Reinforcement-driven spread of innovations and fads

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Reinforcement-driven spread of innovations and fads

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Abstract. We investigate how *social reinforcement* drives the spread of permanent innovations and transient fads. We account for social reinforcement by endowing each individual with $M+1$ possible awareness states $0, 1, 2, \ldots, M$, with state M corresponding to adopting an innovation. An individual with awareness $k < M$ increases to $k+1$ by interacting with an adopter. Starting with a single adopter, the time for an initially unaware population that consists of N individuals to adopt an innovation grows as $\ln N$ for $M = 1$, and as $N^{1-1/M}$ for $M > 1$. When individuals can abandon the innovation at rate λ, the population fraction that remains clueless about the fad undergoes a phase transition at a critical rate λ_c; this transition is second order for $M = 1$ and first order for $M > 1$, with macroscopic fluctuations accompanying the latter. The time for the fad to disappear has an intriguing non-monotonic dependence on λ.

Keywords: interacting agent models, scaling in socio-economic systems

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1. Introduction

Disease propagation [1], the spread of technological innovations [2]–[6], and outbreaks of social and political unrest [7, 8] are all driven by contagion. In this work, we investigate how the mechanism of *social reinforcement* affects the contagion-driven evolution of permanent innovations and transient fads in a simple agent-based model. Social reinforcement means that an individual requires multiple prompts from acquaintances before adopting an innovation. This mechanism was found to foster the adoption of a desired behavior in a controlled online social network [9]. Social reinforcement stands in stark contrast to classical models of epidemics [1], where a susceptible individual can become infected by a single exposure to the infection.

Previous studies of contagion spread have employed a variety of spreading mechanisms, with some that include the possibility of intermediate states before becoming infected [10]–[17]. Other mechanisms that have some connection with reinforcement include the q-voter model [18], where multiple same-opinion neighbors initiate change, the naming game, and the AB model [19]. Perhaps the mechanism that is most analogous to social reinforcement arises in the noise-reduced voter model [20], where a voter keeps a running total of inputs toward changing opinions, and changes opinions each time this counter reaches a predefined threshold. Here we make use of these types of intermediate states in the context of eventually adopting an innovation or a fad.

In our models, individual awareness is assumed to have a finite number of possible states. We quantify this awareness by a variable that ranges over the $M + 1$ values, $0, 1, 2, \ldots, M$. We define an individual with awareness 0 as being susceptible, while an individual moves closer to adopting the innovation as his/her awareness value increases. Adoption of the innovation occurs when an individual reaches the highest awareness value M. The population evolves by repeated interactions between two random individuals. In each interaction with an adopter, someone with awareness $k < M$ advances to awareness $k + 1$, while there are no state changes when two non-adopters interact. For simplicity, and as will be justified in section 2, we restrict opinion-changing interactions to those

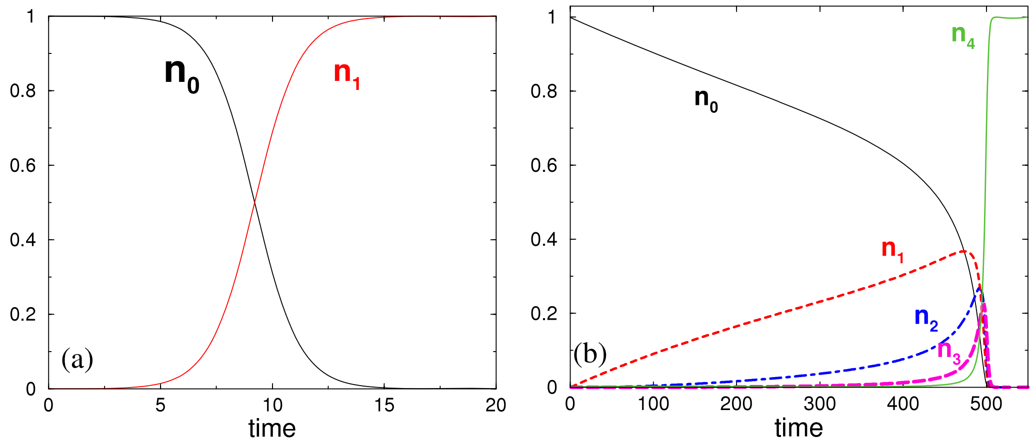

Figure 1. Time dependence of the densities n_k by numerical integration of the rate equations for a population of size $N = 10^4$, in which $n_M(0) = \rho$ and $n_0(0) = 1-\rho$, with $\rho = 1/N$. Shown are the cases (a) $M = 1$ and (b) $M = 4$.

between an adopter and a non-adopter. In our *innovation model*, an innovation is adopted permanently; in our *fad model*, an adopter abandons the fad at a rate λ so that it becomes passé and is eventually forgotten by the entire population.

