Kinetics of Filtration and Clogging

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PRE **58**, R1203 (1998); PRL **84**, 6018 (2000)

What is filtration? What is clogging?

Basic Approach:

minimalist modeling; probabilistic description

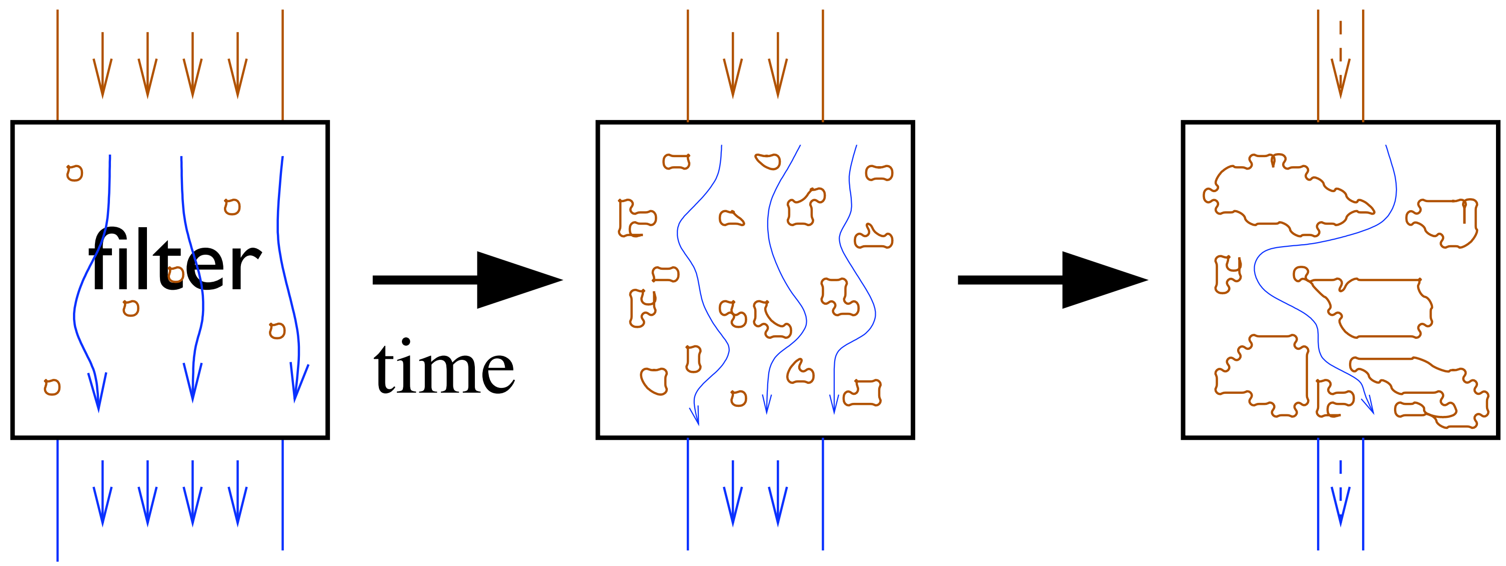
Two Main Results:

density profiles of trapped & escaped particles *gradient driven*

clogging time and its distribution *extreme statistics*

Summary & Outlook

What is filtration? What is clogging?



flow ⇔ trapping

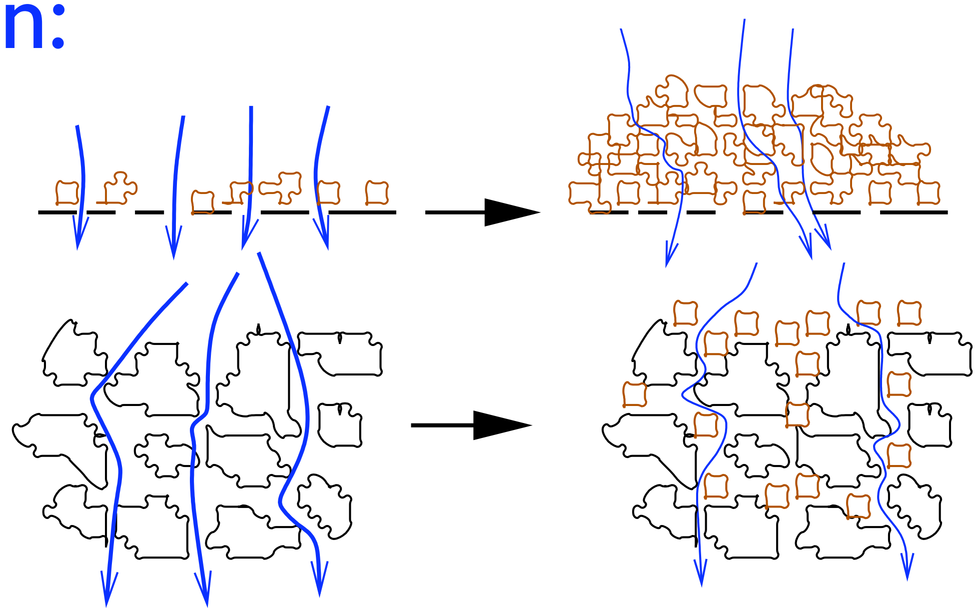
no steady state

→*clogging*

Types of filtration:

cake

depth



Mechanisms of depth filtration:

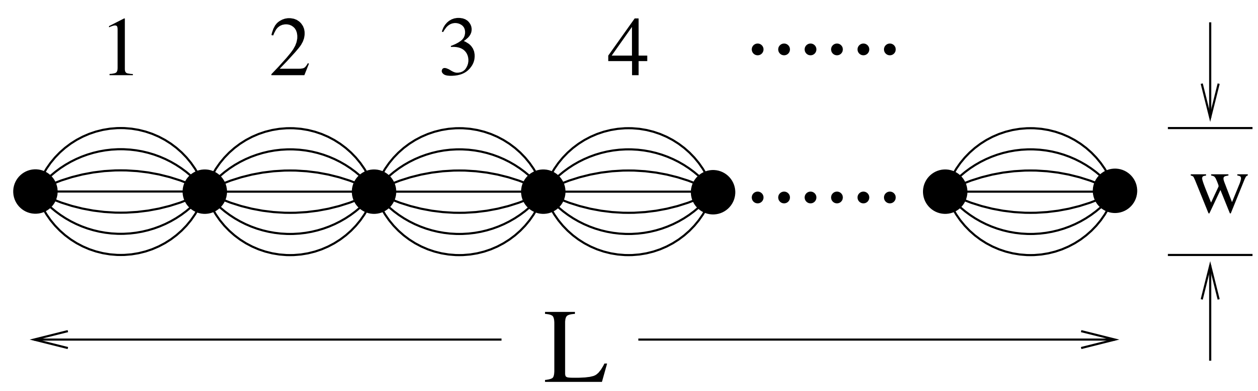
hydrodynamic, gravitation

electrochemical

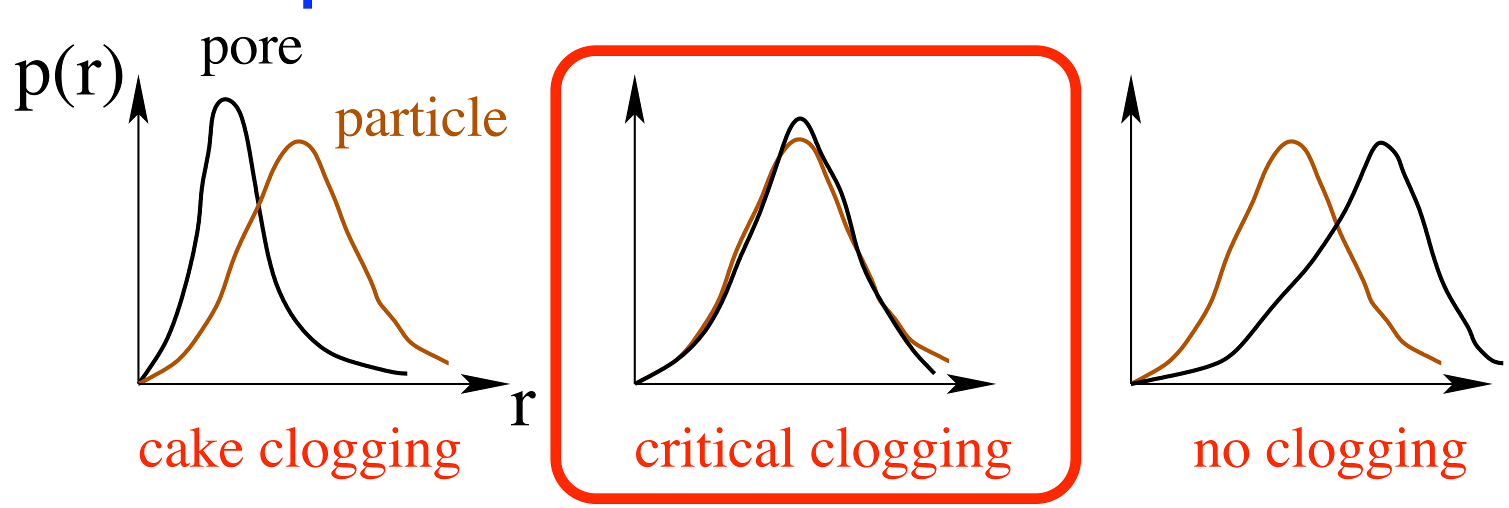
↕ $1\text{–}10\ \mu\mathrm{m}$

Microscopic Modeling

1. Bubble model



2. Particle & pore characteristics



3. Flow characteristics

Poiseuille flow

dynamically neutral particles

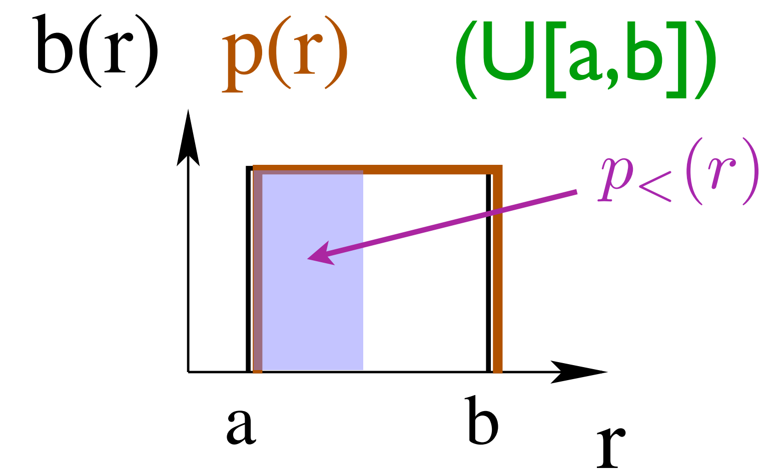
perfect mixing at junctions $p_i = \phi_i / \sum \phi_i$, with $\phi_r \propto r_i^4$

