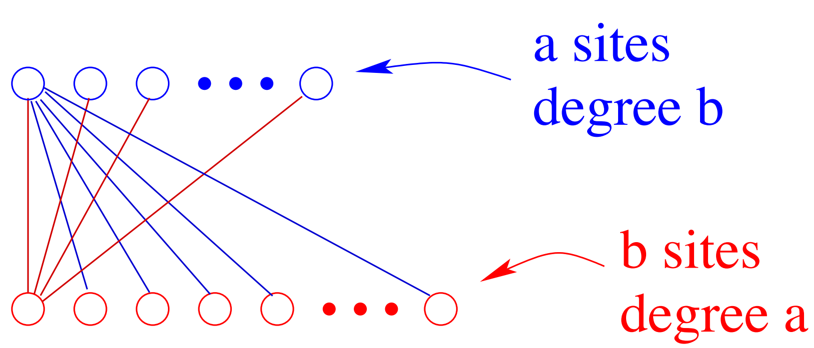


Route to Consensus on Complex Graphs

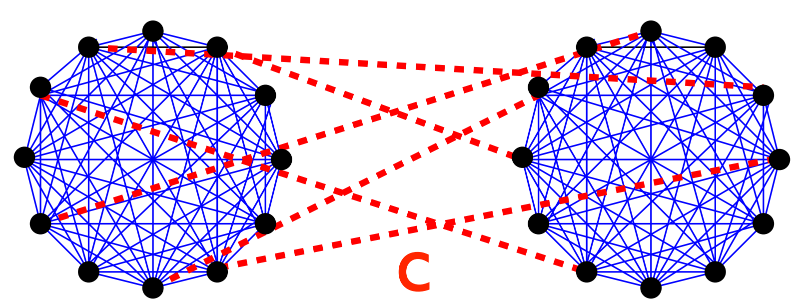




complete bipartite graph



two-clique graph



N=10000, C links/node

Consensus Time Evolution Equation

warmup: complete graph

A Guide to First-Passage Processes
(CUP, 2001)

$T(\rho) \equiv$ av. consensus time starting with density ρ

$$\begin{aligned} T(\rho) &= \mathcal{R}(\rho)[T(\rho+d\rho)+dt] \\ &\quad +\mathcal{L}(\rho)[T(\rho-d\rho)+dt] \\ &\quad +[1-\mathcal{R}(\rho)-\mathcal{L}(\rho)][T(\rho)+dt] \end{aligned}$$

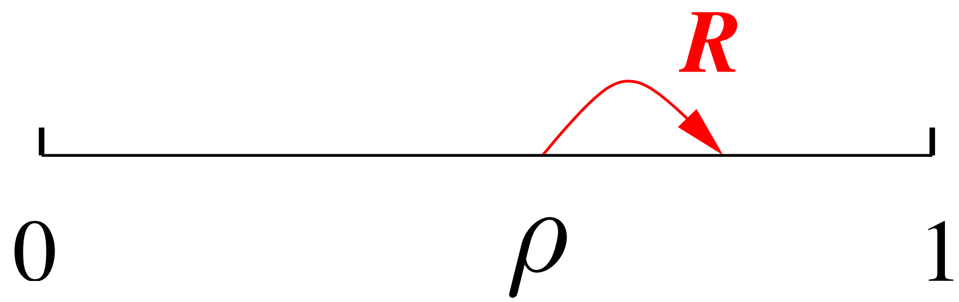
Consensus Time Evolution Equation

A Guide to First-Passage Processes
(CUP, 2001)

warmup: complete graph

$T(\rho) \equiv$ av. consensus time starting with density ρ

$$
\begin{aligned}
T(\rho) \quad &= \quad \mathcal{R}(\rho)[T(\rho + d\rho) + dt] \\
&\quad +\mathcal{L}(\rho)[T(\rho - d\rho) + dt] \\
&\quad +[1 - \mathcal{R}(\rho) - \mathcal{L}(\rho)][T(\rho) + dt]
\end{aligned}
$$



$$
\begin{aligned}
\mathcal{R}(\rho) &\equiv \mathrm{prob}(\downarrow\uparrow\to\uparrow\uparrow) \\
&= \rho(1-\rho)
\end{aligned}
$$

Consensus Time Evolution Equation

A Guide to First-Passage Processes
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warmup: complete graph