2. Permanent innovations

We begin with the simplest situation of no reinforcement [3]–[6], namely, a population with two classes of individuals: susceptible (state 0) and adopters (state 1). Whenever a susceptible individual and an adopter meet, the former is converted to an adopter via $0+1 \rightarrow 1+1$. The rate equations that give the evolution of a homogeneous and well-mixed population (the mean-field limit) are:

$$\dot{n}_0 = -n_0 n_1, \qquad \dot{n}_1 = n_0 n_1. \tag{1}$$

We generically assume that the evolution begins with a small fraction of adopters in an otherwise susceptible population: $n_1(0) = \rho \ll 1$, $n_0(0) = 1-\rho$. For this initial condition the solution to the rate equations is (figure 1)

$$n_0 = \frac{(1-\rho)\mathrm{e}^{-t}}{\rho + (1-\rho)\mathrm{e}^{-t}}, \qquad n_1 = \frac{\rho}{\rho + (1-\rho)\mathrm{e}^{-t}}. \tag{2}$$

We define the *emergence time* t_e of the innovation by the criterion that half the population has become adopters, $n_0(t_\mathrm{e}) = n_1(t_\mathrm{e}) = 1/2$. From equations (2), we have $t_\mathrm{e} \simeq \ln(1/\rho)$. Ultimately everyone is an adopter; we estimate the resulting *completion time* T from $n_1(T) = 1 - 1/N$, corresponding to all but one individual in a population of size N adopting the innovation. This criterion gives $T \simeq \ln(N/\rho)$.[3]

We now implement social reinforcement by requiring individuals to move to progressively higher awareness states before adoption ultimately occurs. Possible examples

[3] If one uses the criterion that the innovation is complete when $n_1 = 1$, one obtains the unphysical result that the completion time is infinite.

of such progressions include: not owning a TV, being aware of TVs, but not owning one, and finally owning a TV [21], or having no cell phone, being aware of cell phones, to finally buying a cell phone, etc. We first treat the simplest example of reinforcement, which is the case of $M = 2$. For this case, there are three classes of individuals: susceptible (state 0), aware (state 1), and adopter (state 2), with respective densities n_0, n_1, and n_2. In an interaction with an adopter, a susceptible agent becomes aware ($2+0 \to 2+1$), while an aware agent adopts the innovation ($2+1 \to 2+2$). It is also reasonable to include interactions between susceptible and aware individuals. In such an interaction, the susceptible individual could be made aware ($0+1 \to 1+1$) or, conversely, the aware individual could be persuaded to no longer think about an innovation ($0+1 \to 0+0$). As long as such interactions between intermediate states are symmetric, they play no role in the dynamics of adoption in the mean-field limit and will be ignored.

When the rates of all processes are the same, the rate equations are:

$$\dot{n}_0 = -n_0 n_2, \qquad \dot{n}_1 = n_0 n_2 - n_1 n_2, \qquad \dot{n}_2 = n_1 n_2. \tag{3}$$

To solve these equations we introduce the internal time $\tau = \int_0^t \mathrm{d}t'\, n_2(t')$ that simplifies equations (3) to a linear system. A generic initial condition is $n_2(0) = \rho$, $n_1(0) = 0$, $n_0(0) = 1 - \rho$, corresponding to an initial population that contains a small group of adopters (perhaps the inventors), while everyone else is susceptible and has no knowledge of the fad. For these initial conditions, the solution is

$$\begin{aligned} n_0 &= (1-\rho)\,\mathrm{e}^{-\tau}, \\ n_1 &= (1-\rho)\,\tau\,\mathrm{e}^{-\tau}, \\ n_2 &= 1-(1-\rho)(1+\tau)\,\mathrm{e}^{-\tau}. \end{aligned} \tag{4}$$

