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Dear Professor Youssef,

Thanks for the accepting and placing my paper (Part 2) on your website.

I would like to submit a revised version of it which corrects my previous treatment of photon's "small jumps". Sorry for the inconvenience.

Sincerely,

John V. Garchin

Photon's Propagator From Maxwell Electrodynamics Part 2: Speculations Regarding Instantaneous Photon Jumps As the Source of Weirdness of Quantum Mechanics

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Photon's trajectories are identified with realizations of the quantum Cauchy process found in the previous paper. Assuming that they survive an analytic continuation in time, they exhibit instanteneous jumps and stops which can explain EPR paradox and other weirdness of Quantum Mechanics.

"Every physicist thinks he knows what a photon is. I spent all my life trying to understand what a photon is, and haven't understood it by now"

A. Einstein

1. INTRODUCTION

I have learned something surprizing about photon's behaviour which follows from the Maxwell equations, as the first-order in time Schrödinger equation for the Riemann-Silberstein vector **f** composed of the electromagnetic free field, [1]. Its motion is described by the quantum **Cauchy process** which is composed of **instantaneous jumps** of a random size intertwined with **stops** of a random duration. Together they can mimic the straight linear shift with the effective velocity $v = c$.
Let us briefly recapitulate the main points of my previous paper referred to as "Part 1":

$$\mathbf{f} = \mathbf{E} + i\mathbf{B} \tag{1.1}$$

$$i\hbar\partial_t \mathbf{f} = c(\mathbf{S}\cdot\hat{\mathbf{p}})\mathbf{f} \tag{1.2}$$

$$\nabla \cdot \mathbf{f} = \mathbf{0} \tag{1.3}$$

Here $\hat{\mathbf{p}} = -i\hbar\nabla$ and S^k , k=1,2,3 are 3x3 spin-one matrices. The Hamiltonian $\hat{H} = c(\mathbf{S}\cdot\hat{\mathbf{p}}) \equiv c\hbar\sqrt{-\triangle}\,\hat{\Lambda}$ ($\hat{\Lambda}$ – helicity operator) generates the time evolution

$$\hat{U}(t) = \exp(-\tfrac{it}{\hbar}\hat{H}) = \exp(-ict\sqrt{-\triangle}\,\hat{\Lambda}) =$$

$$= \cos(ct\sqrt{-\triangle})\mathbf{1}_3 - i\sin(ct\sqrt{-\triangle})\,\hat{\Lambda} \tag{1.4}$$

The photon-antiphoton system is represented by 6 the dimensional vectors

$$F = \frac{1}{\sqrt{2}}\begin{bmatrix} \mathbf{f} \\ \mathbf{f}^* \end{bmatrix} \tag{1.5}$$

which leads to the matrix block structure of the following equations. In particular the evolution matrix becomes

$$\hat{\mathcal{U}}(t) = \begin{bmatrix} \hat{U}(t) & \mathbf{0} \\ \mathbf{0} & \hat{U}^{+}(t) \end{bmatrix} \tag{1.6}$$

The states of a given helicity are given by the vectors

$$|F_\lambda\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} \gamma_\lambda \\ \bar{\gamma}_\lambda \end{bmatrix} = |\lambda\rangle\,, \quad \lambda = \pm 1 \tag{1.7}$$

In particular, the matrix elements of the evolution operator are

$$\langle\lambda|\hat{U}(t)|\lambda'\rangle = \tfrac{1}{2}[\langle\gamma_\lambda|C_t|\gamma_\lambda\rangle + \langle\bar{\gamma}_\lambda|C_t^*|\bar{\gamma}_\lambda\rangle]\delta_{\lambda\lambda'} \tag{1.8}$$

with the notations

$$C_t(\mathbf{r}, \mathbf{r}') \equiv C(\mathbf{r}, t; \mathbf{r}', 0) \equiv ict\{\pi[|\mathbf{r}-\mathbf{r}'|^2 - (ct - i\varepsilon)^2]\}^{-2} \tag{1.9}$$

$$t \geq 0, \quad \mathbf{r}, \mathbf{r}' \in \mathbb{R}^3$$

The right-hand side of (1.8) is given by the formula (3.14) of Part 1. It shows that C_t is an integral operator with the kernel given in (1.9).

One recognizes here the probability amplitude density of the quantum Cauchy stochastic process in $\mathbb{R}^3$. It does not depend on $\hbar$, and also on photon's energy or helicity. It is a function of photon's coordinates and time, only.

The same construction can be carried out for any matrix element of the second-quantized free electromagnetic field

$$\mathbf{f}_{\alpha\beta} \equiv \langle\alpha|\hat{\mathbf{E}}|\beta\rangle + i\langle\alpha|\hat{\mathbf{B}}|\beta\rangle \tag{1.10}$$

hence, the quantum Cauchy process underlies, both, the single photon case as outlined above, and the second-quantized field of an arbitrary collection of them, as well.