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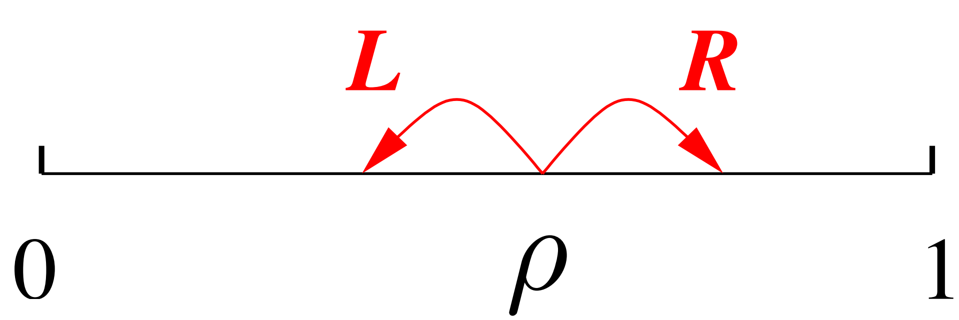

$$\begin{aligned} \mathcal{R}(\rho) &\equiv \text{prob}(\downarrow\uparrow \rightarrow \uparrow\uparrow) \\ \mathcal{L}(\rho) &\equiv \text{prob}(\uparrow\downarrow \rightarrow \downarrow\downarrow) \\ &= \rho(1 - \rho) \end{aligned}$$

Consensus Time Evolution Equation

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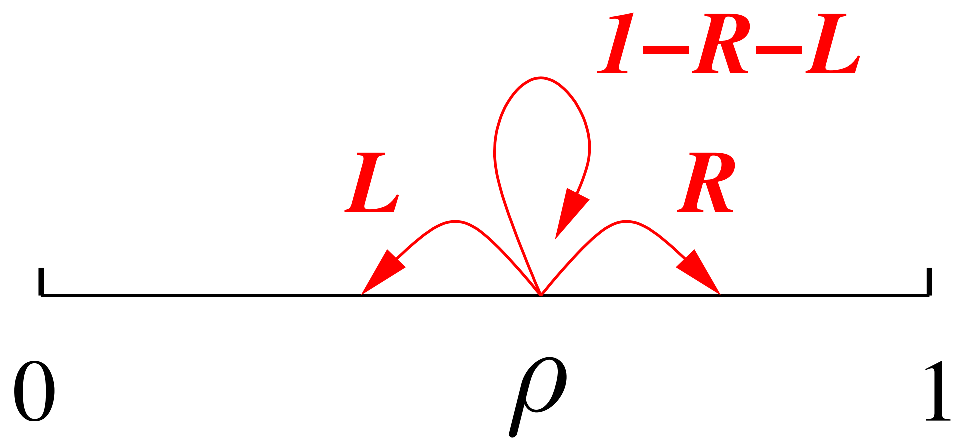


$$\begin{aligned} \mathcal{R}(\rho) &\equiv \text{prob}(\downarrow\uparrow\to\uparrow\uparrow) \\ \mathcal{L}(\rho) &\equiv \text{prob}(\uparrow\downarrow\to\downarrow\downarrow) \\ &= \rho(1-\rho) \end{aligned}$$

Consensus Time on Complete Graph

$$\begin{aligned} T(\rho) &= \mathcal{R}(\rho)[T(\rho + d\rho) + dt] \\ &\quad + \mathcal{L}(\rho)[T(\rho - d\rho) + dt] \\ &\quad + [1 - \mathcal{R}(\rho) - \mathcal{L}(\rho)][T(\rho) + dt] \end{aligned}$$

continuum limit:

$$T'' = -\frac{N}{\rho(1-\rho)}$$

solution:

$$T(\rho) = -N\left[\rho \ln \rho + (1-\rho)\ln(1-\rho)\right]$$

Consensus Time on Heterogeneous Networks

$$T(\{\rho_k\}) \equiv \text{av. consensus time starting with density } \rho_k \text{ on nodes of degree } k$$

$$\begin{aligned}
T(\{\rho_k\}) &= \sum_k \mathcal{R}_k(\{\rho_k\})[T(\{\rho_k^+\}) + dt] \\
&+ \sum_k \mathcal{L}_k(\{\rho_k\})[T(\{\rho_k^-\}) + dt] \\
&+ \left[1 - \sum_k [\mathcal{R}_k(\{\rho_k\}) + \mathcal{L}_k(\{\rho_k\})]\right][T(\{\rho_k\}) + dt]
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}_k(\{\rho_k\}) &= \text{prob}(\rho_k \to \rho_k^+) \\
&= \frac{1}{N} {\sum_x}' \frac{1}{k_x} \sum_y P(\downarrow, \text{---}, \uparrow) \\
&= n_k \omega (1 - \rho_k)
\end{aligned}$$