To characterize the point where the innovation first achieves widespread public awareness, we define the emergence time of the innovation as the point where n_1 passes through a maximum (figure 1(b)). This yields $\tau_{\mathrm{e}} = 1$, from which the corresponding physical emergence time t_{e} is given by

$$t_{\mathrm{e}} = \int_0^1 \frac{\mathrm{d}x}{n_2(x)} = \int_0^1 \frac{\mathrm{d}x}{1-(1-\rho)(1+x)\mathrm{e}^{-x}}. \tag{5}$$

When $\rho \ll 1$, the asymptotic behavior of the integral is

$$t_{\mathrm{e}} \simeq \frac{1}{\sqrt{\rho}} \int_0^{1/\sqrt{\rho}} \frac{\mathrm{d}y}{1+y^2/2} \simeq \frac{\pi}{\sqrt{2\rho}},$$

where $y = x/\sqrt{\rho}$, and sub-leading terms are of order one. For a single innovator in a population of size N (corresponding to initial density $\rho = 1/N$), the N dependence of the emergence time is

$$t_{\mathrm{e}} = \pi\sqrt{N/2} + \mathcal{O}(1). \tag{6}$$

Thus reinforcement changes the emergence time from a logarithmic to a power law N dependence (figure 1). We now estimate the completion time, where the innovation has spread to essentially the entire population, by the criterion $n_2(T) = 1 - 1/N$. This gives the completion time $T = \pi\sqrt{N/2} + \ln N$ to lowest order[4]. Thus, in the presence of social

[4] Details will be given in [22].

reinforcement, once the innovation emerges, it takes little additional time before it is complete.

For an arbitrary number of intermediate states, an individual with awareness k increases to $k+1$ by interacting with a adopter, $[M]+[k] \to [M]+[k+1]$, with $k=0,1,\ldots,M-1$. As discussed above, all other symmetric interactions do not change the average densities of the various states and will be ignored. The corresponding rate equations are

$$\begin{aligned} \dot{n}_0 &= -n_M n_0, \\ \dot{n}_k &= n_M(n_{k-1}-n_k), \qquad k=1,\ldots,M-1, \\ \dot{n}_M &= n_M n_{M-1}. \end{aligned} \tag{7}$$

By again introducing the internal time $\tau = \int_0^t \mathrm{d}t'\, n_M(t')$, we reduce equations (7) to a linear system whose solution is

$$\begin{aligned} n_j &= (1-\rho)\frac{\tau^j}{j!}\,\mathrm{e}^{-\tau}, \qquad j=0,,\ldots,M-1, \\ n_M &= 1-(1-\rho)\sum_{j=0}^{M-1}\frac{\tau^j}{j!}\,\mathrm{e}^{-\tau}. \end{aligned} \tag{8}$$

In analogy with the case of $M=2$, the innovation emerges at $\tau=1$, where n_1 passes through a maximum (generally, each n_j passes through a maximum at $\tau=j$). To obtain explicit time dependences, we recast τ in terms of the physical time via $t=\int_0^\tau \mathrm{d}x/n_M(x)$. Applying the same steps as above and setting $\rho=1/N$, we find the emergence time

$$t_{\mathrm{e}} = \frac{\pi\ (M!)^{1/M}}{M\sin(\pi/M)} N^{1-1/M} \tag{9}$$

in the extreme case of a single initial adopter. Thus increasing the number of intermediate states M progressively delays innovation emergence, as the exponent $1-1/M$ approaches 1 as M becomes large (figure 1(c)).

3. Transient fads

Transient fads arise when adopters can independently abandon the innovation at rate $\lambda>0$. In this case, the innovation can spread to some degree before it is abandoned and fades away. The extent of the fad, as well as its lifetime, are controlled by the abandonment rate. At the end of the process, a discrete population consists of adopters who abandoned the fad and individuals who are forever stuck in intermediate awareness states because of the absence of catalyzing adopters. Of particular interest are the *clueless* individuals, those who were never exposed to the fad while it was active. Their fraction, defined as $c_\infty(\lambda) \equiv n_0(t=\infty)$, characterizes the competing influences of contagion and fad abandonment[5]. This fraction also gives a measure of the penetration of a particular fad into the marketplace. A large clueless fraction indicates that most people were never exposed to the fad (or equivalently, the product) so there was no chance to convert these people to adopters, while a small clueless fraction indicates that the product had

[5] The clueless are analogous to the population susceptible fraction in rumor-spreading models, see [23].