2. AN ANALYTIC CONTINUATION TO THE EUCLIDEAN DOMAIN

We replace the Minkowski time t with the imaginary Euclidean time τ

Since we are working interchangely in, both, Minkowski and Euclidean domains, we will use the Greek alphabet in the latter case while the Latin alphabet in the former, in order to avoid a confusion. We get for the analytically continued transition density

$$C(\mathbf{r}, -i\tau; \mathbf{r}', 0) \equiv \tilde{C}(\mathbf{r}, \tau; \mathbf{r}', 0) \equiv c\tau\{\pi[|\mathbf{r} - \mathbf{r}'|^2 + c^2\tau^2]^2\}^{-2} \tag{2.2}$$

$$C^*(\mathbf{r}, -i\tau; \mathbf{r}', 0) = - \tilde{C}(\mathbf{r}, \tau; \mathbf{r}', 0) \tag{2.3}$$

Alternatively, in the probabilistic terms

$$d^3\mathbf{r}\, \tilde{C}(\mathbf{r}, \tau ; \mathbf{0}, 0)_{|\tau=it} \equiv P\{\mathbf{r} < \mathbf{c}_\tau \leq \mathbf{r} + d\mathbf{r} |\, \mathbf{c}_0 = \mathbf{0}\}_{|\tau=it}$$

$$= d^3\mathbf{r}\, C(\mathbf{r}, t ; \mathbf{0}, 0) \equiv \text{Prob. Ampl.} \{\mathbf{r} < \mathbf{c}_{it} \leq \mathbf{r} + d\mathbf{r} |\, \mathbf{c}_0 = \mathbf{0}\}$$

or (2.4)

$$d^3\mathbf{r}\, C(\mathbf{r}, t ; \mathbf{0}, 0)_{|t=-i\tau} \equiv \text{Prob. Ampl.} \{\mathbf{r} < \mathbf{c}_{it} \leq \mathbf{r} + d\mathbf{r} |\, \mathbf{c}_0 = \mathbf{0}\}_{|t=-i\tau}$$

$$= d^3\mathbf{r}\, \tilde{C}(\mathbf{r}, \tau ; \mathbf{0}, 0) \equiv P\{\mathbf{r} < \mathbf{c}_\tau \leq \mathbf{r} + d\mathbf{r} |\, \mathbf{c}_0 = \mathbf{0})$$

$$t \geqq 0, \quad \tau \geqq 0, \quad \mathbf{r} \in \mathbb{R}^3$$

On the right "P" stands for the probability of the event. The $\mathbf{c}_{it}$ represents the quantum Cauchy process, while $\mathbf{c}_\tau$ represents the classic Cauchy process as is customerily studied in the theory of probability. In the three dimensions it is a vector composed of the three independent components, viz. $\mathbf{c}_\tau = (c^1_\tau, c^2_\tau, c^2_\tau)^T$, and similarly for two-dimensional case.

The vector $\mathbf{c}_{it}$ stands for a position of a point-like photon at time t, while $\mathbf{c}_{\tau}$ represents the same but in the Euclidean time..

The analytic continuation leaves the trajectories unchanged (probability amplitudes of events are being replaced by their probabilities) so they can be studied using a theory of the classic stochastic processes, as it is in the Feynman-Kac formula case.

3. CONNECTION BETWEEM THE PHOTON PROPAGATORS FROM THE FIRST AND THE SECOND QUANTIZED THEORIES

As it is well known, [3] one gets the photon's propagator in terms of the operator valued 4-potentials $A^{\mu}(x)$

$$\langle 0|T[\hat{A}^{\mu}(x)\hat{A}^{\nu}(y)]|0\rangle = ig^{\mu\nu}D_F(x-y) \qquad \text{(Feynman gauge)} \tag{3.1}$$

where $(g^{\mu\nu}) = \mathrm{diag}(1, -1, -1, -1)$, $c = 1$ and

$$D_F(x) = -(2\pi)^{-4}\int d^4k(k^2 + i\varepsilon)^{-1}\exp(-ikx) = [4\pi^2 i(x^2 - i\varepsilon)]^{-1} \tag{3.2}$$

we find from here the connection

$$\tfrac{2}{c}\partial_t D_F(\mathbf{r}, t; \mathbf{r}', 0) = ict\{\pi[|\mathbf{r} - \mathbf{r}'|^2 - c^2t^2 + i\varepsilon]\}^{-2} = C(\mathbf{r}, t; \mathbf{r}', 0) \tag{3.3}$$

Integrating over time and using (3.2), we get from here the relation

$$2D_F(\mathbf{r}, t; \mathbf{r}',0) = i\{2\pi^2|\mathbf{r} - \mathbf{r}'|^2\}^{-1} + c\int_0^t ds C(\mathbf{r}, s; \mathbf{r}', 0), \qquad \mathbf{r} \neq \mathbf{r}' \tag{3.4}$$

The sum of the probability amplitudes on the right indicates the alternatives which all lead to the passage of a single photon from $\mathbf{r}'$ at t = 0, to $\mathbf{r}$ at the time t. The photon can either jump at the instance 0 to its final destination $\mathbf{r}$, and stay there for the duration of t, or can wait at the initial point until the time t and then jump, (the first, static term). The doubling of events, which is also true for the second term, explains the factor of 2 in front of D_F. The photon can jump at the instant s to its final destination and stay there, or can wait until the instant s and then jump and wait at its final destination $\mathbf{r}$ for the remaining time t - s. Covering the distance $|\mathbf{r} - \mathbf{r}'|$ in time s ($0 < s \leq t$) and waiting for t-s, both lead to the effective velocity v = c. This balance of the instantaneous jumps with the waitng periods appears mysterious. It may point out to the very structure of space itself. Since we are dealing here with the pure vacuum, the effective speed is equal c

$$v = c \tag{3.5}$$

This granular, highly discrete type of behaviour of a photon gets completely obscured when dealing with the electromagnetic potentials satisfying the second order d'Alembert equation. The whole process is graphically represented by a segment of straight line connecting the initial $\mathbf{r}'$ and the final points $\mathbf{r}$. This, toghether with well established view that "**light only knows straight lines**" forms a formidable psychological barrier towards understanding photon's connection with the purely jump Cauchy process.