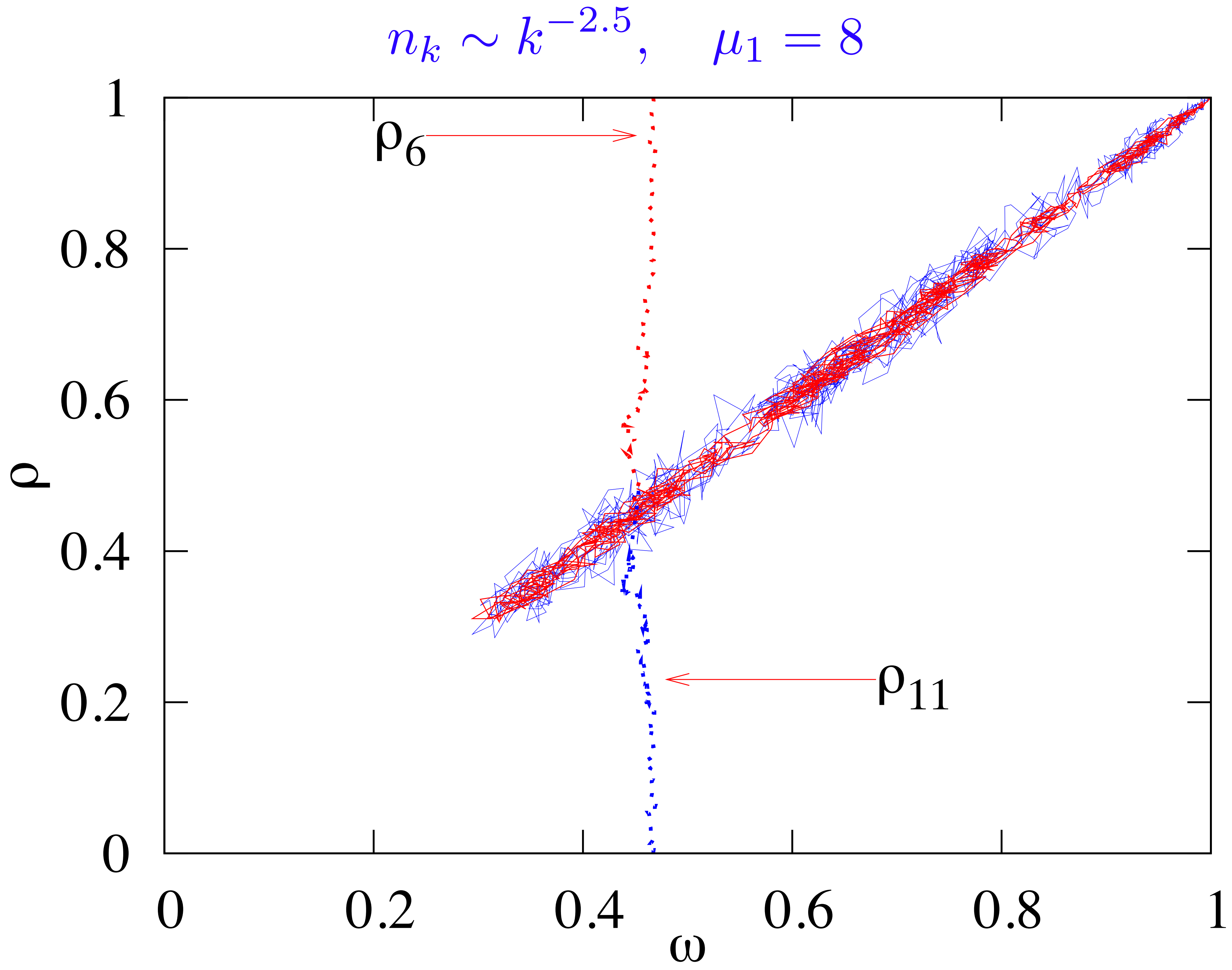
$$\mathcal{L}_k(\{\rho_k\}) = n_k \rho_k (1 - \omega)$$

