Fate of the Kinetic Ising Model

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Basic question:

How does magnetic order emerge in a kinetic Ising model when an initially disordered state is cooled to very low temperature?

The System

Ising Hamiltonian

$$\mathcal{H} = -\sum_{\langle i,j \rangle} \sigma_i \sigma_j \qquad \sigma_i = \pm 1$$

Initial state: disordered, zero magnetization

Final state: ???

Dynamics of the System

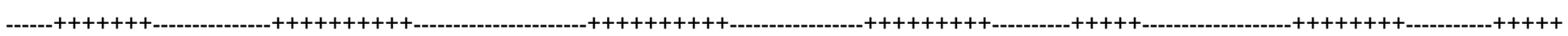
Pick a random spin and compare the outcome after reversing the spin

$$\text{if } \Delta E < 0 \quad \text{flip spin}$$

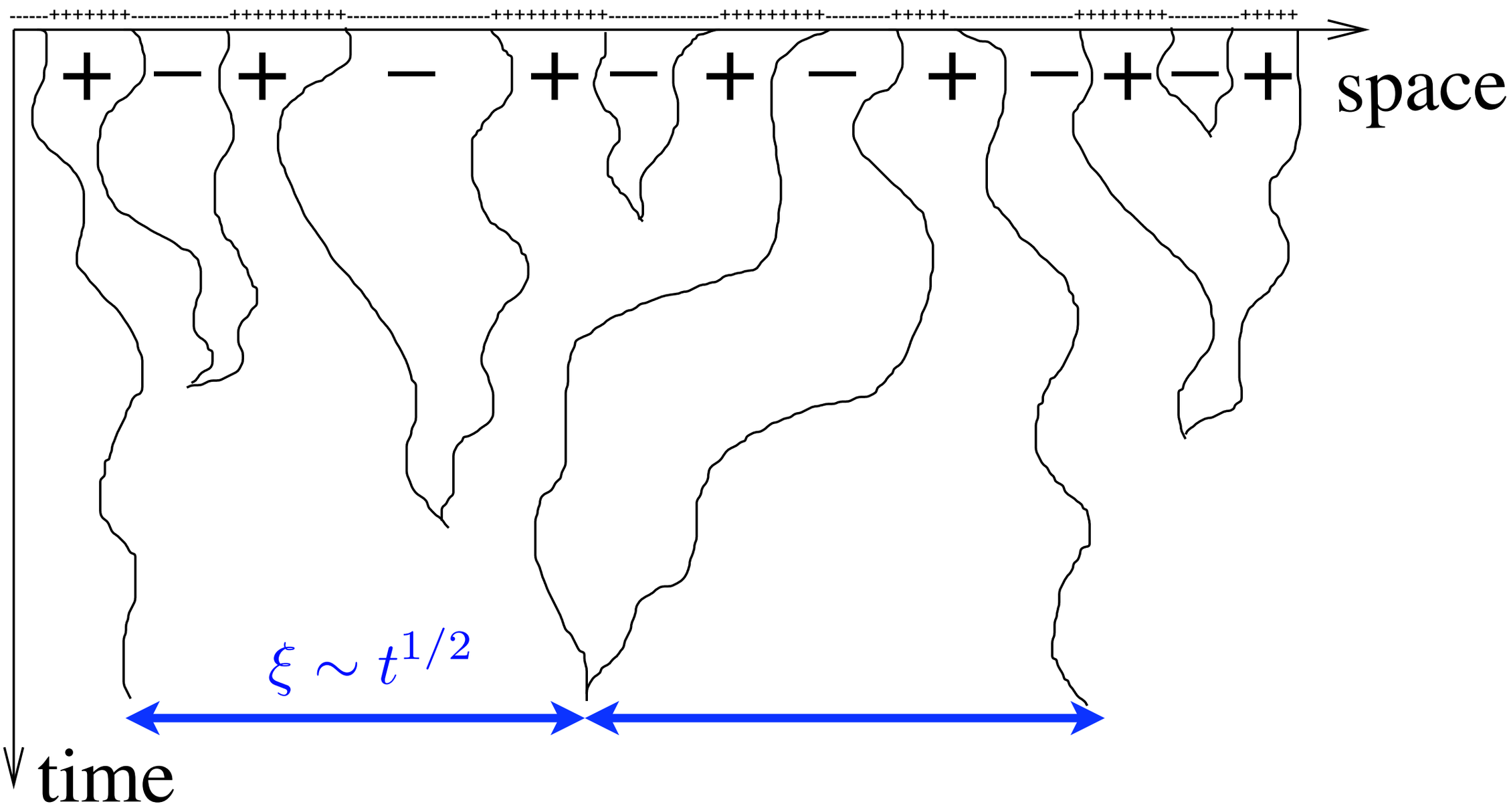
$$\text{if } \Delta E > 0 \quad \text{don't flip}$$