To clarify this further, we pass to the Euclidean time by the replacement

$$t = -i\tau \tag{3.6}$$

In the one-dimensional case, we will have for the transition probability density

$$(d = 1) \qquad \tilde{C}(x, \tau\, ;\, y, 0) = c\tau[\pi(|x - y|^2 + c^2\tau^2]^{-1} \quad , \qquad x, y \in \mathbb{R}, \ \ \tau \geqq 0 \tag{3.7}$$

For the two spatial dimensions, we get

$$(d = 2) \qquad \tilde{C}(x, \tau\, ;\, y, 0) = c\tau\{2\pi[|x - y|^2 + c^2\tau^2]^{3/2}\}^{-1} , \quad x, y \in \mathbb{R}^2 , \ \ \tau \geqq 0 \tag{3.8}$$

For the three-dimensional case

(d = 3) $$\tilde{C}(x, \tau ; y, 0) = c\tau\{\pi^2[|x - y|^2 + c^2\tau^2]^2\}^{-1} \quad , \quad x, y \in \mathbb{R}^3, \ \tau \geqq 0 \qquad (3.9)$$

In the case of a general d-spatial dimensions the density is, [11]

$$\tilde{C}(x, \tau ; y, 0) = \Gamma(\tfrac{d+1}{2})c\tau[\pi(|x - y|^2 + c^2\tau^2)]^{-\frac{d+1}{2}} \quad , \quad x, y \in \mathbb{R}^d \, , \ \tau \geqq 0 \qquad (3.10)$$

Since the paths structure in classical and quantum case are the same, we are going to review well established mathematical facts regarding the classic Cauchy process. We invoke here the Feynman-Kac formula, [16], that deals with probabilities of various paths rather then with probability amplitudes as in the Feynman integral case. The path themselves stay the same, we assume.

We shall to demonstrate the main features of Cauchy process using the simplest one-dimensional case. For example, the mean value is undetermined unless one requires that the principal value of the integral is calculated, which yields zero

$$E\{c_\tau\} = \frac{c\tau}{\pi}\int_{-\infty}^{\infty} \frac{xdx}{x^2+c^2\tau^2} = \text{an arbitrary real number} \qquad (3.11)$$

All the higher-order moments are divergent

$$E\{c_\tau^n\} = \infty \quad , \qquad n \geqq 2 \qquad (3.12)$$

The transition density satisfies the Chapman-Kolmogorov equation which assures the existence a probability space $(\Omega, \mathcal{F}, P)$ on which the Cauchy process $\{c_\tau : \tau \geqq 0\}$ is defined. The process belongs to a larger family of Markovian processes called - **Le'vy processes**

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4. SOME MATHEMATICAL FACTS REGARDING CAUCHY PROCESS

The Cauchy process $\{c_\tau \, ; \tau \geqq 0\}$ with values in $\mathbb{R}$, has the following main properties:

1. $P\{c_0 = 0\} = 1$ (the process starts at zero, almost surely $\equiv$ a. s.) trajectories that do not start from zero are exceptions, they form a set of zero measure.

2. The process has independent increments (for any $0 \le \tau_1 \le \tau_2 \le \ldots \le \tau_n \le \infty$ the random variables $c(\tau_1)$, $c(\tau_2) - c(\tau_1)$, ... , $c(\tau_n) - c(\tau_{n-1})$ are independent).

3. The process has stationary increments $[c(\tau_k) - c(\tau_{k-1})]$ has the same distribution as $c(\tau_k - \tau_{k-1})$

4. The trajectories ; $\{c(\tau, \omega) \in \mathbb{R}\ ;\ \tau \geqq 0,\ \omega$ - fixed $\}$ are a. s. right continuous with left limits. That is, they possess the **cadlag property** (French acronym from: "continu a droite, limites a gauche").