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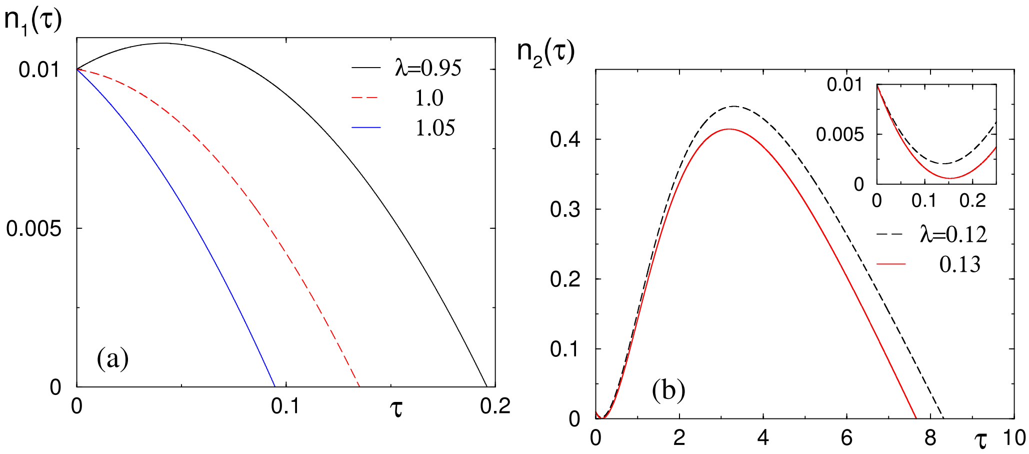

Figure 2. Dependence of $n_M(\tau)$ versus τ for: (a) no reinforcement ($M=1$) and (b) reinforcement, with one intermediate state ($M=2$), equations (10) and (11) respectively. The inset in (b) shows the near tangency of $n_2(\tau)$ versus τ for $\lambda = 0.13 \approx \lambda_c$.

enough exposure to attract a considerable number of adopters. In the management science literature, there is interest in an inverse quantity, the *market penetration*, which is the proportion of a population that a product is ultimately able to reach [24, 25].

For an infinite population, c_∞ undergoes a continuous transition as a function of λ for $M=1$, but a *discontinuous* transition for $M \geq 2$. Moreover, the time to reach the final state varies non-monotonically with λ. This discontinuous transition for $M \geq 2$ implies that there is a region of metastability near the critical value $\lambda = \lambda_c$ where the net rate of fad abandonment and adoption are approximately equal. In this critical region, whether a particular realization of a fad spreads globally or dies out quickly becomes a random process, as we discuss in more detail below. In contrast, without reinforcement, the continuous transition implies that the outcome for any realization is deterministic.

The case of no reinforcement coincides with the classic SIR epidemic model [1] with the identifications: adopter $\leftrightarrow$ infected, abandoner $\leftrightarrow$ recovered, while the meaning of susceptible is the same in both models. The rate equations are $\dot{n}_0 = -n_0 n_1$, $\dot{n}_1 = n_0 n_1 - \lambda n_1$, with solution

$$n_0 = (1-\rho)\,\mathrm{e}^{-\tau}, \qquad n_1 = 1 - \lambda\tau - (1-\rho)\,\mathrm{e}^{-\tau}, \tag{10}$$

where $\tau = \int_0^t \mathrm{d}t'\, n_1(t')$. The evolution ceases at an internal stopping time τ_∞ defined by $n_1(\tau_\infty) = 0$; this corresponds to physical time $t = \infty$. The condition $n_1(\tau_\infty) = 0$ leads to three regimes of behavior for the clueless fraction c_∞ (figure 2(a)). For $\lambda < 1$ (subcritical), adopters abandon the fad sufficiently slowly that the fad can spread globally before dying out. In the supercritical regime of $\lambda > 1$, adopters abandon the fad so quickly that there is little time for the innovation to spread before it is extinguished. In this limit, equations (10) give $\tau_\infty = \rho/(\lambda-1)$ and $c_\infty = 1 - \rho/(\lambda - 1)$ to leading order, while for $\lambda = \lambda_c = 1$, $c_\infty = 1 - \sqrt{2\rho}$. Thus c_∞ undergoes a continuous transition (in the $\rho N \gg 1$ limit) as λ passes through the critical value $\lambda_c = 1$ (figure 3(a)).