$$\text{if } \Delta E = 0 \quad \text{flip with prob. } 1/2$$

Domain Wall Picture in 1d



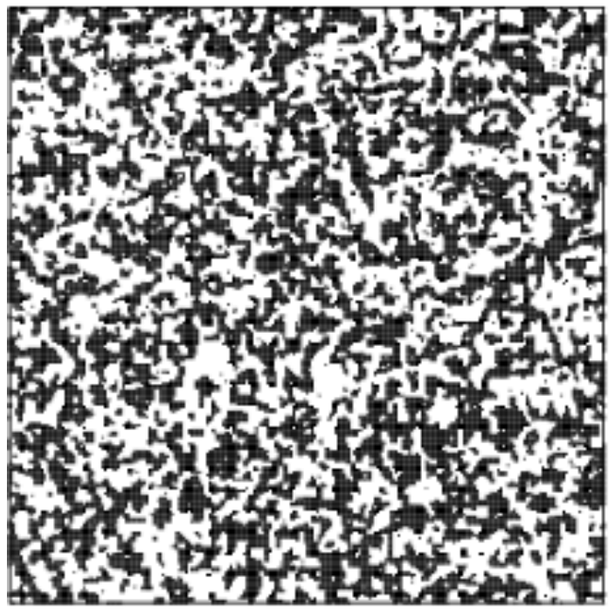
Domain Wall Picture in 1d



- no. interfaces: $\propto t^{-1/2}$
- time to ground state: $T \propto L^2$

Two Dimensions

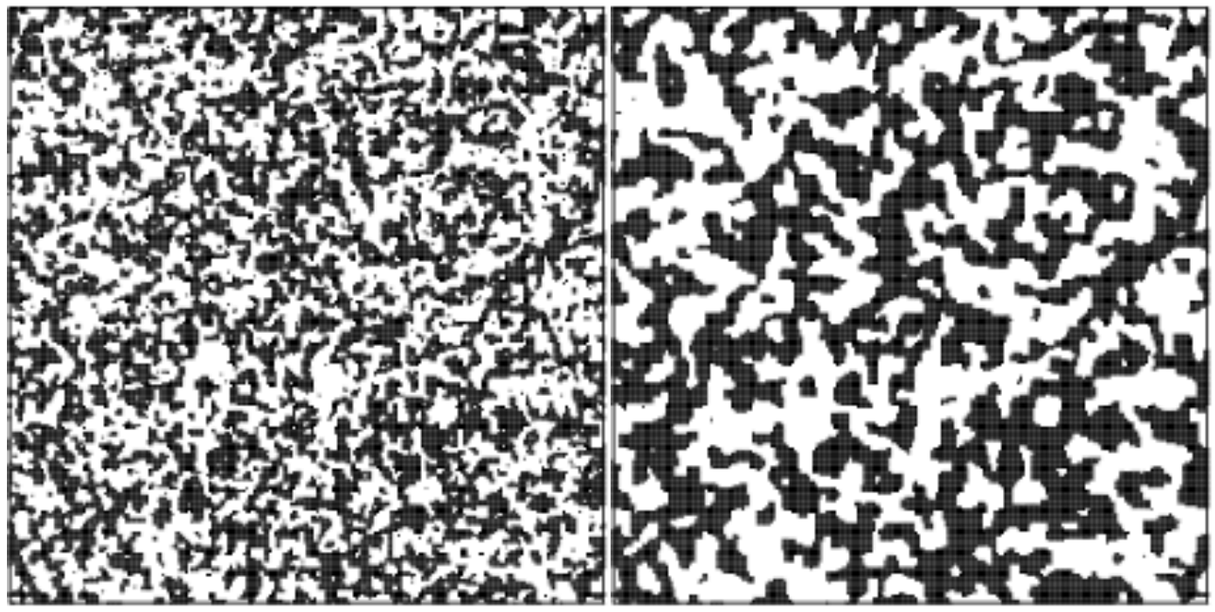
coarsening of 256x256 system



t=4

Two Dimensions

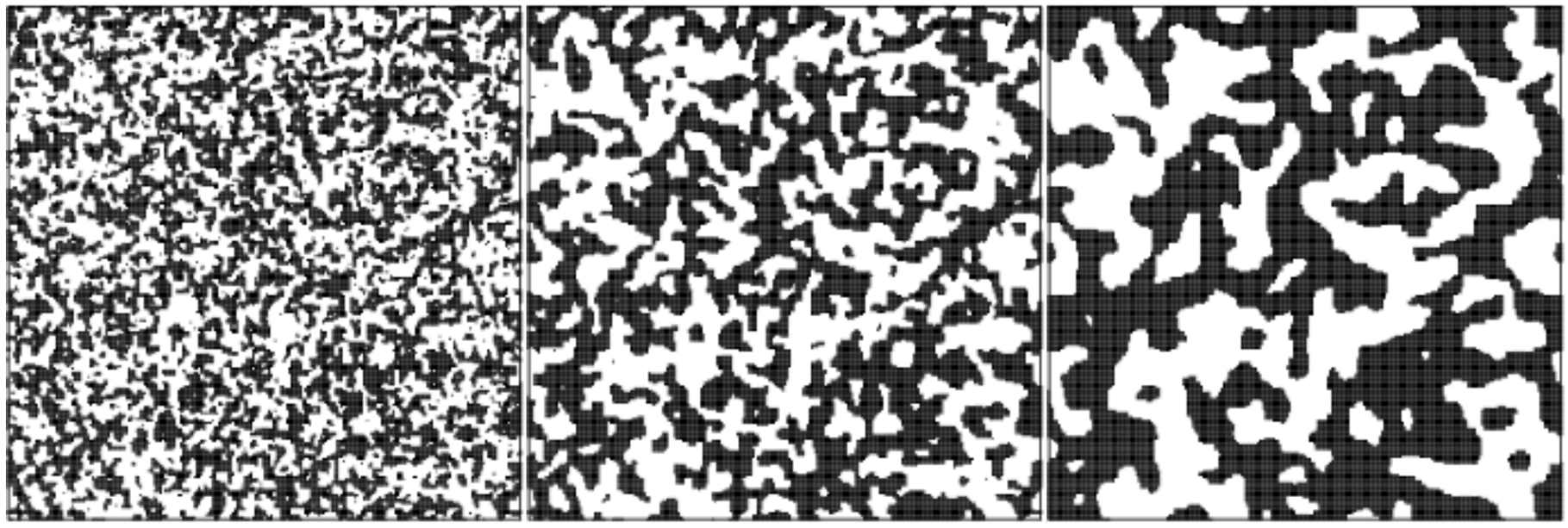
coarsening of 256x256 system



t=4 t=16

Two Dimensions

coarsening of 256x256 system



t=4 t=16 t=64

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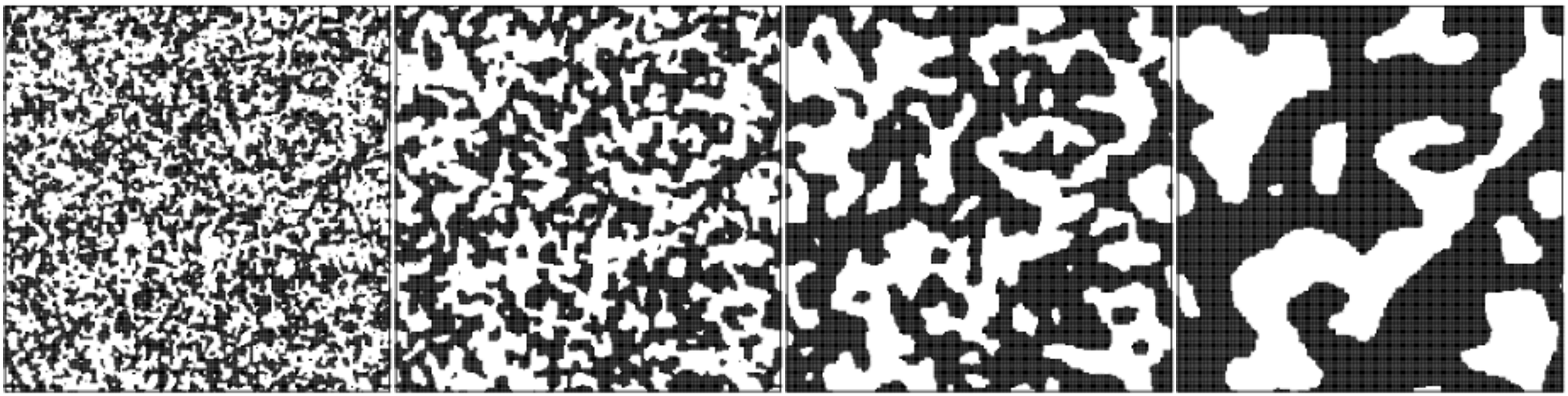
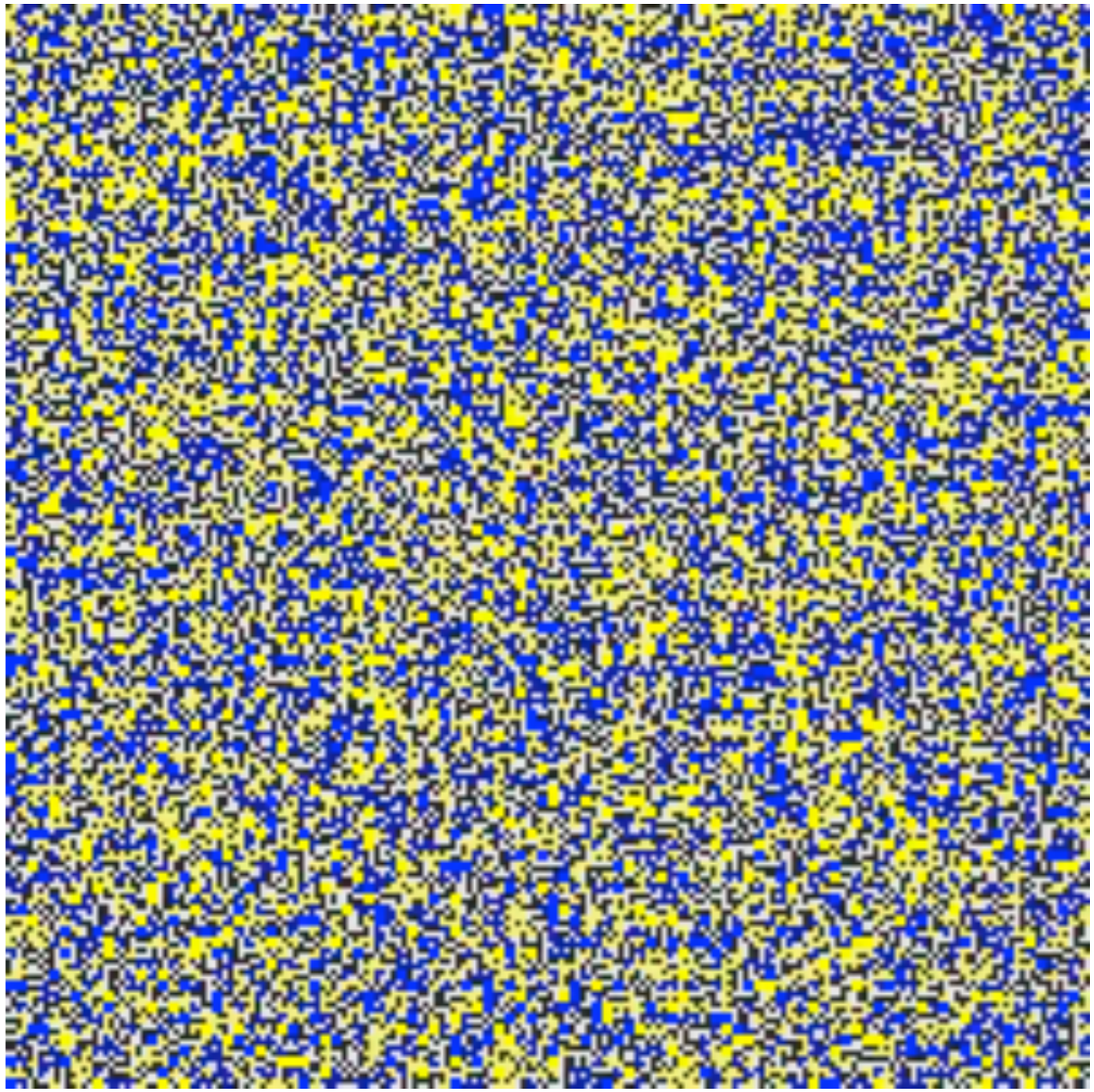
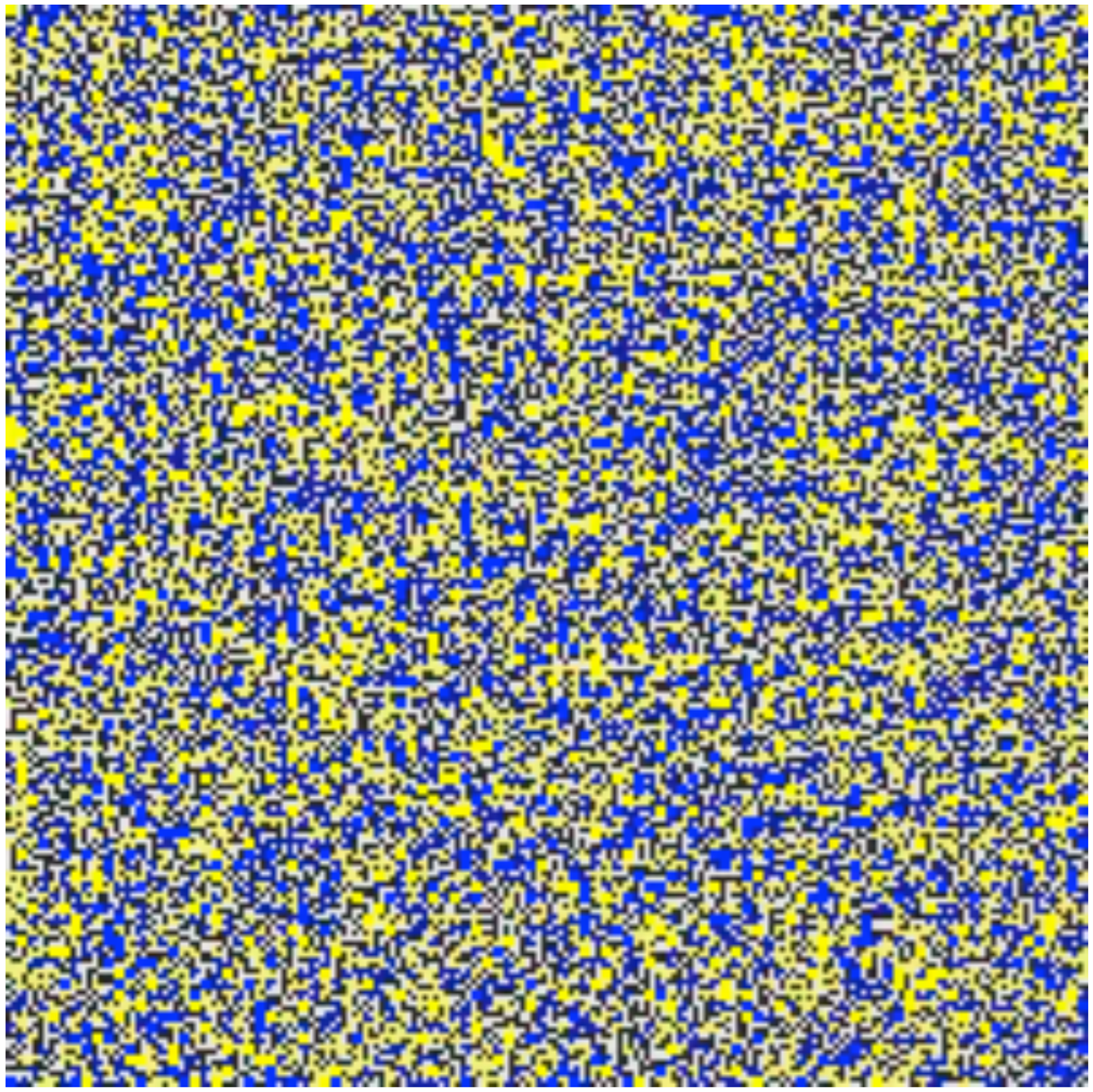