The first three properties are shared with those of the more familiar Brownian motion process (describing a massive particle), the fourth one is characteristic for the massless photon and thus we would like to explain it in some detail,

$$\lim_{\varepsilon\to 0} c(\tau + \varepsilon) = c(\tau), \quad \varepsilon \geqq 0 \qquad \text{(right continuity)} \tag{4.1}$$

$$\lim_{\varepsilon\to 0} c(\tau - \varepsilon) \text{ exists for any } \tau \geqq 0 \qquad \text{(left limits exist)} \tag{4.2}$$

The point is that the last limit **might not be equal** $\mathbf{c}(\tau)$ in which case the trajectory has a jump at the instant τ

$$J_\tau \equiv \Delta c(\tau) \equiv c(\tau) - \lim_{\varepsilon\to 0} c(\tau - \varepsilon), \qquad \varepsilon \geqq 0 \tag{4.3}$$

It is essential to notice that any Cauchy process has the **cadlag version** (which we can substitute for the original process under the E-sign). In plain English it means that the continuous trajectories (as commonly used in the geometric optics) are exeptional and can be omitted from general considerations as they form a set of zero measure. A photon can disappear at one point to be **instantaneously** recreated at some other point. **The spooky action at a distance**, Einstein's nightmare, seems to be real after all. In order to see it more clearly, we need to mention a notion of the infinite divisibility of the c_τ. Using the property #3, we get the equivalence under the E-sign

$$c_\tau \equiv c_\delta + (c_{2\delta} - c_\delta) + \ldots + (c_\tau - c_{(n-1)\delta}) \backsim n c_\delta \tag{4.4}$$

$$\delta = \tfrac{\tau}{n}, \qquad n \in \mathbb{N} \tag{4.5}$$

The probability law of c_τ is n-time convolution of that of c_δ. Since n is arbitrary, it has an interesting analytic consequence in the limit when $n \to \infty$, as encapsulated in the **Le'vy-Khinchine formula**

$$E\{\exp[iu(c\tau + c_\tau)]\} \equiv \exp[-\tau\psi(u)] \tag{4.6}$$

with the characteristic exponent

$$\psi(u) = -i\,cu + \int_{\mathbb{R}} (1 - e^{iux} + iux\mathbf{1}_{(|x|<1)})\nu(dx) \tag{4.7}$$

The **Le'vy measure** - ν has no δ- type singularity at zero and satisfies the conditions

$$\begin{aligned} &\text{a.} \qquad \int_{-1}^{1} x^2 \nu(dx) < \infty \\ &\text{b.} \qquad \nu\{\mathbb{R}\backslash(-1,1)\} = \int_{-\infty}^{-1} \nu(dx) + \int_{1}^{\infty} \nu(dx) < \infty \end{aligned} \tag{4.8}$$

The measure can be found from the trasition density as the weak limit

$$\nu(dx) = \lim_{\tau\to 0} \tfrac{1}{c\tau} \tilde{C}(x, \tau\,;\, 0, 0)dx = \tfrac{dx}{\pi x^2}, \quad x \in \mathbb{R} \tag{4.9}$$

For a general d-dimensional case, we obtain

$$\nu(\mathrm{dx}) = \Gamma(\tfrac{d+1}{2})\{\pi[(x^1)^2+\ldots+(x^d)^2]\}^{-\frac{d+1}{2}}dx^1\ldots dx^d \quad , \quad \mathrm{x}\in\mathbb{R}^d \tag{4.10}$$

The conditions (4.8) are satisfied by the Cauchy process

a. $$\int_{-1}^{1}\mathrm{x}^2\nu(\mathrm{dx}) = \tfrac{2}{\pi} < \infty \tag{4.11}$$

b. $$\nu\{\mathbb{R}\backslash(-1,1)\} = 2\int_{1}^{\infty}\nu(\mathrm{dx}) = \tfrac{2}{\pi} < \infty \tag{4.12}$$

The measure does not contain any terms like $\delta(\mathrm{x})$ or its derivatives (has no "atom" at $\mathrm{x}=0$, in mathematicians parlance) thus $\nu\{0\}=0$ and ν qualifies as the Le$'$vy measure. Finally, we calculate the Le$'$vy measure of the whole space $\mathbb{R}$

$$\nu(\mathbb{R}) = \int_{\mathbb{R}}\nu(\mathrm{dx}) = \tfrac{2}{\pi}\int_0^{\infty}\tfrac{dx}{x^2} = \infty \tag{4.13}$$

This means that the process' **activity** is **high**; the amount of jumps in any finite time interval is **countably infinite**. Almost all trajectories of c_τ have the countable infinite number of jumps in any compact time interval. In the sequel we shall consider the sum of a relativistic drift, $X_\tau^{(1)} = c\tau$ and the Cauchy process, $c_\tau = X_\tau^{(3)}$. The second component, $X_\tau^{(2)} = \sigma B_\tau$ is usually reserved for the Brownian motion which is absent in our case. Finally, the last piece in the Le$'$vy-Itô decomposition, $X_\tau^{(4)}$, is reserved for, so called, **compensated compound Poisson process**. In order to see it more clearly, it is customery to rewrite the Le$'$vy - Khinchine formula (4.7) as follows

$$\psi(\mathrm{u}) = \psi^{(1)}(\mathrm{u}) + \psi^{(3)}(\mathrm{u}) + \psi^{(4)}(\mathrm{u}) \tag{4.14}$$

where each part represents specific process

$$\psi^{(1)}(u) = -i\,cu \qquad \text{(the drift)} \qquad (4.15)$$

$$\psi^{(3)}(u) = \int_{\mathbb{R}} (1 - e^{iux}) \mathbf{1}_{(|x| \geq 1)}(x) \nu(dx) \qquad \text{(big jumps)} \qquad (4.16)$$

$$\psi^{(4)}(u) = \int_{\mathbb{R}} (1 - e^{iux} + iux) \mathbf{1}_{(|x| < 1)}(x) \nu(dx) \quad \text{(small jumps)} \qquad (4.17)$$

