

Dynamics of Voter Models on Heterogeneous Networks

Sid Redner (physics.bu.edu/~redner)

Complex Network Program SAMSI Aug. 29-Sept. 1, 2010

T. Antal (Edinburgh), V. Sood (NBI)

NSF DMR0535503 & DMR0906504

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The classic voter model

3 basic results

Voting on complex networks

T. Antal, V. Sood

new conservation law & fixation probabilities

two time-scale route to consensus

short consensus time

Partisan voting

can truth be reached?

N. Gibert (Paris)

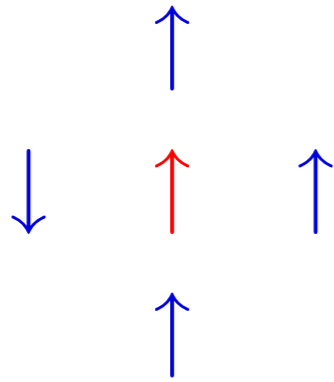
N. Masuda (Tokyo)

Classic Voter Model

Clifford & Sudbury (1973)
Holley & Liggett (1975)

Classic Voter Model

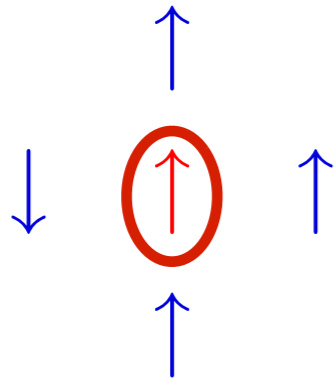
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0. Binary voter variable at each site i

Classic Voter Model

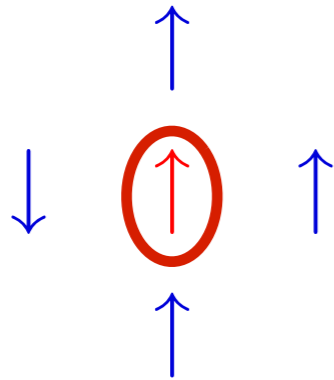
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1. Pick a random voter

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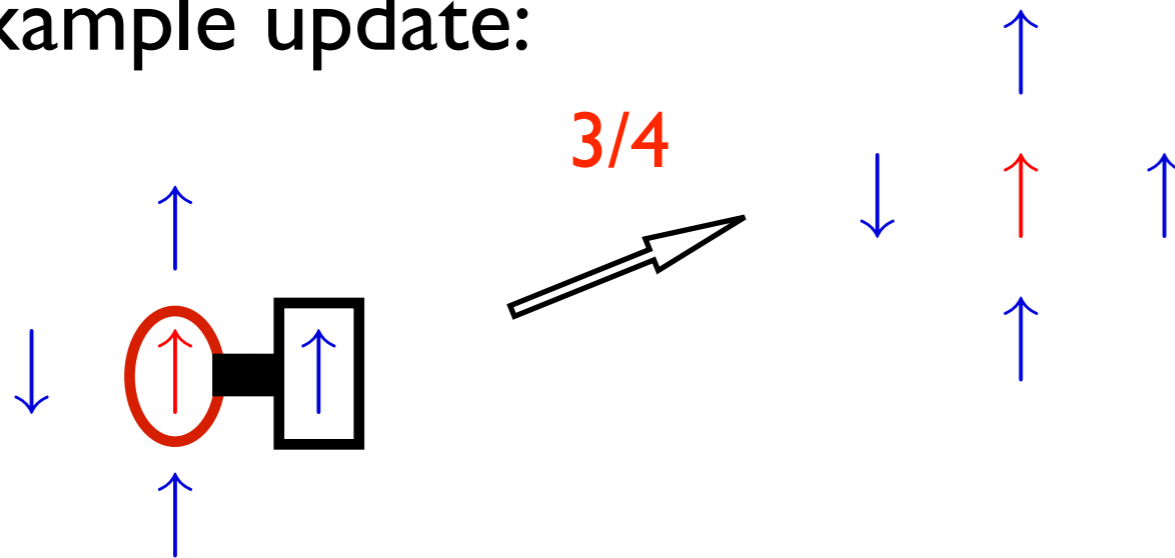
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2. Assume state of randomly-selected neighbor
individual has no self-confidence & adopts neighbor's state

Classic Voter Model

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Example update:



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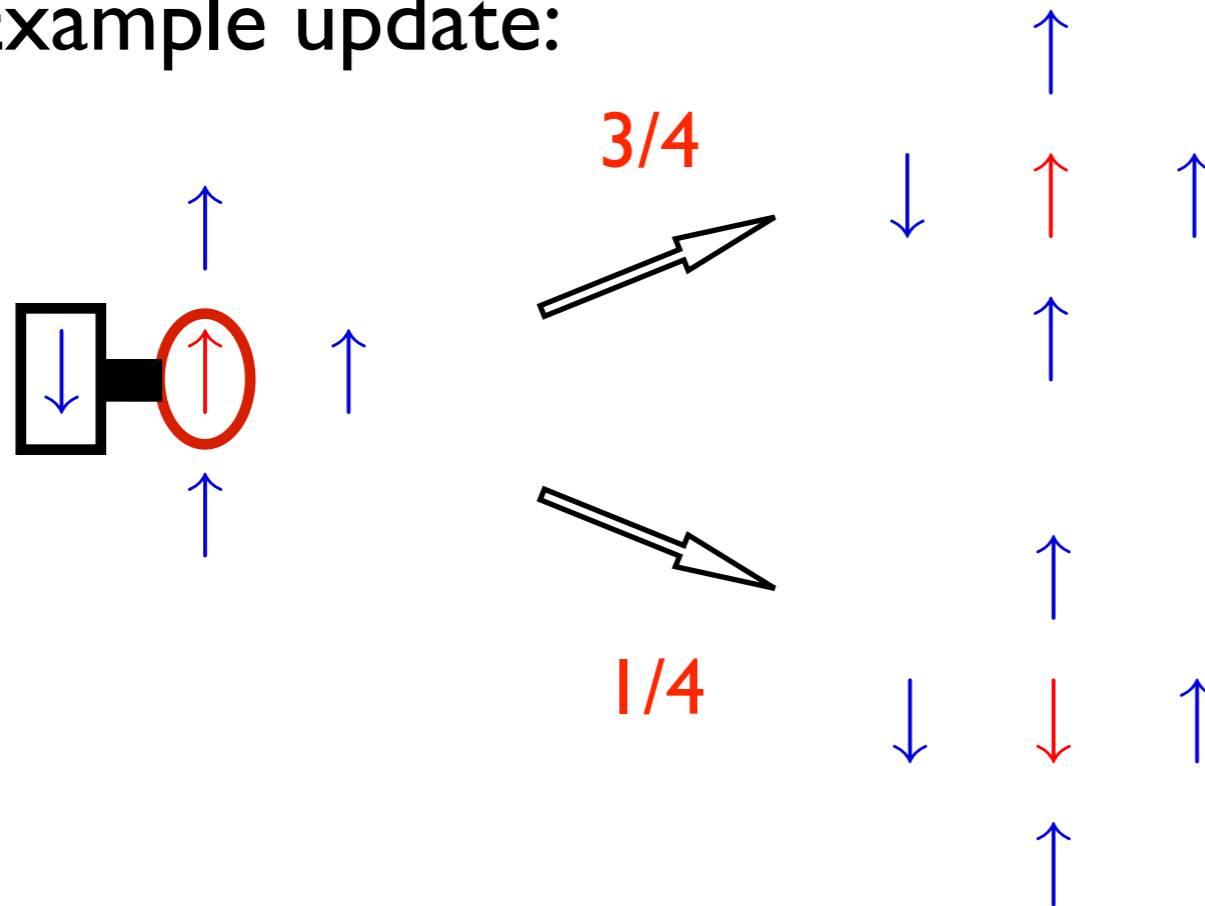
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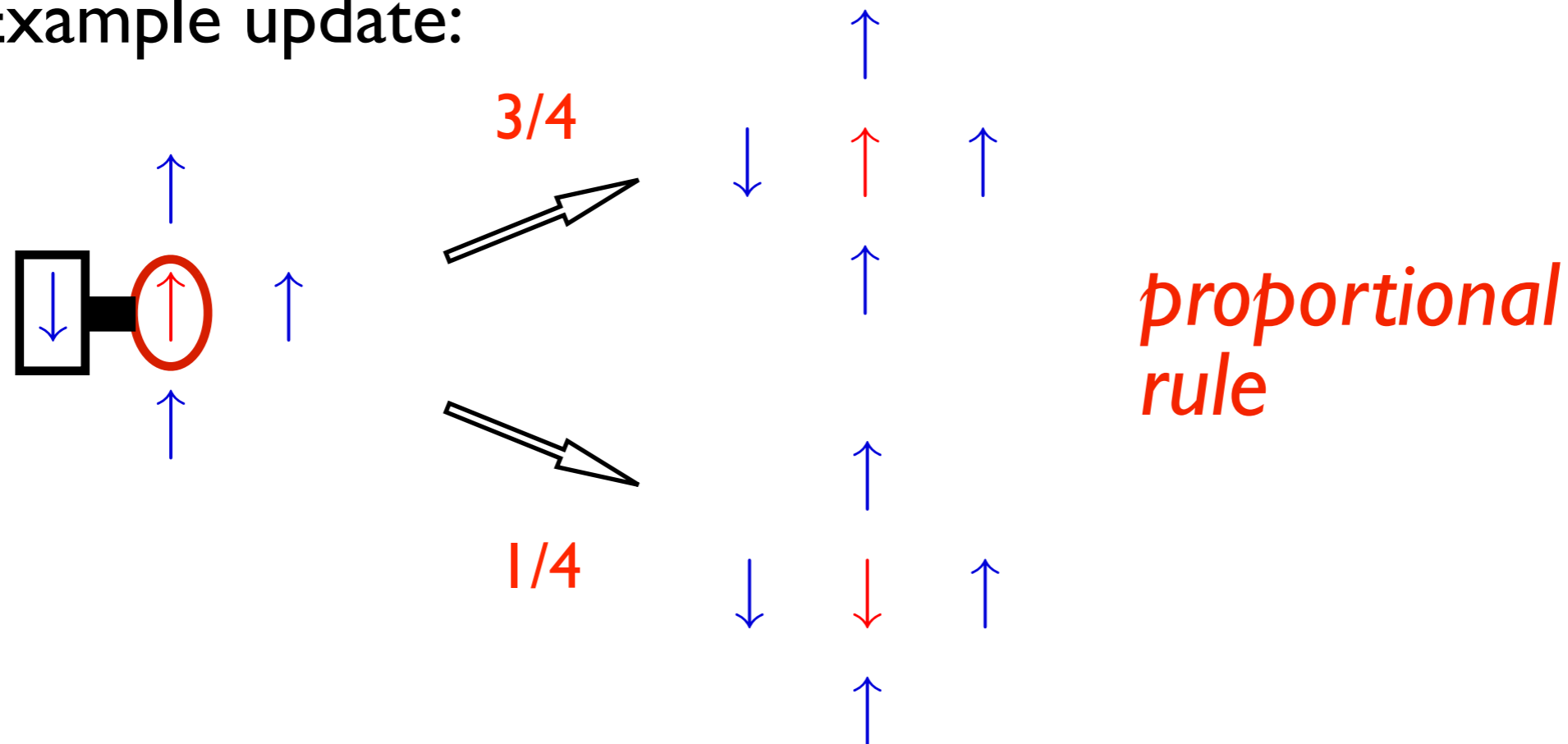
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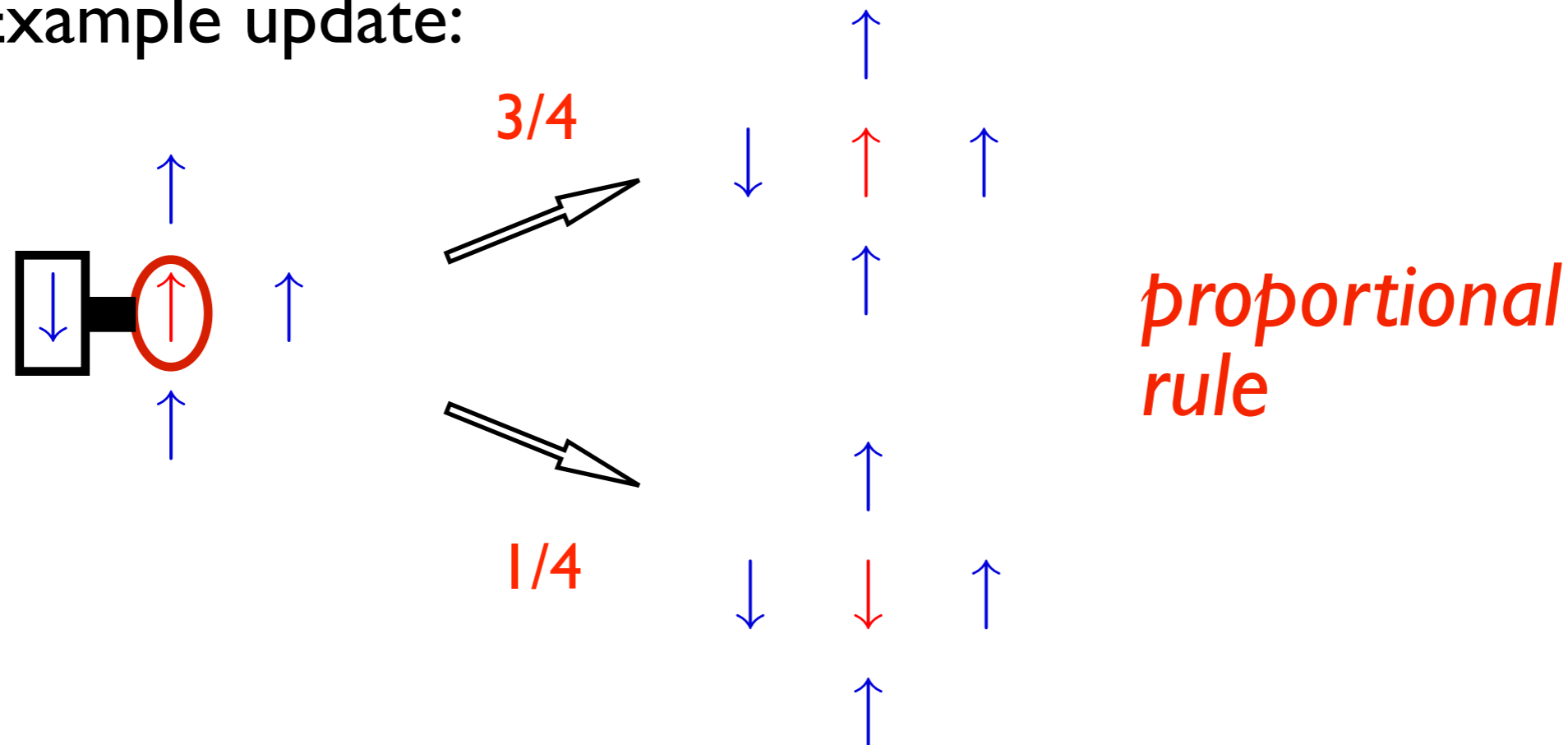
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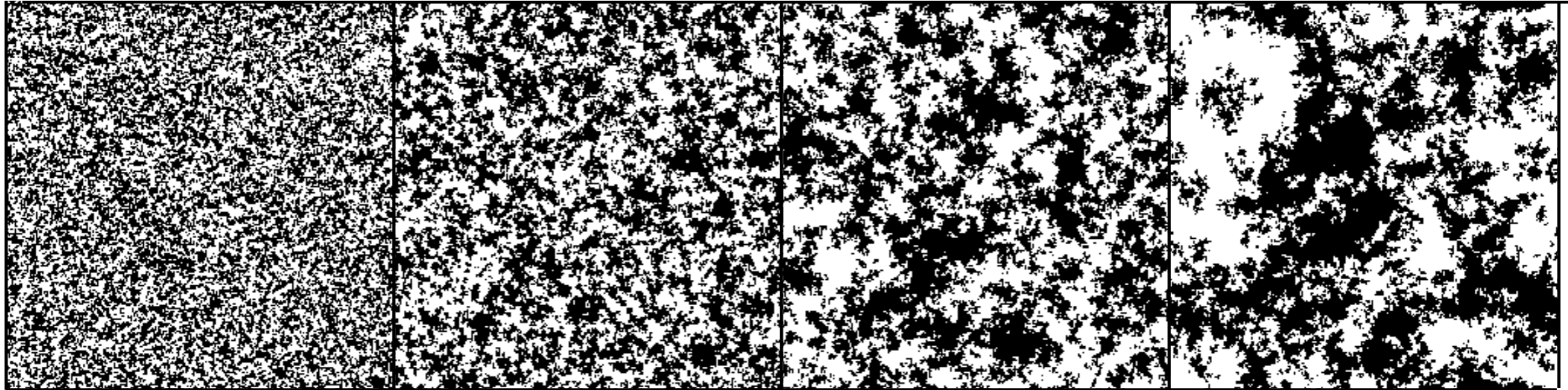


