Errata to: A Guide to First-Passage Processes

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Chapter 1:

1. Page 2. The second paragraph should read: "To make these questions precise and then answer them, note that the price performs a symmetric random walk in n, where $n = N_- - N_+$ is the difference in the number of "down" and "up" days, N_- and N_+, respectively.

   I thank Ronny Straube for pointing out this error.

2. Page 5. The second sentence of Sec. 1.2 should read: "An important starting fact is that the survival probability $S(t)$ in an absorbing domain is closely related to the time integral of the first-passage probability up to time t over the spatial extent of the boundary (see Eq. (1.5.7))".

3. Page 8. The integral in Eq. (1.3.5) is over the range $-\pi \le k \le \pi$. This is not a contour integral.

4. Page 9. Third line after Eq. (1.3.6) should not have the word "gives". Eq. (1.3.8) should read:

$$P(x,N) \to \frac{1}{\sqrt{2\pi Npq}}\, e^{-[x-N(p-q)]^2/8Npq}$$

   I thank David Waxman for pointing out the above three errors and Tibor Antal for pointing out a second error in (1.3.8).

5. Page 9. The second line of the un-numbered formula after Eq. (1.3.8) should read:

$$\sim e^{ik\langle x\rangle - \frac{1}{2}k^2(\langle x^2\rangle - \langle x\rangle^2)}, \quad k \to 0.$$

6. Page 9. Eq. (1.3.9) should read:

$$P(x,N) \to \frac{1}{\sqrt{2\pi N(\langle x^2\rangle - \langle x\rangle^2)}}\, e^{-(x-\langle x\rangle)^2/[2N(\langle x^2\rangle - \langle x\rangle^2)]}.$$

   I thank Paul Krapivsky for pointing out these two errors.

7. Page 10. In Eq. (1.3.11), the denominator inside the square brackets should be $2zq$ for $x > 0$ and $2zp$ for $x < 0$.

   I thank David Waxman for pointing out this error.

8. Page 11. In Eq. (1.3.13), the left-hand side should read $\frac{\partial P(n,t)}{\partial t}$.

9. Pages 12 & 13. Three lines after Eq. (1.3.19), the inequality should read $n \geq q0$. Two lines after Eq. (1.3.22), there should be a prefactor $\frac{1}{2\pi i}$ before the integral. In the next line, the variable transformation should be $u = \sqrt{-s}$.

   I thank Tibor Antal for pointing out these errors.

10. Page 22. In Eq. (1.5.6), the factor $(1 - R)$ should read $(1 - \mathcal{R})$.

11. Page 31. In Eq. (1.6.22), the argument on the left-hand side should be $\vec{r}$, not $\vec{r}'$.

Chapter 2:

1. Page 46. The statement of "complete parallelism" in the 4^{th} line is a bit misleading. While the splitting probabilities and the exit times for the discrete random walk and continuum diffusion agree, this correspondence does not extend to all higher moments.

   I thank Tibor Antal for clarifying this point.

2. Page 47. The factor in the second of Eqs. (2.2.16) should be $(1 - u_0^2)$ not $(1 - u_0)^2$.

3. Page 49. In the second-to-last line: $j_-(s; x_0)$ instead of $j(s; x_0)_-$.

   I thank Shai Carmi for this correction.

4. Page 52. The prefactor in the last line of (2.2.26) is upside down and the sign is wrong. This prefactor should read $(-1)^n \, \frac{(2n+1)\pi D}{L^2}$.

   I thank Maciej Dobrzynski and Gleb Oshanin for these corrections.

5. Pages 53 & 54. There are several errors. The last line of text on page 53 should read: "and the Laplace transform of the initial condition, $j(x = 0, s) = 1$, fixes the constant to be". Second, the line immediately after the formula at the bottom of page 53 should be followed by the statement: "where we have used the simplification $v - D\alpha_\pm = D\alpha_\mp$". The statement immediately after (2.2.29) should then be removed. Third, there should not be a D in the denominator of Eq. (2.2.29). Finally, the 5^{th} line of text should read: "where $Pe = \frac{vL}{2D}$ is again the Péclet number and $P_s \equiv \sqrt{v^2 + 4Ds} \, \frac{L}{2D}$".

   I thank Shai Carmi for some of these corrections.

6. In Eq. (2.2.29), the last term in the denominator should read $\alpha_- \, e^{-\alpha_+ L}$.

   I thank Eli Ben-Naim for this correction.