t=4 t=16 t=64 t=256

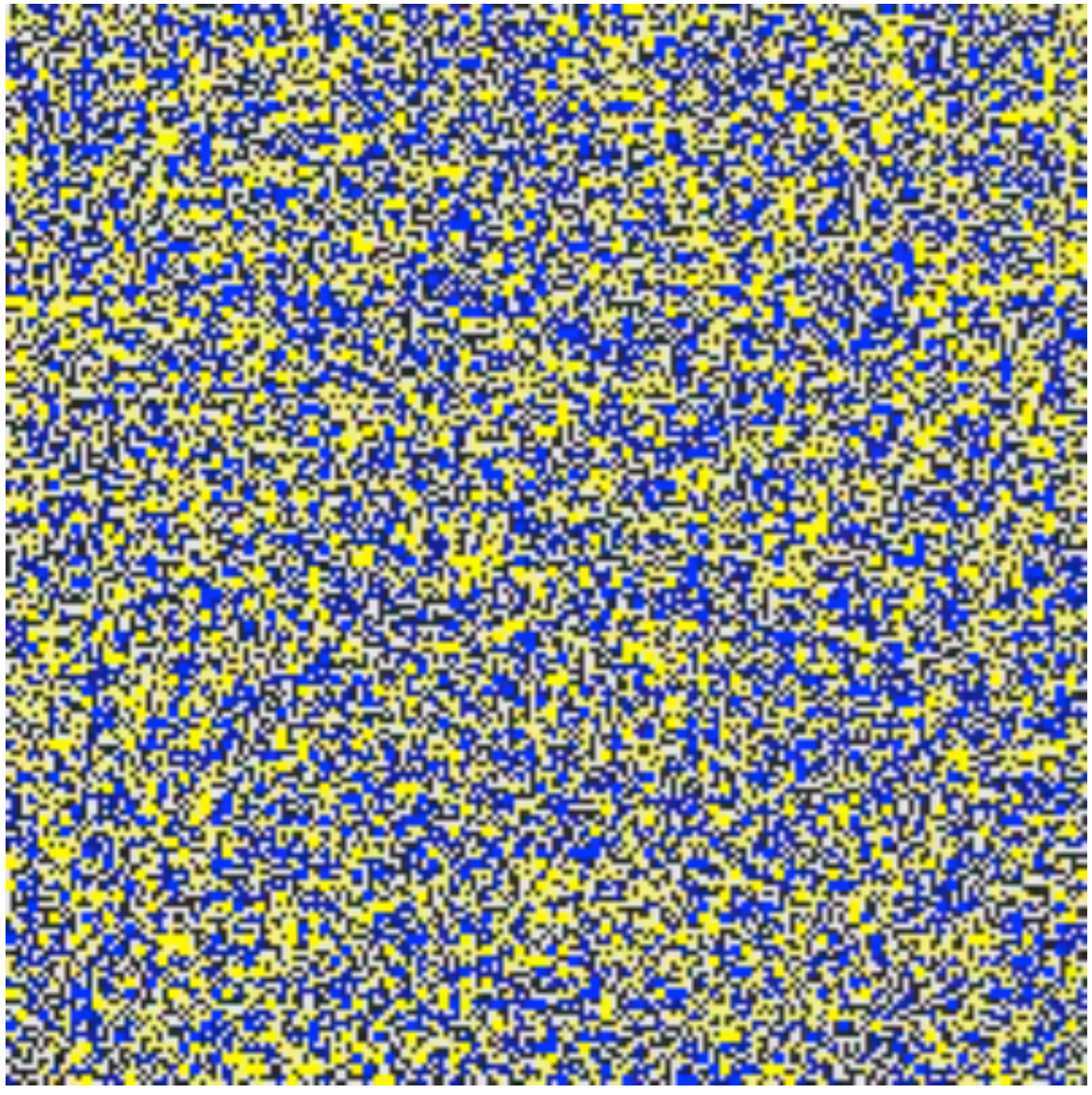
Evolution to the Ground State



Evolution to Stripe State



Evolution to Diagonal Stripe State

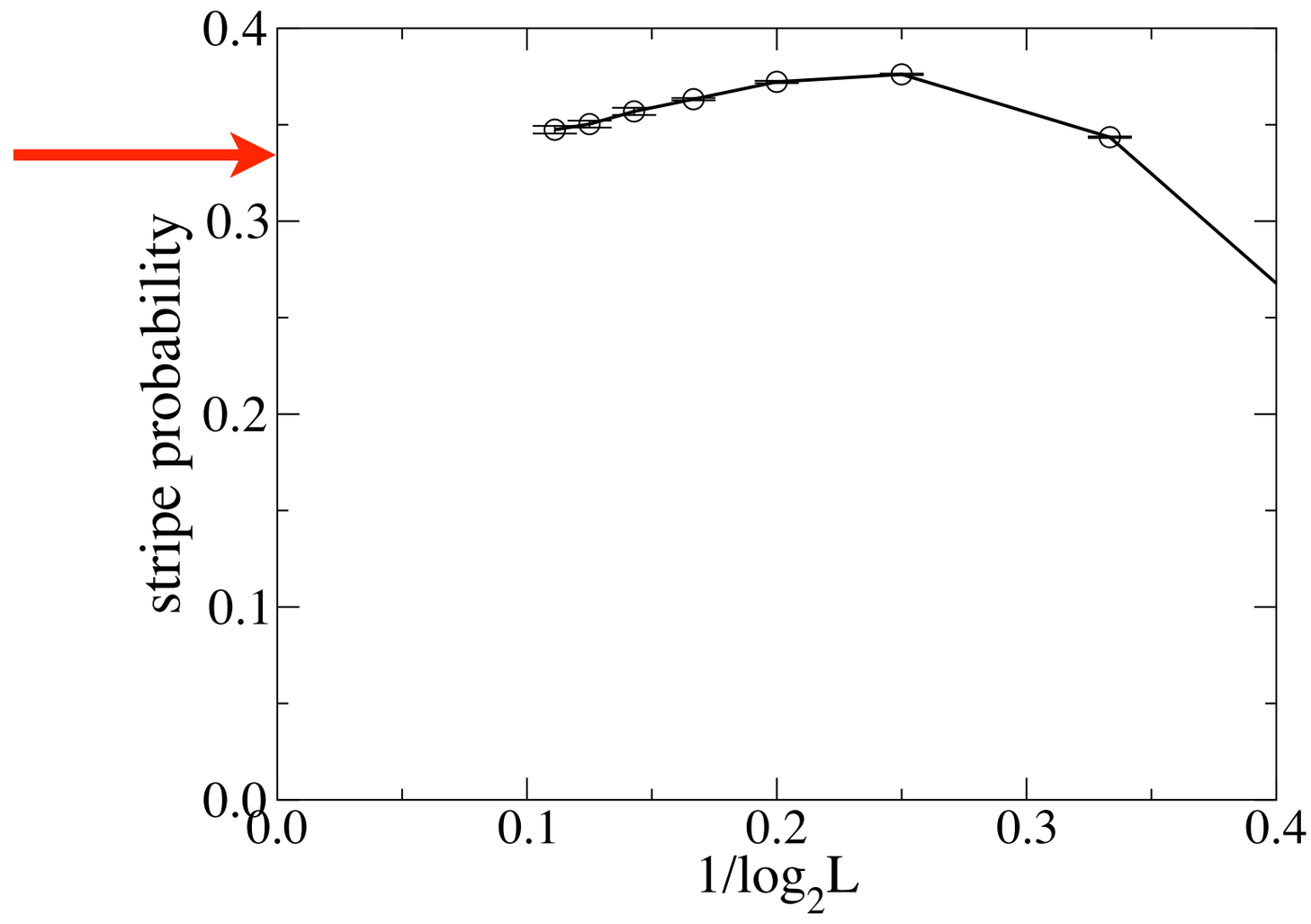


Two Dimensions

Question: what ***is*** the final state?

Answer from simulations:

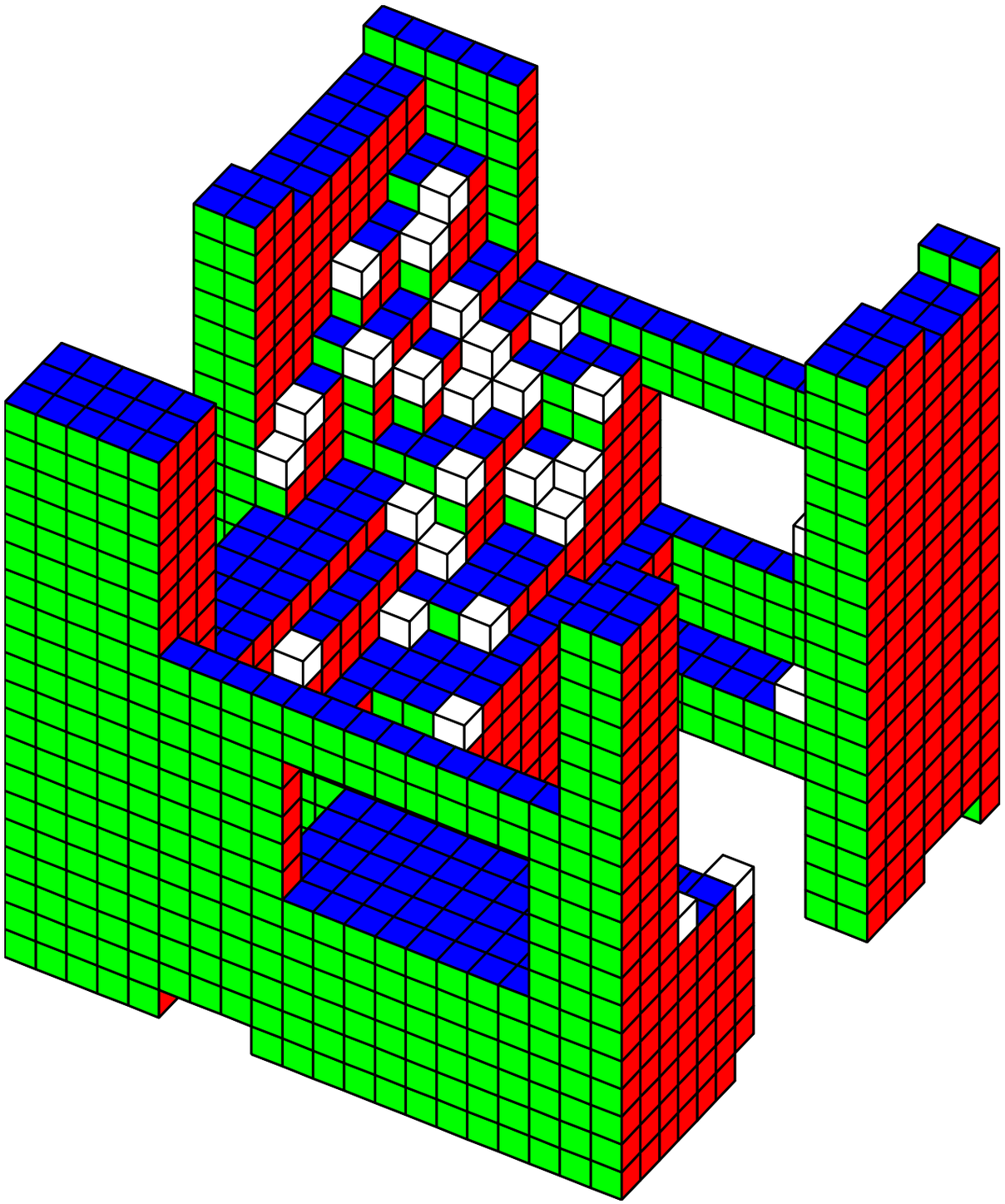
ground state with probability ≈ 2/3
stripe state with probability ≈ 1/3