0. Binary voter variable at each site i
1. Pick a random voter
2. Assume state of randomly-selected neighbor
individual has no self-confidence & adopts neighbor's state
3. Repeat 1 & 2 until consensus *necessarily* occurs in a finite system

Voter Model Evolution

Dornic et al. (2001)

random initial condition, 256 x 256 square:



t=4

t=16

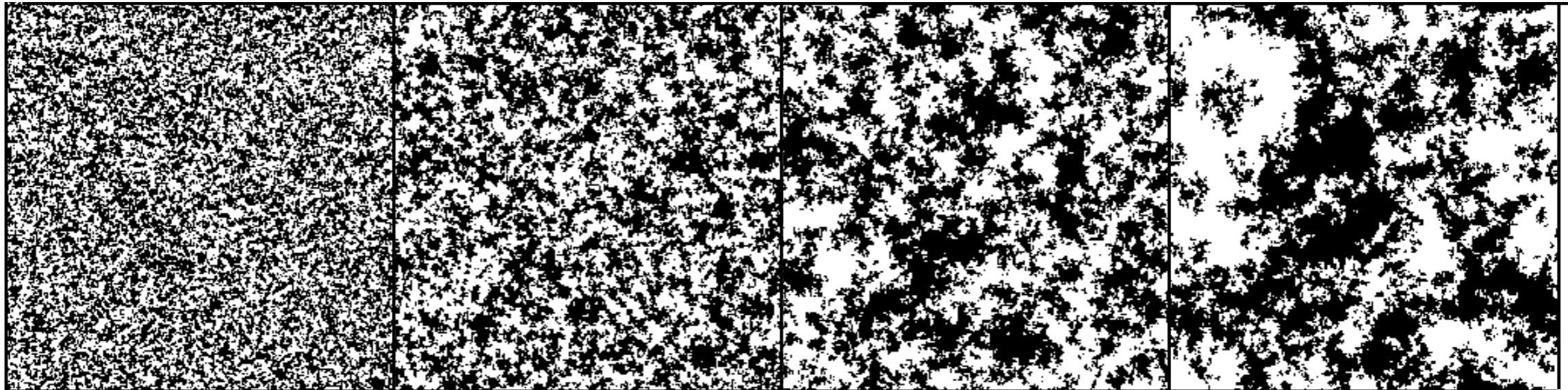
t=64

t=256

Voter Model Evolution

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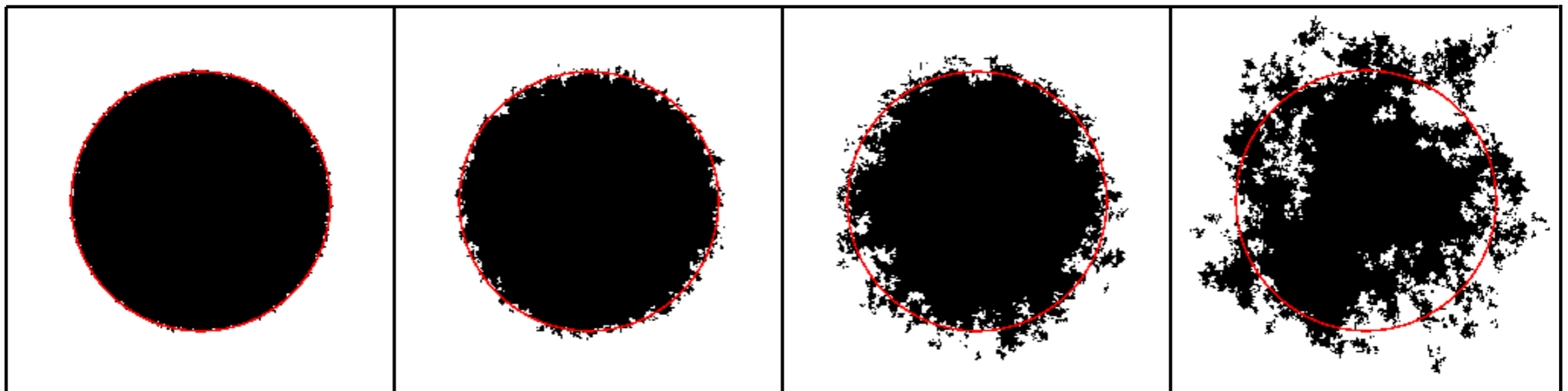
$t=4$

$t=16$

$t=64$

$t=256$

droplet initial condition:



no surface tension

Voter Model & Cousins

Voter Model:

Tell me how to vote

lemming



Voter Model & Cousins

Voter Model:

Tell me how to vote

lemming



Invasion Process:

I tell you how to vote

persuasive "friend"



Voter Model & Cousins

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Link Dynamics:

Pick two disagreeing agents and change one at random



Voter Model & Cousins

Voter Model: Tell me how to vote



Invasion Process: I tell you how to vote



Link Dynamics: Pick two disagreeing agents and change one at random



identical on regular lattices, *distinct* on random graphs

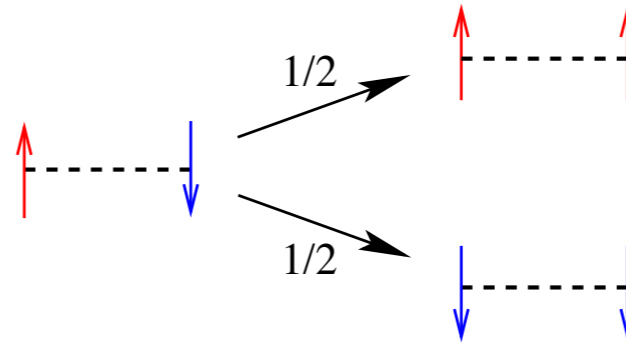
Suchecki, Eguiluz & San Miguel (2005), Castellano (2005), Sood & SR (2005)

Lattice Voter Model: 3 Basic Properties

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I. Final State (Exit) Probability $\mathcal{E}(\rho_0)$

Evolution of a single active link:

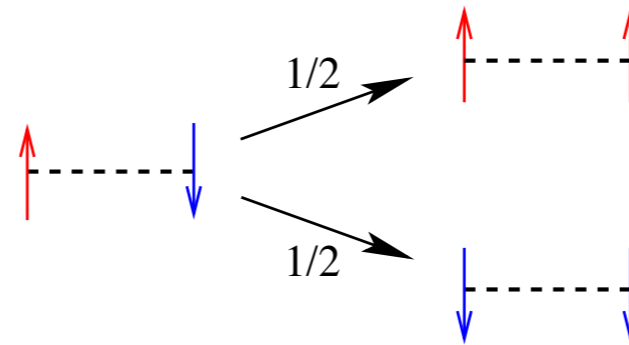


average
magnetization
conserved

Lattice Voter Model: 3 Basic Properties

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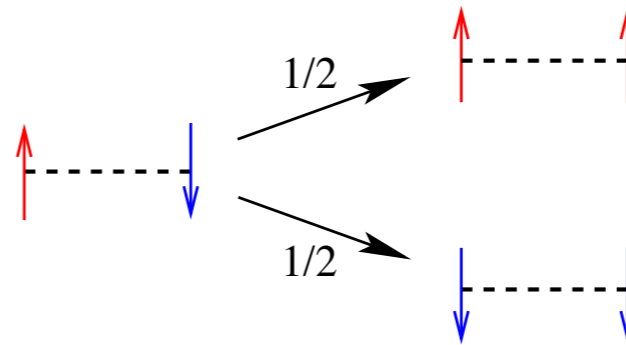


average
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Lattice Voter Model: 3 Basic Properties

1. Final State (Exit) Probability $\mathcal{E}(\rho_0) = \rho_0$

Evolution of a single active link:



average magnetization conserved

2. Two-Spin Correlations

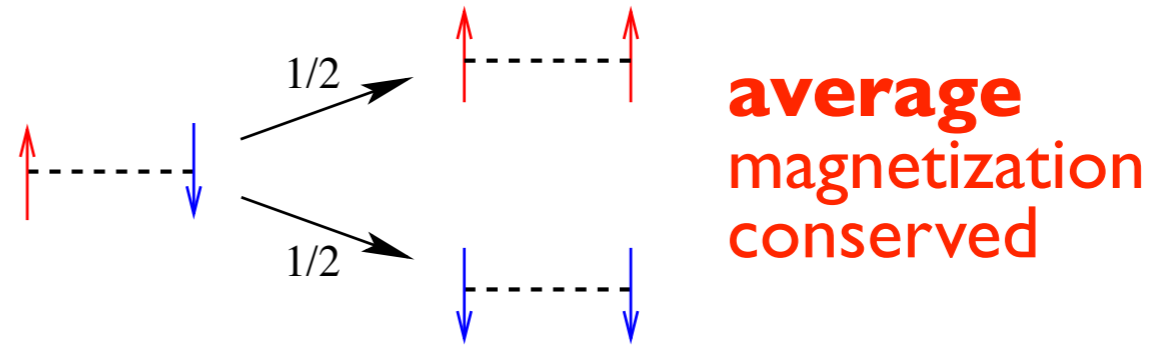
$$\frac{\partial c_2(\mathbf{r}, t)}{\partial t} = \nabla^2 c_2(\mathbf{r}, t)$$

$$\begin{aligned} c_2(r=0, t) &= 1 \\ c_2(r > 0, t=0) &= 0 \end{aligned}$$

Lattice Voter Model: 3 Basic Properties

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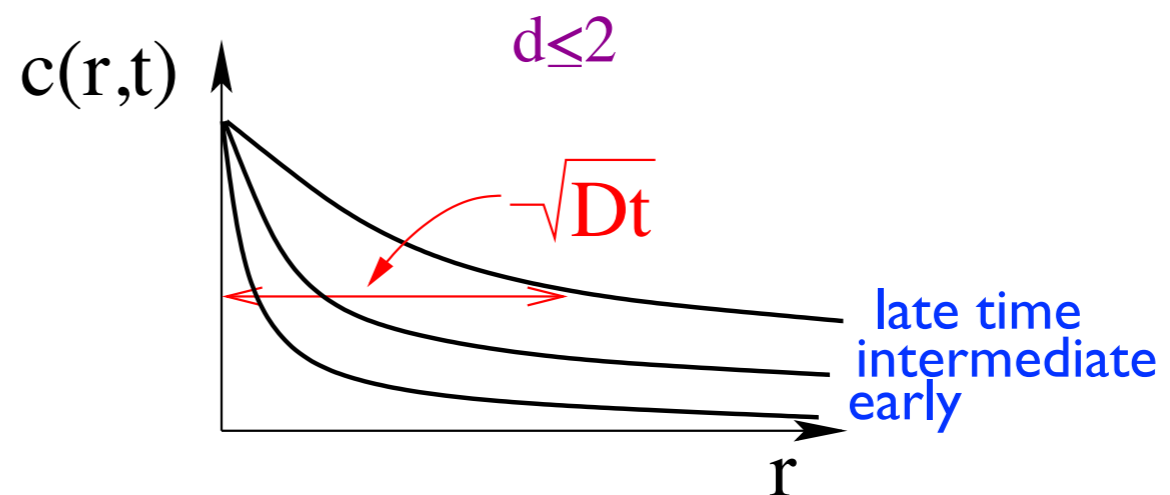
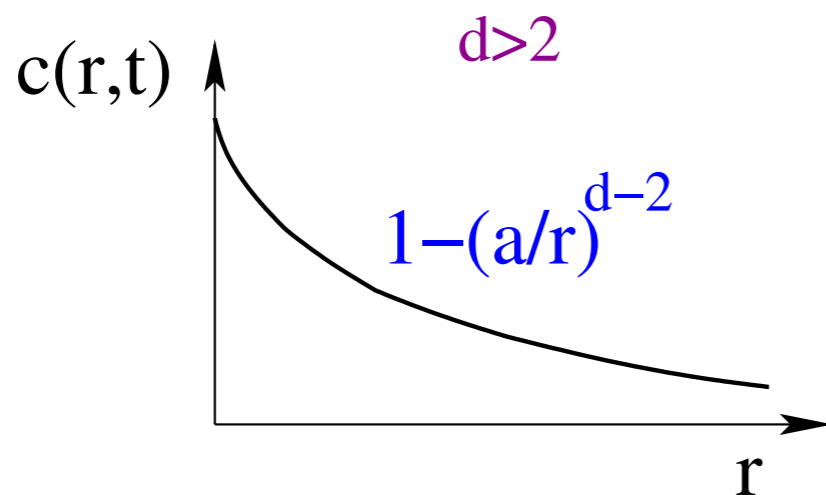


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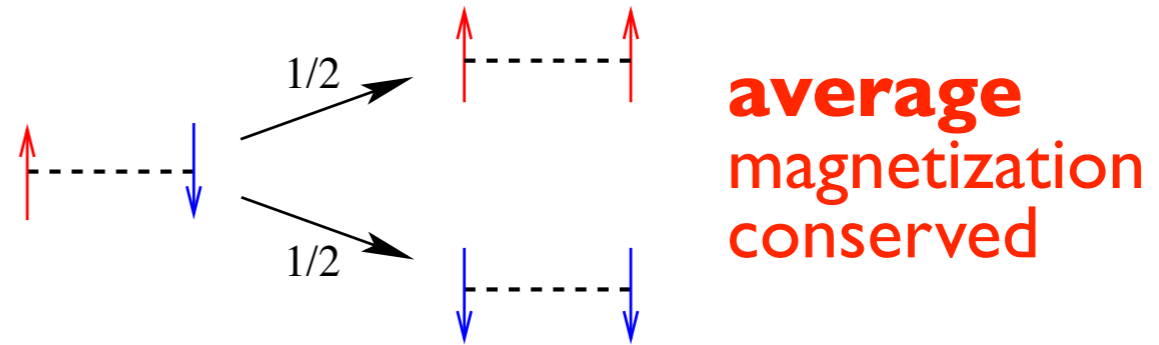
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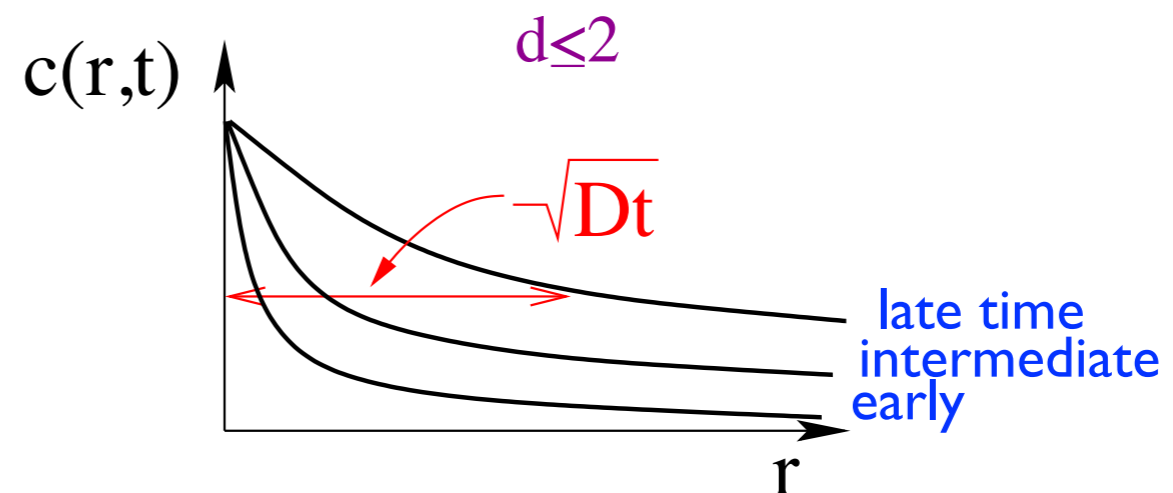
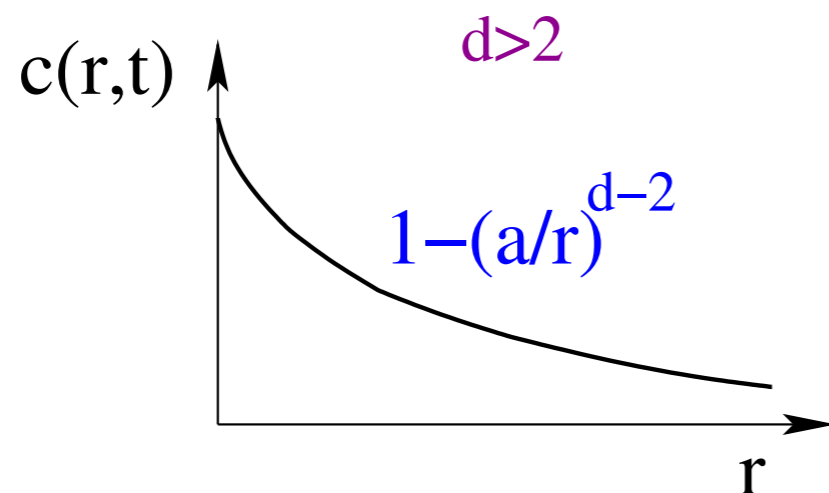