The split into the big jumps and small jumps follows the argument of the indicator function $\mathbf{1}_A$. All three pieces are generated by three independent processes: the linear (non-random) drift, $X_\tau^{(1)} = c\tau$, the **compound Poisson process**, $\tilde{N}_\tau$(for the big jumps), and the **compensated compound Poisson process**, $\check{N}_\tau$(for the small jumps).
The corresponding **Le′vy - Itô decomposition**: $X_\tau = X_\tau^{(1)} + X_\tau^{(3)} + X_\tau^{(4)}$splits the process into independent from one another pieces. In general, both, $X_\tau^{(1)}$ and $X_\tau^{(2)}$ (if present) have continuous trajectories, while $X_\tau^{(3)}$ and $X_\tau^{(4)}$ are pure jump processes with discontinuous trajectories.

So called **stable** processes display the **fractal structure** of their trajectories manifested in their scalling properties

$$b_{a\tau} \backsim \sqrt{a}\; b_\tau \qquad (4.18)$$

$$c_{a\tau} \backsim a\, c_\tau, \qquad a \geqq 0 \qquad , \tau \geqq 0 \qquad (4.19)$$

It is perhaps worth mentioning that in the quantum case, when a $= i$, the (4.18) actually expresses the process in terms of the familiar classic Brownian motion. The right covariance is reproduced which leads to the right higher-order momenta, [15].

For a general Le′vy process with the stability parameter $\alpha \in [0, 2)$, we have

$$L_{a\tau}^{(\alpha)} \backsim a^{1/\alpha} L_\tau^{(\alpha)} \qquad a \geqq 0 \qquad (4.\,20)$$

To clarify the structure of the $X^{(3)}$ and $X^{(4)}$ processes, we will review some basics regarding the Poisson process. We start with the **Poisson random variable** N, first. It assumes equidistantly spaced discrete values, only. Its distribution function is given by the formula

$$P\{N \leq x\} = \begin{cases} e^{-\lambda}\frac{\lambda^n}{n!} & x = n = 0, 1, 2, \ldots, \\ 0 & \text{otherwise} \end{cases} \qquad (4.21)$$

So the distribution function vanishes for $x < 0$, and it is supported on the naturals. The only parameter $\lambda \geqq 0$ is called the random variable **intensity**. The random variable N possesses the characteristic function

$$E\{e^{iuN}\} = \sum_{n=0}^{\infty} E\{e^{iuN}|N = n\}e^{-\lambda}\frac{\lambda^n}{n!} = \sum_{n=0}^{\infty} e^{iun}e^{-\lambda}\frac{\lambda^n}{n!} = \exp[-\lambda(1 - e^{iu})] \qquad (4.22)$$

$$u \in \mathbb{R}$$

We can find from here any moment of N by differentiating and equating the parameter u to zero. For example, the mean value of N is

$$E\{N\} = -i\frac{d}{du}\exp[-\lambda(1 - e^{iu})]_{||u=0} = \lambda \qquad (4.23)$$

Next, we shall consider the **Poisson process** $\{N_\tau : \tau \geqq 0\}$ where each N_τ is the Poisson variable. It shares the properties 1- 4 of the Cauchy process listed above. In addition, for each $\tau \geqq 0$, N_τ is equal in distribution to a Poisson random variable with the linear intensity $\lambda c\tau$. Therefore, its characteristic function is given by

$$E\{e^{iuN_{c\tau}}\} = \exp[-c\tau\lambda(1 - e^{iu})], \qquad u \in \mathbb{R}, \tau > 0 \qquad (4.24)$$

Next, we would like to introduce the **compound Poisson process** which is defined as a random sum of the form

$$\tilde{N}_{c\tau} \equiv \sum_{k=1}^{N_{c\tau}} J_k , \qquad \tau \geqq 0 \tag{4. 25}$$

where $\{N_{c\tau} ; \tau \geqq 0\}$ is the Poisson process with intensity $\lambda c\tau$ and $\{J_k ; k=1, 2, ...\}$ is a familly independent, identically distributed random variables that are also independent of the Poisson random variables $N_{c\tau}$. Their common probability law is denoted F and describes the distribution of the size of jumps, (and it is assumed further that it has no "atom" at zero). The compound Poisson process $\{\tilde{N}_{c\tau} ; \tau \geqq 0\}$ has stationary independent increments, and its characteristic function, for $\tau \geqq 0$, can be found as follows:

$$E\{e^{iu\tilde{N}_{c\tau}}\} = \sum_{n=0}^{\infty} E\{\exp[iu\sum_{k=1}^{N_{c\tau}} J_k | N_\tau = n\} e^{-\lambda c\tau} \frac{(\lambda c\tau)^n}{n!} =$$

$$= \sum_{n=0}^{\infty} (E\{e^{iuJ_1}\})^n e^{-\lambda c\tau} \frac{(\lambda c\tau)^n}{n!}$$

$$= \sum_{n=0}^{\infty} [\int_{\mathbb{R}} e^{iux} F(dx)]^n e^{-\lambda c\tau} \frac{(\lambda c\tau)^n}{n!}$$

$$= \exp\{-c\tau\lambda[\int_{\mathbb{R}} (1 - e^{iux}) F(dx)]\} \tag{4. 26}$$

The Poisson process, much like the Brownian motion process for a massive particle, both are main processes out of which various other processes can be constructed.