Let us now investigate the role of reinforcement on the fad dynamics. We consider the simplest situation of a single intermediate state ($M=2$) or, equivalently, three possible

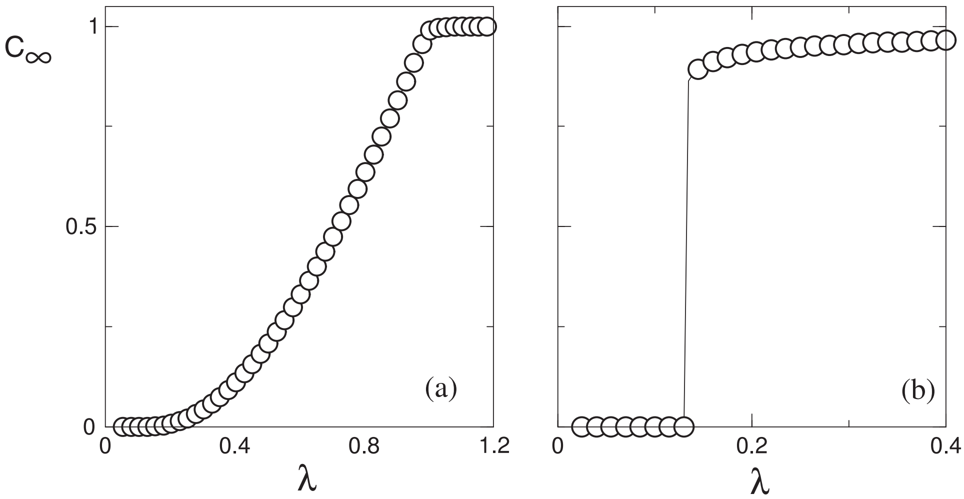

Figure 3. Clueless fraction c_∞ versus abandonment rate λ for: (a) two-state and (b) three-state models. The initial adopter fraction is $n_1(0) = 10^{-4}$ in (a) and $n_2(0) = 10^{-2}$ in (b).

states for each individual. In this case, the evolutions of n_0 and n_1 are again given by equations (4), while the solution of n_2 is

$$n_2 = 1 - (1-\rho)(1+\tau)\mathrm{e}^{-\tau} - \lambda\tau. \tag{11}$$

A curious feature of this result for n_2 is that the density of fad adopters $n_2(\tau)$ can first decrease, then increase, before ultimately vanishing (figure 2(b)). This unusual behavior stems from the delicate interplay between fad abandonment and the creation of new adopters from the remaining reservoir of susceptible individuals. As a result of the two extrema in n_2 as a function of τ, the stopping condition $n_2(\tau_\infty) = 0$ can have one, two, or three roots, depending on λ. This change in the number of roots underlies the discontinuity in the clueless fraction c_∞ as a function of λ.

To locate this transition in the supercritical case, $\lambda > \lambda_c$, notice that (11) has three roots as a function of τ. We are interested in the smallest root and therefore expand the left-hand side of equation (11) for small τ. Keeping the leading terms gives

$$n_2(\tau_\infty) \approx \rho + \tfrac{1}{2}\tau_\infty^2 - \lambda\tau_\infty = 0. \tag{12}$$

From this quadratic equation, we see that the interesting behavior arises when $\lambda = \mu\sqrt{\rho}$ where $\mu = \mathcal{O}(1)$. With this convenient parameterization, the solution for τ_∞ is $\tau_\infty = \sqrt{\rho}\,[\mu \pm \sqrt{\mu^2 - 2}]$. Using the physically relevant smaller solution, we find, for $\mu > \mu_c$ (which equals $\sqrt{2}$ to lowest order)

$$c_\infty = (1-\rho)\,\mathrm{e}^{-\tau_\infty} \simeq 1 - \sqrt{\rho}\,(\mu - \sqrt{\mu^2 - 2}); \tag{13}$$

i.e. the clueless fraction is close to one (figure 3(b)). In the subcritical case, $\mu < \sqrt{2}$, the relevant root of $n_2(\tau_\infty) = 0$ is $\tau_\infty = 1/(\mu\sqrt{\rho})$ to leading order. Consequently, the clueless fraction is

$$c_\infty = \mathrm{e}^{-\tau_\infty} = \mathrm{e}^{-1/(\mu\sqrt{\rho})}, \tag{14}$$

which is close to zero for $\rho \to 0$. Thus the clueless fraction undergoes a first-order transition as λ passes through λ_c.