Three Dimensions

Basic result: ground state is *never* reached!

typical 20x20x20 system



Features:

1. Swiss cheesy
2. Zero *average* curvature

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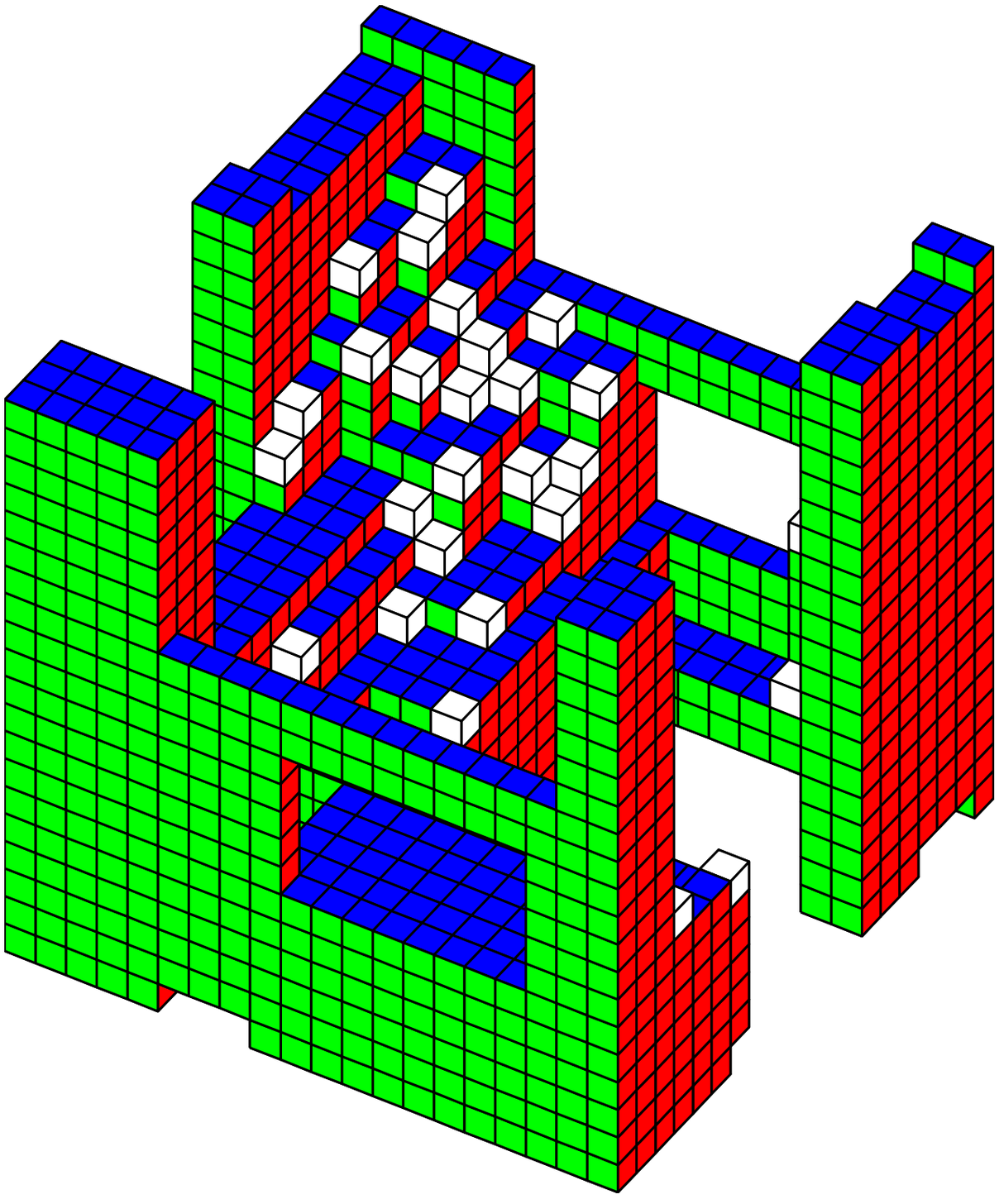

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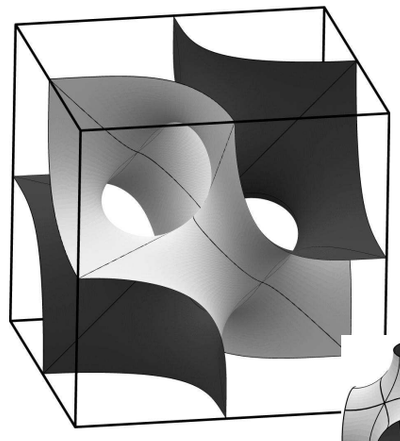
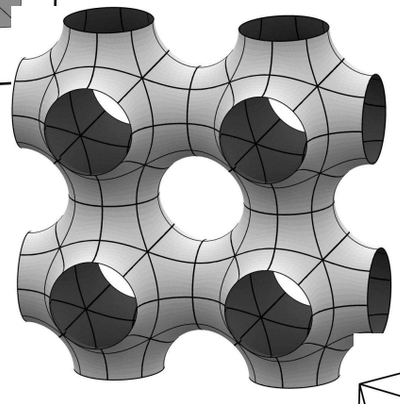
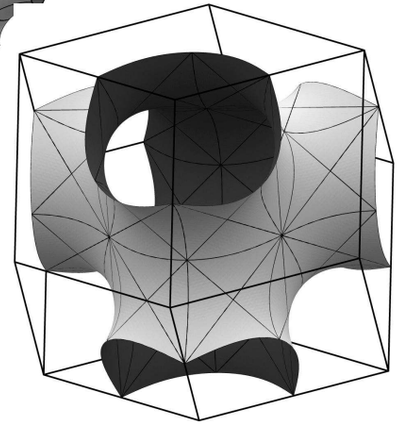
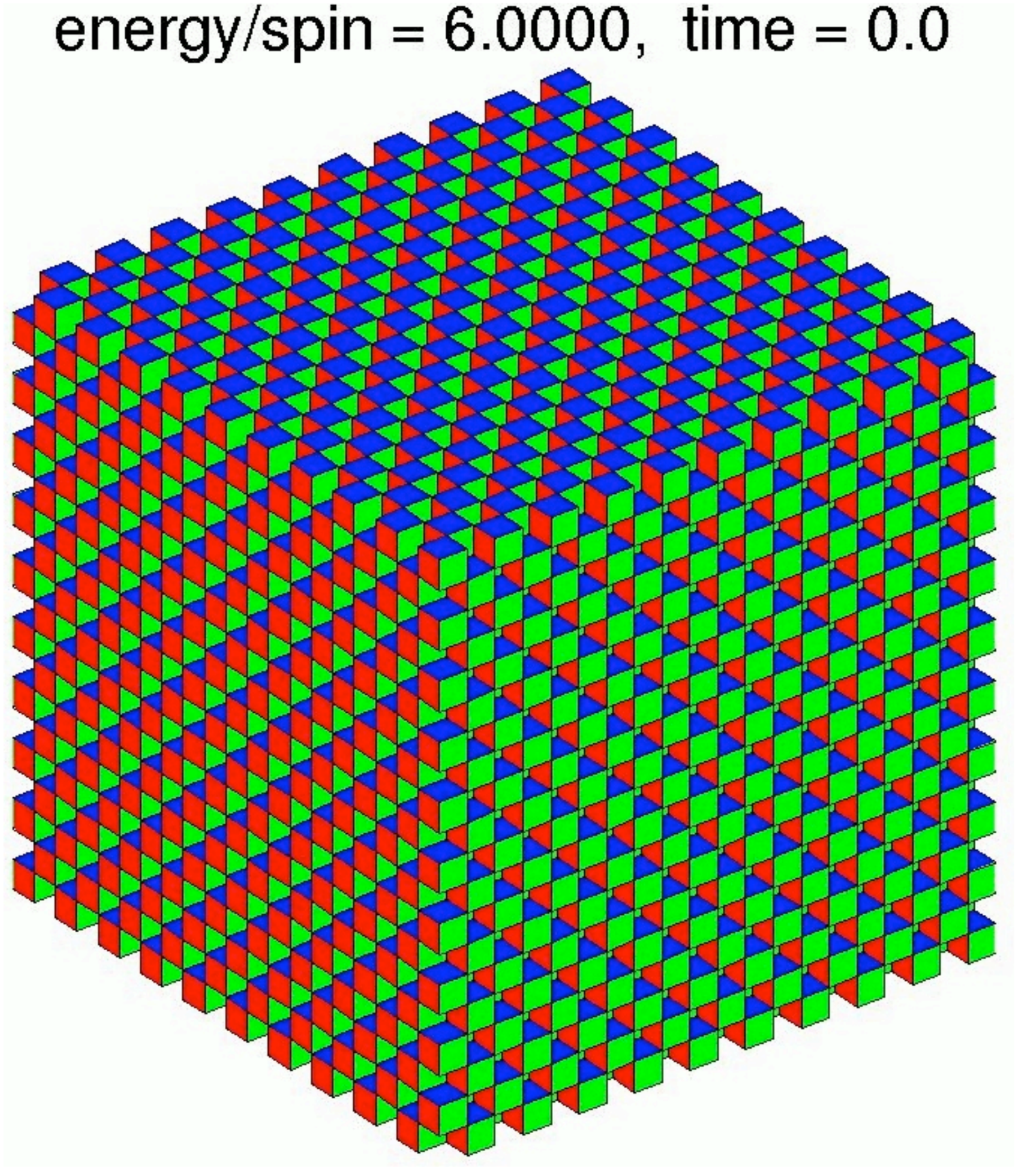