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3. Consensus Time

$$\int^{\sqrt{Dt}} c(r, t) r^{d-1} dr = N$$

dimension	consensus time
1	N^2
2	$N \ln N$
> 2	N

Voter Model on Complex Networks

C. Castellano, D. Vilon, A. Vespignani, EPL **63**, 153 (2003)

K. Suchecki, V. M. Eguiluz, M. San Miguel, EPL **69**, 228 (2005)

V. Sood & SR, PRL **94**, 178701 (2005); T. Antal, V. Sood, SR, PRE **77**, 041121 (2008)

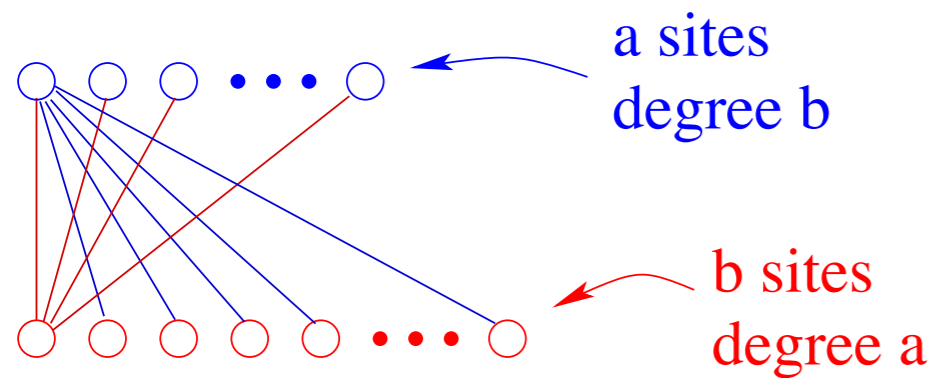
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illustrative example:
complete bipartite graph



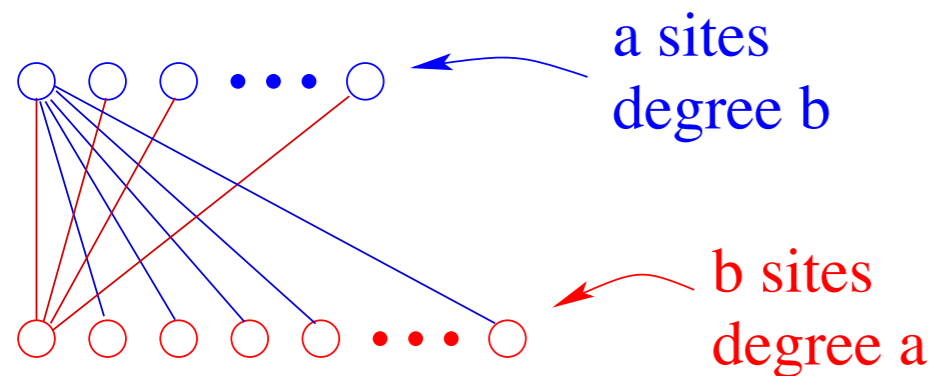
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rate equation

$$dN_{\uparrow,a} = \frac{N_{\downarrow,a}N_{\uparrow,b} - N_{\uparrow,a}N_{\downarrow,b}}{(a+b)b}$$

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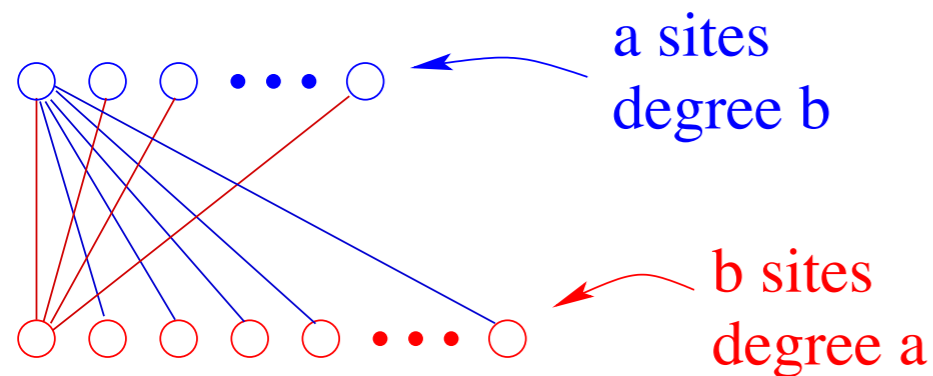
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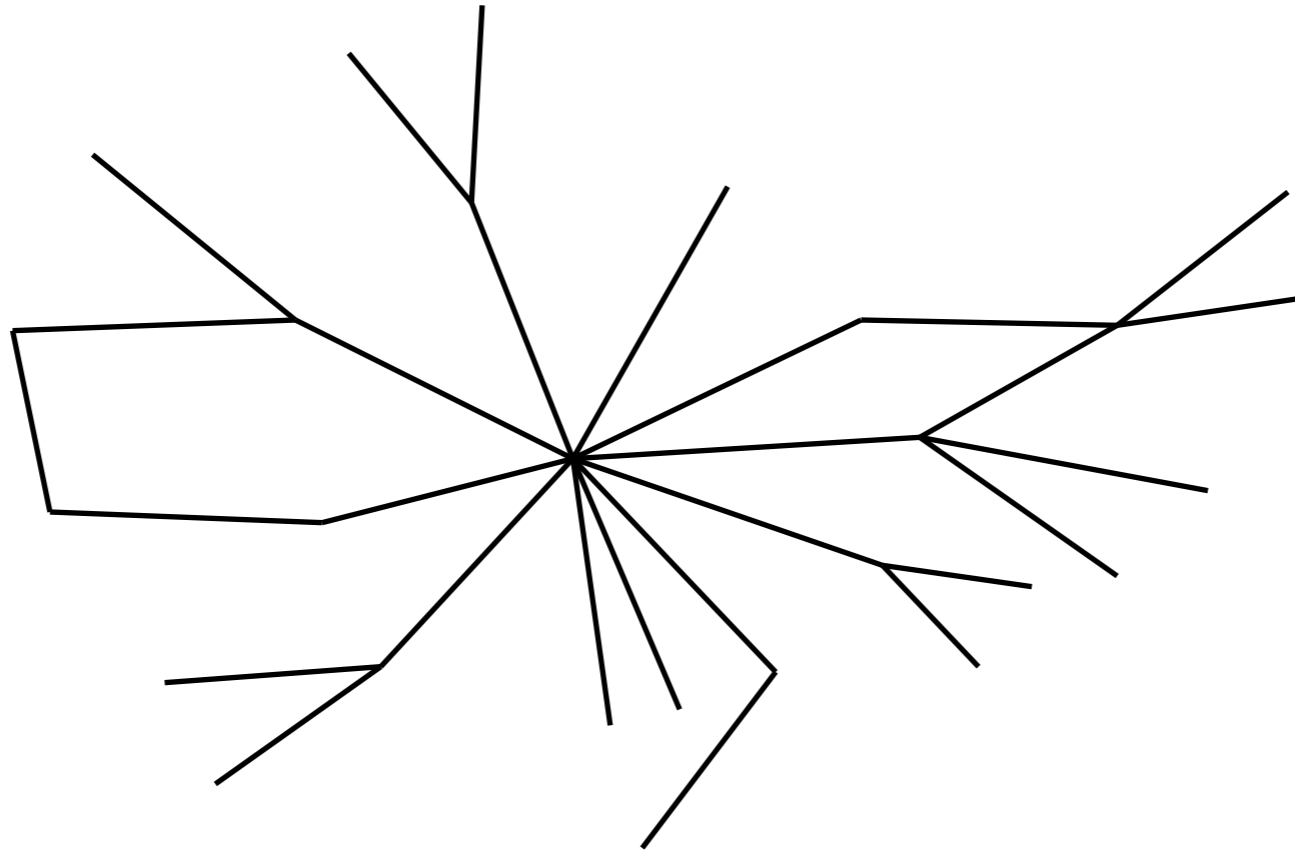
Subgraph densities: $\rho_a = N_{\uparrow,a}/a$, $\rho_b = N_{\uparrow,b}/b$ $dt = 1/(a+b)$

$$\rho_{a,b}(t) = \frac{1}{2}[\rho_{a,b}(0) - \rho_{b,a}(0)] e^{-2t} + \frac{1}{2}[\rho_a(0) + \rho_b(0)]$$

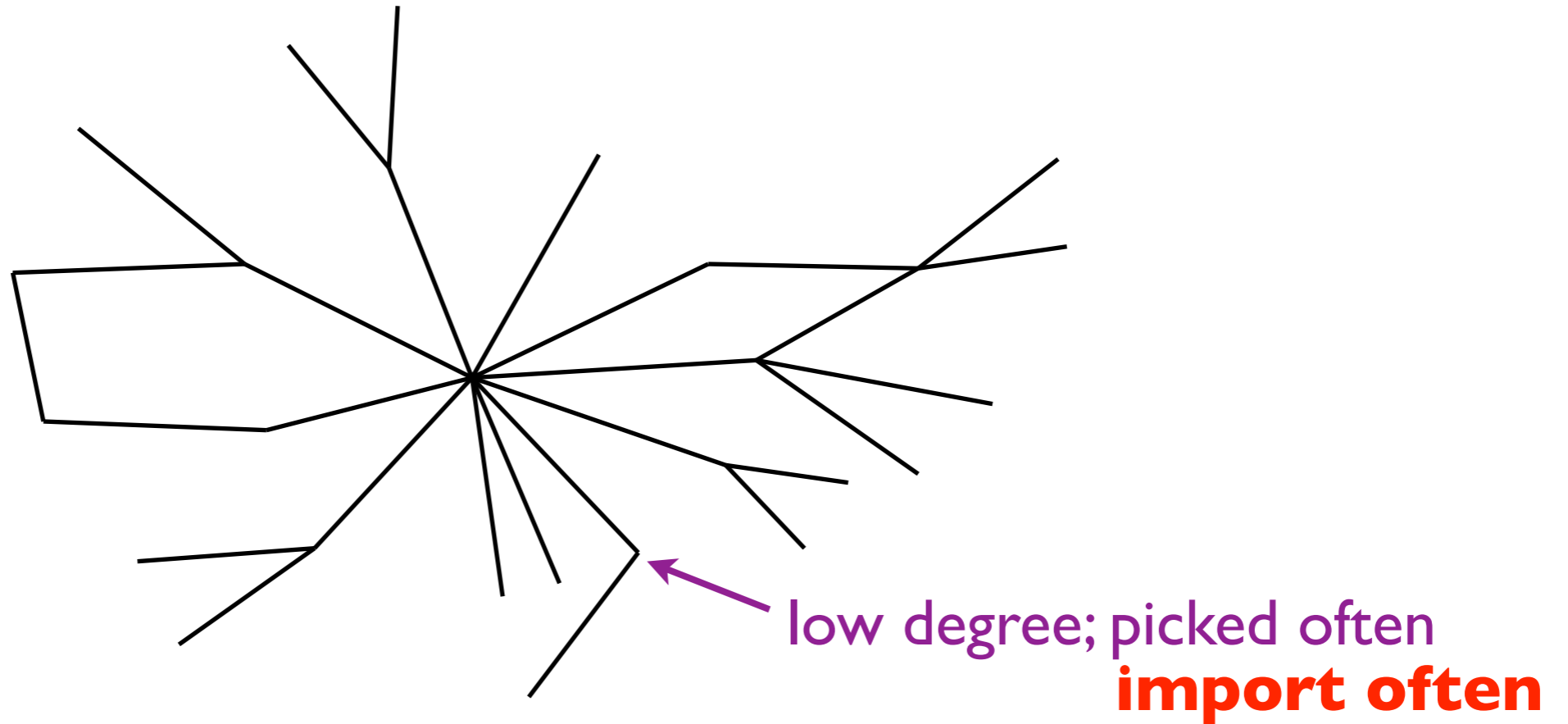
$$\rightarrow \frac{1}{2}[\rho_a(0) + \rho_b(0)]$$

