Nonlinearity meets Quantum Mechanics:

Intrinsic Localized Modes (ILMs) in a Bose-Einstein Condensate (BEC) trapped in an Optical Lattice (OL)

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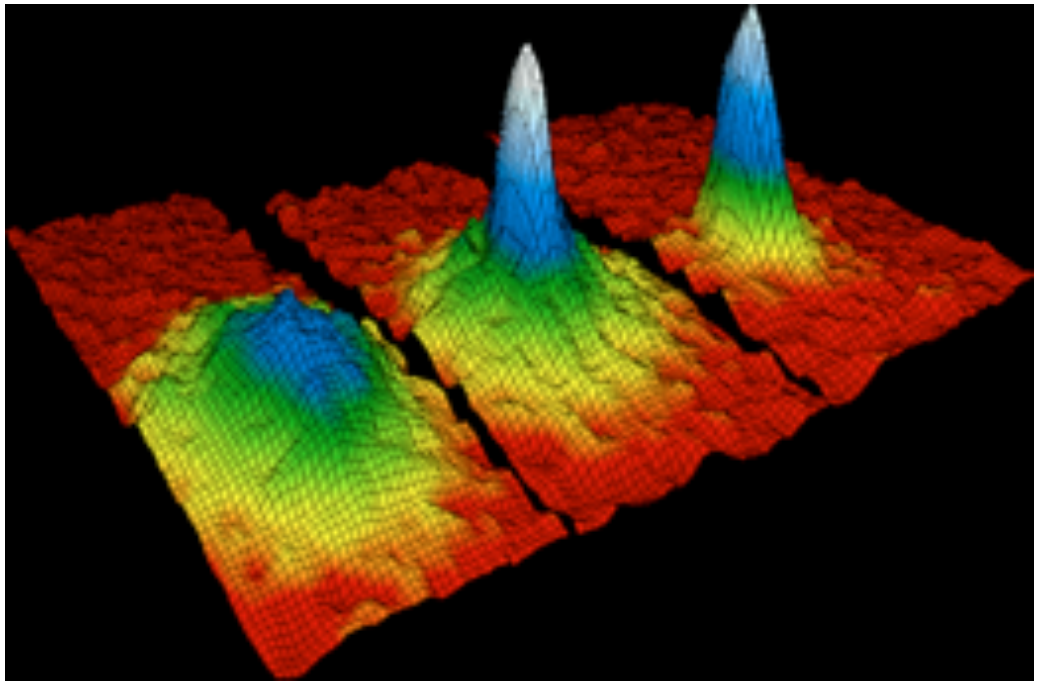
Physics 482, February 14, 2013



- Whoa! Lots of TLAs (Three Letter Acronyms): will I understand any of this??
- Nonlinear: $(a+b)^2 \neq a^2 + b^2$

  $= a^2 + b^2 + 2ab$

  - Chaos (pictures)
  - "Solitons" (pictures)
- ILM = Intrinsic Localized Mode ~ soliton (pictures)
- BEC = Bose-Einstein Condensate ~ weird state of matter at ultralow temperature, > 100,000 atoms all in a single (macroscopic) quantum state !!! (picture)
- OL = Optical Lattice (picture)

"Chaos"

The "Standard" Map

$$p_{n+1} = p_n - \frac{k}{2\pi}\sin 2\pi q_n \qquad \text{(MOD 1)}$$

Nonlinearity Parameter

$$q_{n+1} = q_n + p_{n+1}$$

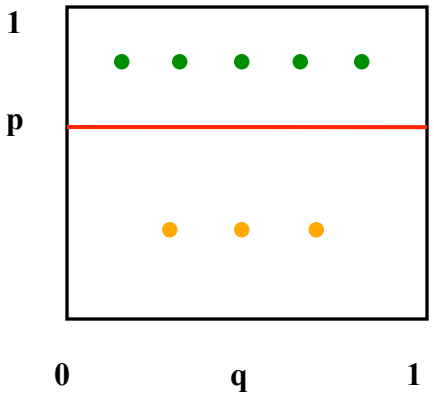
For $k = 0$ (i.e. no nonlinearity)

$$p_{n+1} = p_n \equiv p_0$$

Momentum Conserved

$$q_{n+1} = q_n + p_0$$

So orbits are straight lines of constant $p=p_o$ and q simply "rotating" around interval