Comparing this expression with (4.16), we find that the probabilistic measure F(dx) and the intensity λ are given by (cf. (4.12))

$$\lambda = \nu\{\mathbb{R}\backslash(-1, 1)\} = \frac{2}{\pi} \tag{4. 27}$$

and the distribution F(dx) is given by

$$F(dx) = \lambda^{-1}\nu(dx)\mathbf{1}_{(|x|\geqq 1)}(x) = \frac{dx}{2x^2}\mathbf{1}_{(|x|\geqq 1)}(x) \tag{4. 28}$$

This is indeed the probabilistic measure since it has the correct normalization

$$\int_{-\infty}^{\infty} F(dx) = \int_{-\infty}^{-1} \frac{dx}{2x^2} + \int_{1}^{\infty} \frac{dx}{2x^2} = \int_{1}^{\infty} \frac{dx}{x^2} = 1 \tag{4. 29}$$

Finally, the last term of (4.17) that corresponds to the small jumps, can be written as follows

$$\psi^{(4)}(u) = \int_{|x|<1} (1 - e^{iux} + i\, ux)\nu(dx) = \sum_{n=0}^{\infty} \lambda_n \int_{D_n} (1 - e^{iux}) F_n(dx) +$$

$$+ i\, u \sum_{n=0}^{\infty} \lambda_n \int_{D_n} x\, F_n(dx) \tag{4. 30}$$

where the domains D_n are covering the region of integration $\{|x| \geqq 1\}$, and are disjoint for the different indecies, and are symmetric with respect to zero, viz.

$$D_n \equiv \{x : 2^{-(n+1)} \leq |x| < 2^{-n}\}, \qquad n = 0, 1, 2, \ldots \tag{4. 31}$$

$$\bigcup_{n=0}^{\infty} D_n = (-1, 1) \tag{4. 32}$$

$$D_m \cap D_n = \varnothing, \qquad m \neq n \tag{4. 33}$$

The intensities λ_n are defined as

$$\lambda_n \equiv \nu(D_n), \qquad n = 0, 1, 2, \ldots \tag{4.34}$$

Finally the small jump distributions $F_n(dx)$ are supported on $D_n^{'}$s

$$F_n(dx) \equiv \lambda_n^{-1} \nu(dx)\, \mathbf{1}_{D_n}(x), \qquad n = 0, 1, 2, \ldots \tag{4. 35}$$

It is easy exercise to verify that they are probabilistic measures (normalized to unity).
In the case at hand of the Cauchy process with the Le'vy measure given in (4. 9), we find that the last term in the formula (4. 30), which gives the name "compensating", vanishes. As the result, we get a sum of compound Poisson processes of the same type as for the big jumps. The intensities are

$$\lambda_n = \frac{2^{n+1}}{\pi}, \qquad n = 0, 1, 2, \ldots \tag{4. 36}$$

and the distributions of the jum sizes are

$$F_n(dx) = \frac{dx}{2^{n+1}x^2} \mathbf{1}_{D_n}(x), \qquad n = 0, 1, 2, \ldots \tag{4. 37}$$

Result is that the fourth component of the Le'vy - Itô decomposition is given by the infinite sum of the compound Poisson processes of the same type as that given in (4. 25) for the big jumps

$$X^{(4)}_\tau = \sum_{n=0}^{\infty} \tilde{N}^{(n)}_{c\tau}, \qquad \tau \geqq 0 \tag{4. 38}$$

Hence, the Cauchy process has the following decomposition into independent parts

$$c_\tau = X^{(3)}_\tau + X^{(4)}_\tau = \tilde{N}_{c\tau} + \sum_{n=0}^{\infty} \tilde{N}^{(n)}_{c\tau}, \qquad \tau \geqq 0 \tag{4.39}$$

Neglecting the linear in u term in the formula (4. 17) can be justified as it leads to a correct value of the integral. We have

$$\psi(u) = -i\, cu + \frac{2}{\pi} \int_{\mathbb{R}} (1 - e^{iux}) \frac{dx}{2x^2} \tag{4.40}$$

Using Euler formula, discarding the sin(ux) term due to its antisymmetry, and applying the basic trigonometric identities, the last integral's principal value is of the form

$$\frac{2}{\pi}\int_{\mathbb{R}}(1 - e^{iux})\frac{dx}{2x^2} = |u|\frac{2}{\pi}\int_0^\infty \frac{\sin^2 x}{x^2}dx \qquad (4.41)$$

Integrating by parts, we find, [14]

$$\int_0^\infty \frac{\sin^2 x}{x^2}dx = \int_0^\infty \frac{\sin x}{x}dx = \frac{\pi}{2} \qquad (4.42)$$

Hence, we obtain for the characteristic exponent

$$\psi(u) = -i\, cu + |u| \qquad (4.43)$$

which corresponds to the sum of the relativistic drift and the Cauchy process, [9].

$$X_\tau = c\tau + c_\tau, \qquad \tau \geqq 0 \qquad (4.44)$$

where c_τ is given in (4.39). The first term representing a ray coming from outside while the second describs instataneous jumps (both big and small). In the three-dimensional case

$$\mathbf{X}_\tau = c\tau\mathbf{n} + \mathbf{c}_\tau, \qquad \tau \geqq 0 \qquad (4.45)$$

where $\mathbf{n}$ is a unit vector showing ray's direction while the vector $\mathbf{c}_\tau$ is composed of three independent processes $c_\tau^{(k)}$, k = 1, 2, 3 as given in (4.39), with all the random components independent from each other. This leads to a **broadening of a ray in the vacuum**. It would be interesting to see how it mixes up with the diffraction phenomenon.