magnetization **not** conserved

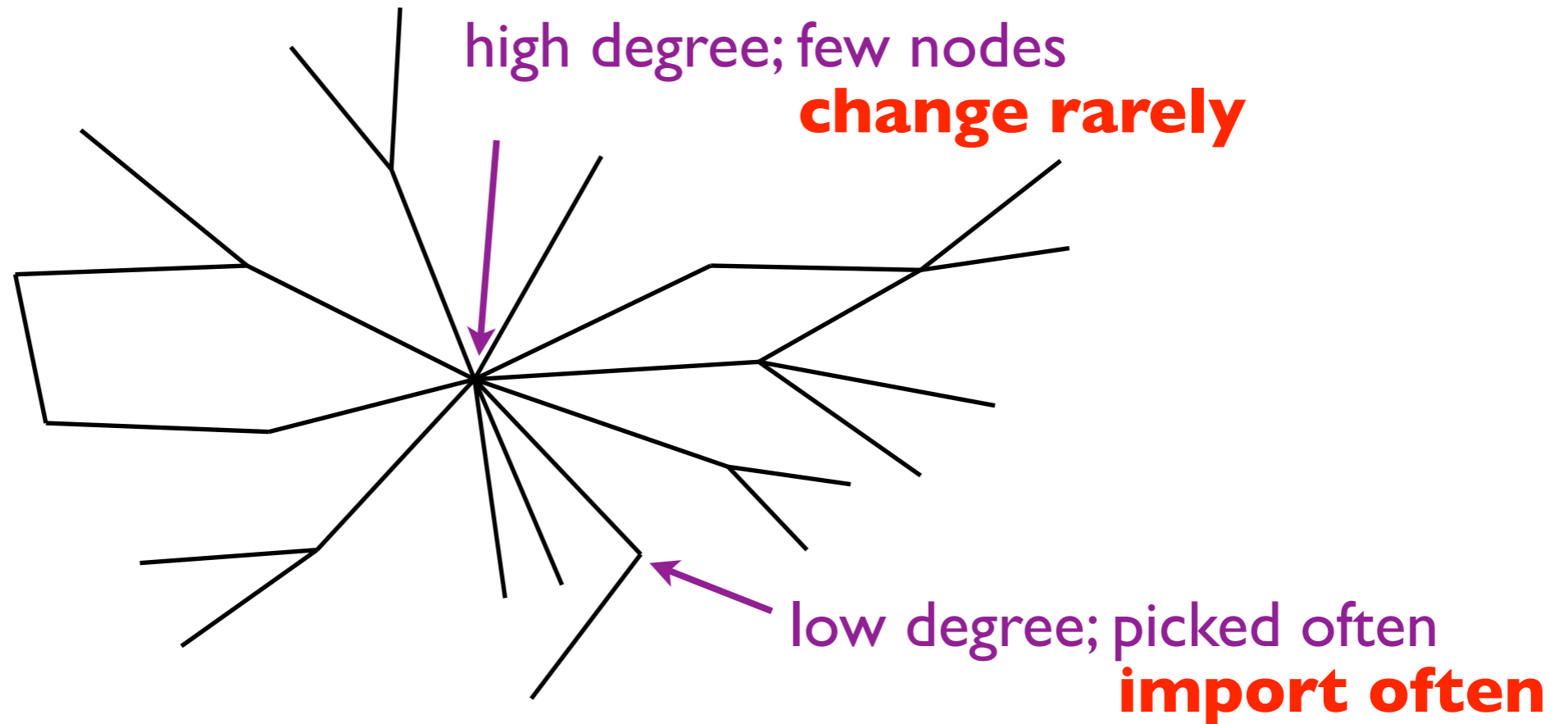
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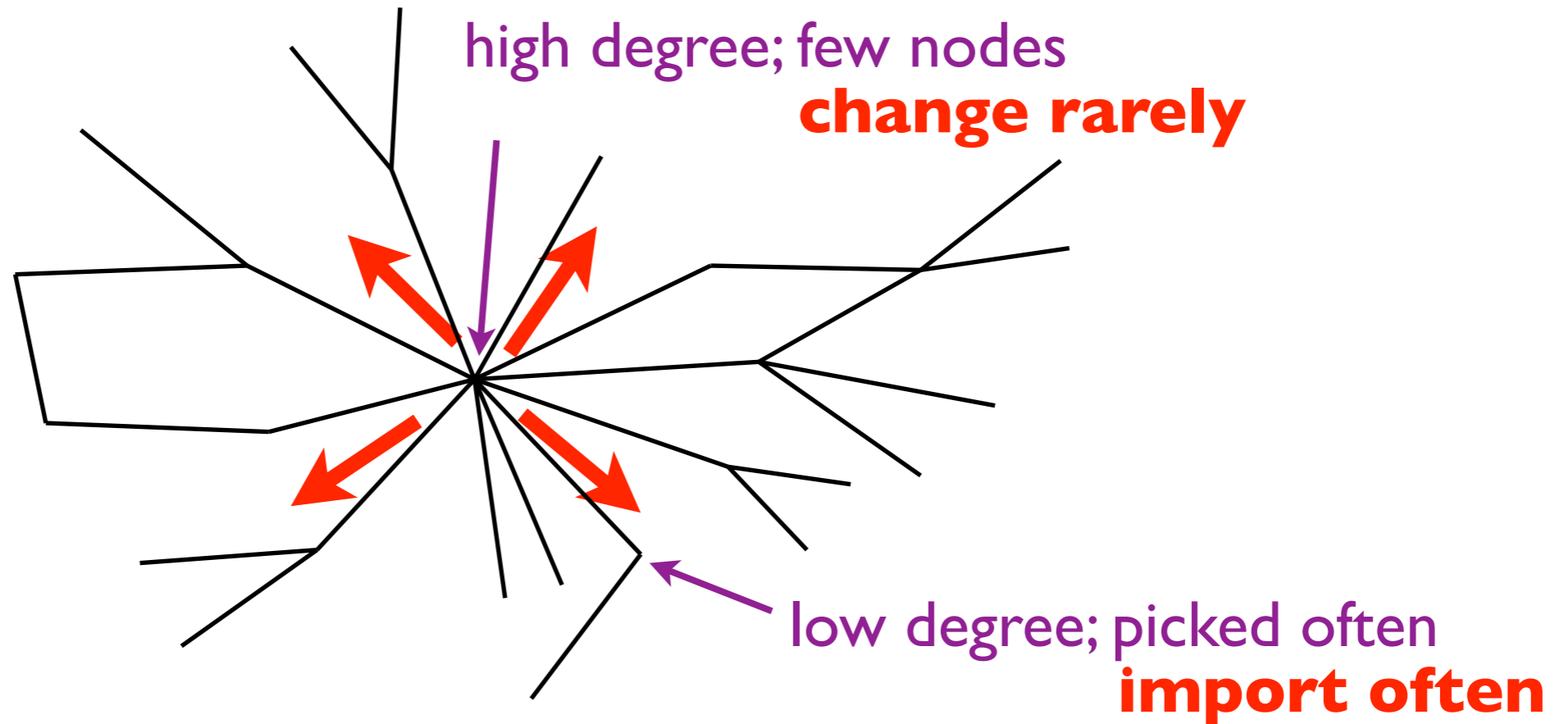
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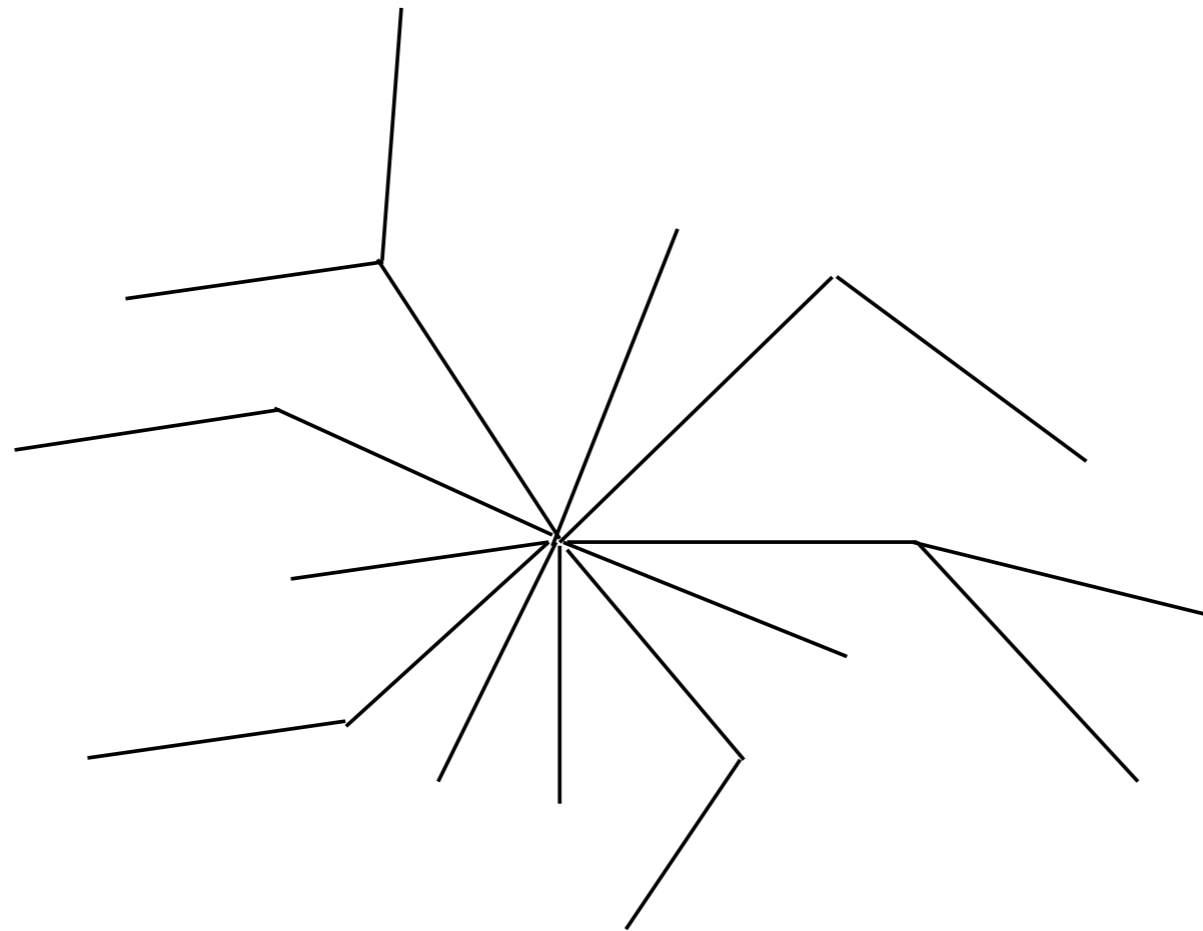
Voter Model on Complex Networks



“flow” from **high** degree to **low** degree

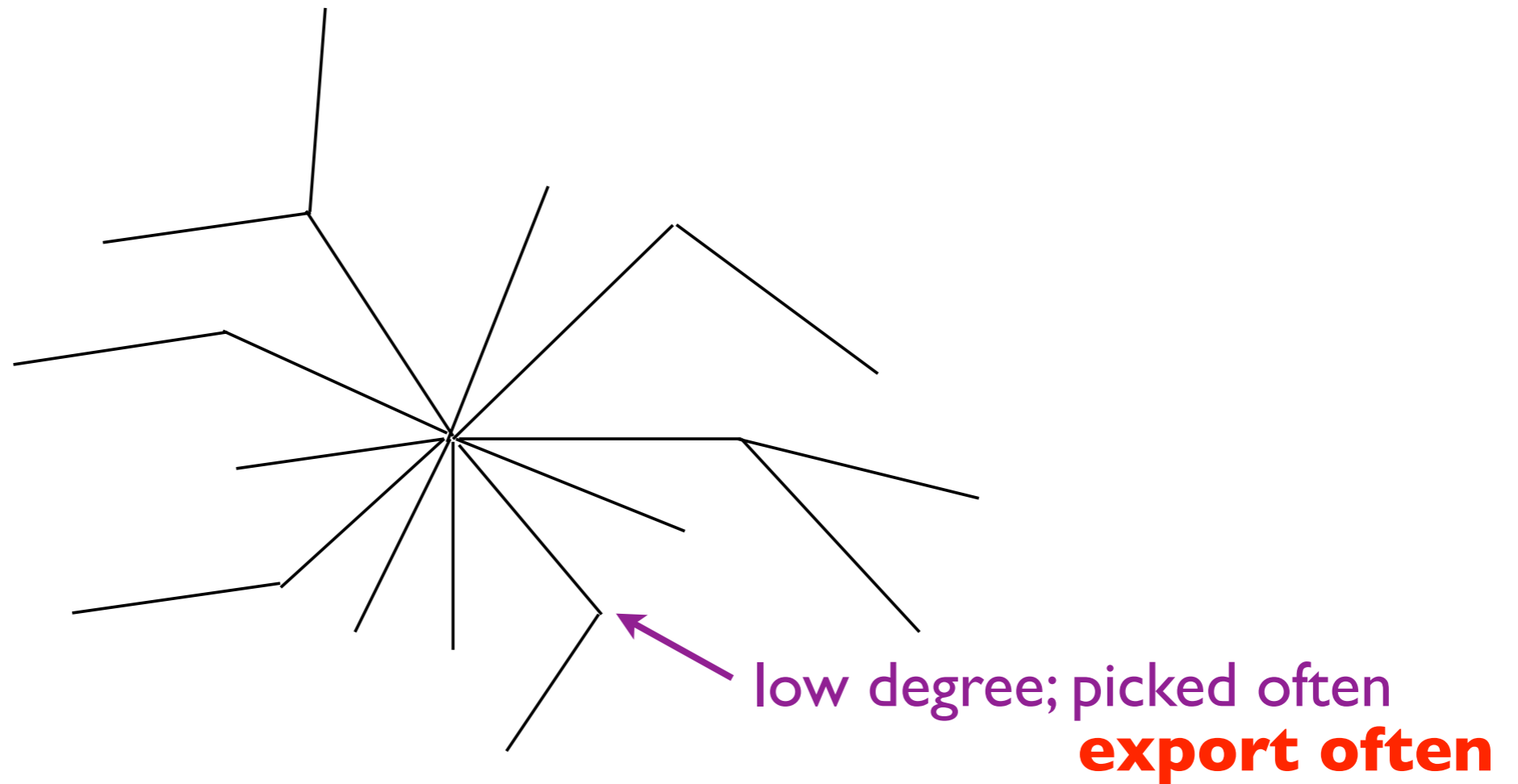
Invasion Process on Complex Networks

Castellano, AIP Conf Proc **779**, 114 (2005)



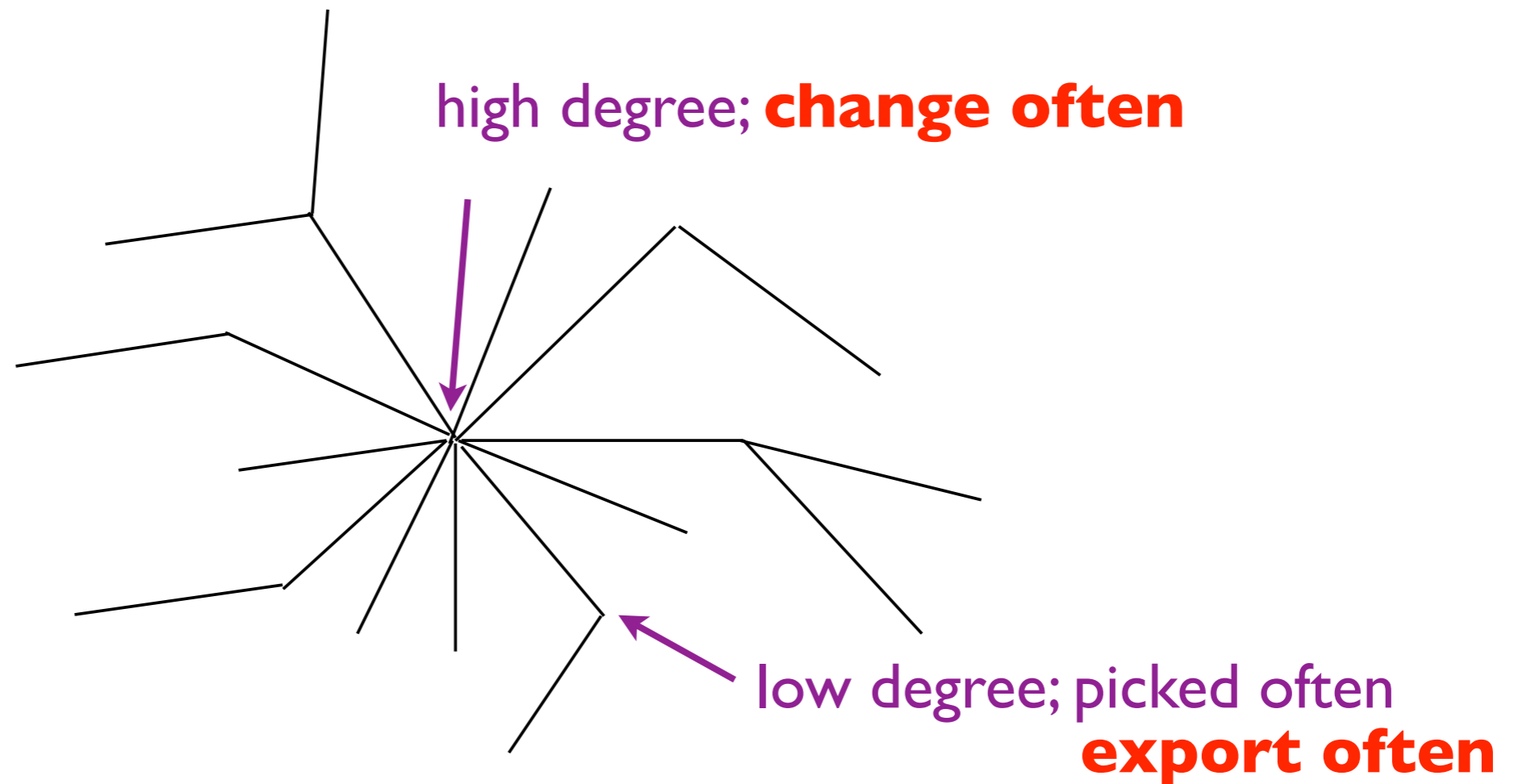
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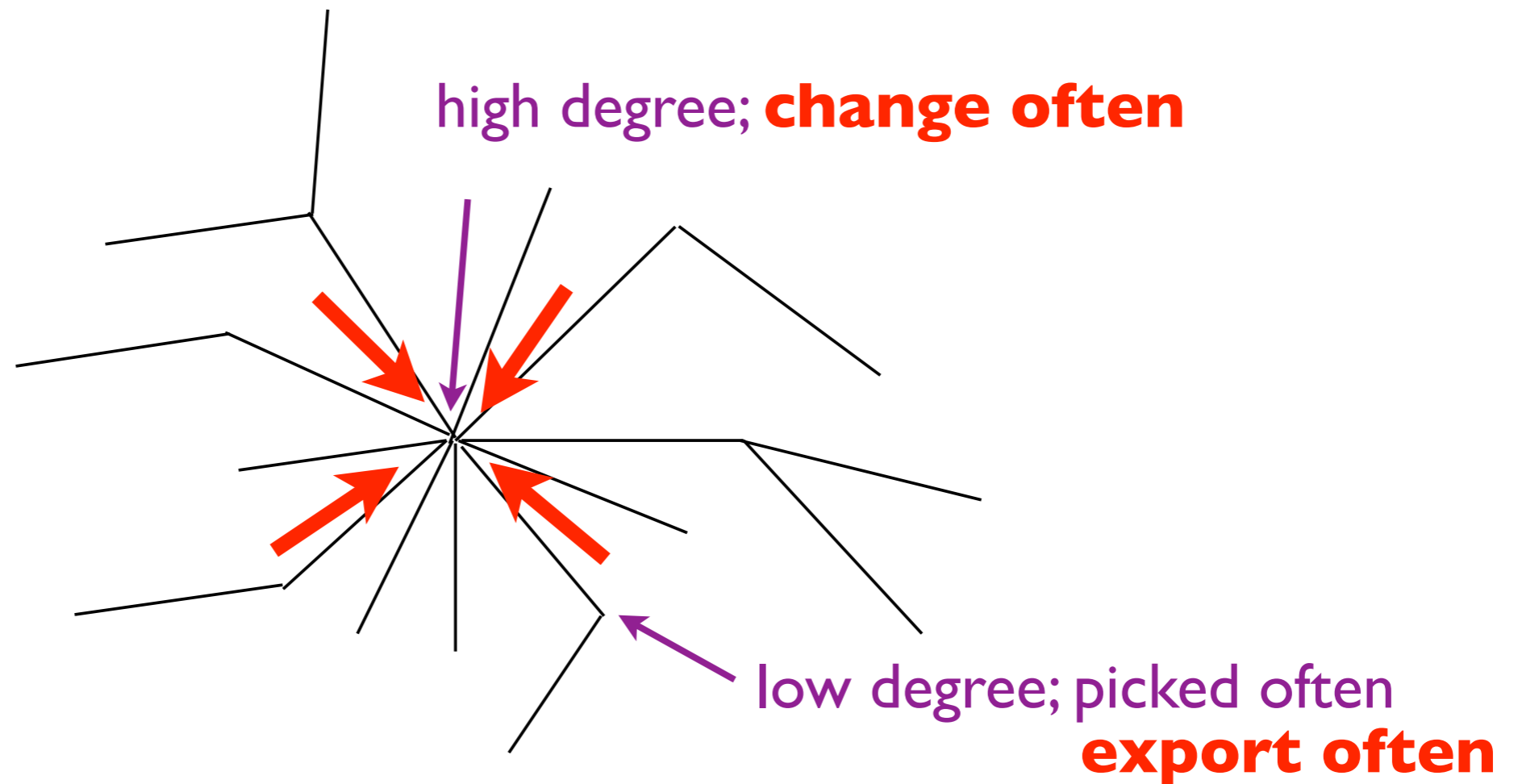
Invasion Process on Complex Networks