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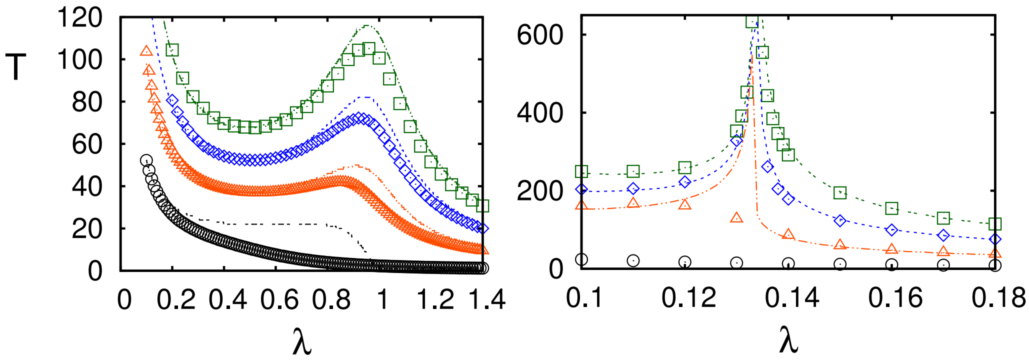

Figure 4. Completion time T versus λ for the fad model, for $M = 1$ (left) and $M = 2$ (right) with $\rho = 10^{-2}$. Points are simulation results for $N = 10^2, 10^4, 10^6, 10^8$ (bottom to top). Dashed lines are the corresponding results from numerically integrating the rate equations. For $M = 1$, the width of the peak at $\lambda_c = 1$ scales as $\sqrt{\rho}$.

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4. Fad completion time

A striking aspect of our fad model is that the time for a fad to die out has a non-monotonic dependence on the abandonment rate λ (figure 4). This non-monotonicity has a simple qualitative origin. If the abandonment rate is large, then the initial adopters abandon before they can recruit new adopters. Thus the fad quickly disappears. Conversely, if the abandonment rate is small, essentially the entire population adopts the innovation *en masse*, after which the fad disappears in a time that scales as $1/\lambda$. Between these two limits, the fad slowly 'smolders' because new adopters are replenished at nearly the same rate as other adopters abandon the fad. In this situation, the fad can be extremely long lived.

In a population of size N, we determine the time for a fad to end by the criterion $n_M(\tau^*) = 1/N$. Namely, only a single adopter remains in a finite population. (Notice that this is distinct from the criterion $\tau = \tau_\infty$, where the number of adopters vanishes in an infinite population.) This internal time corresponds to the value of the physical stopping time $T = \int_0^{\tau^*} \mathrm{d}\tau / n_M(\tau)$ at which the fad disappears. The actual determination of the completion time is very different for the cases $M = 1$ and $M > 1$, and we investigate these two cases in turn.

4.1. No reinforcement, $M = 1$

Generically, T is proportional to $\ln N$ because n_1 goes to zero with a finite slope as τ approaches τ_∞ (figure 2(a)). As a consequence, the integral for T is logarithmically divergent in N. However, the details of this dependence depend on the value of the abandonment rate λ.

In the subcritical regime ($\lambda < 1$), we determine T by expanding n_1 about τ_∞ and using the condition $\mathrm{e}^{-\tau_\infty} + \lambda\tau_\infty = 1$ to obtain

$$T = \frac{1}{\lambda + \lambda\tau_\infty - 1} \int_{1/N} \frac{\mathrm{d}y}{y} = \frac{\ln N}{\lambda + \lambda\tau_\infty - 1}. \tag{15}$$

The lower limit in equation (15) follows from the stopping criterion $n_1(\tau^*) = 1/N$, while the upper limit is immaterial for the asymptotic behavior.

In the supercritical regime ($\lambda > 1$), the density of adopters n_1 decreases almost linearly in τ over the entire range for which n_1 is positive. In this case[6], an expansion of n_1 about τ_∞ leads to $T = \ln(\rho N)/(\lambda - 1)$.