$p_0 = 4/5$, $q_0 = 0.1$

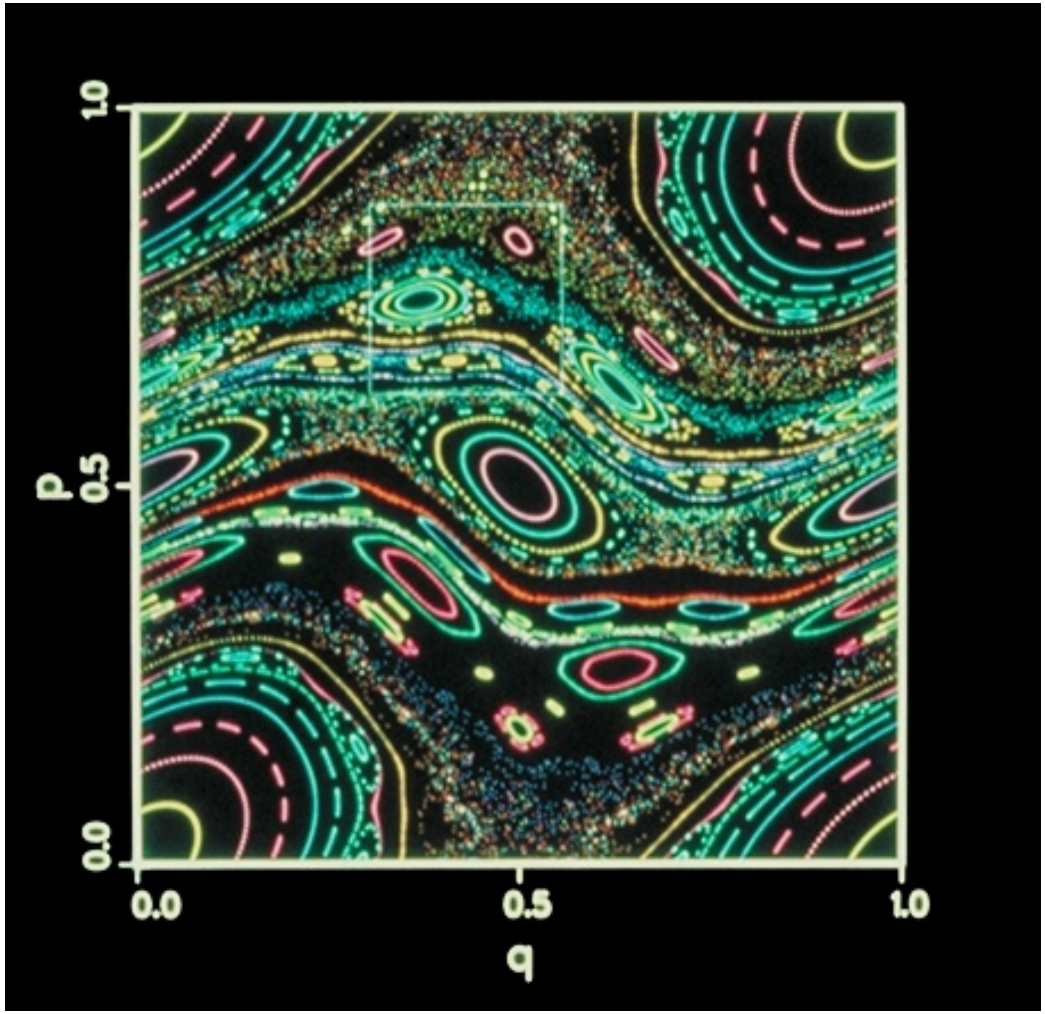
$p_0 = 1/\sqrt{2}$, q_0 = arbitrary

$p_0 = 1/3$, $q_0 = 0.5$

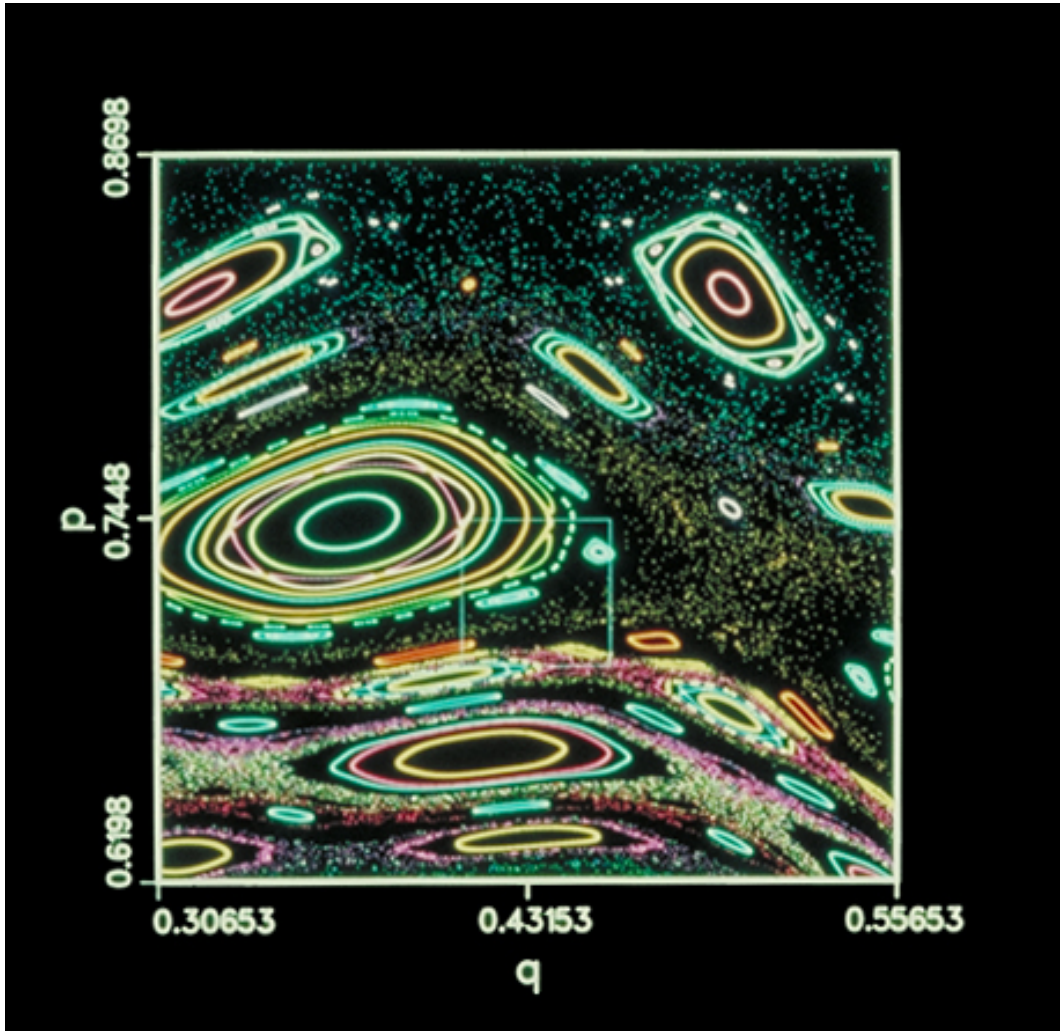
What about $k \neq 0$? => Chaos !