→ *large pores entered preferentially*

4. Trapping mechanism:

particle > pore → *permanent, complete blockage*

1. Distribution of Trapped Particles



assumptions:

overlapping particle and pore sizes *critical clogging*

large w *(unperturbed)*

for particle of radius r:

$$p_< \equiv \text{prob. of getting stuck in a bubble}$$

$$= \int_a^r r'^4 dr' \Big/ \int_a^b r'^4 dr' = \frac{r^5 - a^5}{b^5 - a^5}$$

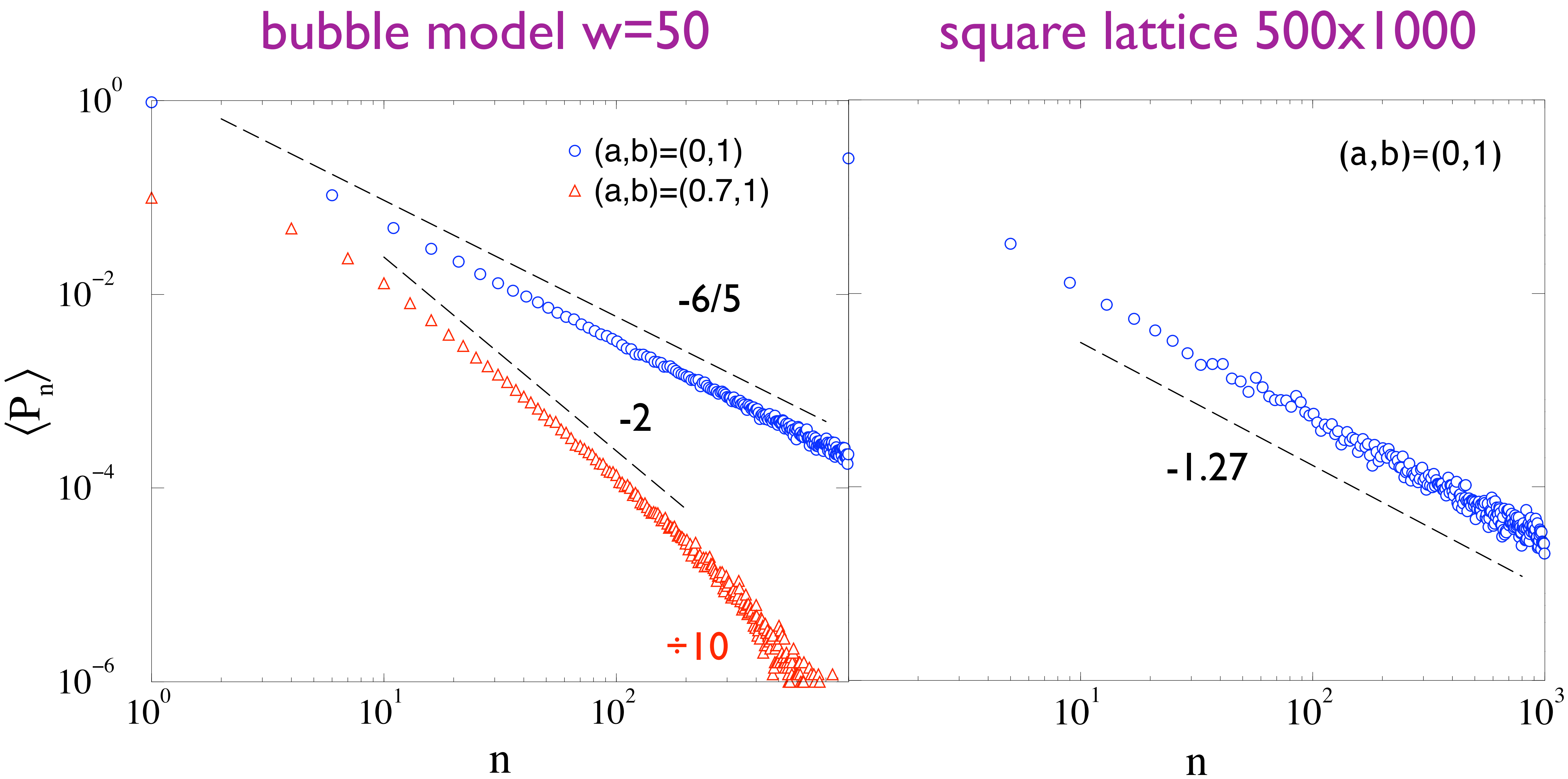
$$P_n \equiv \text{prob. of getting stuck in bubble } n$$

$$= (1 - p_<)^{n-1} p_<$$

$$\langle P_n \rangle = \int_a^b \left(1 - \frac{r^5 - a^5}{b^5 - a^5}\right)^{n-1} \frac{r^5 - a^5}{b^5 - a^5} \frac{dr}{b-a} \qquad u = r - a$$

$$\simeq \begin{cases} \frac{b^5 - a^5}{5a^4(b-a)} n^{-2}, & a \neq 0; \quad \int e^{-nu} \, n \, u \, d \, nu \quad n^{-2} \\ 0.1836 \ldots n^{-6/5} & a = 0. \quad \int e^{-nr^5} \, n \, r^5 d \, n^{1/5} r \quad n^{-6/5} \end{cases}$$