In the critical case of $\lambda = 1$, n_1 decreases quadratically with τ and the same expansion procedure as outlined above gives $T = \ln(\rho N)/\sqrt{2\rho}$ in the asymptotic limit. Consistent with the logarithmic dependence at the critical point for the case $\rho = 1/N$, we find the completion time distribution has a power law tail, $P(T) \sim T^{-2}$. From this distribution, the average completion time was found, in the context of epidemic dynamics, to be $T = \frac{1}{3} \log N$ [26], a behavior that we confirmed numerically.

4.2. Reinforcement, $M > 1$

When reinforcement is operative, the fad evolution in the supercritical regime closely mirrors the behavior of the non-reinforced model ($M = 1$). In particular, the time dependence of $n_M(\tau)$ is similar to $n_1(\tau)$ in the case of no reinforcement: n_M approaches zero with finite slope, from which the ending time of the fad again scales as $\ln N$. However, in contrast to the case of $M = 1$, the value of τ where $n_M(\tau)$ first reaches zero changes discontinuously as λ passes through λ_c (figure 2(b)). More interestingly, when $\lambda \approx \lambda_c$, n_M approaches zero with a quadratic minimum, as illustrated in the inset to figure 2(b). This property is the mechanism that gives an algebraic, rather than a logarithmic, dependence of the completion time on N. Finally for $\lambda < \lambda_c$, n_M again reaches zero with a finite slope, leading to a logarithmic dependence of the ending time on N. Thus the time for the fad to disappear has a local maximum at the critical point (figure 4). Monte Carlo simulations of the fad dynamics in a finite population mirror our analytic predictions, except near the first-order transition, where large fluctuations arise.

Let us now focus on the properties of the completion time at the first-order transition point where fluctuations are particularly strong. There are two independent and natural scenarios for which to define the lifetime of the fad: (i) a fixed *number* of initial adopters (generally we treat the case of one adopter) or (ii) a fixed *fraction* ρ of initial adopters. To find the fad lifetime in the former case of $\rho = 1/N$, it is again convenient to use parameterization $\lambda = \mu\sqrt{\rho}$ because the critical value of μ is $\mathcal{O}(1)$. We therefore substitute the lowest-order approximation for the critical value $\mu_c = \sqrt{2}$ into the expansion (12) for n_2 to obtain $n_2 = \frac{1}{2}(\sqrt{2\rho} - \tau)^2$. The ending time for the fad is now given by

$$T = 2 \int_0^{\tau^*} \frac{d\tau}{(\sqrt{2\rho} - \tau)^2}, \tag{16}$$

with $\tau*$ determined from the criterion $n_2(\tau^*) = 1/N$. The latter gives $\tau^* = \sqrt{2\rho} - \sqrt{2/N}$ and using this upper limit in (16) gives $T = \sqrt{2N}$. However, the prefactor $\sqrt{2}$ arises from using the imprecise criterion $n_2(\tau^*) = 1/N$ to define the ending time. Simulations instead give $T \sim 0.56\sqrt{N}$. For a fixed fraction of initial adopters ρ, our simulations show that the average fad lifetime grows with N roughly as $N^{1/4}$ for $M = 2$, a result for which we do not yet have an explanation.

[6] Here we assume $\rho \ll 1$ and $\rho N \gg 1$. In the extreme case of $\rho = N^{-1}$, fluctuations are artificially large and the rate equation approach becomes invalid.

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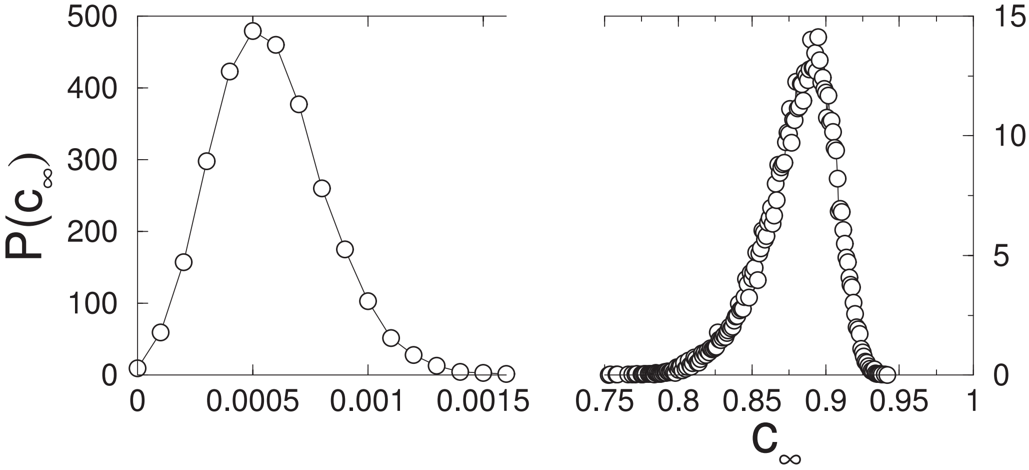