The next images show behavior for k =1.1

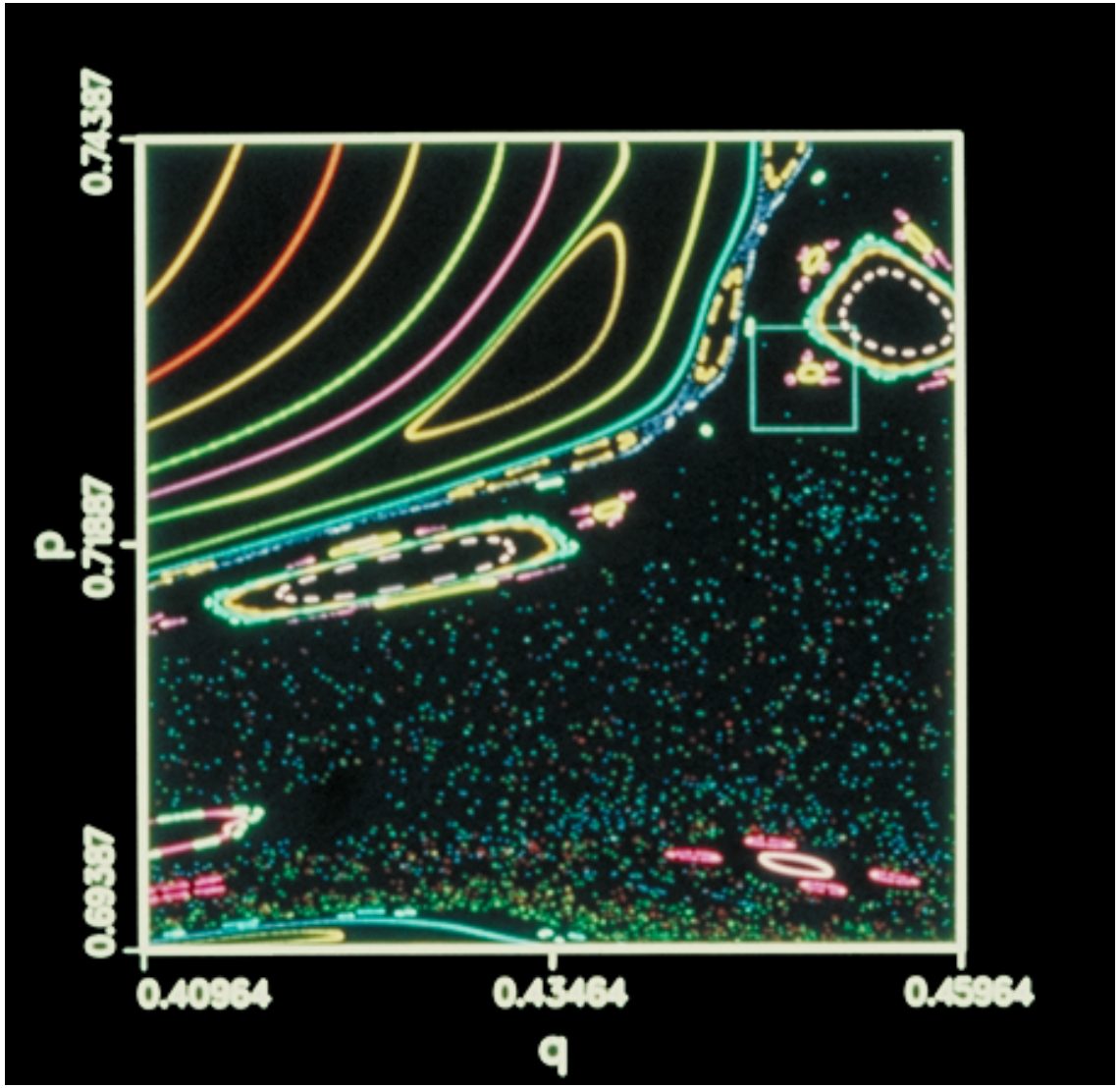
Chaos in Standard Map



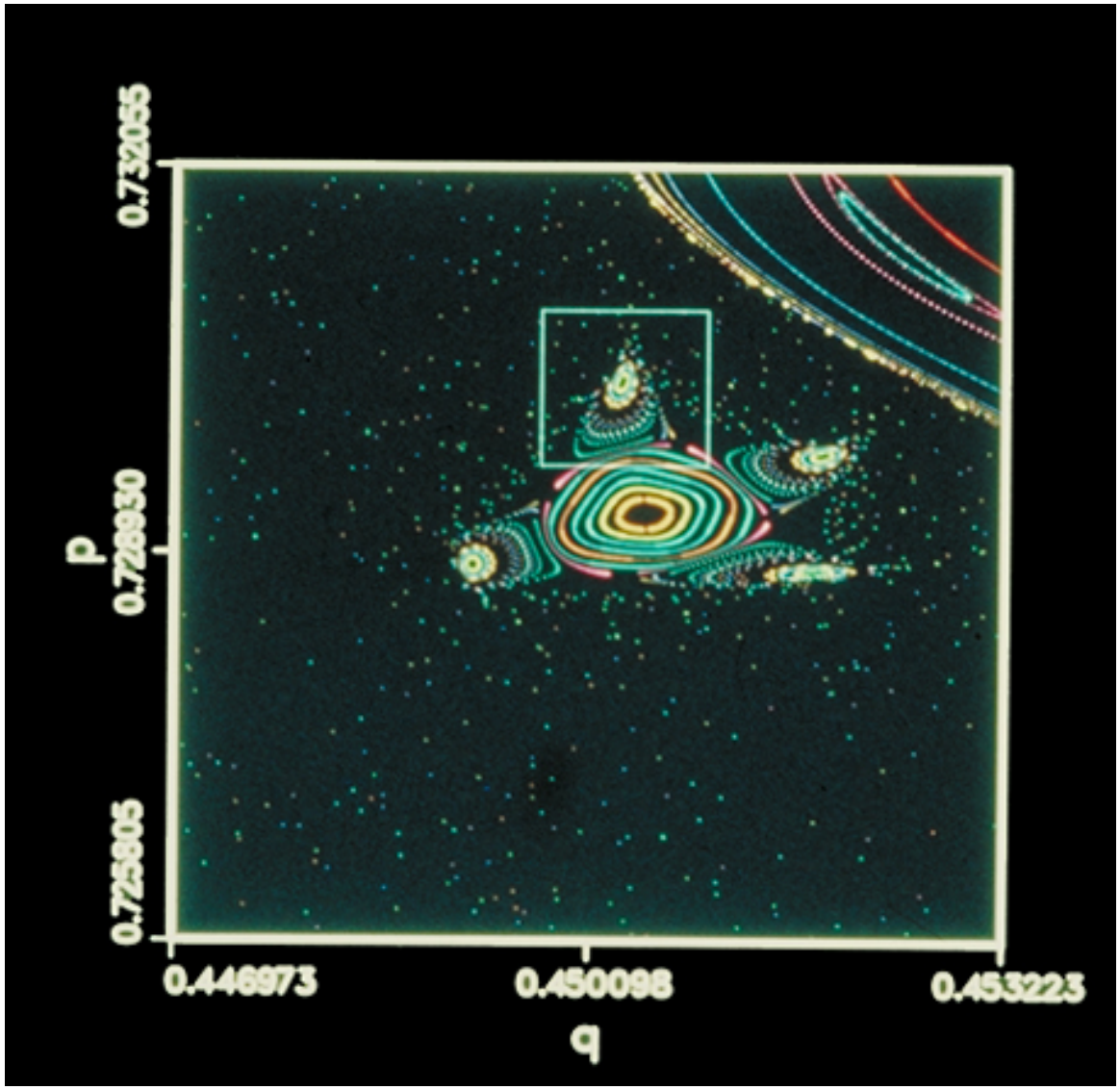
Chaos in Standard Map