7. Page 55. The displayed equation and the line following should read:

$$\tanh \sqrt{Pe^2 + sL^2/D} = \sqrt{1 + sL^2/(D\, Pe^2)}$$

   which, in the limit $Pe \to \infty$, gives the criterion $s \approx -(4DPe^2/L^2)\, e^{-2|Pe|} \equiv 1/\tau$.

   I thank Shai Carmi for pointing out the erroneous formula.

8. Page 56. Near the bottom of the page, the formula should be $\langle t(x_0)^2 \rangle = 2 \int_0^L C_1(x) dx$, *i.e.*, a factor 2 is missing).

   I thank Shai Carmi for this correction.

9. Page 57. In the second line, the signs of the v and D terms are *not* opposite to the sense of the corresponding terms in the convection-diffusion equation. Also, 5 lines from the bottom, the length scale is D/v not D/v^2.

   I thank Shai Carmi for the latter correction.

10. Page 58, first equation, second line, the factor D equals $\delta x^2/2\delta t$, *i.e.*, a factor 2 is missing.

    I thank Shai Carmi for this correction.

11. Page 59. The solid curve in Fig. 2.5 should be labeled as $2.5t_{\max}/\tau$ and the last parenthetical phrase of the caption should read "(multiplied by 2.5 for visibility)". Finally, the sentence just before (2.3.13) should read: "The corresponding first-passage time is (with $z \equiv x_{\max}/L$)", while the factor Pe at the end of the first line of (2.3.13) should be smaller. I thank Shai Carmi for pointing out these errors.

12. Page 63. Just above (2.3.23), the reference is to (1.6.27) not (1.6.29).

13. Page 64. Two lines above (2.3.24), the boundary conditions should be $g_+(0) = g_+(2Pe) = 0$.

    I thank Shai Carmi for this correction.

14. Page 74. The second-to-last of Eqs. (2.4.10) should read $P_N = \frac{a}{1-z} P_{N-1}$; *i.e.*, the letter z should be lower case.

15. Page 75. While the final result (2.4.12) is correct, some of the text leading to the result is sloppy. Two lines about (2.4.11), the text should read "This form is valid for $n = 2, 3, 4, \ldots, N$, whereas the equations for P_0 and P_1 are distinct." Parenthetically, to reach the un-numbered displayed equation after (2.4.11) one can use tricks like $2a\lambda_\pm - 1 = \pm\sqrt{1-4a^2}$ as well as $\sqrt{1-4a^2} = a(\lambda_+ - \lambda_-)$.

    I thank Jakub Otwinowski for clarifying these points.

16. Page 75. In the line after (2.4.12), the word "completely" should be replaced by "asymptotically".

    I thank Tibor Antal for clarifying this point.

17. Page 76. The line after (2.4.14) should read: "Continuing this procedure to site $N-2$ gives" ($N-2$ instead of $N-1$), and in the next equation replace P_{N-1} on the left-hand side with P_{N-2} and P_N in the second term of the right-hand side with P_{N-1}. The next line of text should be: "Then the equation for P_{N-1} gives", and the following equations should read: $P_{N-1} = aP_{N-2} = \ldots + f_{N-1}P_{N-1}$ or $P_{N-1} = \ldots$".

    I thank Shai Carmi for providing these corrections.

Chapter 3:

1. Page 83. In Eq. (3.2.4) the derivative should be with respect to x not t.

   I thank Carl Gold for pointing out this error.

2. Page 84. In Eq. (3.2.6) there is no factor of t in the square root.

   I thank Robin Groenevelt for pointing out this error.

3. Page 86, Eq. (3.2.11). Oy! Is this formula messed up! It should read:

$$
\begin{aligned}
\langle x\rangle &= \frac{1}{S(t)}\frac{1}{\sqrt{4\pi Dt}}\int_0^\infty x\,\left[e^{-(x-x_0)^2/4Dt}-e^{-(x+x_0)^2/4Dt}\right]dx \\
&= \frac{1}{S(t)}\frac{1}{\sqrt{\pi}}\left[\int_{-\frac{x_0}{\sqrt{4Dt}}}^\infty (u\sqrt{4Dt}+x_0)\,e^{-u^2}du-\int_{\frac{x_0}{\sqrt{4Dt}}}^\infty (u\sqrt{4Dt}-x_0)\,e^{-u^2}du\right] \\
&= \frac{1}{\sqrt{\pi}\,S(t)}\left[\int_{-\frac{x_0}{\sqrt{4Dt}}}^{\frac{x_0}{\sqrt{4Dt}}} u\sqrt{4Dt}\,e^{-u^2}\,du+\left(\int_{\frac{-x_0}{\sqrt{4Dt}}}^{\frac{x_0}{\sqrt{4Dt}}}+\int_{\frac{x_0}{\sqrt{4Dt}}}^\infty\right)x_0\,e^{-u^2}\,du\right] \\
&\sim \frac{x_0}{S(t)}\sim\sqrt{\pi Dt} \qquad\qquad \text{as}\ \ t\to\infty.
\end{aligned}
$$

I thank Shai Carmi for pointing out this error.