Figure 5. Probability density $P(c_\infty)$ for the fraction of clueless individuals at the end of the process at the critical point, $\lambda_c = \sqrt{2\rho}$, for the $M = 2$ fad model, with initial density $\rho = 10^{-2}$ and population size $N = 10^4$. (Note the horizontal scale break.)

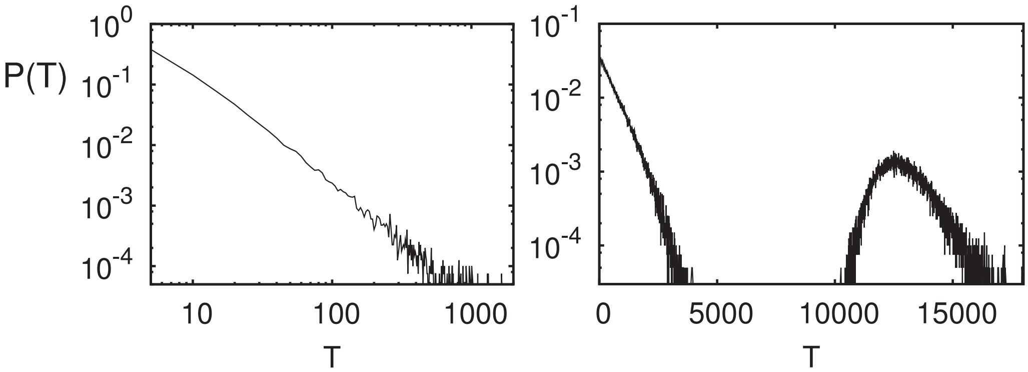

Figure 6. Probability distribution of completion time T for the fad model at the critical point, for $M = 1$ (left) and $M = 2$ (right) with $\rho = 1/N$. We use $N = 10^9$ for $M = 1$ and $N = 10^6$ for $M = 2$.

As a result of the large fluctuations near the transition, the completion time distribution consists of two distinct components. One component corresponds to realizations where the fad quickly dies out so that the population at infinite time consists almost entirely of clueless individuals (figure 5). In contrast, for the remaining fraction of realizations, nearly everyone adopts and then abandons the fad. Corresponding to this dichotomy in the fate of individual realizations, the distribution of times at which the fad disappears has distinct short-lived and long-lived contributions (figure 6).

5. Summary

We have shown how social reinforcement plays an essential role in determining how permanent innovations and transient fads are adopted in a socially interacting population.

For permanent innovations, we modeled the effect of reinforcement by endowing each individual with $M+1$ levels of awareness $0, 1, 2, \ldots, M$. An individual increases his/her level of awareness by one unit as a result of interacting with an adopter, and adoption occurs when an individual reaches the highest awareness level M. In the mean-field limit, we found that the time for the innovation to be adopted universally scales as $N^{1-1/M}$, so that increasing M delays the onset of the innovation.

For transient fads, the fad quickly disappears for $\lambda > \lambda_c$, while for $\lambda < \lambda_c$ the fad is nearly universally adopted before finally being forgotten. Curiously, the fad lasts the longest for the intermediate case where $\lambda = \lambda_c$. At this point, new adopters are replenished at nearly the same rate as others abandon, so that the fad slowly smolders rather than igniting and quickly burning out. The transition in the fraction of clueless individuals—those who have no knowledge of the fad before it disappears—is second order as a function of λ when there is no reinforcement, but first order in λ with reinforcement. Near the first-order transition, the dynamics exhibits macroscopic sample-to-sample realizations in the evolution of a fad. As a consequence, the clueless fraction may be either close to zero or close to one in different realizations of the dynamics with the same initial condition.

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