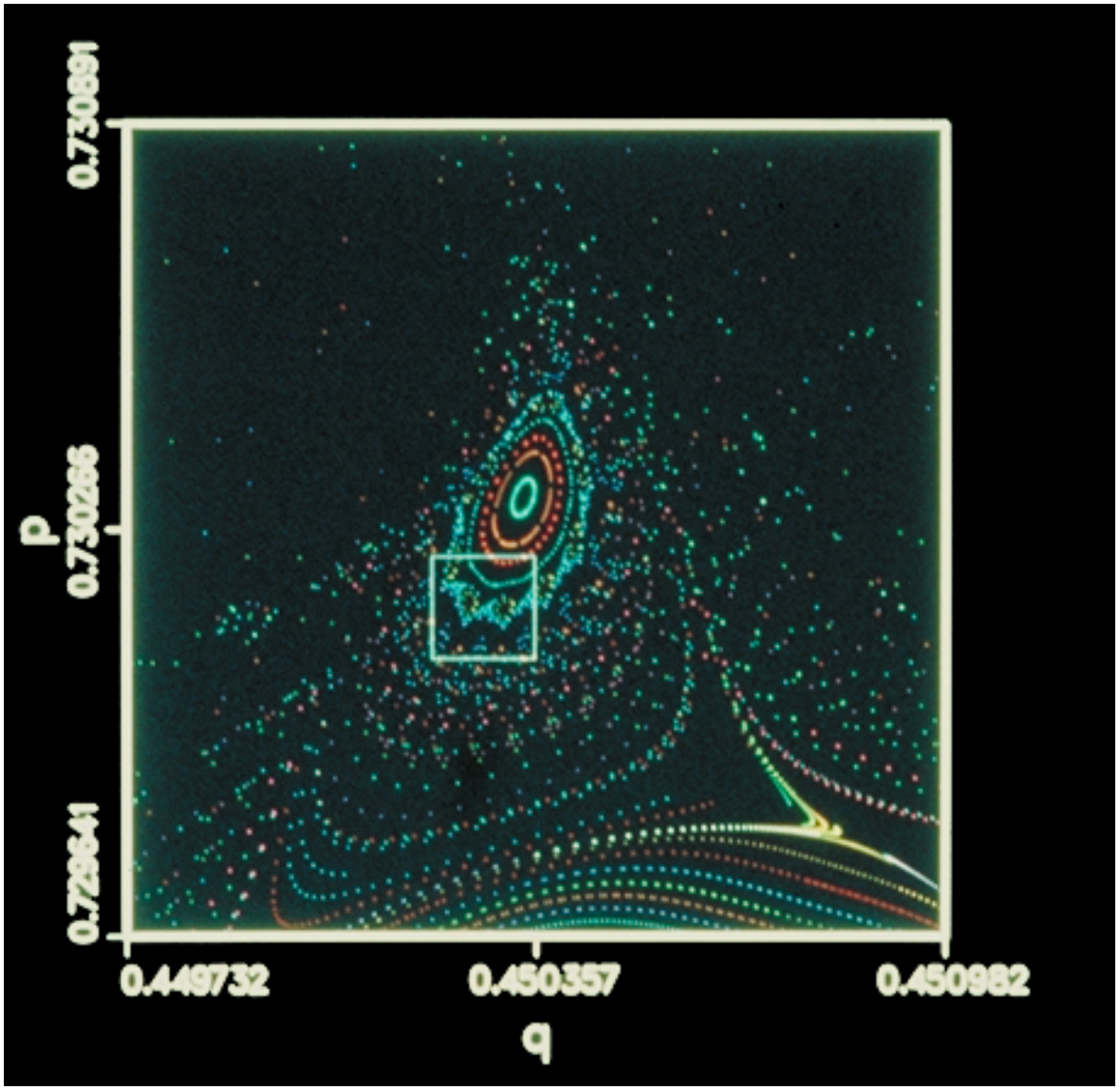
Chaos in Standard Map



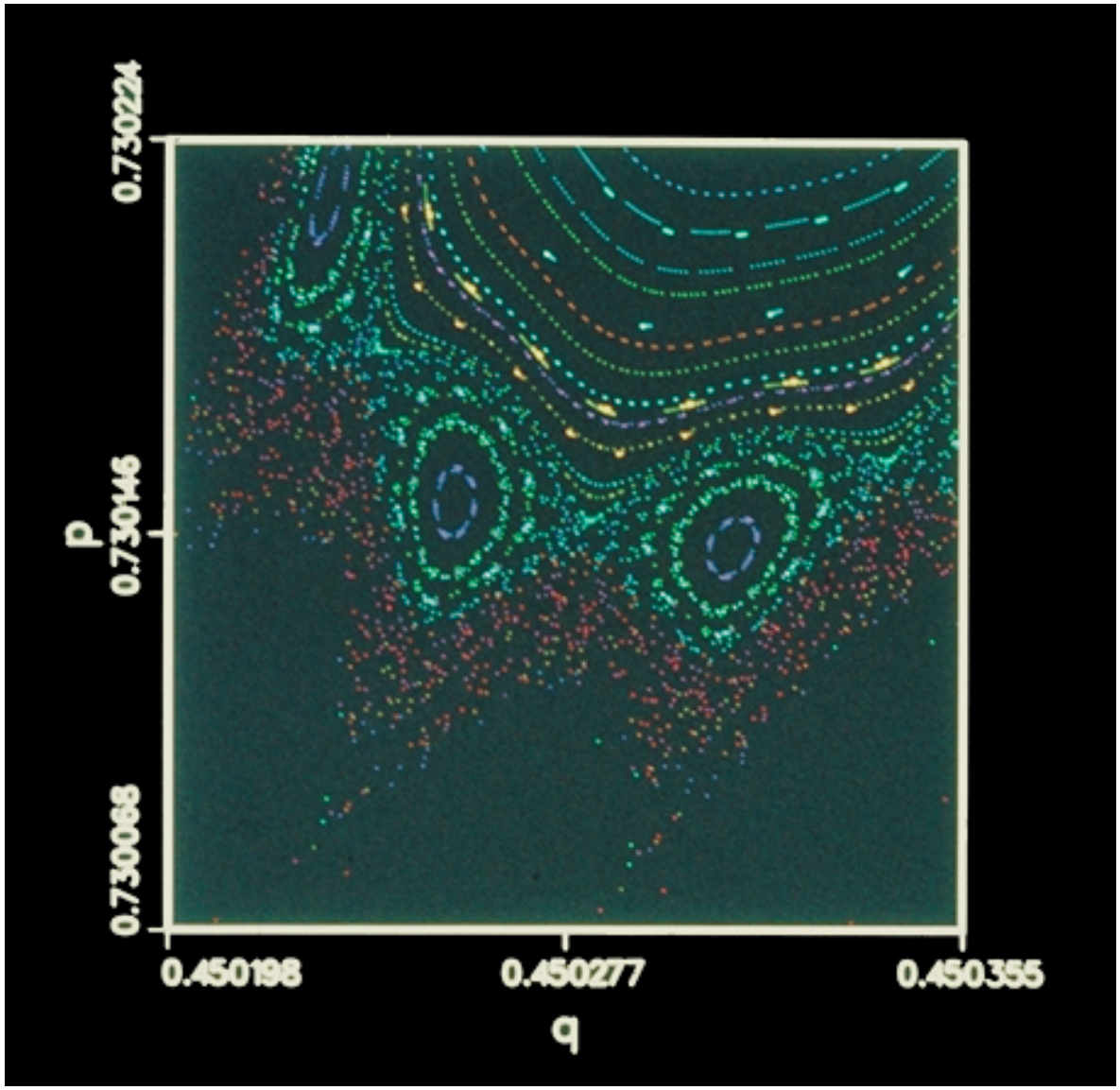
Chaos in Standard Map



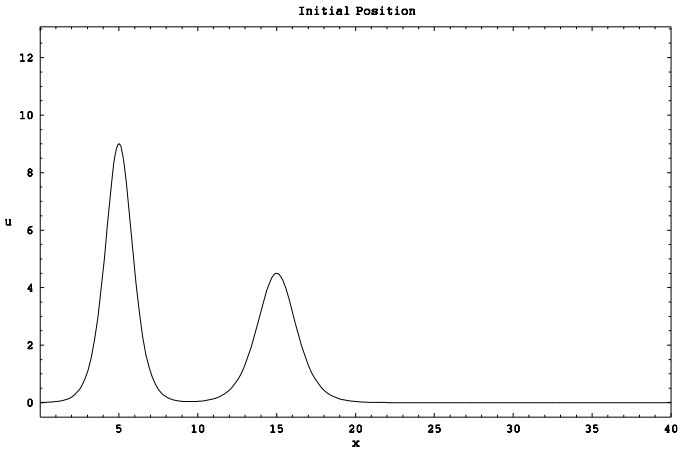