Consensus Time on Heterogeneous Networks

continuum limit:

$$\sum_k \left[(\omega - \rho_k) \frac{\partial T}{\partial \rho_k} + \frac{\omega + \rho_k - 2\omega\rho_k}{2Nn_k} \frac{\partial^2 T}{\partial \rho_k^2} \right] = -1$$

(Molloy-Reed) Configuration Model



Consensus Time on Heterogeneous Networks

continuum limit:

$$\sum_k \left[(\omega - \rho_k) \frac{\partial T}{\partial \rho_k} + \frac{\omega + \rho_k - 2\omega\rho_k}{2Nn_k} \frac{\partial^2 T}{\partial \rho_k^2} \right] = -1$$

now use $\rho_k \longrightarrow \omega \quad \forall k$

and $\frac{\partial}{\partial \rho_k} = \frac{\partial \omega}{\partial \rho_k} \frac{\partial}{\partial \omega} = \frac{k n_k}{\mu_1} \frac{\partial}{\partial \omega}$

Consensus Time on Heterogeneous Networks

continuum limit:

$$\sum_k \left[(\omega - \rho_k) \frac{\partial T}{\partial \rho_k} + \frac{\omega + \rho_k - 2\omega\rho_k}{2Nn_k} \frac{\partial^2 T}{\partial \rho_k^2} \right] = -1$$

now use $\rho_k \longrightarrow \omega \quad \forall k$

and $\frac{\partial}{\partial \rho_k} = \frac{\partial \omega}{\partial \rho_k} \frac{\partial}{\partial \omega} = \frac{k n_k}{\mu_1} \frac{\partial}{\partial \omega}$

to give $\frac{\partial^2 T}{\partial \omega^2} = -\frac{N \mu_1^2/\mu_2}{\omega(1-\omega)}$ same as $T'' = -\frac{N}{\rho(1-\rho)}$

with *effective* size $N_{\text{eff}} = N\, \mu_1^2/\mu_2$

Consensus Time for Power-Law Degree Distribution $n_k \sim k^{-\nu}$

Voter model: $T_N \sim N_{\text{eff}} = N\mu_1^2/\mu_2$

$$T_N \sim \begin{cases} N & \nu > 3, \\ N/\ln N & \nu = 3, \\ N^{(2\nu-4)/(\nu-1)} & 2 < \nu < 3, \\ (\ln N)^2 & \nu = 2, \\ \mathcal{O}(1) & \nu < 2. \end{cases}$$

Consensus Time for Power-Law Degree Distribution $n_k \sim k^{-\nu}$

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Invasion process: $T_N \sim N_{\rm eff} = N\mu_1\,\mu_{-1}$

$$T_N \sim \begin{cases} N & \nu > 2 \\ N\ln N & \nu = 2 \\ N^{3-\nu} & \nu < 2 \end{cases}$$