Trapped Particle Distributions



Large-w Approximation

prob. that particle of radius r is trapped
in a bubble of w bonds:

$$\begin{aligned} p_< &= w \int_0^r dr_1 \int_0^1 dr_2 \ldots \int_0^1 \frac{r_1^4 \; dr_w}{r_1^4 + r_2^4 + \ldots + r_w^4} \qquad u_i = r_i/r_1 \\ &= w \int_0^r r_1^{w-1} dr_1 \int_0^{1/r_1} du_2 \cdots \int_0^{1/r_1} \frac{du_w}{1 + u_2^4 + \ldots + u_w^4} \\ &\sim w \int_0^r r_1^{w-1} dr_1 \int_{0+}^{1/r_1} \frac{u^{w-2}}{u^4}\, du \end{aligned}$$

$$p_< \propto \begin{cases} r^w, & w < 5; \\ r^5 \ln r, & w = 5; \\ r^5, & w > 5. \end{cases}$$

2. Size Distribution of Escaping Particles

assume particle and pore sizes (U[0,1]) $p_< = r^5$

to escape nth bubble, get trapped at n'>n

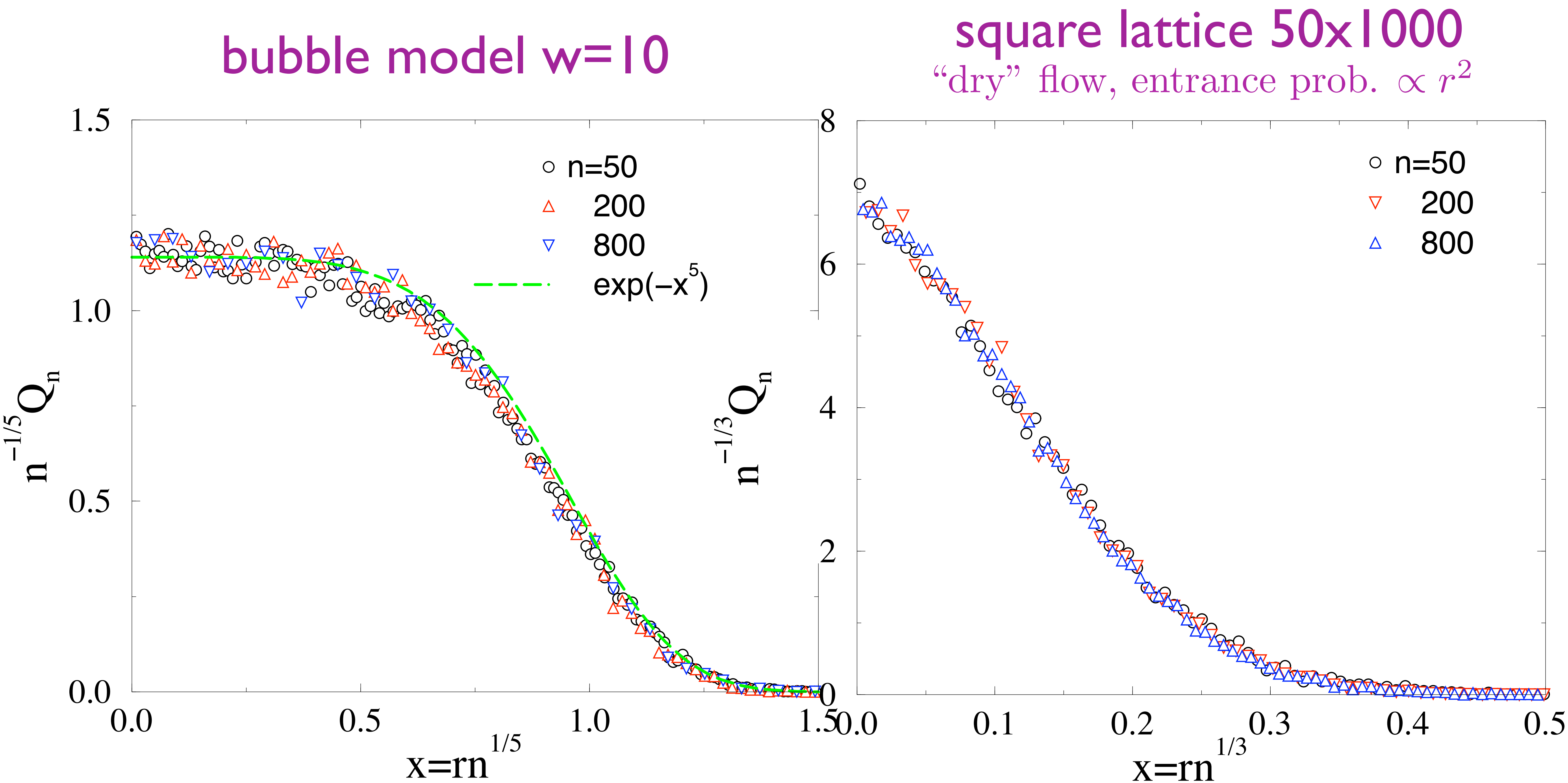
$$\sum_{n'>n}^{\infty} P_{n'} = \sum_{n'>n}^{\infty} r^5 (1 - r^5)^{n'-1}$$