4. Page 87. In Eq. (3.2.12) the prefactor in the second term should be $e^{-vx_0/D}$. Similarly, in the first and also the third line of text below this equation the factor should read $e^{-vx_0/D}$.

5. Page 88. On line 9, the flux should read $-(vc-Dc')$. In the line after Eq. (3.2.14), $u^2 = x^2/4Dt'$. I thank Shai Carmi for pointing out these errors.

6. Page 88. Replace x by x_0 in the line after Eq. (3.2.14) and in the first line of (3.2.15). Also, the fontsize for the first factor of Pe in the 3rd line of this formula should be larger.

7. Page 89. The second half of of Eq. (3.2.16) is incorrect. The correct result is

$$\left(\frac{x_0}{\sqrt{Dt}}\right)^3\frac{1}{\sqrt{4\pi}}\,\frac{1}{Pe^2}\,e^{-Pe^2\,Dt/x_0^2}=\sqrt{\frac{4}{\pi}}\,\frac{x_0\sqrt{Dt}}{(vt)^2}\,e^{-v^2t/4D}$$

I thank David Mukamel for pointing out the above two errors.

8. Page 93. The prefactor in Eq. (3.3.1) should be $\frac{1}{\sqrt{4Ds}}$.

9. Page 93. In the line below Eq. (3.3.1), $c_>$ and $c_<$ should be interchanged. I thank Shai Carmi for pointing out this error.

10. Page 94. In the first line of Eq. (3.3.3), the last factor should be $e^{-\alpha_+ x_0}$. I thank Shai Carmi for pointing out this error.

11. Page 99. In the last two lines of Eq. (3.4.4), the factor in the denominator should be $2(t-1)$ not $2t-1$.

12. Page 99. The last factor in Eq. (3.4.5) should be $\sqrt{\frac{2}{\pi t}}\,2^t$. I thank Shai Carmi for pointing out this error.

13. Page 102. The first line of Eq. (3.4.10) should read

$$\frac{P(x,z)}{P(0,z)}\left[1-\frac{1}{P(0,z)}\right]^{m-1}$$

I thank Shai Carmi for pointing out this error.

14. Page 103. The second argument of F in Eq. (3.4.12) should be j, not n. In the line above Eq. (3.3.14), the references should be to Eq. (1.2.3), not (1.2.1). In Eq. (3.4.14), the exponent should be m, not $m-1$. Finally, the equivalence between Eqs. (3.4.14) and (1.3.11) mentioned at the bottom of the page holds *only* in the limit $z\to 1$ (and also up to an overall factor of $\sqrt{2}$). I thank Shai Carmi for pointing out these errors.

15. Page 103. Eq. (3.4.15) should read

$$G^{(m)}(0,n) \sim \sqrt{\frac{2}{\pi n}}\, e^{-m^2/2n}$$

16. Page 105. Three lines above Eq. (3.4.17), the reference should be to Eq. (3.4.7). The second line of Eq. (3.4.17) should read

$$\frac{2}{\pi\sqrt{k(n-k)}}.$$

Also, the limits of integration in Eq. (3.4.19) should be from k to n. I thank Shai Carmi for pointing out these errors.

17. Page 108. Add the word "with" in the 9$^{\text{th}}$ line. It should read "... returned to site 1 with probability r ...". I thank Ronny Straube for pointing out this error.

18. Page 114. In the second to last line, the slope is $c_0/\sqrt{Dt}$ I thank Shai Carmi for pointing out this error.

Chapter 4:

1. Page 123. Sec. 4.3.2, third line: "Flyvbjerg", not "Flyvbjery".

2. Page 130. The last term in the equation at the top of the page should read $\frac{1}{2}(r_{\text{in}} + r_{\text{out}})\frac{\partial^2 P}{\partial n^2}$. I thank Paul Krapivsky for pointing out this error.