Partisan Voting and Truth

N. Masuda, N. Gibert, SR
PRE **82**, 010103(R) (2010)

↑ prefers truth & in T state — density T_+

↓ prefers truth & in F state — density T_-

↑ prefers false & in T state — density F_+

↓ prefers false & in F state — density F_-

Partisan Voting and Truth

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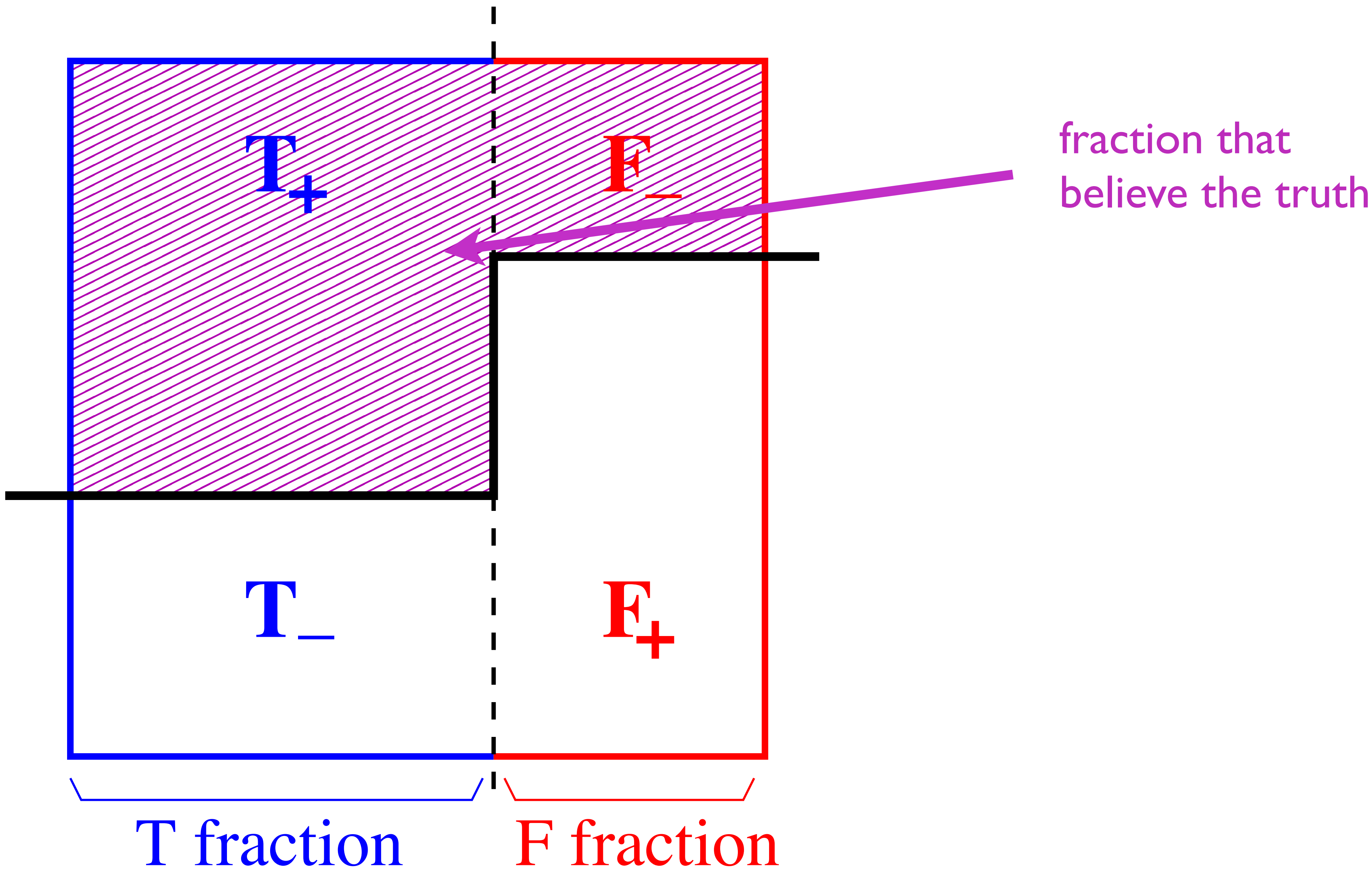
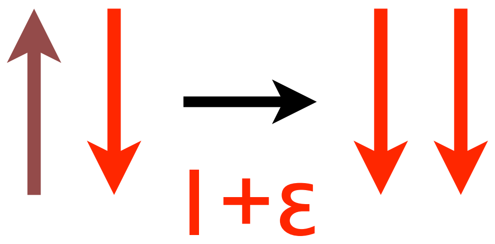


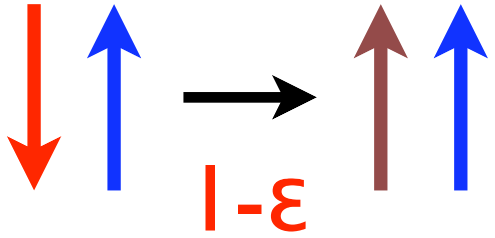
partisan voting update:

1. Pick voter, pick neighbor (as in usual voter model);

2a. If initial voter becomes *concordant* by adopting neighboring state, change occurs with rate 1+ε;



2b. If initial voter becomes *discordant* by adopting neighboring state, change occurs with rate 1-ε.