$$\sim \exp(-nr^5)$$

$$Q_n \equiv \text{size distribution of escapees at } n$$

$$\propto n^{1/5} \exp(-nr^5)$$

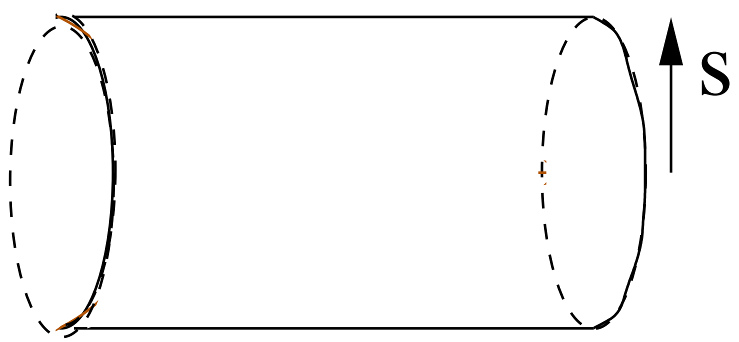
Escapee Size Distribution



Clogging Time and its Distribution

Single bond clogging:

assume $b(r) = 2\alpha r e^{-\alpha r^2} \rightarrow s = 1/\sqrt{\alpha}$



clogged!

Clogging Time and its Distribution

Single bond clogging (cont):

prob. that bond of radius R is blocked by Nth particle:

$$Q_N(R) = \left[\int_0^R p(r)\,dr\right]^{N-1} \int_R^\infty p(r)\,dr \quad \equiv p_<(r)^{N-1}\,p_>(r)$$

$$\langle Q_N(R)\rangle \;=\; \int_0^\infty Q_N(R)\,b(R)\,dR \qquad p(r) = 2re^{-r^2}$$