5. CONCLUDING REMARKS

Bell's theorem and quantum entanglement seemed to suggest that one could use quantum theory to act at a distance, instantly. Nudge a particle here and its partner would instantaneously dance over there, regardless of whether it was nano-meters or light-years away.

J. S. Bell

The instantaneously jumping photon is acting here. Baas, A. et al., [25], testing the speed of *spooky action at a distance* concluded that if the photons had communicated, they must have done so at least **100,000 times faster than the speed of light**.
The teleportation has been established over large distances (over 150 km).Needless to say that we feel encouraged in our speculations by these mind boggling experimental findings.

Our analysis of the corpuscular aspect of a single photon relies on Maxwell equations and employs rather advanced (Le′vy-Itô decomposition) **probabilistic tools**. Not quite Einstein′s favored, one might add ("God does not play dice"). Getting out of the long shadow casted by the master will be facilitated by better teaching practices in explaining probability theory mainly to physicists, coupled with further honning of the experimental techniques, particularly those of reliably handling the single quanta.

Omitted proofs of various statements can be find in the cited mathematical literature concerning Le′vy processes. The relevant Internet websites are also recommended.

It goes without saying that one needs to investigate further the matter outlined above. At may age (of nearly 77), I have decided to make my thoughts public hoping that they are "crazy enough" to much the recent experimental findings.

APPENDIX

In view of their possible practical relevance, we would like to work out the basic formulae for the two and three spatial dimensions. Using the formula (4.9) we get from (3.8)

$$\nu(dx, dy) = \frac{dxdy}{2\pi(x^2+y^2)^{3/2}} = F(dx, dy), \qquad x, y \in \mathbb{R} \qquad (A.1)$$

We need to check the conditions:

$$\text{a. } \int_{x^2+y^2<1}(x^2+y^2)\nu(dx,dy) = \frac{1}{2\pi}\int_0^{2\pi}d\theta\int_0^1 d\rho = 1 < \infty \tag{A.2}$$

$$\text{b. } \nu\{\mathbb{R}^2\backslash(x^2+y^2)^{1/2}< 1\} \equiv \lambda = \frac{1}{2\pi}\int_0^{2\pi}d\theta\int_1^{\infty}\frac{d\rho}{\rho^2} = 1 < \infty \tag{A.3}$$

Since, in addition, ν does not contain of any δ- type singularities at zero, it qualifies as a Le'vy measure. The resulting Cauchy process on the plane is of **high activity** since the following integral diverges

$$\nu\{\mathbb{R}^2\} = \int_{\mathbb{R}^2}\nu(dx, dy) = \frac{1}{2\pi}\int_0^{2\pi}d\theta\int_0^{\infty}\frac{d\rho}{\rho^2} = \infty \tag{A.4}$$

The third term in (4.16) does not contribute due to the anti-symmetry of the integrand. We shall consider the superposition of a relativistic shift in a direction of a unit vector **n**, and the (symmetric) Cauchy random jump process. We look for its characterictic function

$$E\{\exp[i\mathbf{u}\cdot(cnt + \mathbf{c}_t)]\} = \exp[-t\psi(\mathbf{u})] , \quad \mathbf{u} \in \mathbb{R}^2 \tag{A.5}$$

$$\psi(\mathbf{u}) = -i\, c\mathbf{u}\cdot\mathbf{n} + \int_{\mathbb{R}^2}[1 - \exp(i\mathbf{u}\cdot\mathbf{z})]\nu(d^2\mathbf{z}) \tag{A.6}$$

$$\mathbf{z} \equiv (x, y)^T$$

Again, discarding the sin($\mathbf{u}\cdot\mathbf{z}$) term on account of the symmetry considerations, we find the last integral using polar coordinates

$$\int_{\mathbb{R}^2}[1 - \cos(\mathbf{u}\cdot\mathbf{z})]\nu(d^2\mathbf{z}) = \frac{4}{\pi}\int_0^{\frac{\pi}{2}}d\theta\int_0^{\infty}d\rho\frac{\sin^2(\rho\frac{u}{2}\cos\theta)}{\rho^2} \tag{A.7}$$

where we denoted $||\mathbf{u}|| \equiv u$, and we replaced the integral over the full period 2π

with four integrals over the first quadrant. This is permissible since the sign of cosθ plays no role under the $\sin^2$ function. Changing the variables of integration over ρ

$$\rho \frac{u}{2} \cos\theta = x \tag{A.8}$$

we find with the help of (4.28)

$$\psi(\mathbf{u}) = -i\, c\mathbf{u}\cdot\mathbf{n} + u, \qquad \mathbf{u} \in \mathbb{R}^2 \tag{A.9}$$

$$u \equiv \sqrt{(u^1)^2 + (u^2)^2} = \sqrt{u_x^2 + u_y^2} \tag{A.10}$$

which validates our calculations and shows independence of the relativistic shift and the superimposed pure jump Cauchy process.