Chaos in Standard Map



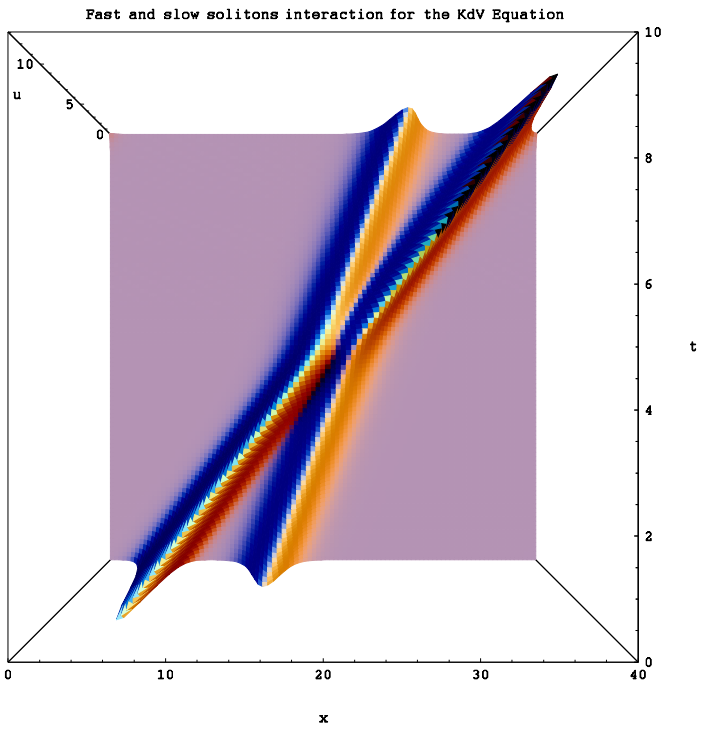
Chaos in Standard Map

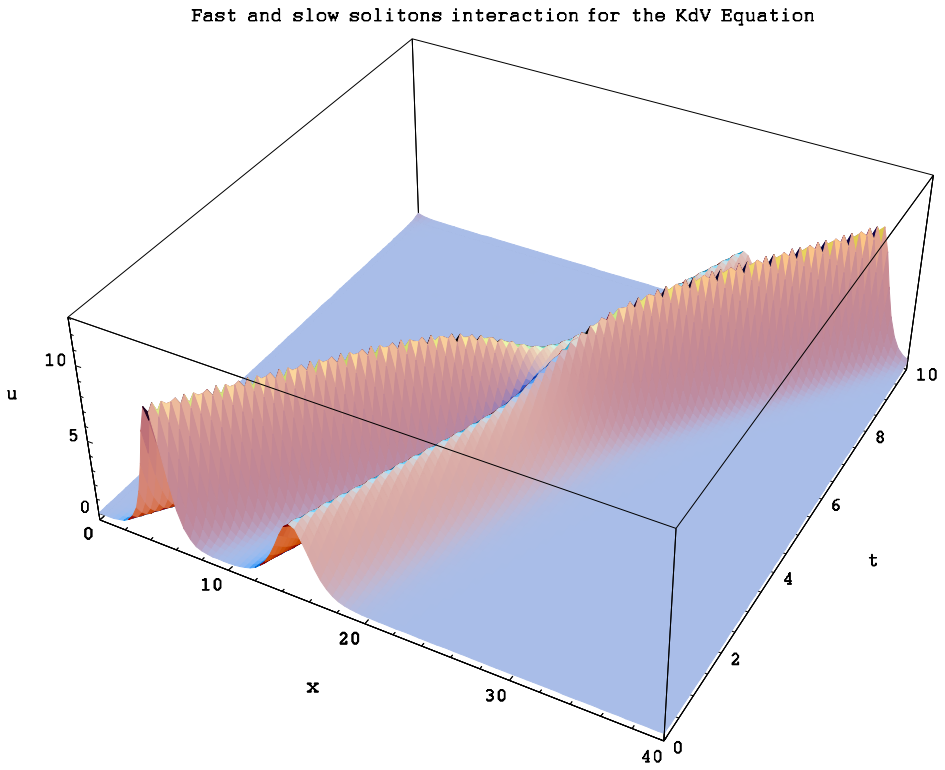


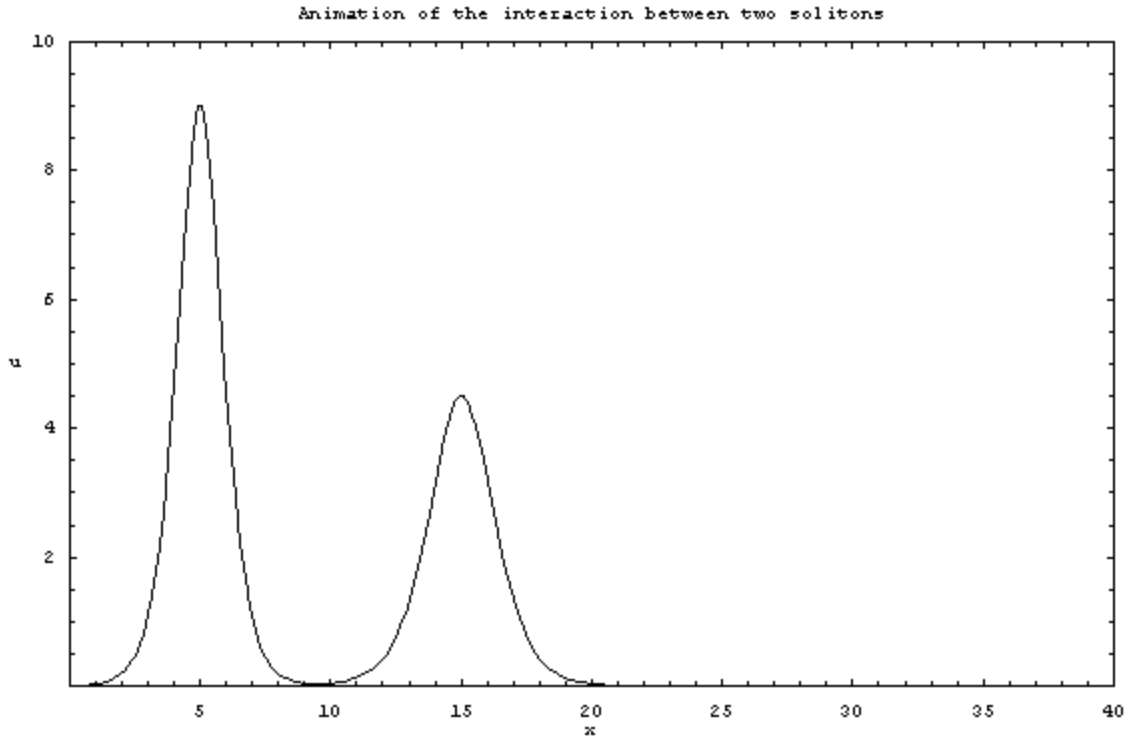
Solitons: Localized *nonlinear* waves



Soliton collision: $V_l = 3$, $V_s = 1.5$






Animation of the interaction between two solitons
10
8
6
4
2
u
5
10
15
20
25
30
35
40
x

