MIDTERM EXAM PY542 November 13, 2008

No notes or books are allowed or needed. Please be sure to **explain your work clearly** to maximize the probability of receiving the appropriate partial credit.

1. A narrow pipe of radius 1 cm and length 1 m connects two identical and large containers of ideal gas at room temperature – one at pressure $P_1 = 1$ atm and the other at pressure $P_2 = 1.1$ atm. Due to the density difference in the containers, gas flows in the pipe. This flow can be treated as constant because the containers are large.
   (a) Using elementary kinetic theory, determine the coefficient of self-diffusion. Give a numerical estimate of this quantity for a typical gas at room temperature.
   (b) Numerically estimate the gas flow in the pipe (*i.e.*, number of molecules per unit time). If each container has a volume of 1 m^3, estimate how long would it take for the densities to become approximately equal.
   (c) Suppose that the temperature is increased by a factor of 2. By what factor would the gas flux change?

2. Consider a particle whose motion is described by the classical Langevin equation in which there is Gaussian white noise. That is,

$$\dot{v}(t) = -\gamma v(t) + \eta(t),$$

   with $\langle \eta(t) \rangle = 0$, $\langle \eta(t)\eta(t') \rangle = \Gamma\delta(t - t')$, and all higher cumulants of the noise vanish.
   (a) Write the exact expression for $v(t)$.
   (b) Compute the exact time-dependent behavior of $\langle v(t)^4 \rangle$. (*Hint:* For a Gaussian process, since the fourth cumulant vanishes, there is a simple relation between the fourth and second moments.) For partial credit, determine the dependence of $\langle v(t)^4 \rangle$ on γ and Γ for $t \to \infty$.

3. Consider a diffusing particle on the infinite half line $x > 0$ that also experiences a constant bias to the right, with bias velocity v. If the particle happens to reach the origin, it is absorbed. The particle is initially at x_0.
   (a) Write the equation of motion for this particle, as well as the initial condition and the boundary conditions.
   (b) Using any method you like compute the probability that this particle eventually reaches the origin, $x = 0$.

   For partial credit, work this part of the problem for the case of zero bias.