Rate Equations

$$\dot{T}_+ = (1+\epsilon)T_- \left[T_+ + F_-\right] - (1-\epsilon)T_+ \left[T_- + F_+\right]$$

$$\dot{T}_- = (1-\epsilon)T_+ \left[T_- + F_+\right] - (1+\epsilon)T_- \left[T_+ + F_-\right]$$

$$\dot{F}_+ = (1+\epsilon)F_- \left[F_+ + T_-\right] - (1-\epsilon)F_+ \left[F_- + T_+\right]$$

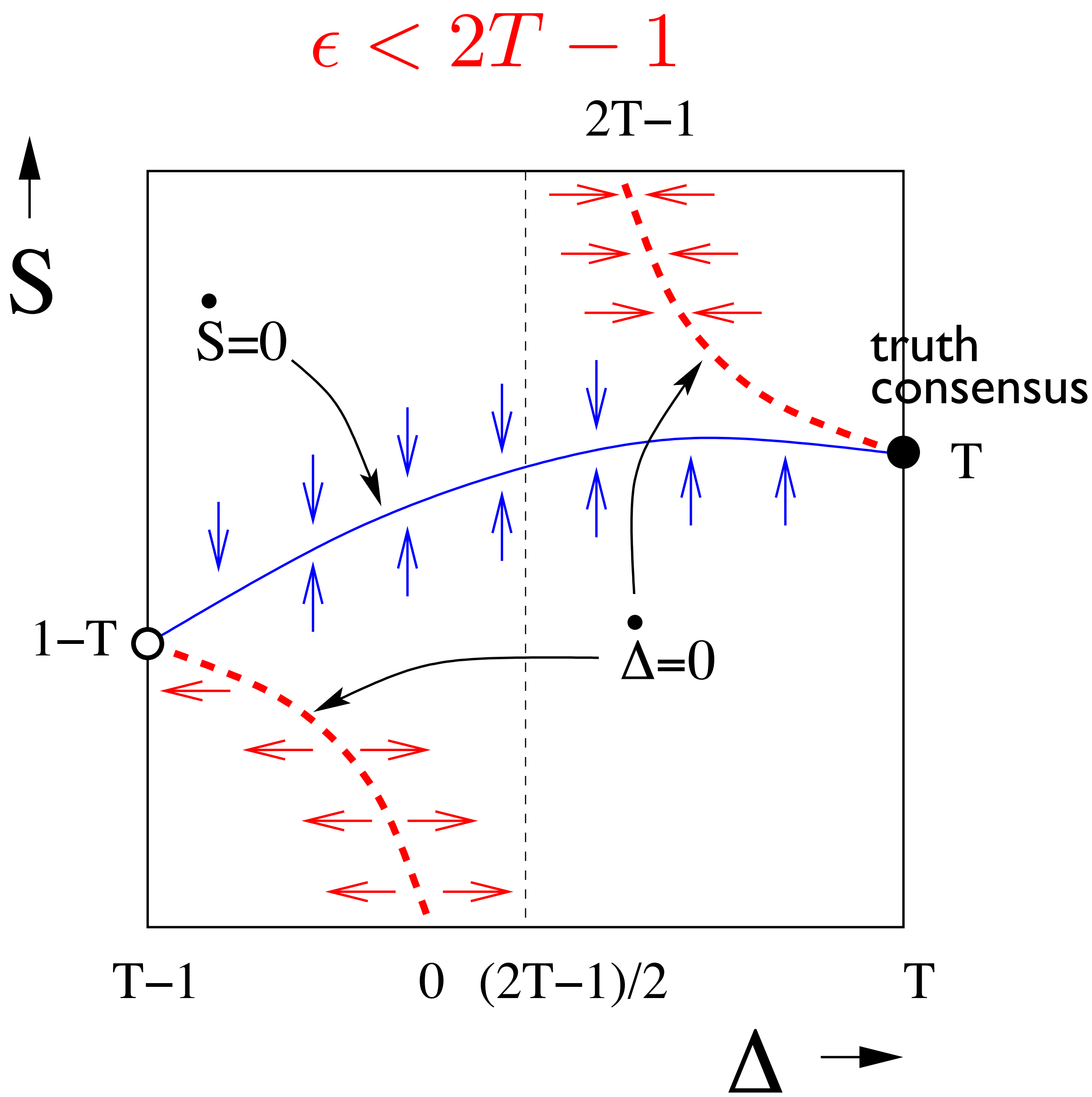
$$\dot{F}_- = (1-\epsilon)F_+ \left[F_- + T_+\right] - (1+\epsilon)F_- \left[F_+ + T_-\right]$$