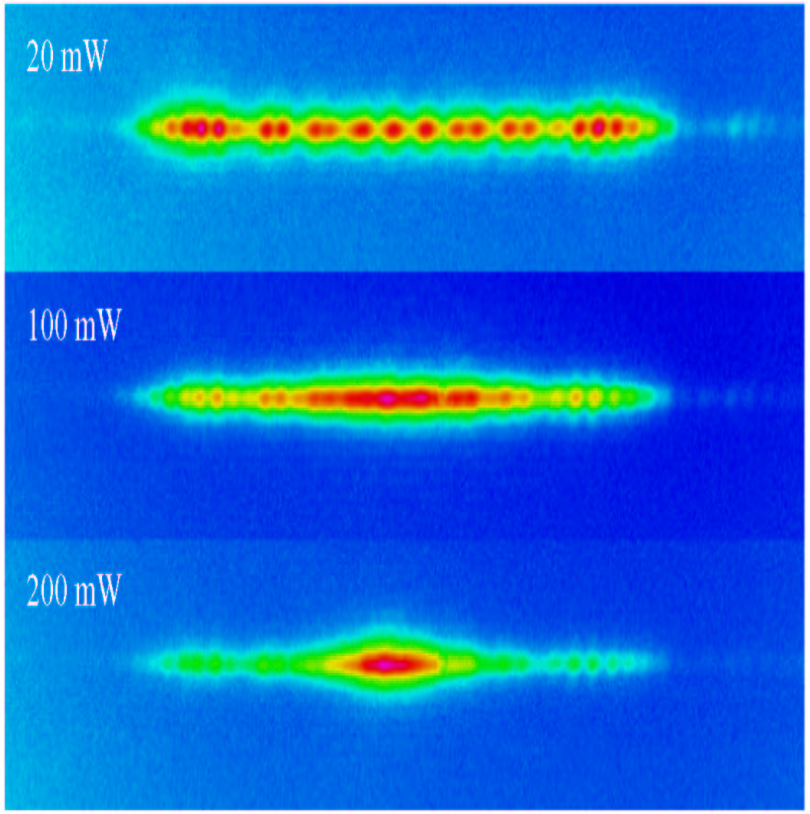
Surfin’ the Severn River

Definition of ILMs and Bottom Line

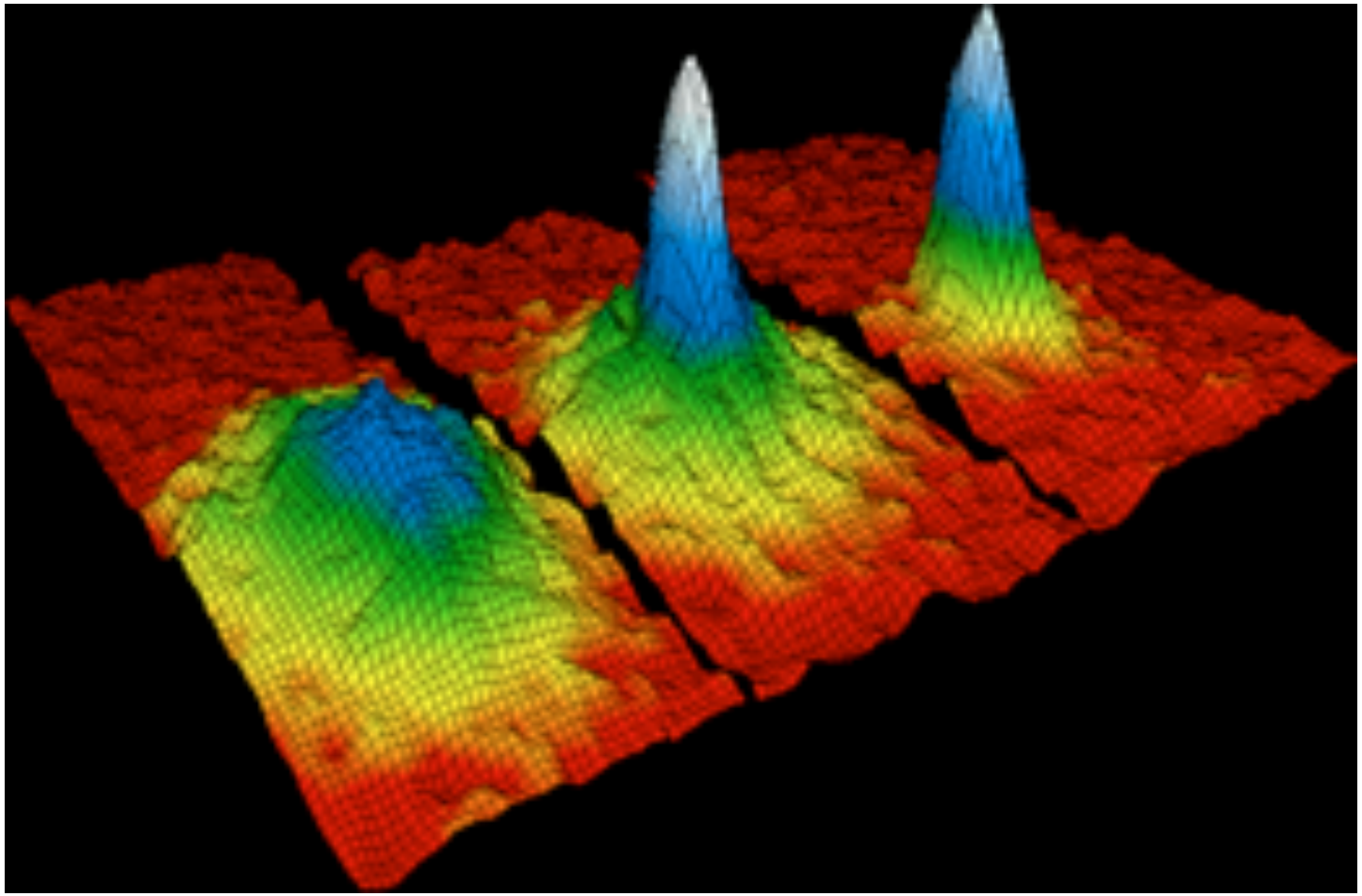
- Definition: an "intrinsic localized mode"—or "discrete breather"—is a spatially localized, time-periodic, stable (or at least very long-lived) excitation in a spatially extended, perfectly periodic, discrete system.
- Bottom Line: The mechanism that permits the existence of ILMs/DBs has been understood theoretically for more than a decade. Only recently have they been observed in physical systems as distinct as charge-transfer solids, Josephson junctions, photonic structures, and micromechanical oscillator arrays.

ILMs in Optical Waveguides

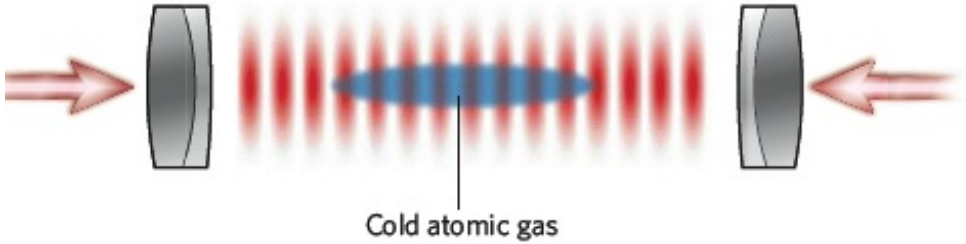
Edge-on view of the output facet of the coupled optical waveguide array shown on previous slide. The input pulse is localized at the center of the array. At low power, pulses propagate linearly and "diffract" across entire array. At intermediate