In the three-dimensional case we use the formula (4.10) at d = 3

$$\nu(dx, dy, dz) = \frac{dxdydz}{\pi^2(x^2+y^2+z^2)^2}, \qquad x, y, z \in \mathbb{R} \tag{A.11}$$

Checking the conditions (A.2) and (A.3), we find in the spherical coordinates

$$x = r\sin\theta\cos\phi,\ y = r\sin\theta\sin\phi,\ z = r\cos\theta$$

$$0 \le r \le \infty, \quad 0 \le \theta \le \pi, \quad 0 \le \phi \le 2\pi$$

$$dx\, dy\, dz = r^2 dr \sin\theta\, d\theta\, d\phi$$

$$\text{a. } \int_{r<1} (x^2 + y^2 + z^2)\nu(dx, dy, dz) = 4 < \infty \tag{A.12}$$

$$\text{b. } \nu\{\mathbb{R}^3 \backslash (r < 1)\} = \nu\{r \geq 1\} \equiv \lambda = \frac{4}{\pi} < \infty \tag{A.13}$$

These, together with the absence of an "atom" at the origin, assures that ν is indeed a Le$'$vy measure. The Cauchy process is of high activity since

$$\nu\{\mathbb{R}^3\} = \int_{\mathbb{R}^3} \nu(\mathrm{dx, dy, dz}) = \infty \tag{A.14}$$

As before, we consider the superposition of the relativistic drift in a direction given by the unit vector **n** and the pure jump Cauchy process $\mathbf{c}_t$

$$\mathrm{X}_\tau = \mathrm{c}\tau\mathbf{n} + \mathbf{c}_\tau \,, \qquad \tau \geqq 0 \tag{A.15}$$

We would like to find the characteristic function of this process as given by the Le$'$vy-Khinchine formula

$$\mathrm{E}\{\exp[i\mathbf{u}\cdot(\mathrm{c}\tau\mathbf{n} + \mathbf{c}_\tau)]\} = \exp[-\tau\psi(\mathbf{u})] \tag{A.16}$$

where the characteristic exponent is given by the formula like (A.6)

$$\psi(\mathbf{u}) = -i\,\mathrm{c}\mathbf{u}\cdot\mathbf{n} + \int_{\mathbb{R}^3}[1 - \exp(i\mathbf{u}\cdot\mathbf{v}) + i\mathbf{u}\cdot\mathbf{v}\,\mathbf{1}_{(|\mathrm{x}|<1)}(\mathbf{v})]\nu(\mathrm{d}^3\mathbf{v}) \tag{A.17}$$

where we have denoted

$$\mathbf{v} \equiv (\mathrm{x, y, z})^T, \qquad \mathrm{r} \equiv ||\mathbf{v}|| \equiv (\mathrm{x}^2 + \mathrm{y}^2 + \mathrm{z}^2)^{1/2} \tag{A.18}$$

The last term can be dropped on the account of the symmetry considerations while the remaining integral can be brought to the form

$$\int_{\mathbb{R}^3}[1 - \exp(i\mathbf{u}\cdot\mathbf{v})]\nu(\mathrm{d}^3\mathbf{v}) = \frac{2}{\pi^2}\int_{\mathbb{R}^3}\sin^2(\tfrac{1}{2}\mathbf{u}\cdot\mathbf{v})\frac{\mathrm{d}^3\mathbf{v}}{r^4} \tag{A.19}$$

Choosing the z-axis along the vector **u**, we obtain without the loss of generality

$$u_x = u_y = 0 \text{ and } u_z = ||\mathbf{u}|| \equiv u \tag{A.20}$$

Passing to the spherical coordinates and using the substitution in the integral over r

$$\mathbf{u}\cdot\mathbf{v} = ur\cos\theta \equiv 2\xi \tag{A.21}$$

, and the result (4.28), we find easily that the integral (A.19) equals u. Hence,

$$\psi(\mathbf{u}) = -ic\mathbf{u}\cdot\mathbf{n} + u \tag{A.22}$$

as expected. Finally, the probability distribution of photon's jumps becomes

$$F(dx, dy, dz) = \lambda^{-1}\nu(dx, dy, dz) = \frac{1}{4\pi}\frac{dxdydz}{(x^2+y^2+z^2)^2} \tag{A.23}$$

Using the measure F, we can estimate the average size of a jump. In the all three cases (4.24), (A.1), and (A.23) we find that mean jump is an arbitrary real number

$$E_F\{J\} = \int_{-\infty}^{\infty} xF(dx) = \frac{1}{2}\int_{-\infty}^{\infty}\frac{dx}{x} = \text{an arbitrary real number} \tag{A.24}$$

and similarly for the jumps mean size in the y and z directions. This is another way of saying that an issue of a position operator for a photon is not a simple matter. An evolution of the notion of the localizability of quantum systems can be found in in [26], [27], [28], and [29].

Recent review of a role of the Riemann-Silberstein vector in electromagnetism ,and especially for the wave function aspect of a photon see [30].

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