Castellano, AIP Conf Proc **779**, 114 (2005)



Invasion Process on Complex Networks

Castellano, AIP Conf Proc **779**, 114 (2005)



“flow” from **low** degree to **high** degree

Formal Approach for Conservation Law

flip rate: $\mathbf{P}[\eta \rightarrow \eta_x] = \sum_y \frac{A_{xy}}{\mathcal{Z}} [\Phi(x, y) + \Phi(y, x)]$

$\eta = \{1, 1, 0, 0, \dots, 1\}$ system state

$\eta_x =$ system state when voter at x flips

$\eta(x) =$ state of voter at x

Formal Approach for Conservation Law

$$\Phi(x, y) \equiv \eta(x)[1 - \eta(y)]$$

A_{xy} = adjacency matrix

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Formal Approach for Conservation Law

2 connected nodes
in different states

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η_x = system state when voter at x flips

$\eta(x)$ = state of voter at x

$$\mathcal{Z} \equiv \begin{cases} Nk_x & \text{VM} & \text{choose } x, \text{ choose neighbor of } x \text{ with prob. } (Nk_x)^{-1} \\ Nk_y & \text{IP} & \text{choose } y \text{ (neighbor of } x), \text{ choose of } x \text{ with prob. } (Nk_y)^{-1} \\ N\mu_1 & \text{LD} & \text{choose link \& update } x \text{ with prob. } (N\mu_1)^{-1} \end{cases}$$

Formal Approach for Conservation Law

$$\langle \Delta \eta(x) \rangle = [1 - 2\eta(x)] \mathbf{P}[\eta \rightarrow \eta_x] = \sum_y \frac{A_{xy}}{\mathcal{Z}} [\eta(y) - \eta(x)]$$

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degree-weighted moments

$$\langle \omega_m \rangle \equiv \frac{1}{N\mu_m} \sum_x k_x^m \eta(x) = \frac{1}{\mu_m} \sum_k k^m n_k \rho_k$$

Formal Approach for Conservation Law

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$$\Delta \langle \omega_1 \rangle = \sum_{x,y} \frac{A_{xy}}{Nk_x} \textcircled{k_x} [\eta(y) - \eta(x)] = 0$$

voter
model

$$\Delta \langle \omega_{-1} \rangle = \sum_{x,y} \frac{A_{xy}}{Nk_x} \textcircled{k_y} [\eta(y) - \eta(x)] = 0$$

invasion
process

$$\langle \Delta \omega_0 \rangle = \langle \Delta \rho \rangle = \sum_{x,y} \frac{A_{xy}}{N\mu_1} [\eta(y) - \eta(x)] = 0$$

link
dynamics

Exit Probability on Complex Networks

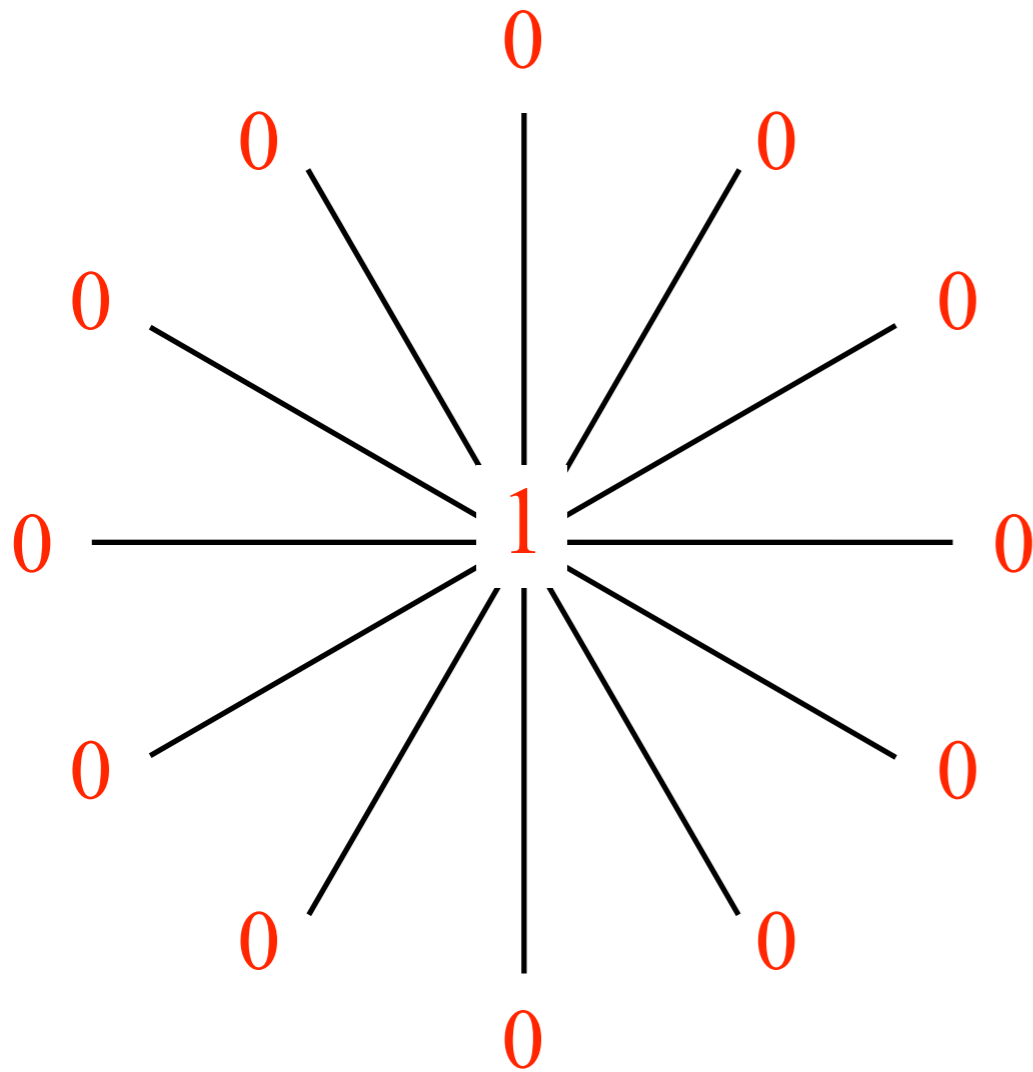
Voter model: $\mathcal{E}(\omega) = \omega$

Exit Probability on Complex Networks

Voter model: $\mathcal{E}(\omega) = \omega$

Extreme case: star graph

N nodes: degree 1
1 node: degree N

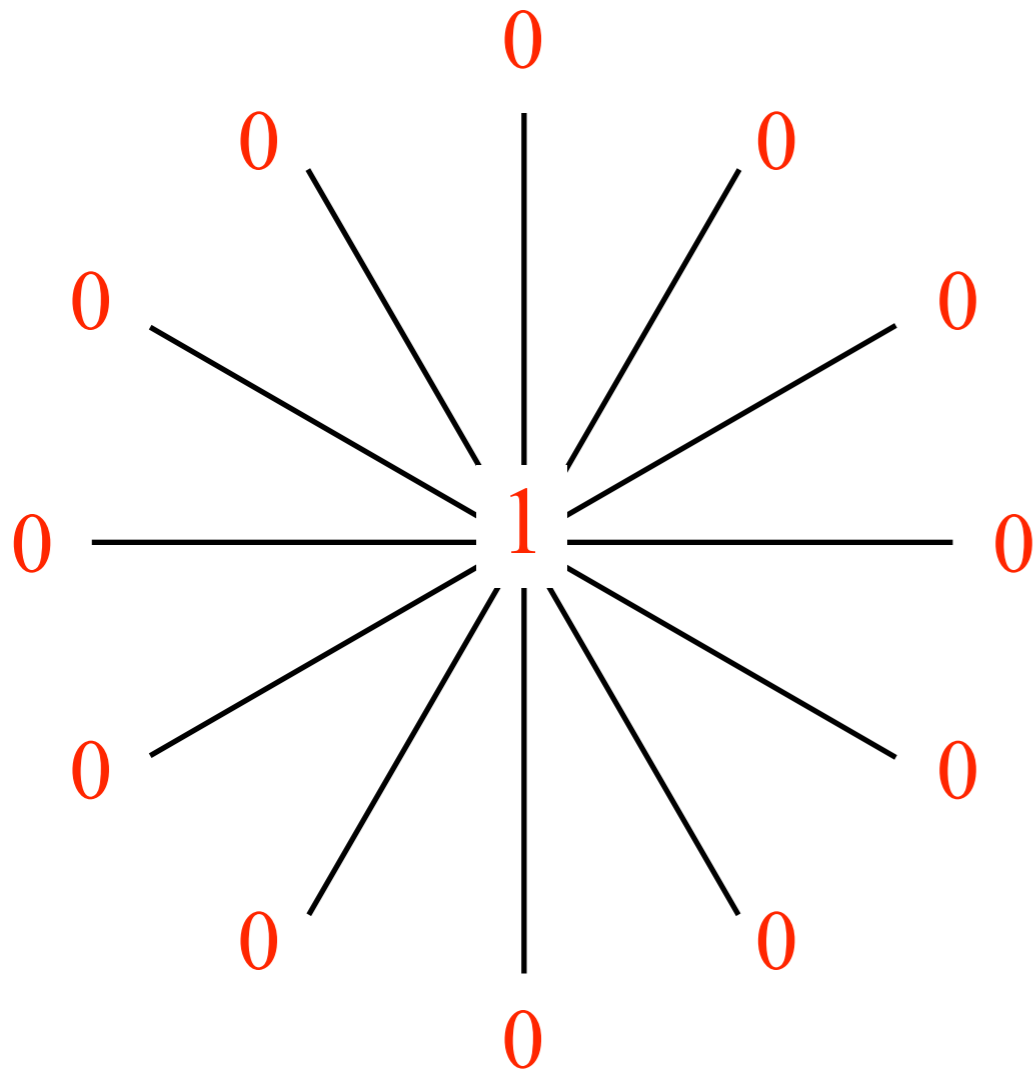


Exit Probability on Complex Networks

Voter model: $\mathcal{E}(\omega) = \omega$

Extreme case: star graph

N nodes: degree 1
1 node: degree N



$$\omega = \frac{1}{\mu_1} \sum_k k n_k \rho_k = \frac{1}{2}$$

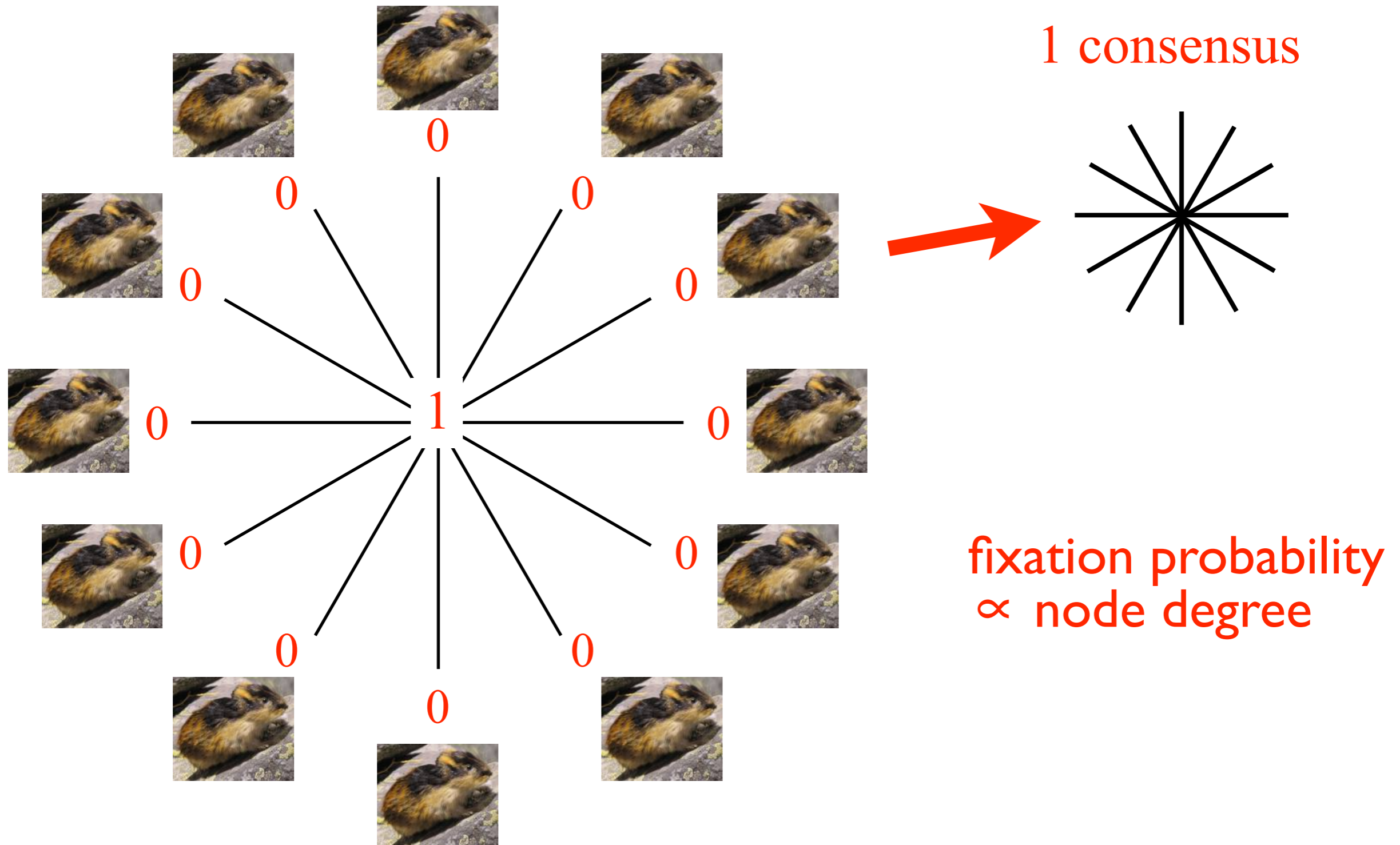
Final state: all 1 with prob. 1/2!