Evolution from Antiferromagnetic State



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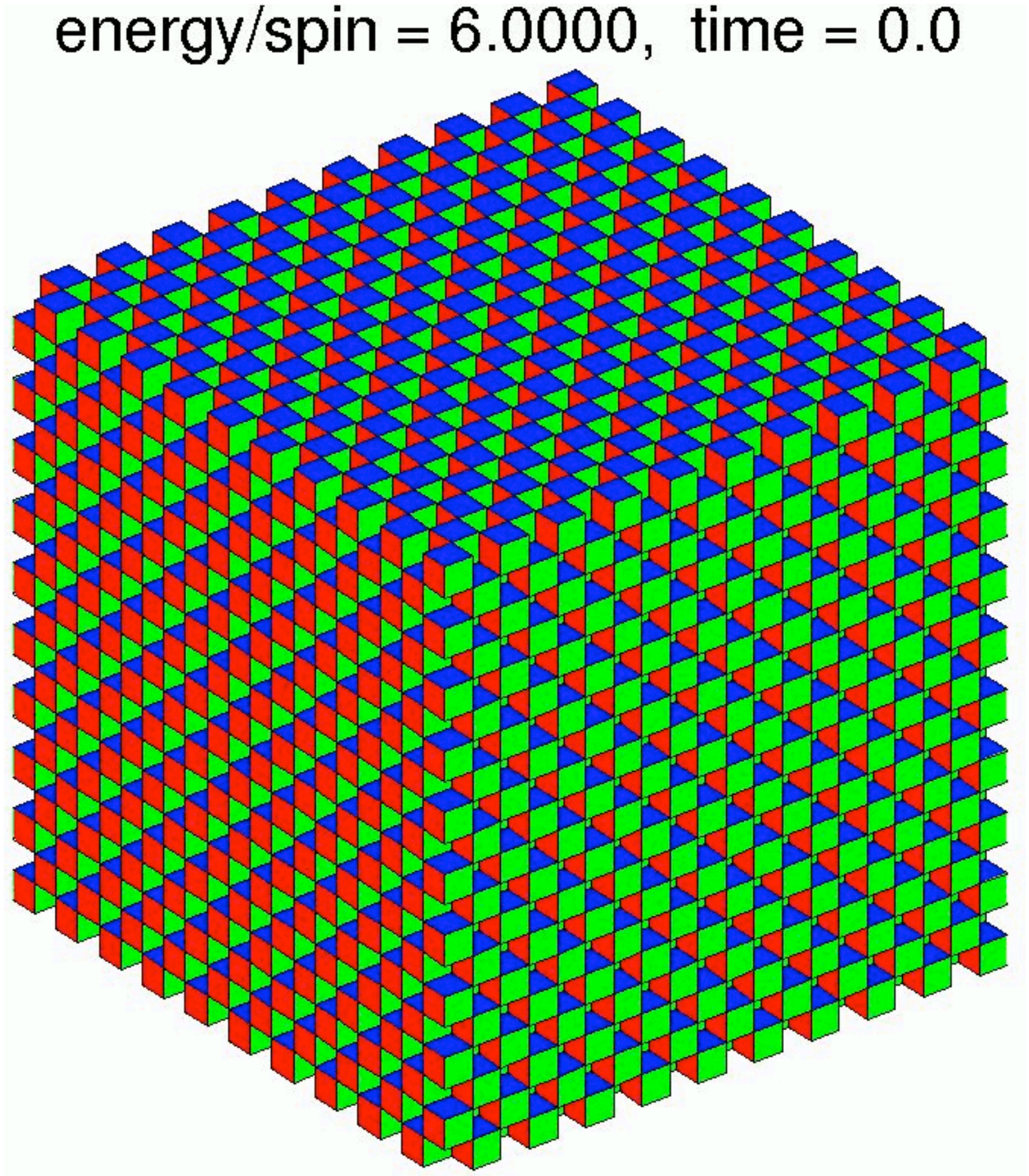

Three Dimensions (Olejarz, Krapvisky, & SR)

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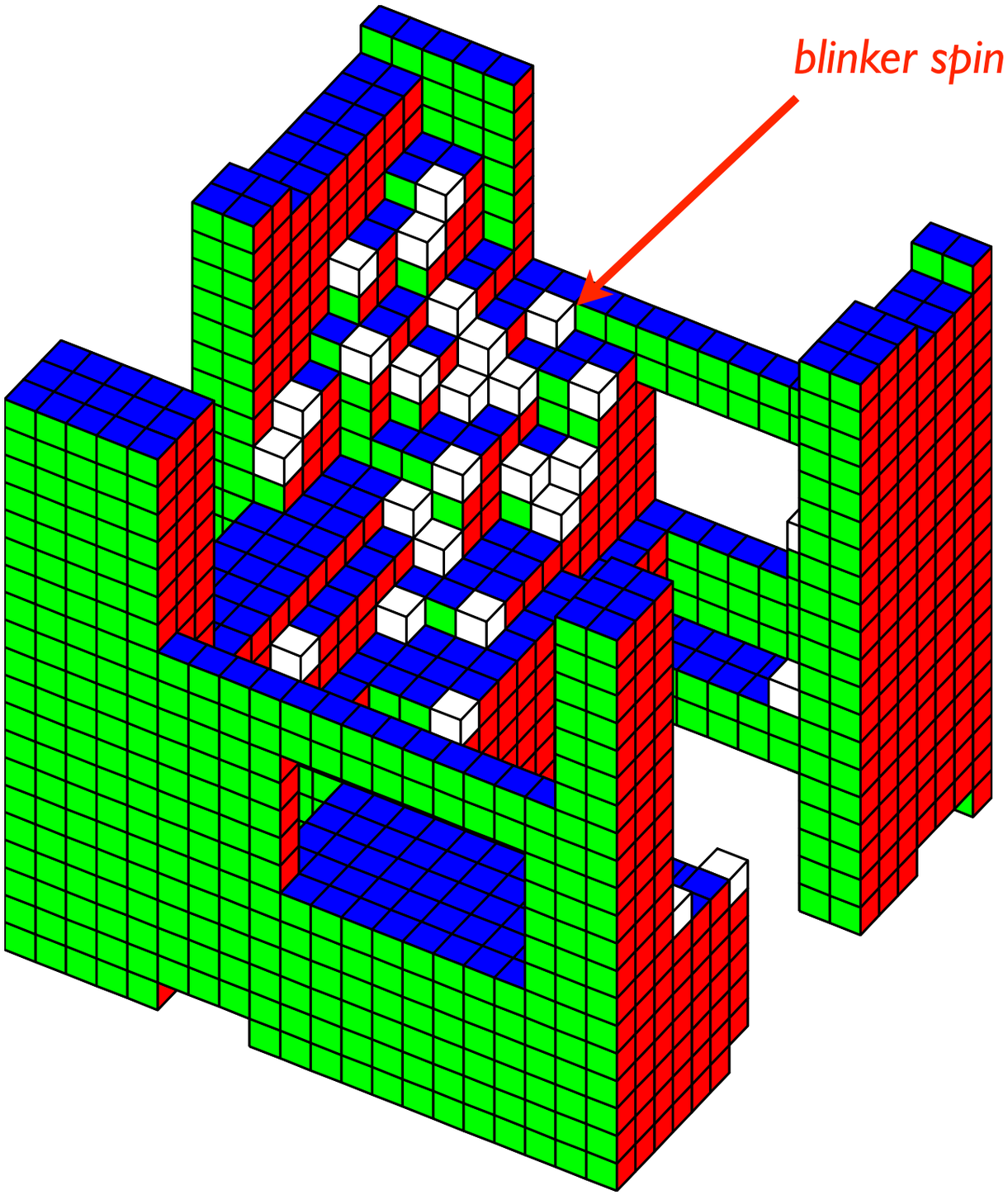
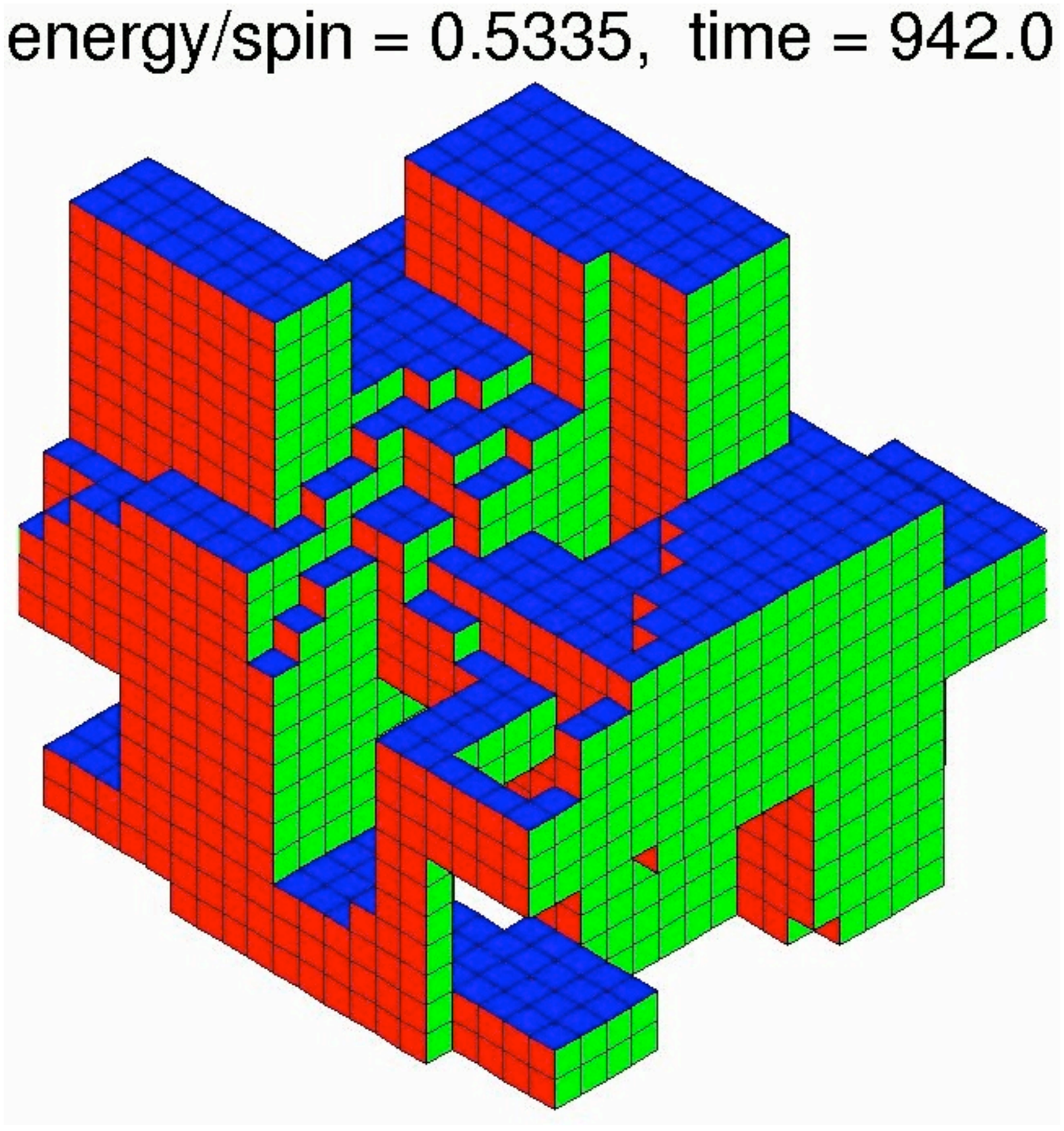
typical 20x20x20 system