3. Page 142. Eqs. (4.5.5) and (4.5.6) are incorrect. The correct form of (4.5.5) is:

$$\begin{aligned}\langle t\rangle &= \frac{D}{v_1 v_2}\left\{(1-e^{-v_1 x_1/D})\left[1-e^{v_2(x_1-1)/D}\right]+\frac{v_2 x_1}{D}+\frac{v_1(1-x_1)}{D}\right\}\\ &\quad+\frac{D}{v_1^2 v_2^2}\left\{v_2^2\left[e^{-v_1 x_1/D}-1\right]+v_1^2\left[e^{v_2(x_1-1)/D}-1\right]\right\},\end{aligned}$$

while the correct form of (4.5.6) is:

$$\langle t\rangle \to \begin{cases}\dfrac{x_1}{v_1}+\dfrac{1-x_1}{v_2}, & v_1, v_2 \to +\infty,\\ \dfrac{D}{|v_1 v_2|}\, e^{|v_1|x_1+|v_2|(1-x_1)} & v_1, v_2 \to -\infty.\end{cases}$$

I thank Nan Shi for alerting me to the errors in these equations.

4. Page 151. The sign of the third term on the right-hand side of Eq. (4.6.1) should be plus, not minus.
I thank Alex Petersen for pointing out this error.

5. Page 151/152. The prefactor on the right side of Eq. (4.6.5) should be 1 not 4, and the prefactor on the right side of Eq. (4.6.6) should be $\frac{1}{2}$ not 2.
I thank Alex Petersen for pointing out these errors.

6. Page 152. Remove the word "a" four lines after Eq. (4.6.6).

7. Page 166. The double subscript y_{y1} should simply be y_1.

   I thank Robin Groenevelt for pointing out this error.

Chapter 5:

1. Page 202. Seventh line in section 5.5.3.1, a factor should read $1/(3+2\epsilon)$, not $1(3+2\epsilon)$.

Chapter 6:

1. Page 209. The result in the second of Eq. (6.2.2) actually gives $\mathcal{E}_-(r)$. The correct result is:

$$\mathcal{E}_+(r) = \frac{\ln(r/R_-)}{\ln(R_+/R_-)}.$$

   I thank Ronny Straube for pointing out this error.

2. Page 211. In Eq. (6.2.5b), the last line is for $d > 2$; also the last factor should be R_-, not R_+.

   I thank Keith Cheveralls for pointing out the first error and Mika Pruikkonen for noticing the second.

3. Page 215. In the line after Eq. (6.3.3b), the factor should read $(a/r_0)^{d-2}$.

4. Page 219. The formulae for the derivatives of the Bessel functions before Eq. (6.4.3) are wrong. They should read: $I_\nu^{'} = -\frac{\nu}{x} I_\nu + I_{\nu-1}$ and $K_\nu^{'} = -\frac{\nu}{x} K_\nu - K_{\nu-1}$.

5. Page 220. Eq. (6.4.5) has some small errors. The first line should contain a minus sign after the equivalence. In the third line, the leading $\pm$ should be replaced by $\mp$ and the exponent ν should be replaced by $-\nu$.

   I thank Enrique Abad for pointing out these errors.

6. Pages 226–227. In the first formula in subsection 6.5.2.2 and in the first un-numbered formula after Eq. (6.5.6), the prefactor should be $\sqrt{\frac{D}{s}}$ rather than $\frac{D}{s}$.

   I thank Ronny Straube for pointing out these two errors.

Chapter 7:

1. Page 236. In the first sentence of the third paragraph, the phrase "We thus define $c(r, \theta, t = 0)\ldots$" is missing an equal sign.

2. Page 240. The exponent in Eq. (7.3.2) should be ν, not ν_0.

3. Page 243. In the first line of Eq. (7.4.3), the fraction $\frac{1}{2\pi D}$ should be replaced by D.

   I thank Robin Groenevelt for pointing out these two errors.

Chapter 8:

1. Page 261. In Eq. (8.2.10) the factor $\ln w$ in the exponential should read $\ln q$ and not $\ln w$.

2. Page 266. The expression for Θ_{end} should read:

$$\Theta_{\text{end}} = \pi - \cos^{-1} \frac{D_3}{\sqrt{(D_1 + D_3)(D_2 + D_3)}},$$

   while the final expression for β_{end} should read

$$\left[2 - \frac{2}{\pi} \cos^{-1} \frac{D_3}{\sqrt{(D_1 + D_3)(D_2 + D_3)}} \right]^{-1}.$$

   I thank Alan Bray for pointing out this error.

3. Page 269. In figure 8.6(a) the labels $1 = 3$ and $2 = 3$ should be transposed.

4. Page 284. In Eq. (8.4.24), the expression for $c_k(t)$ should have a $t^{-3/2}$ time dependence in the prefactor, not $t^{-1/2}$.

   I thank Pu Chen for pointing out this error.