power, nonlinear effects induce some localization. At high power, the pulse remains truly localized and is an example of an ILM in these systems. (From Eisenberg et al).



Bose-Einstein Condensates: velocity distribution "collapses" to a single wave function



BECs in an Optical Lattice



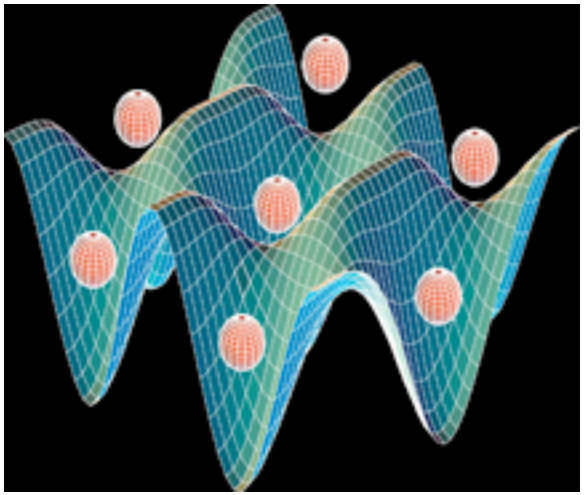
Counter-propagating laser pulses creating standing wave that interacts with atoms, so that the BEC experiences a periodic potential of the form:

$$V_{\text{ext}}(\vec{r}) = U_L(x,y)\sin^2[2\pi z/\lambda]$$

$U_L(x,y)$ is transverse confining potential, λ is the laser wavelength (typically 850 nm) and "z" is the direction of motion.

Image from I. Bloch *Nature* **453** 1016-1022 (2008)

Image of 2D optical lattice trapping Bose particles



Egg carton/marble model

- Now DO you understand the TLAs and the basic ideas ?? Let's see:
- Nonlinear: $(a+b)^2 \neq a^2 + b^2$

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QUESTIONS ??