$$\sim \int e^{-Nx}\,x^{1/\alpha}dx, \quad x = e^{-\alpha R^2}$$

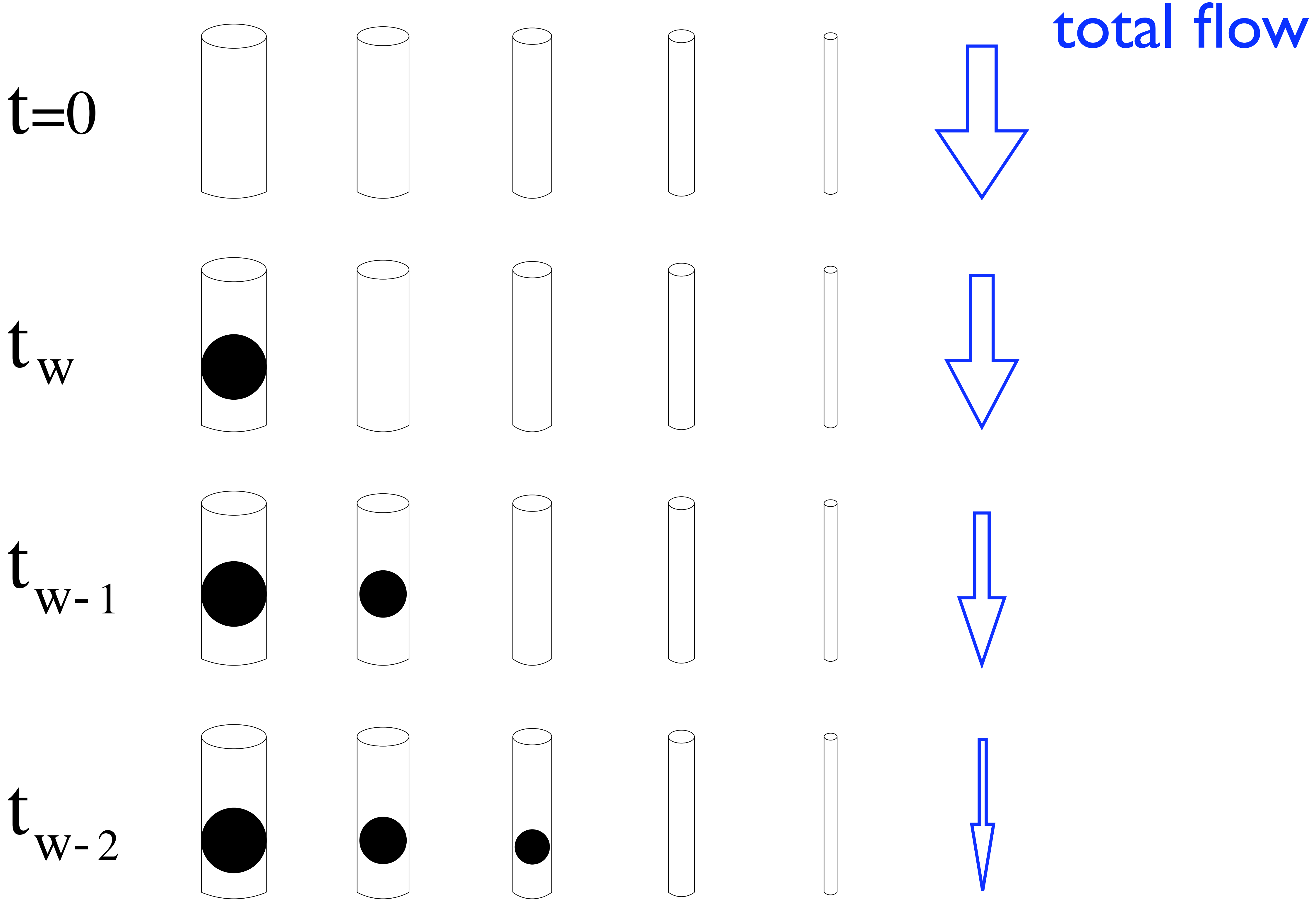
$$\sim \alpha/N^{1+\alpha}$$

$$\langle N\rangle \simeq \int_1^\infty \frac{\alpha N\,dN}{N^{1+\alpha}} \sim \frac{\alpha}{1-\alpha} \qquad \langle N\rangle = \begin{cases} \infty & \text{for } s = \frac{1}{\sqrt{\alpha}} \geq 1 \\ \text{finite} & \text{for } s = \frac{1}{\sqrt{\alpha}} < 1 \end{cases}$$

Cartoon for Network Clogging

size-ordered blocking

(replace entrance prob. $\propto r^4$ with r^∞)



Estimate of the Clogging Time

size-ordered clogging: *biggest clogged first smallest clogged last*

radius of kth smallest bond:

$$\int_0^{r_k} 2\alpha r\, e^{-\alpha r^2}\, dr = \frac{k}{w}, \qquad r_k = s\sqrt{\frac{k}{w}}$$

flow rate when k bonds still open:

$$\kappa(k) = \sum_{j=1}^{k} r_j^4 \approx s^4 \sum_{j=1}^{k} \left(\frac{j}{w}\right)^2 \sim \frac{s^4 k^3}{w^2}$$

$$\kappa(w) = s^4 w$$

and corresponding time scale:

$$\rightarrow t_k = \frac{\kappa(w)}{w\kappa(k)} \sim \frac{w^2}{k^3}$$

clogging time:

$$T = t_1 + t_2 + t_3 + \ldots \approx w^2 \left[1 + \frac{1}{2^3} + \frac{1}{3^3} + \ldots\right] \propto w^2$$

Clogging Time Distribution

hypothesis: *clogging time determined by smallest bond*

smallest bond distribution:

$$\text{If } b(r) = 2\alpha r\, e^{-\alpha r^2} \to b_>(r) = e^{-\alpha r^2} \qquad s \sim 1/\sqrt{\alpha}$$

$$\begin{aligned} S_w(r) &\equiv \text{prob. smallest bond out of } w \text{ has radius } r \\ &= w\, b(r)\, [b_>(r)]^{w-1} \\ &= 2\alpha\, wr\, e^{-\alpha w r^2} \end{aligned}$$