Byproduct: Voter Model Fixation Probability

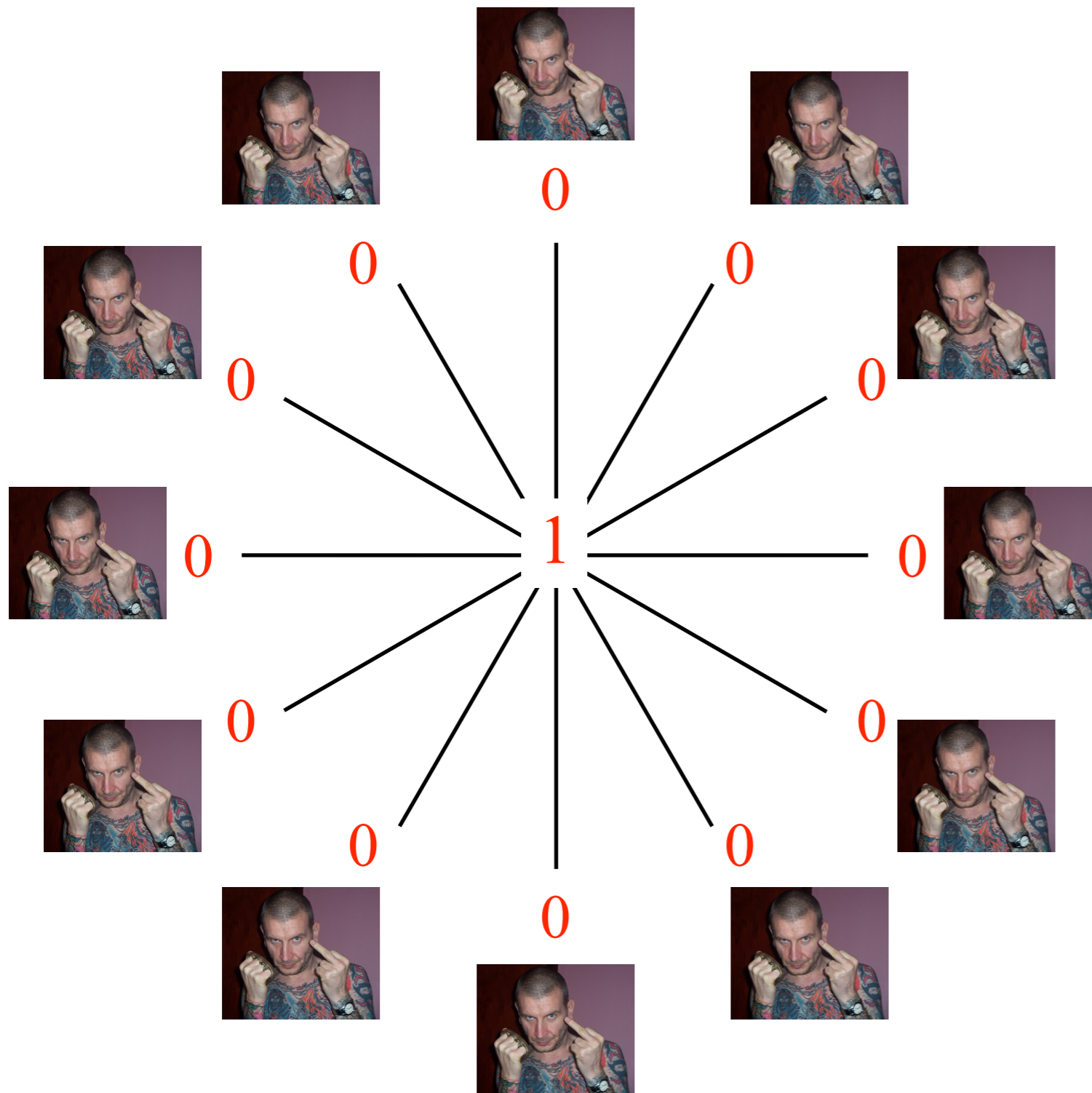
What is the probability that a single mutant “takes over” a population?

Byproduct: Voter Model Fixation Probability

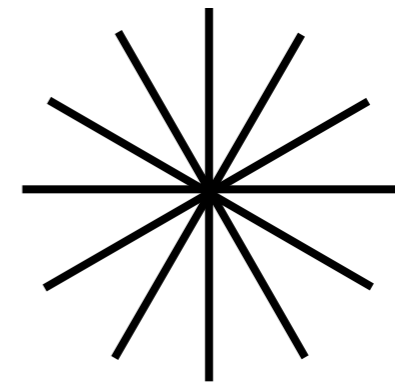
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Invasion Process Fixation Probability

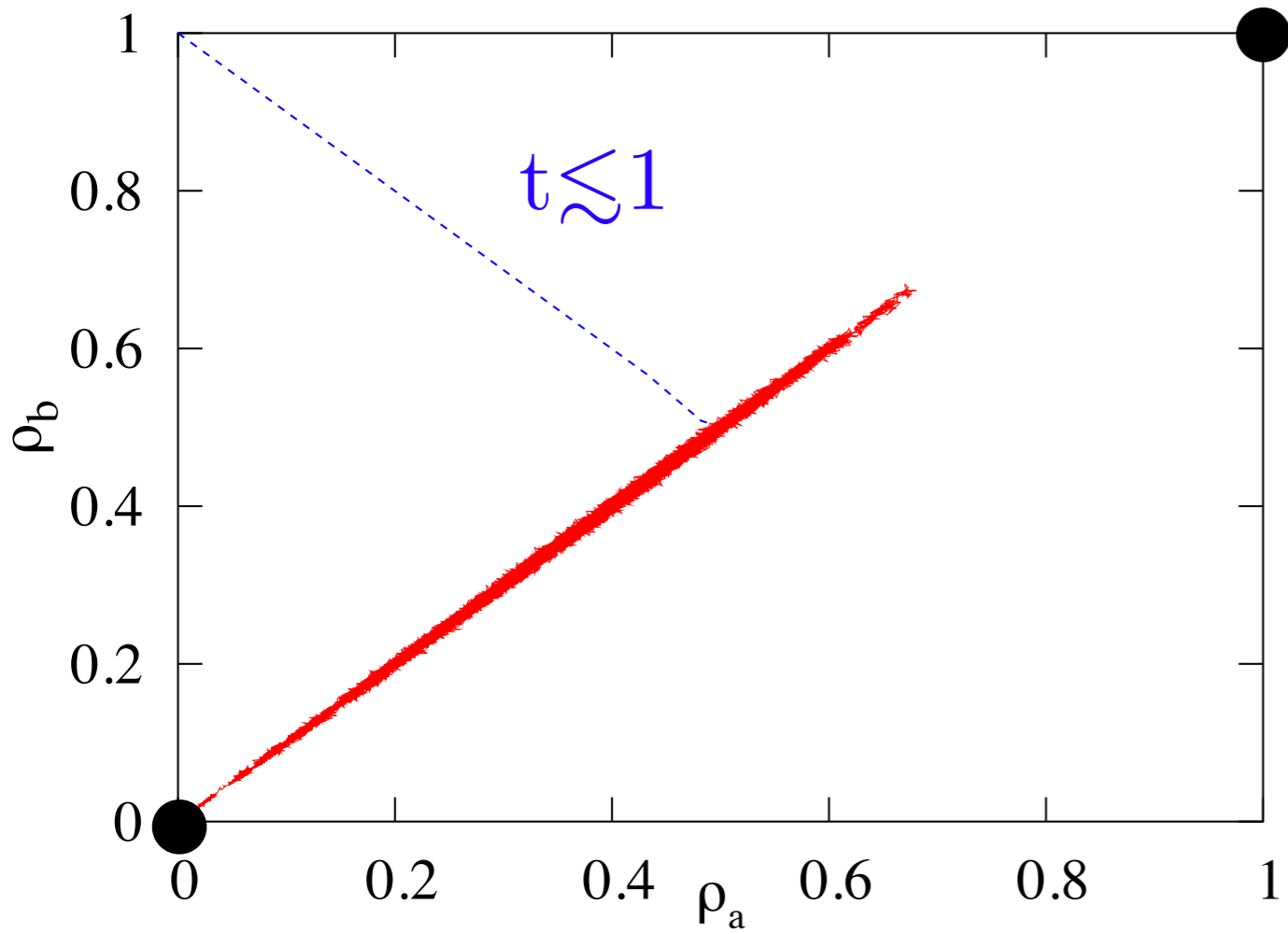


0 consensus

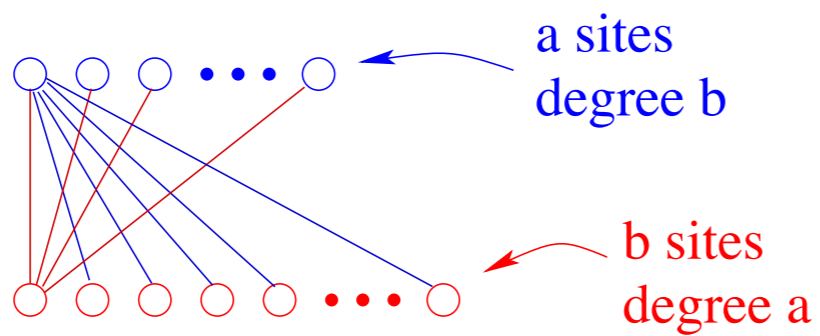


fixation probability
 $\propto 1/(\text{node degree})$

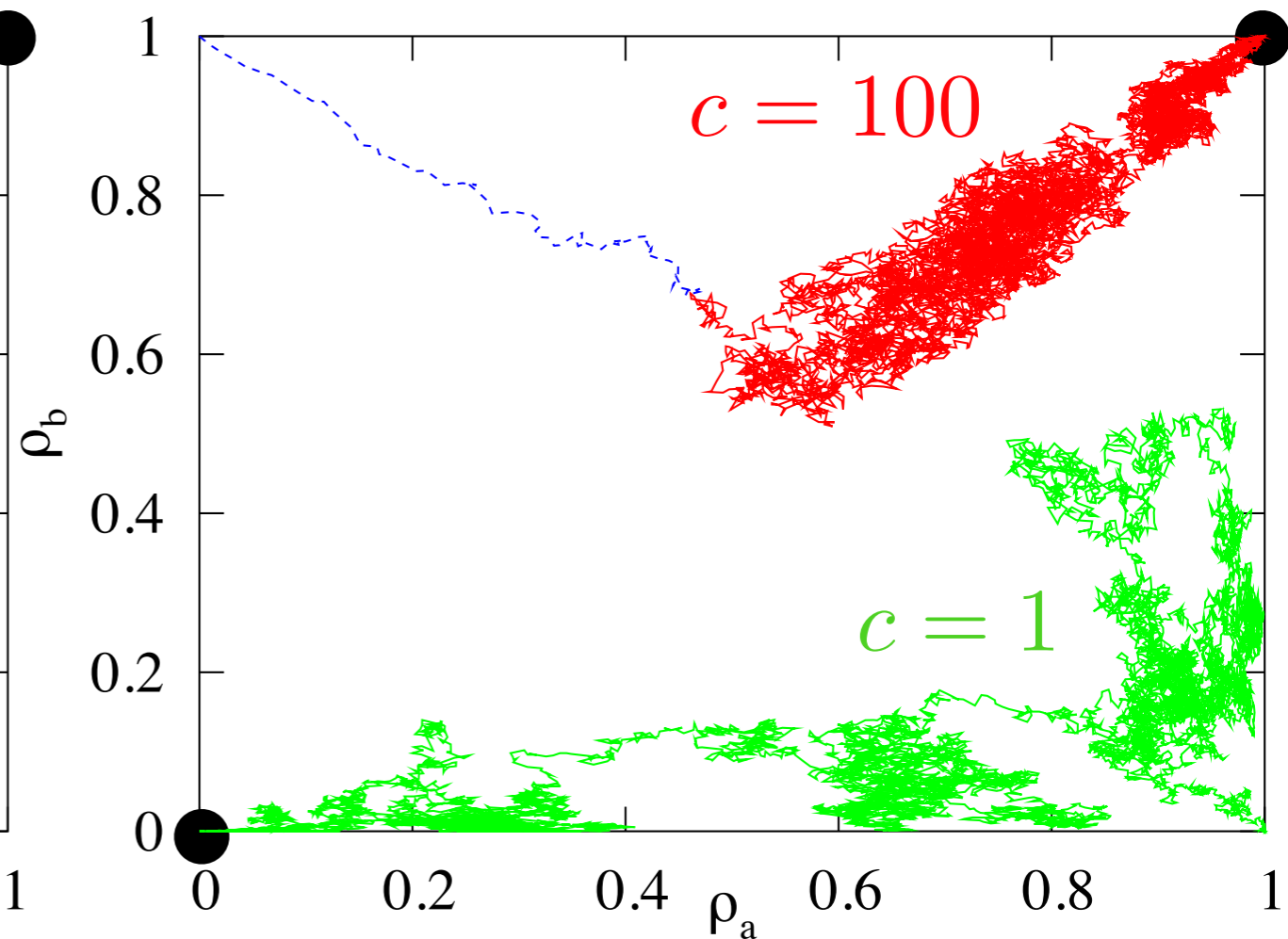
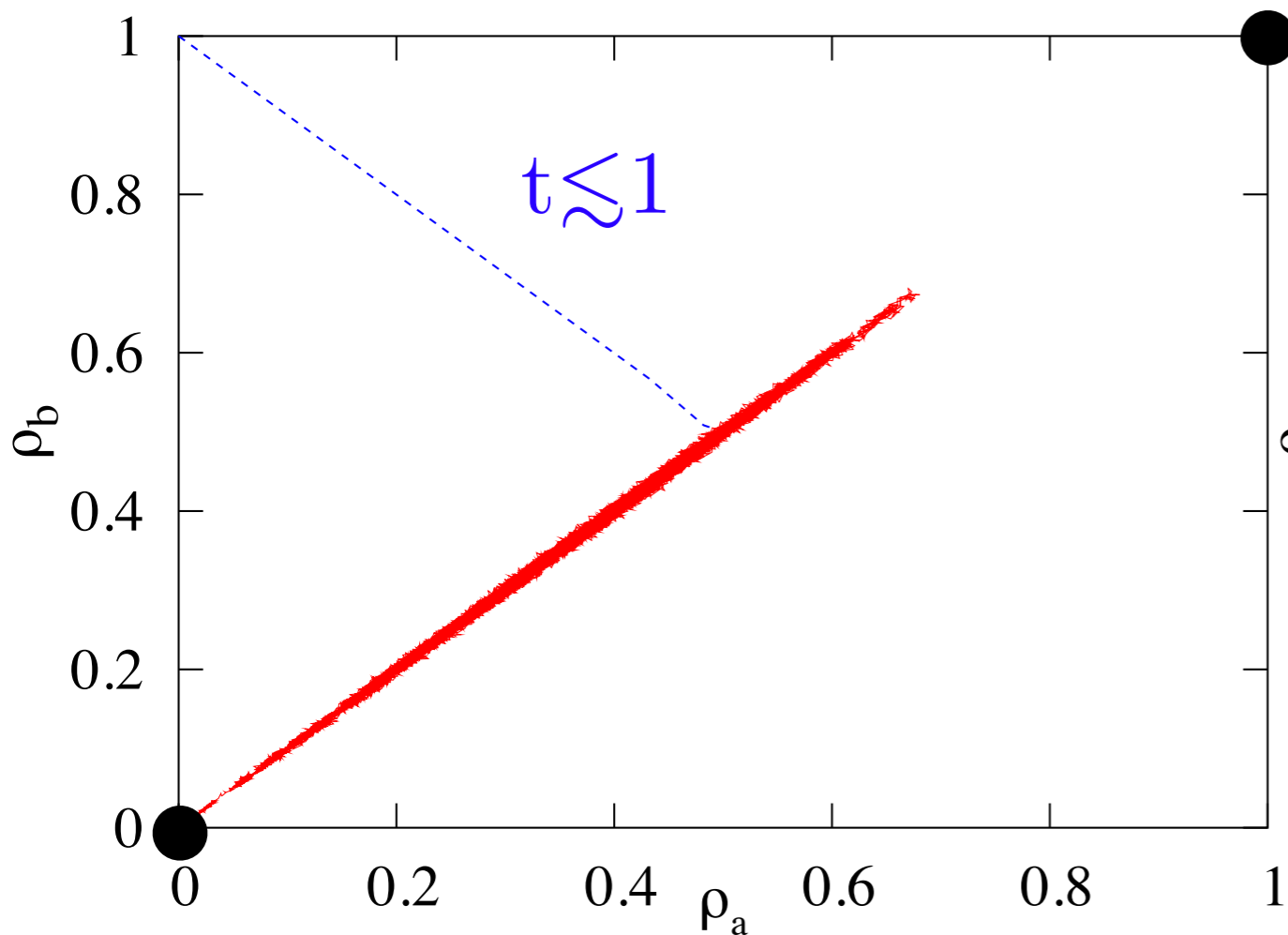
Route to Consensus on Complex Graphs