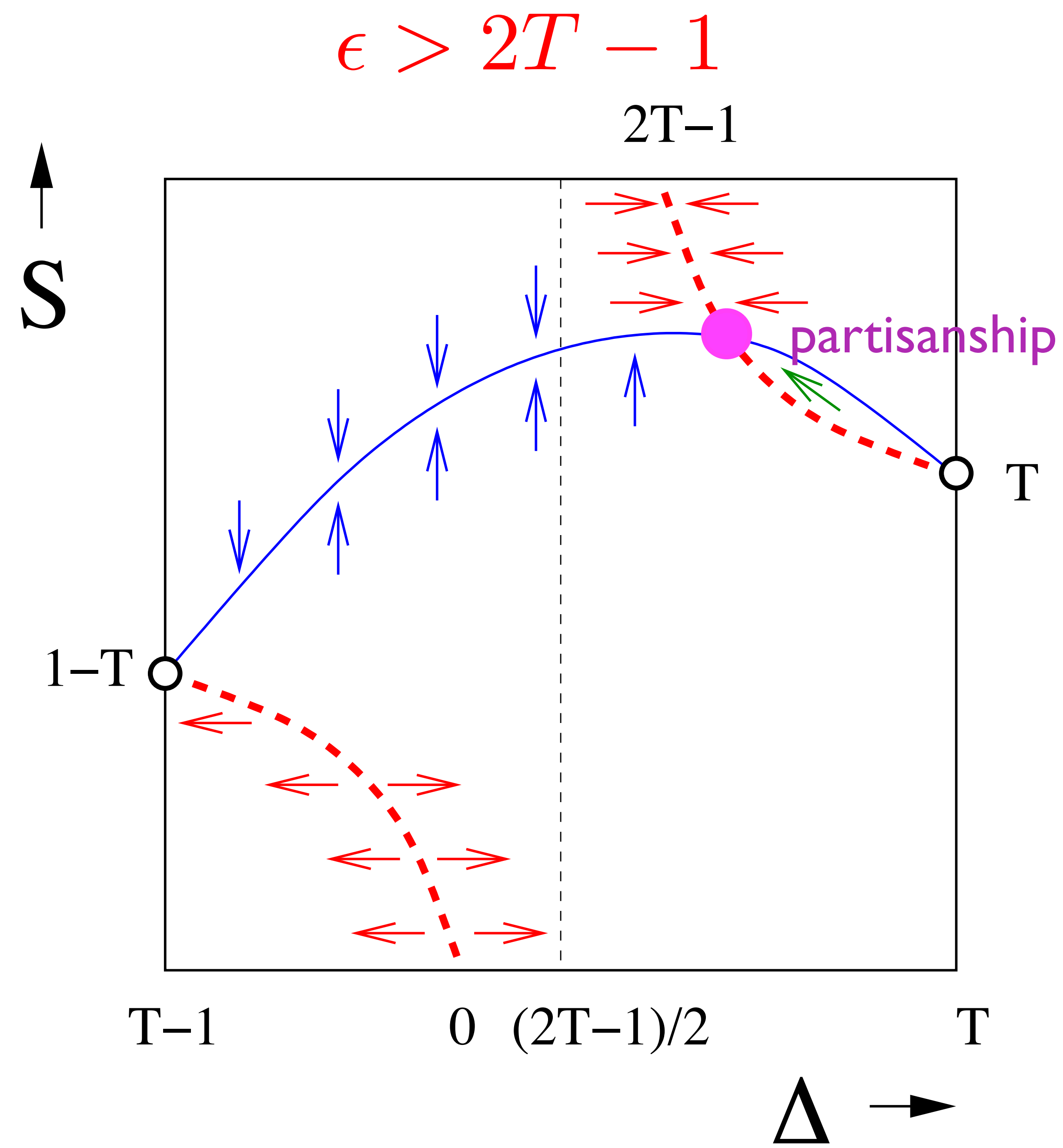
$$T_- = T - T_+ \qquad S = T_+ + F_+$$

$$F_- = F - F_+ \qquad \Delta = T_+ - F_+$$

Flow Diagram

$$S = T_+ + F_+$$
$$\Delta = T_+ - F_+$$





Summary & Outlook

Voter model:

paradigmatic, soluble, (but hopelessly naive)

Voter model on complex networks:

new conservation law
two time-scale route to consensus
fast consensus for broad degree distributions