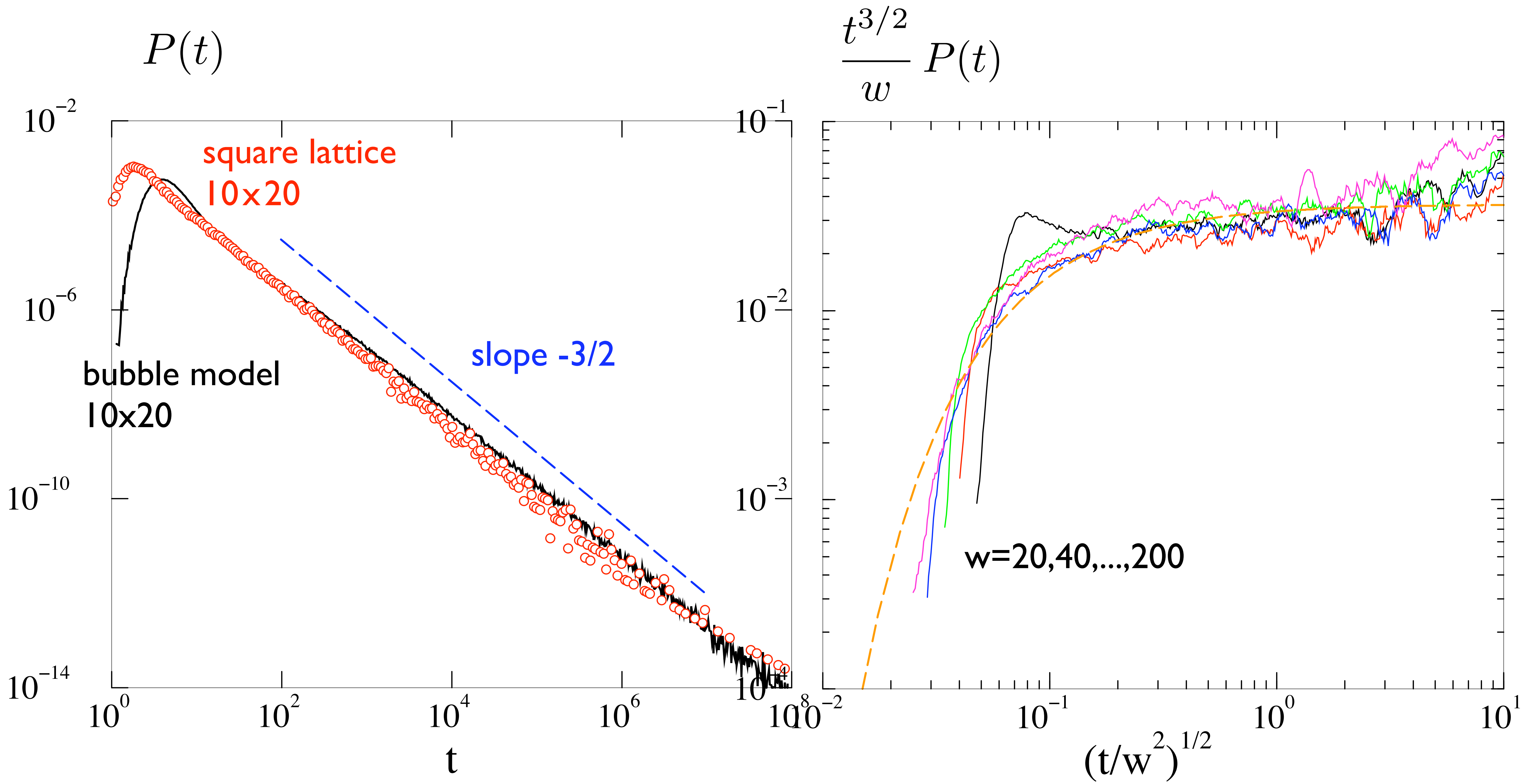
$s^4 w$ $\quad r_1^4$

connection between t and r: $T \sim t_1 \approx \dfrac{1}{w} \dfrac{\kappa(w)}{\kappa(1)} = \dfrac{s^4}{r_1^4}$

clogging time distribution: $P_w(t)\, dt = S_w(r)\, dr$

$$\to P_w(t) \sim \frac{w}{t^{3/2}}\, e^{-w/t^{1/2}} \qquad w^2 < t < w^2 N^2$$

Clogging Time Distribution $P_w(t) \sim \frac{w}{t^{3/2}} \, e^{-w/t^{1/2}}$



Moments of the Clogging Time

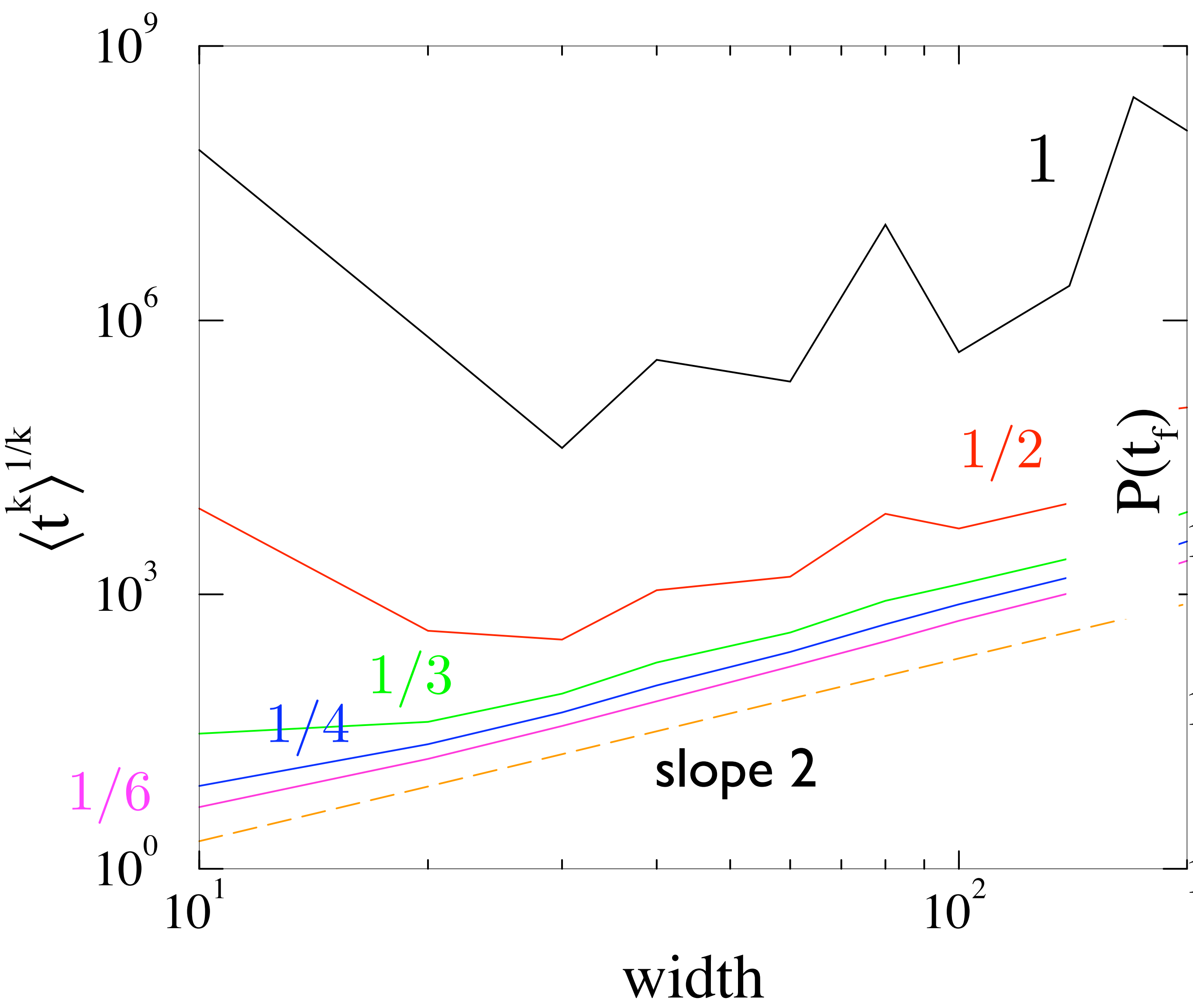
$$P_w(t) \sim \frac{w}{t^{3/2}}\; e^{-w/t^{1/2}} \qquad w^2 < t < w^2 N^2$$