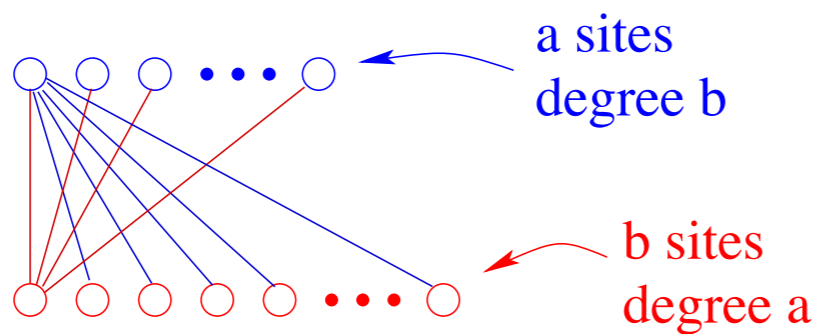
complete bipartite graph



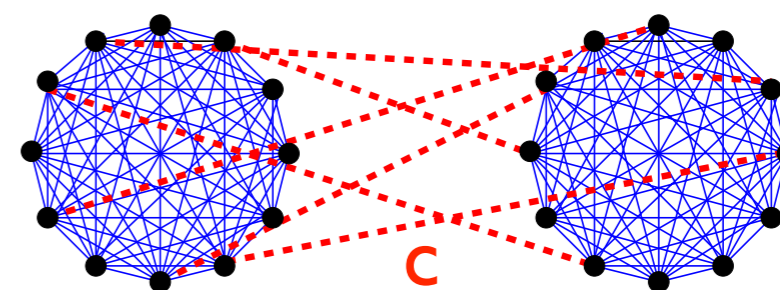
Route to Consensus on Complex Graphs



complete bipartite graph



two-clique graph



$N=10000$, C links/node

Consensus Time Evolution Equation

A Guide to First-Passage Processes
(CUP, 2001)

warmup: complete graph

$T(\rho) \equiv$ av. consensus time starting with density ρ

$$\begin{aligned} T(\rho) &= \mathcal{R}(\rho)[T(\rho + d\rho) + dt] \\ &\quad + \mathcal{L}(\rho)[T(\rho - d\rho) + dt] \\ &\quad + [1 - \mathcal{R}(\rho) - \mathcal{L}(\rho)][T(\rho) + dt] \end{aligned}$$

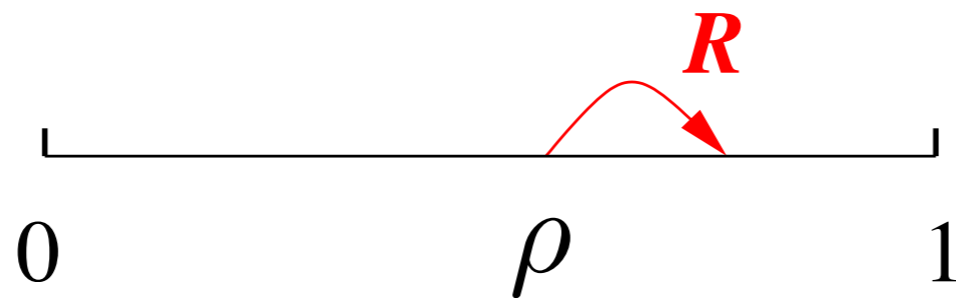
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$$\begin{aligned} \mathcal{R}(\rho) &\equiv \text{prob}(\downarrow \uparrow \rightarrow \uparrow \uparrow) \\ &= \rho(1 - \rho) \end{aligned}$$

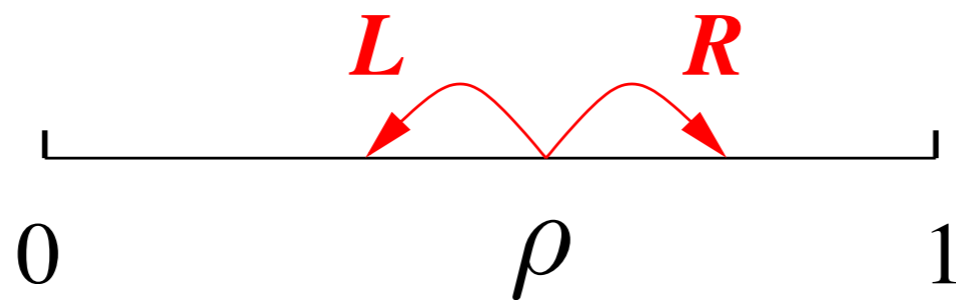
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warmup: complete graph

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$$\begin{aligned} T(\rho) &= \mathcal{R}(\rho)[T(\rho + d\rho) + dt] \\ &\quad + \mathcal{L}(\rho)[T(\rho - d\rho) + dt] \\ &\quad + [1 - \mathcal{R}(\rho) - \mathcal{L}(\rho)][T(\rho) + dt] \end{aligned}$$



$$\mathcal{R}(\rho) \equiv \text{prob}(\downarrow\uparrow \rightarrow \uparrow\uparrow)$$

$$\mathcal{L}(\rho) \equiv \text{prob}(\uparrow\downarrow \rightarrow \downarrow\downarrow)$$

$$= \rho(1 - \rho)$$

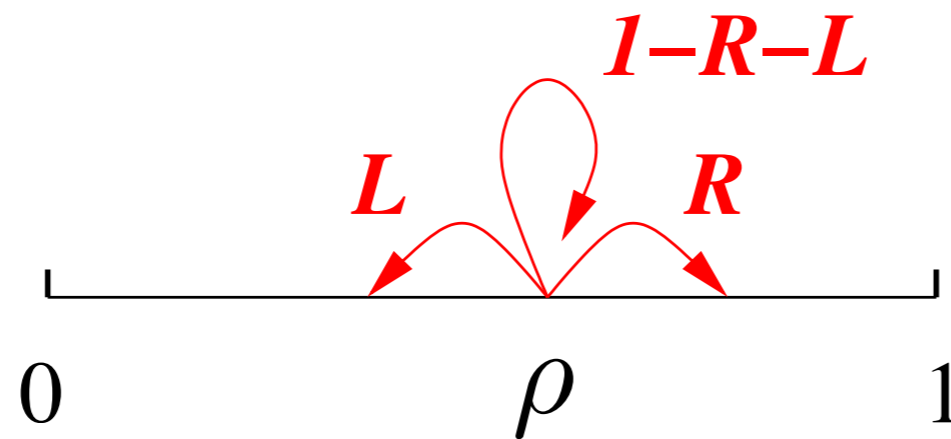
Consensus Time Evolution Equation

A Guide to First-Passage Processes
(CUP, 2001)

warmup: complete graph

$T(\rho) \equiv$ av. consensus time starting with density ρ

$$\begin{aligned} T(\rho) &= \mathcal{R}(\rho)[T(\rho + d\rho) + dt] \\ &\quad + \mathcal{L}(\rho)[T(\rho - d\rho) + dt] \\ &\quad + [1 - \mathcal{R}(\rho) - \mathcal{L}(\rho)][T(\rho) + dt] \end{aligned}$$



$$\begin{aligned} \mathcal{R}(\rho) &\equiv \text{prob}(\downarrow\uparrow \rightarrow \uparrow\uparrow) \\ \mathcal{L}(\rho) &\equiv \text{prob}(\uparrow\downarrow \rightarrow \downarrow\downarrow) \\ &= \rho(1 - \rho) \end{aligned}$$

Consensus Time on Complete Graph

$$\begin{aligned} T(\rho) &= \mathcal{R}(\rho)[T(\rho + d\rho) + dt] \\ &\quad + \mathcal{L}(\rho)[T(\rho - d\rho) + dt] \\ &\quad + [1 - \mathcal{R}(\rho) - \mathcal{L}(\rho)][T(\rho) + dt] \end{aligned}$$

continuum limit:

$$T'' = -\frac{N}{\rho(1-\rho)}$$

solution:

$$T(\rho) = -N [\rho \ln \rho + (1 - \rho) \ln(1 - \rho)]$$

Consensus Time on Heterogeneous Networks

$T(\{\rho_k\}) \equiv$ av. consensus time starting with density ρ_k
on nodes of degree k

$$\begin{aligned} T(\{\rho_k\}) &= \sum_k \mathcal{R}_k(\{\rho_k\}) [T(\{\rho_k^+\}) + dt] \\ &+ \sum_k \mathcal{L}_k(\{\rho_k\}) [T(\{\rho_k^-\}) + dt] \\ &+ \left[1 - \sum_k [\mathcal{R}_k(\{\rho_k\}) + \mathcal{L}_k(\{\rho_k\})] \right] [T(\{\rho_k\}) + dt] \end{aligned}$$

$$\begin{aligned} \mathcal{R}_k(\{\rho_k\}) &= \text{prob}(\rho_k \rightarrow \rho_k^+) & \mathcal{L}_k(\{\rho_k\}) &= n_k \rho_k (1 - \omega) \\ &= \frac{1}{N} \sum_x' \frac{1}{k_x} \sum_y P(\downarrow, \text{---}, \uparrow) \\ &= n_k \omega (1 - \rho_k) \end{aligned}$$

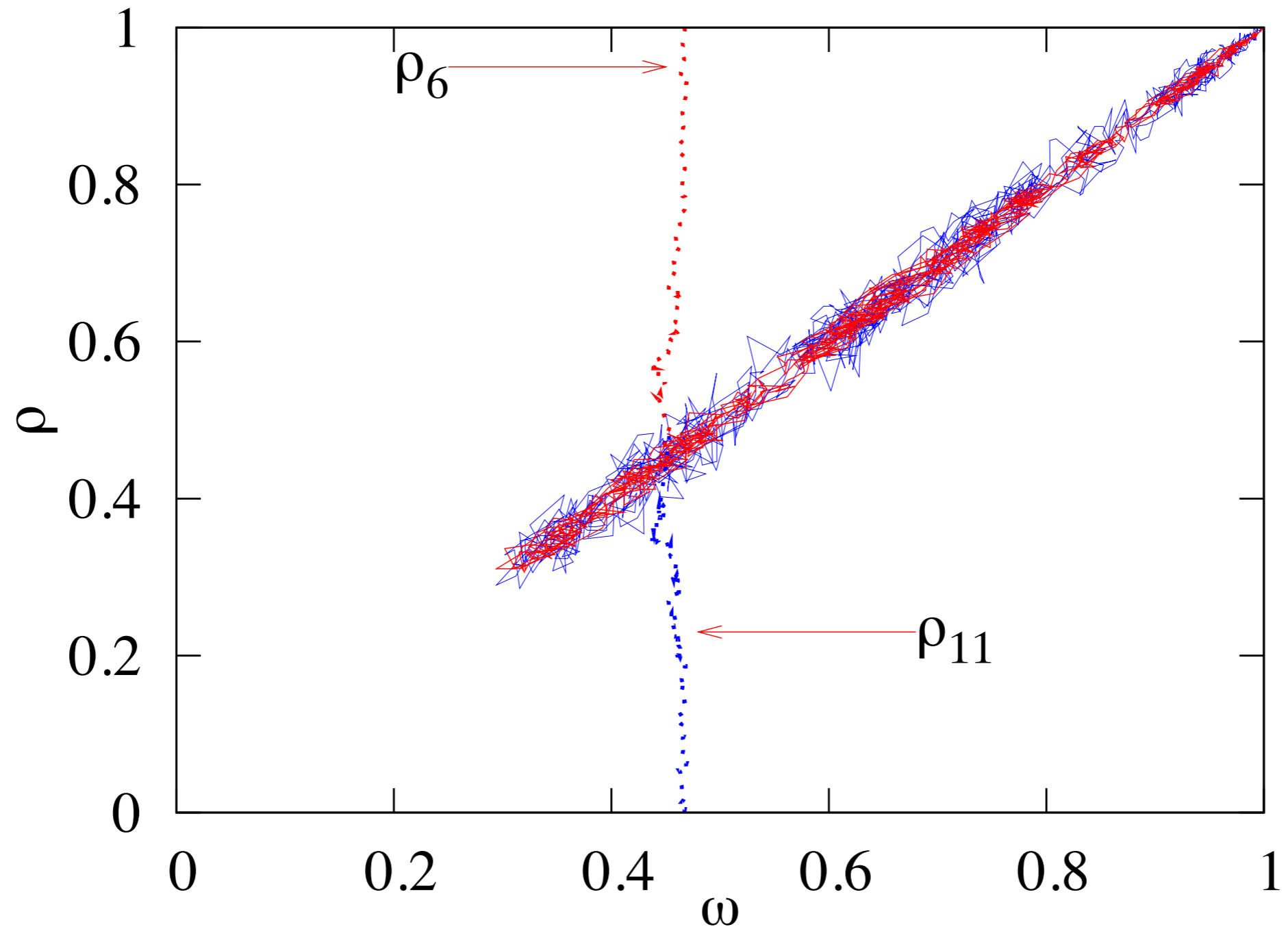
Consensus Time on Heterogeneous Networks

continuum limit:

$$\sum_k \left[(\omega - \rho_k) \frac{\partial T}{\partial \rho_k} + \frac{\omega + \rho_k - 2\omega\rho_k}{2Nn_k} \frac{\partial^2 T}{\partial \rho_k^2} \right] = -1$$

(Molloy-Reed) Configuration Model

$$n_k \sim k^{-2.5}, \quad \mu_1 = 8$$



Consensus Time on Heterogeneous Networks

continuum limit:

$$\sum_k \left[(\omega - \rho_k) \frac{\partial T}{\partial \rho_k} + \frac{\omega + \rho_k - 2\omega\rho_k}{2Nn_k} \frac{\partial^2 T}{\partial \rho_k^2} \right] = -1$$

now use $\rho_k \rightarrow \omega \quad \forall k$

and $\frac{\partial}{\partial \rho_k} = \frac{\partial \omega}{\partial \rho_k} \frac{\partial}{\partial \omega} = \frac{kn_k}{\mu_1} \frac{\partial}{\partial \omega}$

Consensus Time on Heterogeneous Networks

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and $\frac{\partial}{\partial \rho_k} = \frac{\partial \omega}{\partial \rho_k} \frac{\partial}{\partial \omega} = \frac{kn_k}{\mu_1} \frac{\partial}{\partial \omega}$

to give $\frac{\partial^2 T}{\partial \omega^2} = -\frac{N\mu_1^2/\mu_2}{\omega(1-\omega)}$ same $T'' = -\frac{N}{\rho(1-\rho)}$
as

with effective size $N_{\text{eff}} = N \mu_1^2 / \mu_2$

Consensus Time for Power-Law Degree Distribution $n_k \sim k^{-\nu}$

Voter model: $T_N \sim N_{\text{eff}} = N\mu_1^2/\mu_2$

$$T_N \sim \begin{cases} N & \nu > 3, \\ N/\ln N & \nu = 3, \\ N^{(2\nu-4)/(\nu-1)} & 2 < \nu < 3, \\ (\ln N)^2 & \nu = 2, \\ \mathcal{O}(1) & \nu < 2. \end{cases}$$

Consensus Time for Power-Law Degree Distribution $n_k \sim k^{-\nu}$

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Invasion process: $T_N \sim N_{\text{eff}} = N \mu_1 \mu_{-1}$

$$T_N \sim \begin{cases} N & \nu > 2 \\ N \ln N & \nu = 2 \\ N^{3-\nu} & \nu < 2 \end{cases}$$