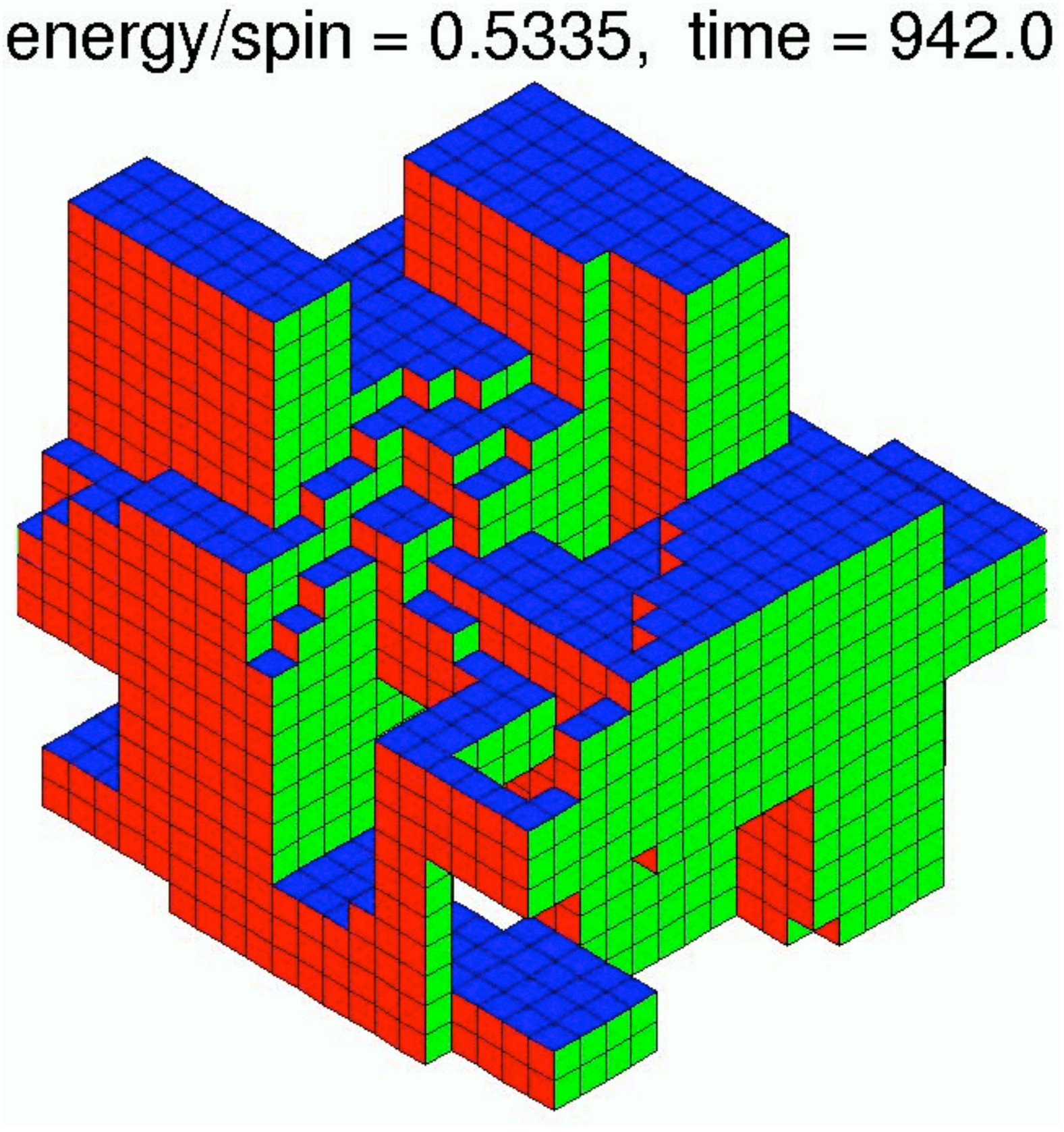
Features:

1. Swiss cheesy
2. Zero *average* curvature
3. Non-static

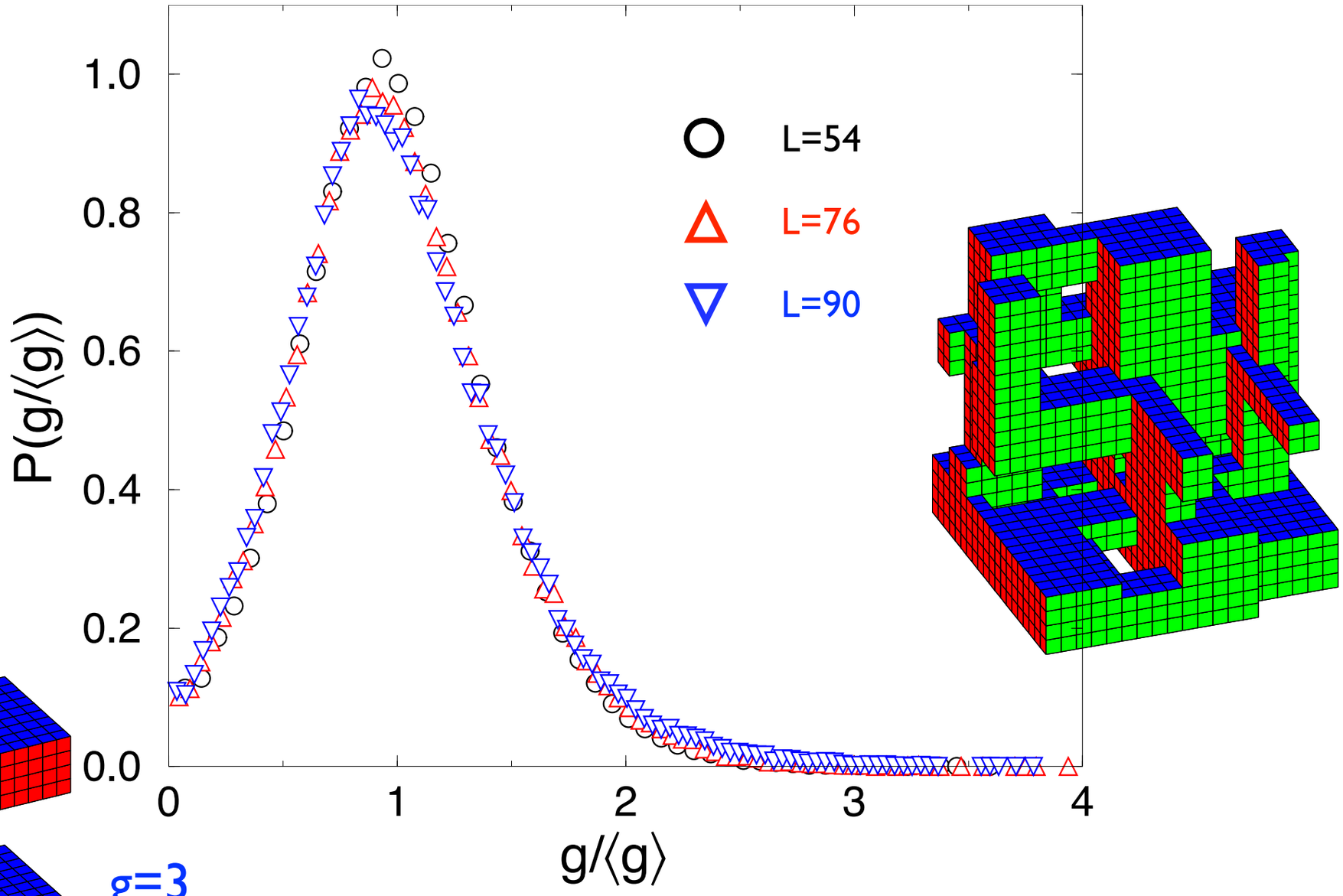
Blinker Evolution in Three Dimensions



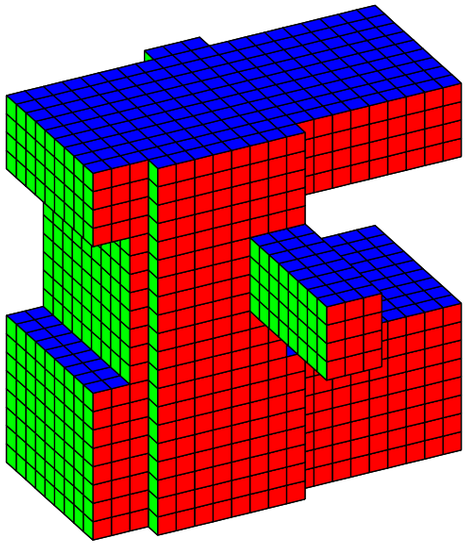
Blinker Evolution in Three Dimensions



Genus Distribution



g=16

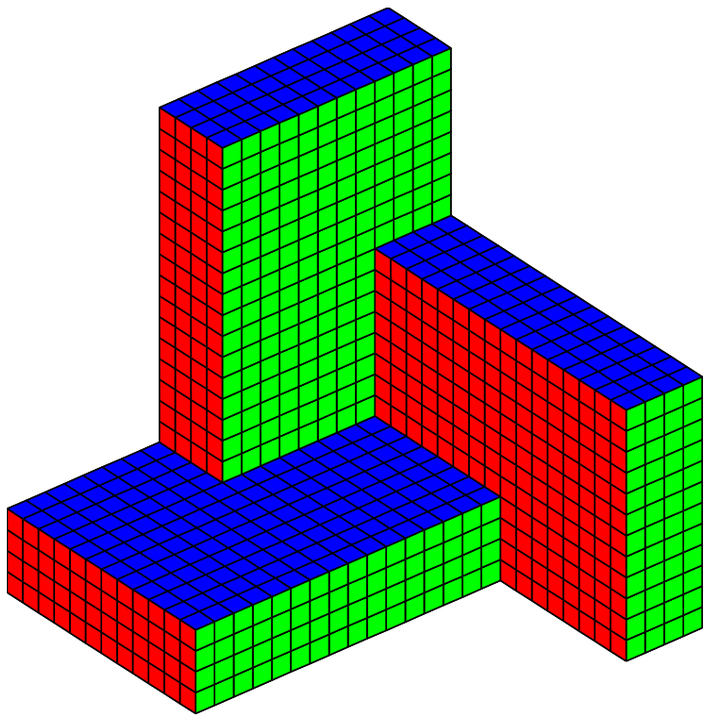


g=3

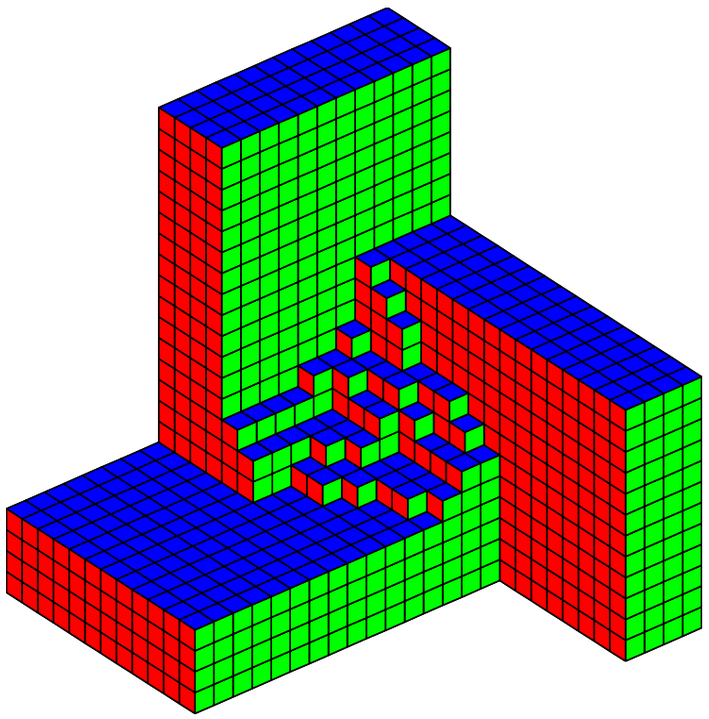
$$\langle g \rangle \sim L^{1.7}$$

Slow Relaxation of Blinkers

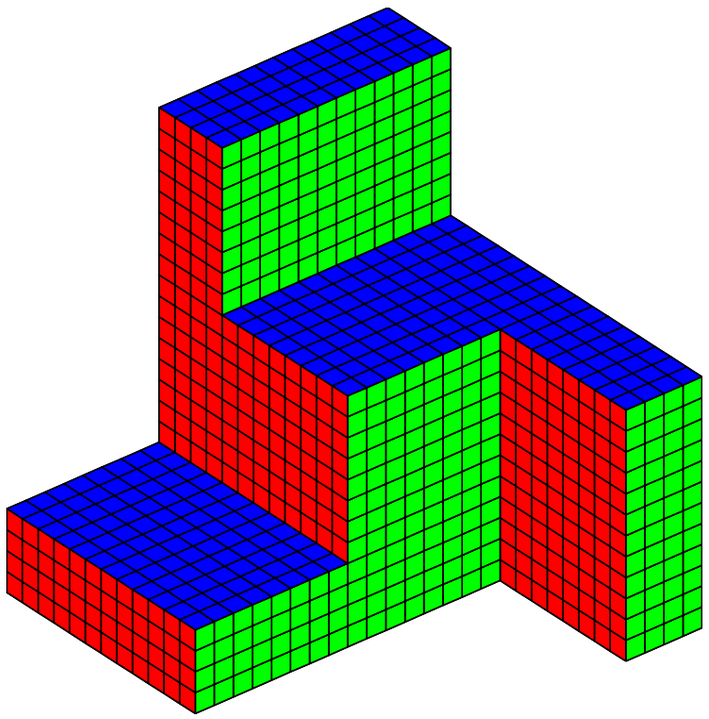
synthetic blinker configuration



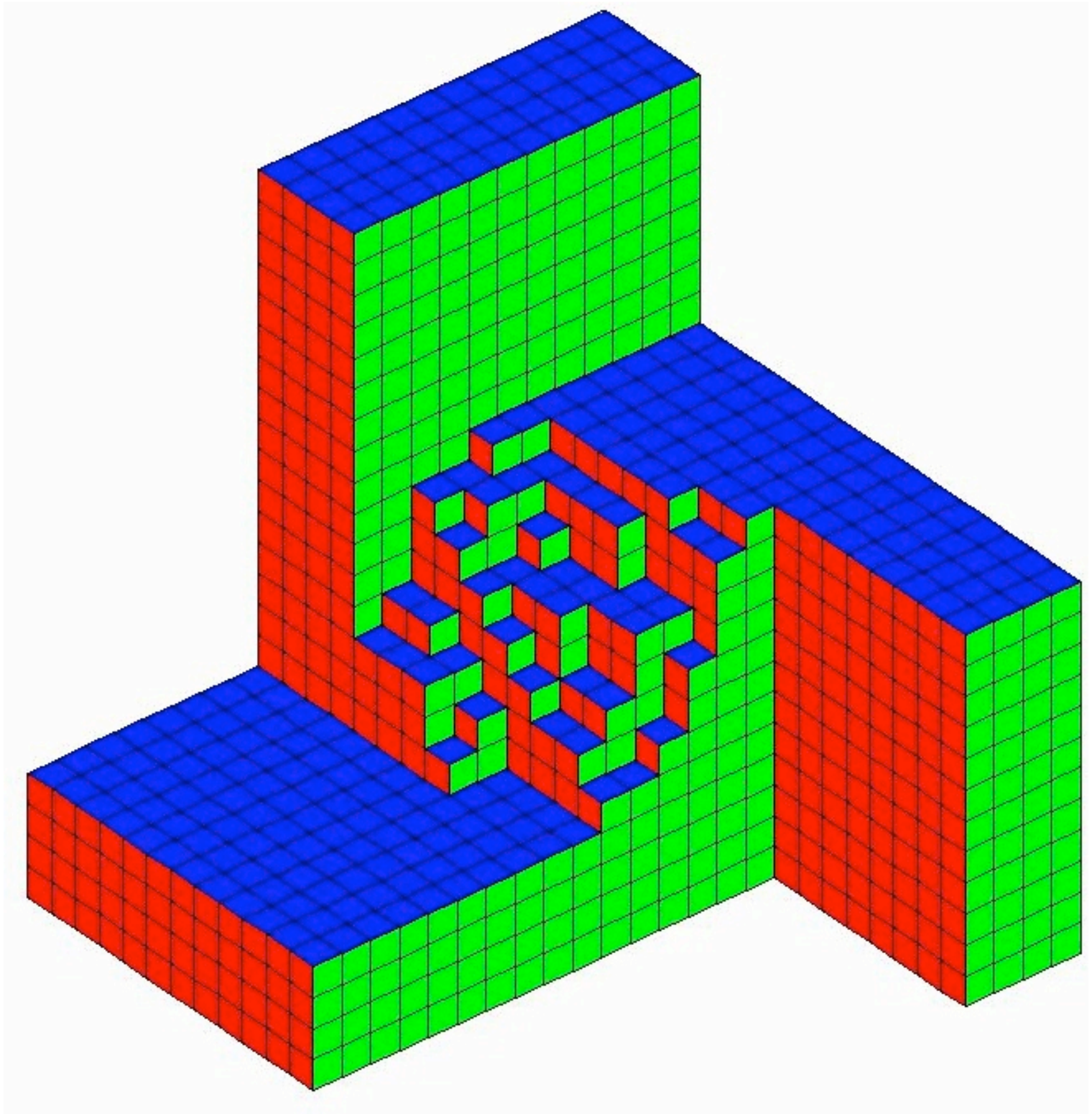
deflated

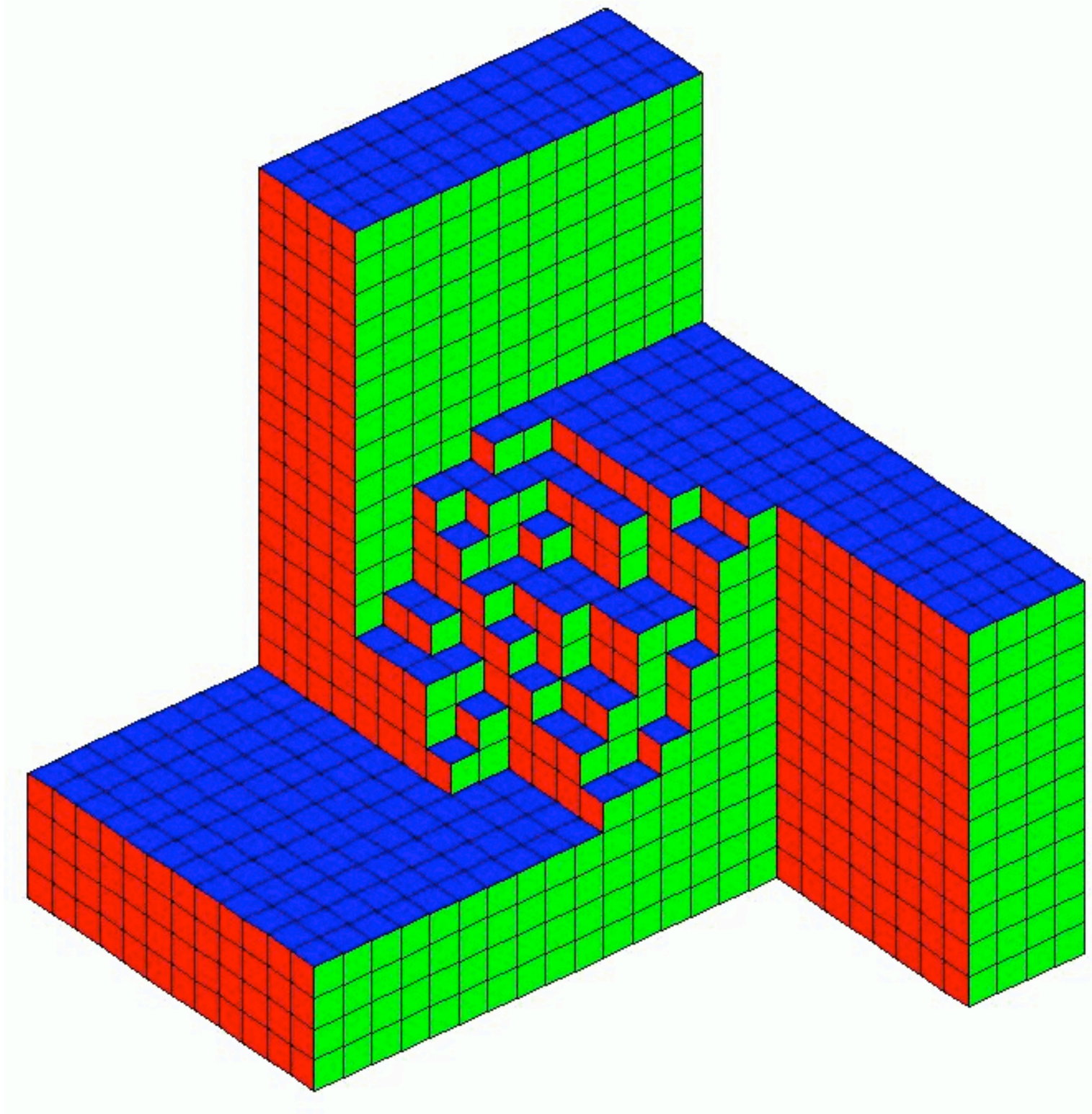


intermediate

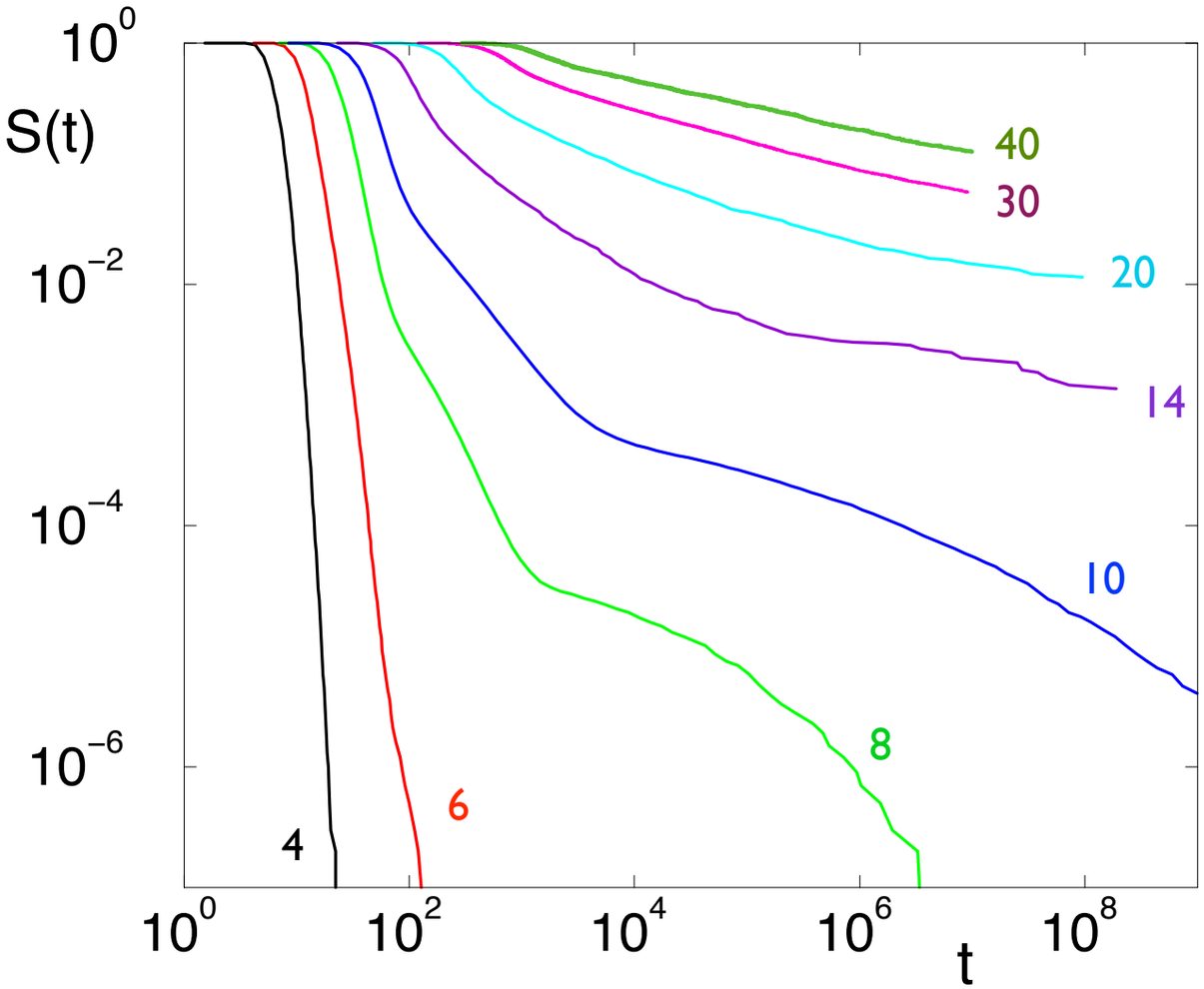


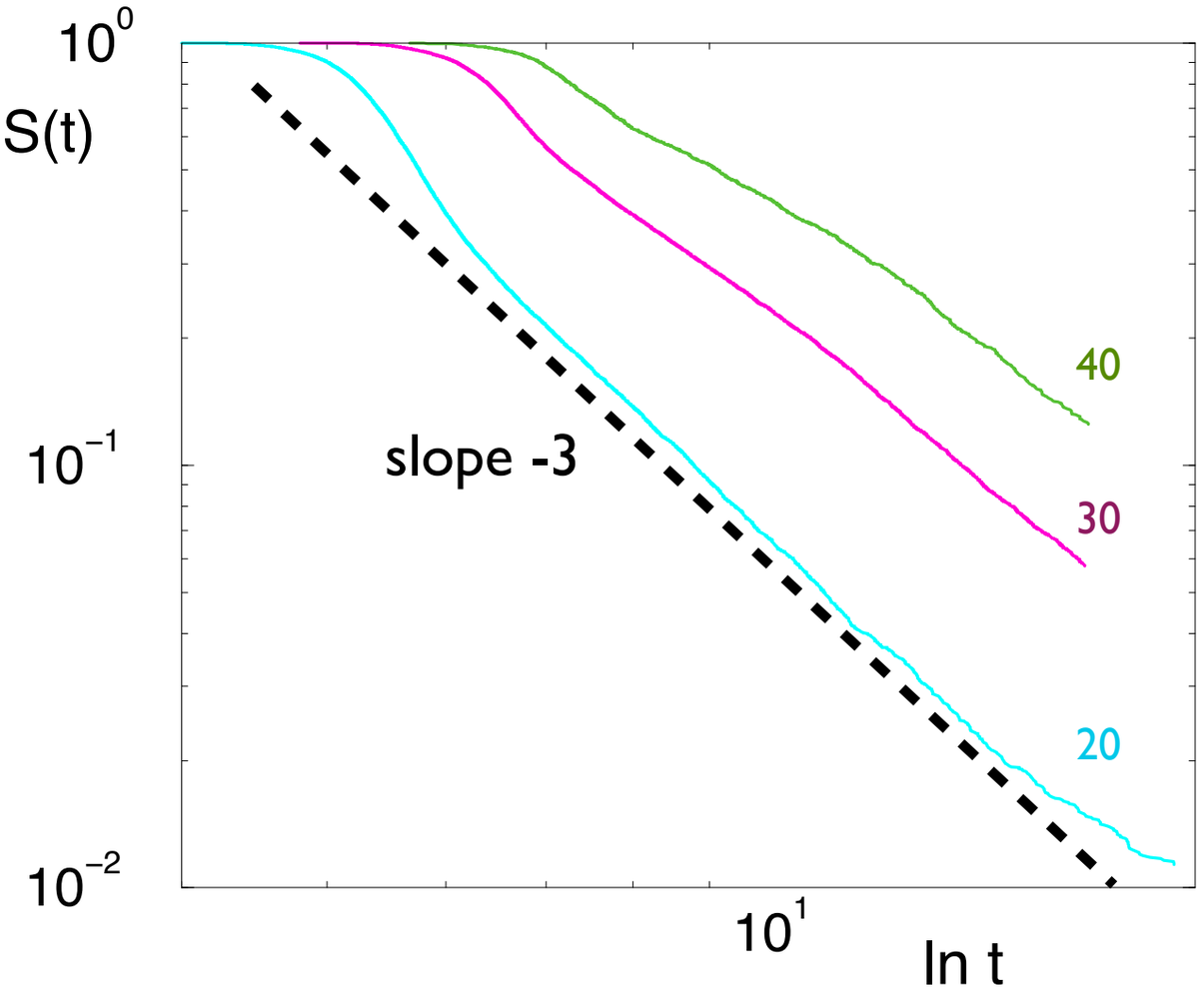
inflated





Slow Relaxation in 3d





$$S(t) \sim (\ln t)^{-3} \ ?$$

Summary & Open Problems

d=1: *almost*, but not quite, completely soluble

final state: the ground state
completion time: L^2
domain length distribution still unsolved

d=2: ground state/metastable stripe states

final state: *usually* the ground state
connection to percolation crossing probabilities
completion time: usually L^2, sometimes $L^{3.5}$
finite temperature
corner geometry: *exactly soluble*

d≥3: rich state space structure

topologically complex final state
topological connection between energy & genus
perpetually blinking spins
ultra-slow relaxation *whose functional form is unknown*
finite temperature
corner geometry: *partial result*