$$\langle t^k \rangle = \int_0^\infty t\, P_w(t)\, dt \approx \int_{w^2}^{N^2 w^2} w\, t^{k-3/2}\, dt$$

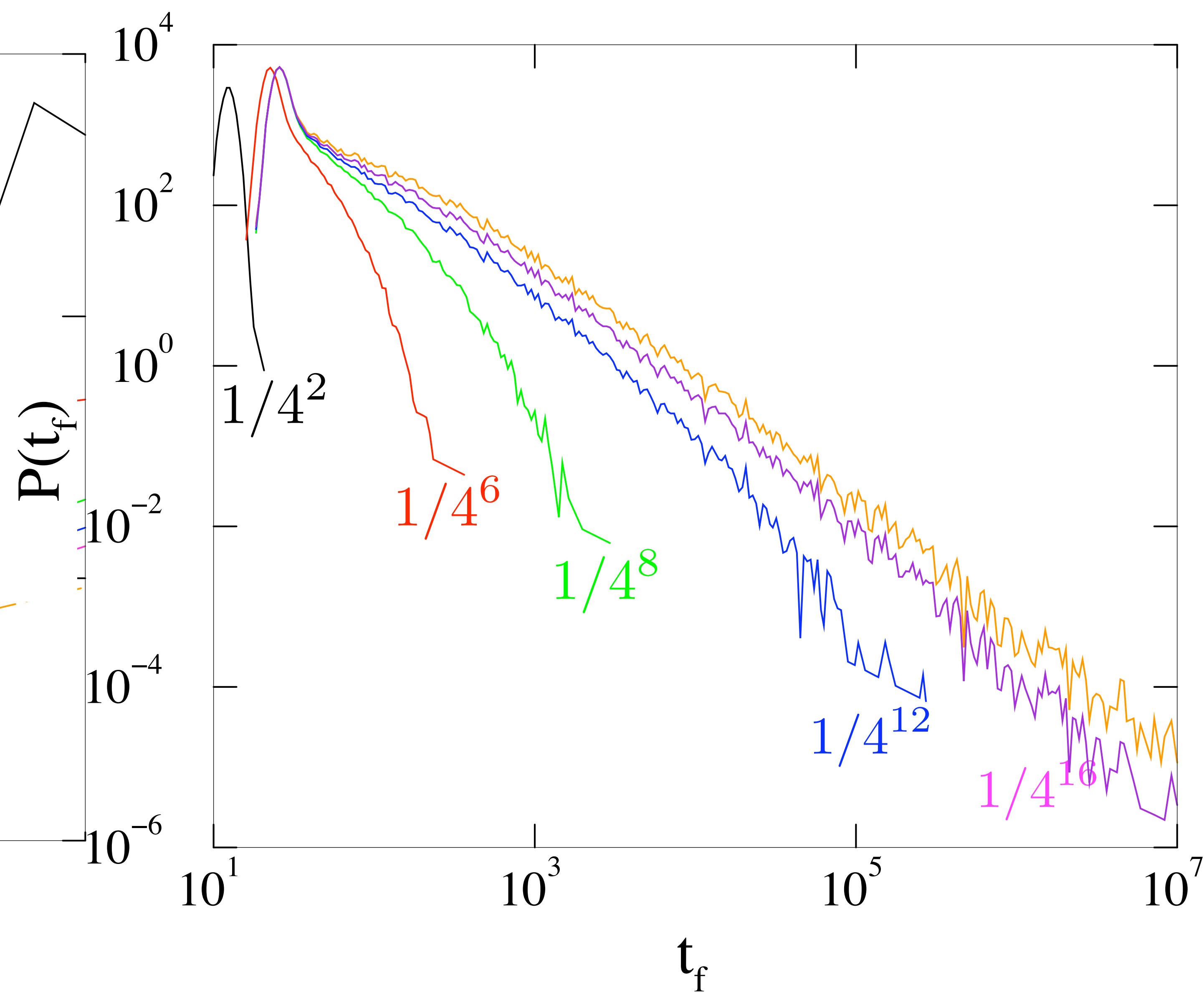
$$M_k(w) \equiv \langle t^k \rangle^{1/k} \sim \begin{cases} w^2 N^{2-1/k} & k > 1/2 \\ w^2 (\ln N)^2 & k = 1/2 \\ w^2 & k < 1/2 \end{cases}$$

Measures of Clogging Time

moments of clogging time



partial clogging time distribution



Summary & Outlook

Gradients drive depth filtration breakdown

Basic contradiction of filters:

A filter should be long for good filtering
short to be active

Clogging time governed by extreme events
→ *power law clogging time distribution*

Open questions:

When is a filter "dead"?

How to described gradient-driven percolation?

Sclerosis