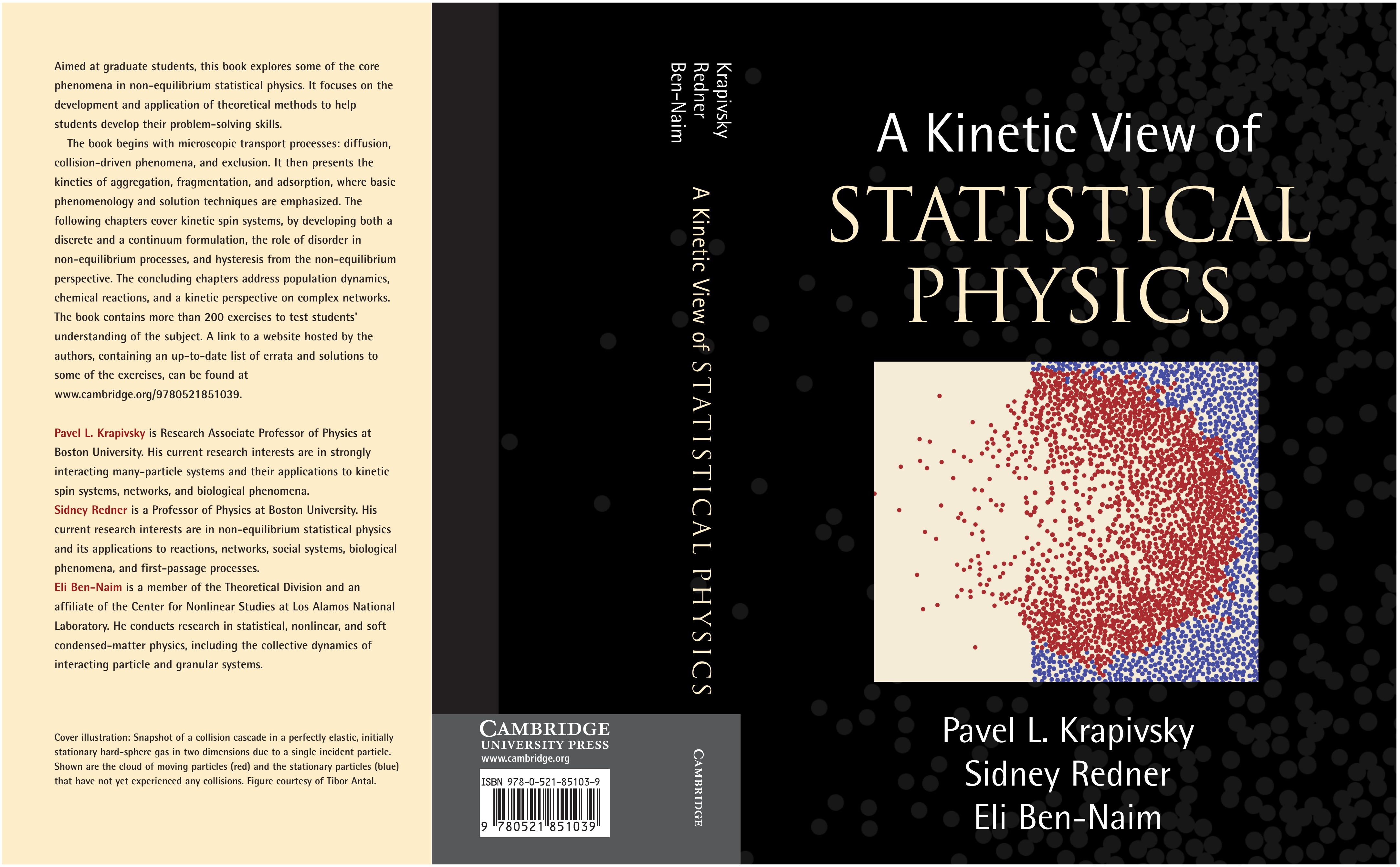
Extension to Partisanship:

partisanship forestalls consensus to the truth

Future:

“churn” rather than consensus
heterogeneity of real people
positive and negative social interactions

Crass Commercialism



to appear this October

1. Aperitifs
2. Diffusion
3. Collisions
4. Exclusion
5. Aggregation
6. Fragmentation
7. Adsorption
8. Spin Dynamics
9. Coarsening
10. Disorder
11. Hysteresis
12. Population Dynamics
13. Diffusion Reactions
14. Complex Networks