Partisan Voting and Truth

N. Masuda, N. Gibert, SR
PRE **82**, 010103(R) (2010)

↑ prefers truth
& in \bar{T} state

density T_+

↓ prefers truth
& in F state

density T_-

↑ prefers false
& in \bar{T} state

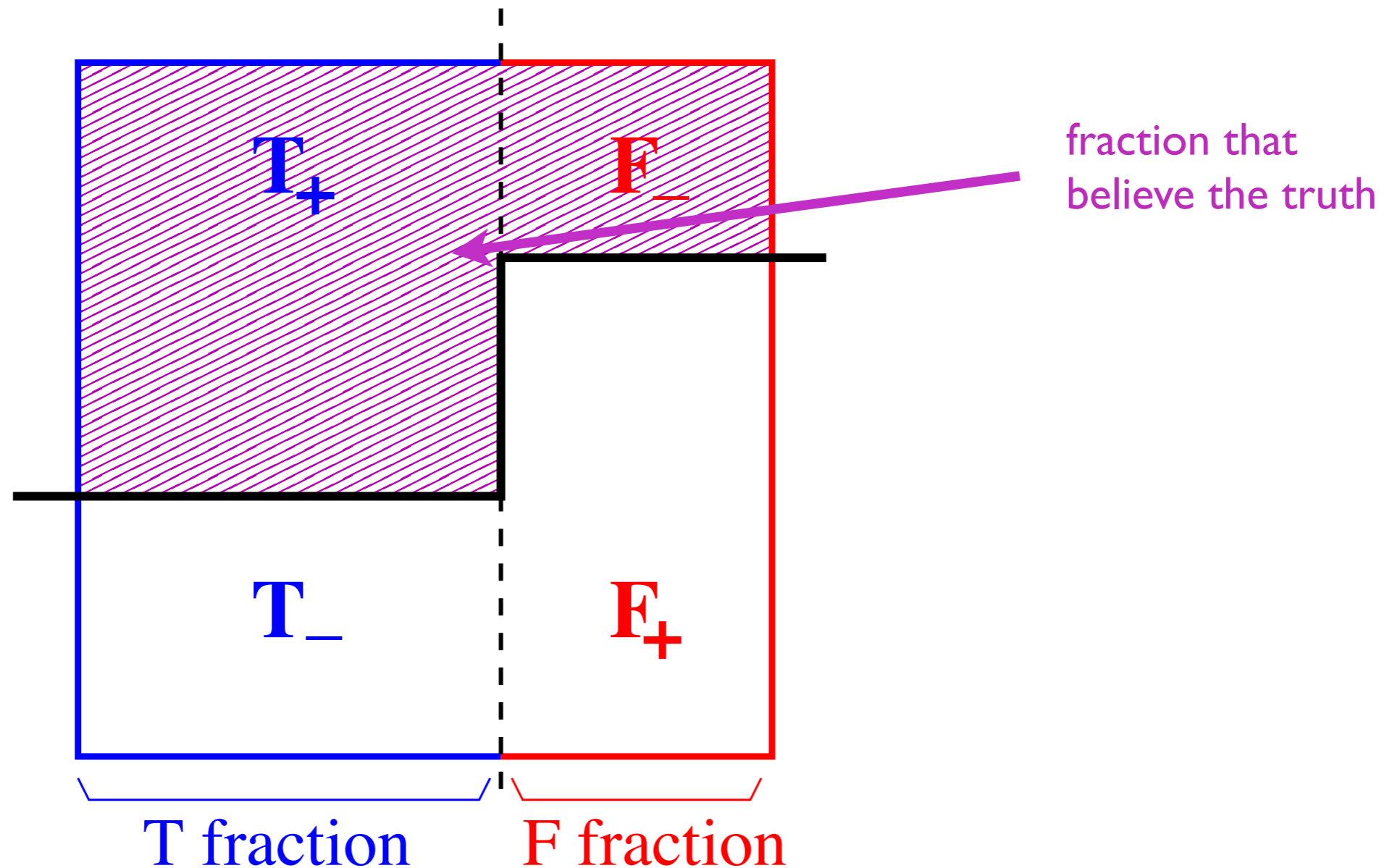
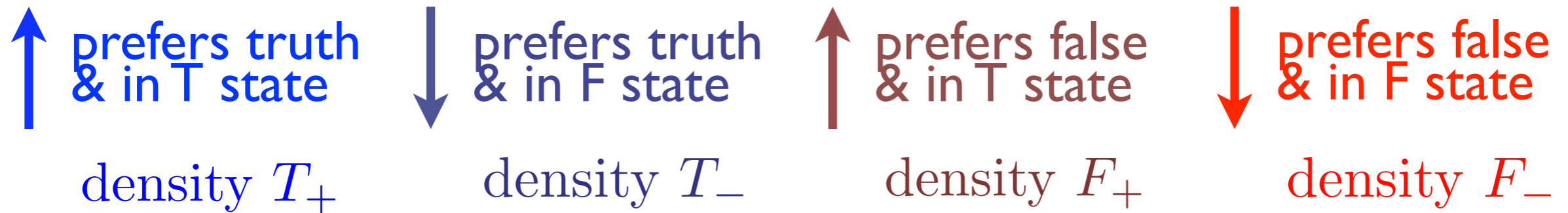
density F_+

↓ prefers false
& in F state

density F_-

Partisan Voting and Truth

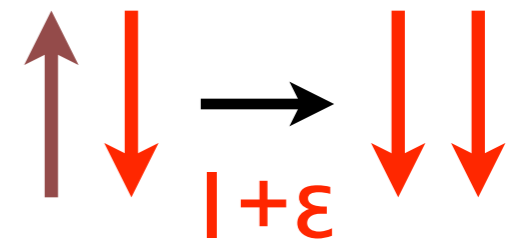
N. Masuda, N. Gibert, SR
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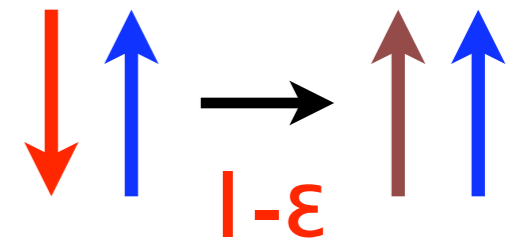
partisan voting update:

1. Pick voter, pick neighbor (as in usual voter model);

2a. If initial voter becomes *concordant* by adopting neighboring state, change occurs with rate $1+\epsilon$;



2b. If initial voter becomes *discordant* by adopting neighboring state, change occurs with rate $1-\epsilon$.



Rate Equations

$$\dot{T}_+ = (1 + \epsilon)T_- [T_+ + F_-] - (1 - \epsilon)T_+ [T_- + F_+]$$

$$\dot{T}_- = (1 - \epsilon)T_+ [T_- + F_+] - (1 + \epsilon)T_- [T_+ + F_-]$$

$$\dot{F}_+ = (1 + \epsilon)F_- [F_+ + T_-] - (1 - \epsilon)F_+ [F_- + T_+]$$

$$\dot{F}_- = (1 - \epsilon)F_+ [F_- + T_+] - (1 + \epsilon)F_- [F_+ + T_-]$$

$$T_- = T - T_+$$

$$S = T_+ + F_+$$

$$F_- = F - F_+$$

$$\Delta = T_+ - F_+$$

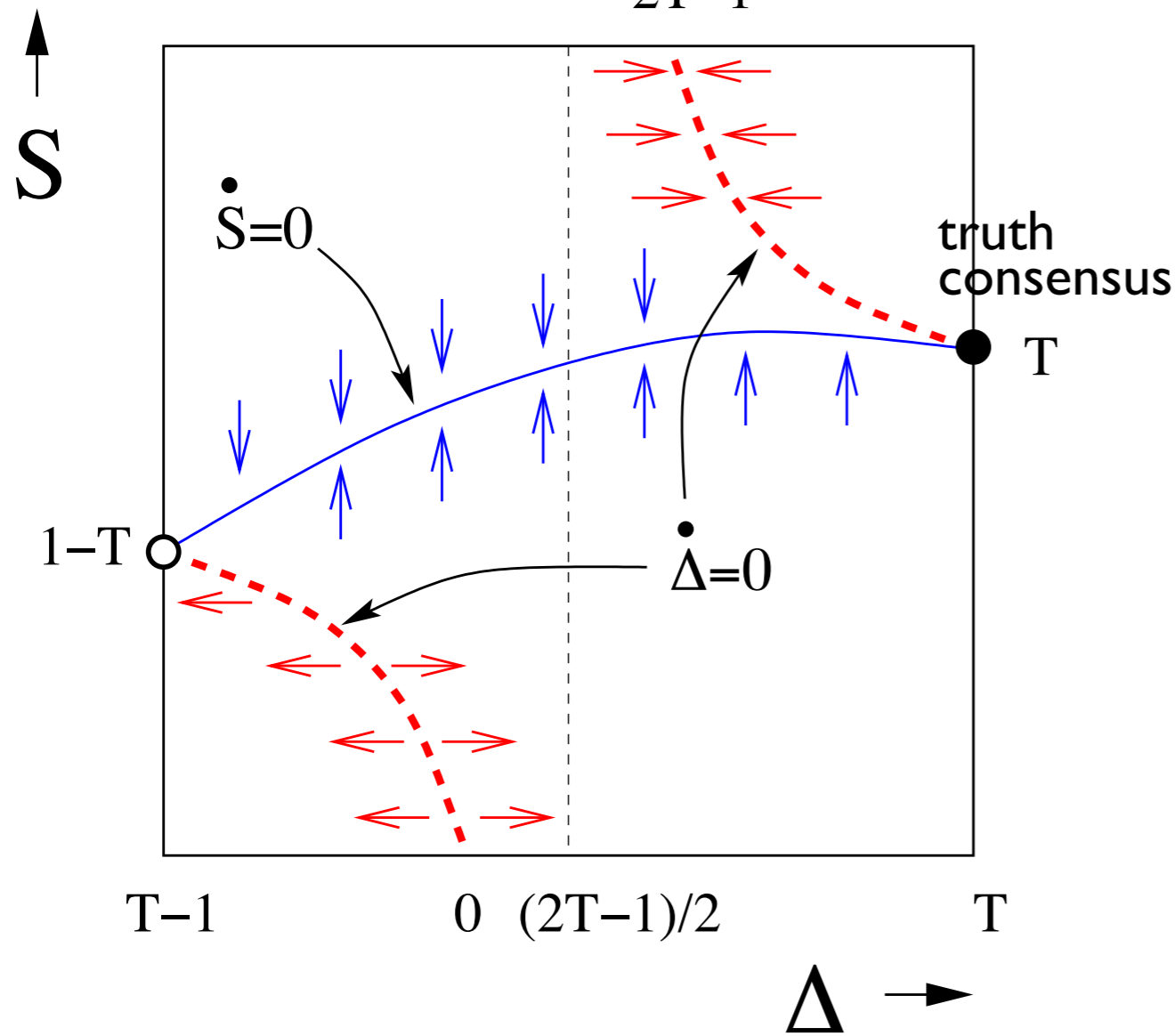
Flow Diagram

$$S = T_+ + F_+$$

$$\Delta = T_+ - F_+$$

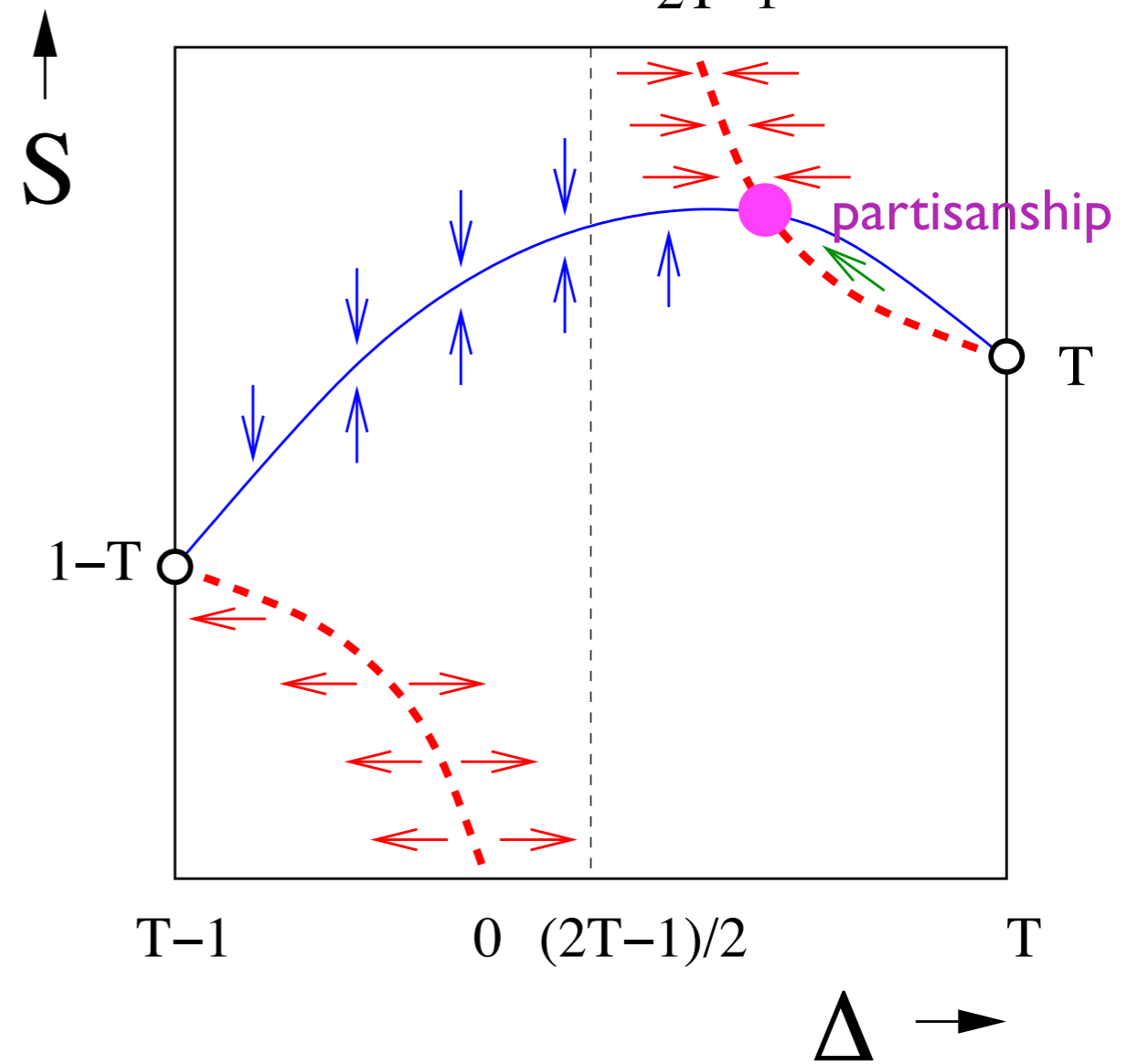
$$\epsilon < 2T - 1$$

$$2T-1$$



$$\epsilon > 2T - 1$$

$$2T-1$$



Summary & Outlook

Voter model:

paradigmatic, soluble, (but hopelessly naive)

Voter model on complex networks:

new conservation law

two time-scale route to consensus

fast consensus for broad degree distributions

Extension to Partisanship:

partisanship forestalls consensus to the truth

Future:

“churn” rather than consensus

heterogeneity of real people

positive and negative social interactions

Crass Commercialism

Aimed at graduate students, this book explores some of the core phenomena in non-equilibrium statistical physics. It focuses on the development and application of theoretical methods to help students develop their problem-solving skills.

The book begins with microscopic transport processes: diffusion, collision-driven phenomena, and exclusion. It then presents the kinetics of aggregation, fragmentation, and adsorption, where basic phenomenology and solution techniques are emphasized. The following chapters cover kinetic spin systems, by developing both a discrete and a continuum formulation, the role of disorder in non-equilibrium processes, and hysteresis from the non-equilibrium perspective. The concluding chapters address population dynamics, chemical reactions, and a kinetic perspective on complex networks. The book contains more than 200 exercises to test students' understanding of the subject. A link to a website hosted by the authors, containing an up-to-date list of errata and solutions to some of the exercises, can be found at www.cambridge.org/9780521851039.

Pavel L. Krapivsky is Research Associate Professor of Physics at Boston University. His current research interests are in strongly interacting many-particle systems and their applications to kinetic spin systems, networks, and biological phenomena.

Sidney Redner is a Professor of Physics at Boston University. His current research interests are in non-equilibrium statistical physics and its applications to reactions, networks, social systems, biological phenomena, and first-passage processes.

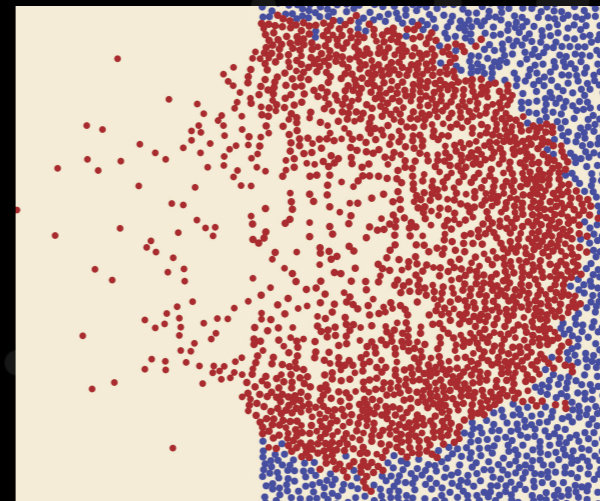
Eli Ben-Naim is a member of the Theoretical Division and an affiliate of the Center for Nonlinear Studies at Los Alamos National Laboratory. He conducts research in statistical, nonlinear, and soft condensed-matter physics, including the collective dynamics of interacting particle and granular systems.

Cover illustration: Snapshot of a collision cascade in a perfectly elastic, initially stationary hard-sphere gas in two dimensions due to a single incident particle. Shown are the cloud of moving particles (red) and the stationary particles (blue) that have not yet experienced any collisions. Figure courtesy of Tibor Antal.

Krapivsky
Redner
Ben-Naim

A Kinetic View of STATISTICAL PHYSICS

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Eli Ben-Naim

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1. Aperitifs
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14. Complex Networks

to
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