Dynamics of Heterogeneous Voter Models

Sid Redner (physics.bu.edu/~redner)

Nonlinear Dynamics of Networks UMD April 5-9, 2010

T. Antal (BU→Harvard), N. Gibert (ENSTA), N. Masuda (Tokyo),
M. Mobilia (BU→Leeds), V. Sood (BU→NBI), D. Volovik (BU)

NSF DMR0535503 & DMR0906504

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The classic voter model

3 basic results

Voting on complex networks

T. Antal, V. Sood

new conservation law
two time-scale route to consensus
short consensus time

Strategic voting (>2 states)

M. Mobilia, D. Volovik

long time-scale switching

Partisan voting

N. Masuda, N. Gibert

selfishness vs. collectiveness
ultraslow evolution

Classic Voter Model

Clifford & Sudbury (1973)
Holley & Liggett (1975)

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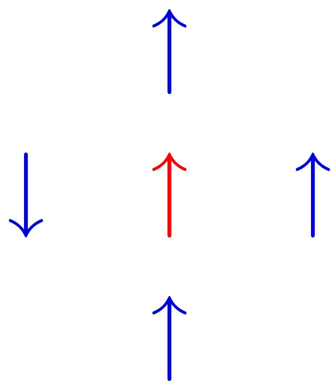

0. Binary voter variable at each site i

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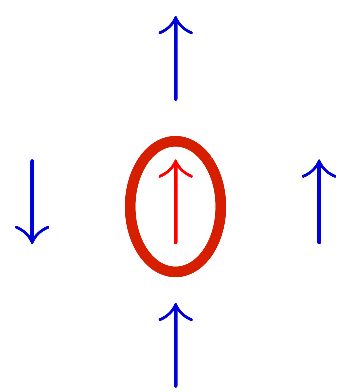

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1. Pick a random voter

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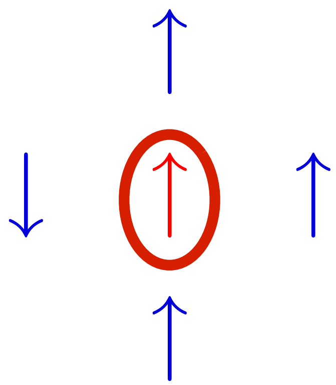

0. Binary voter variable at each site i
1. Pick a random voter
2. Assume state of randomly-selected neighbor
   individual has no self-confidence & adopts neighbor's state

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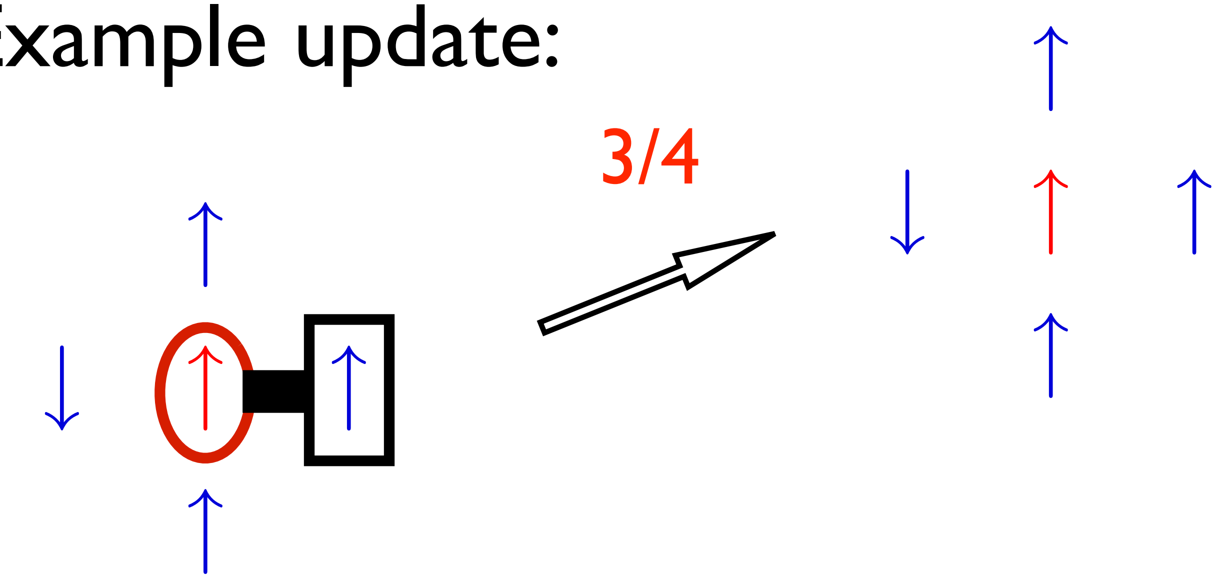
Example update:



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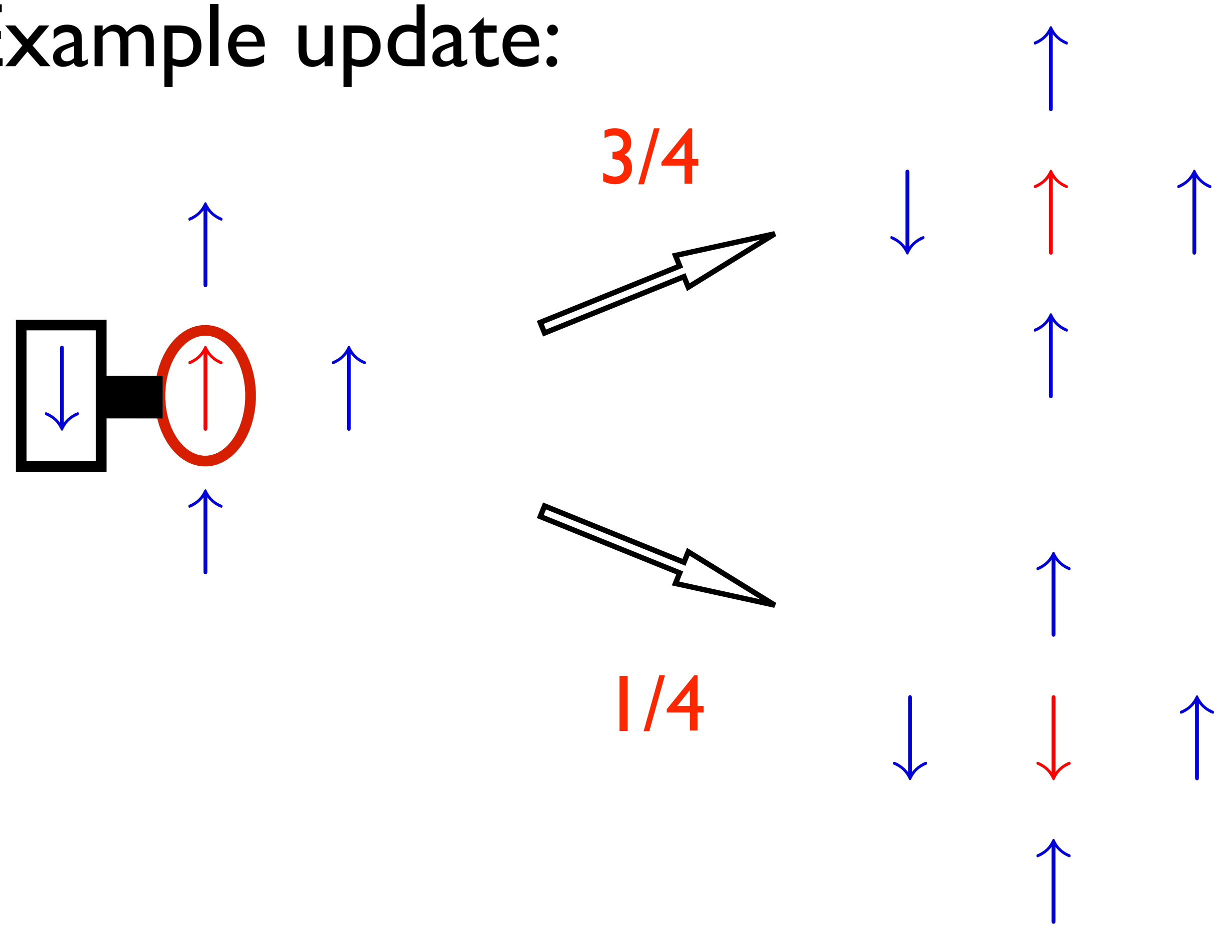

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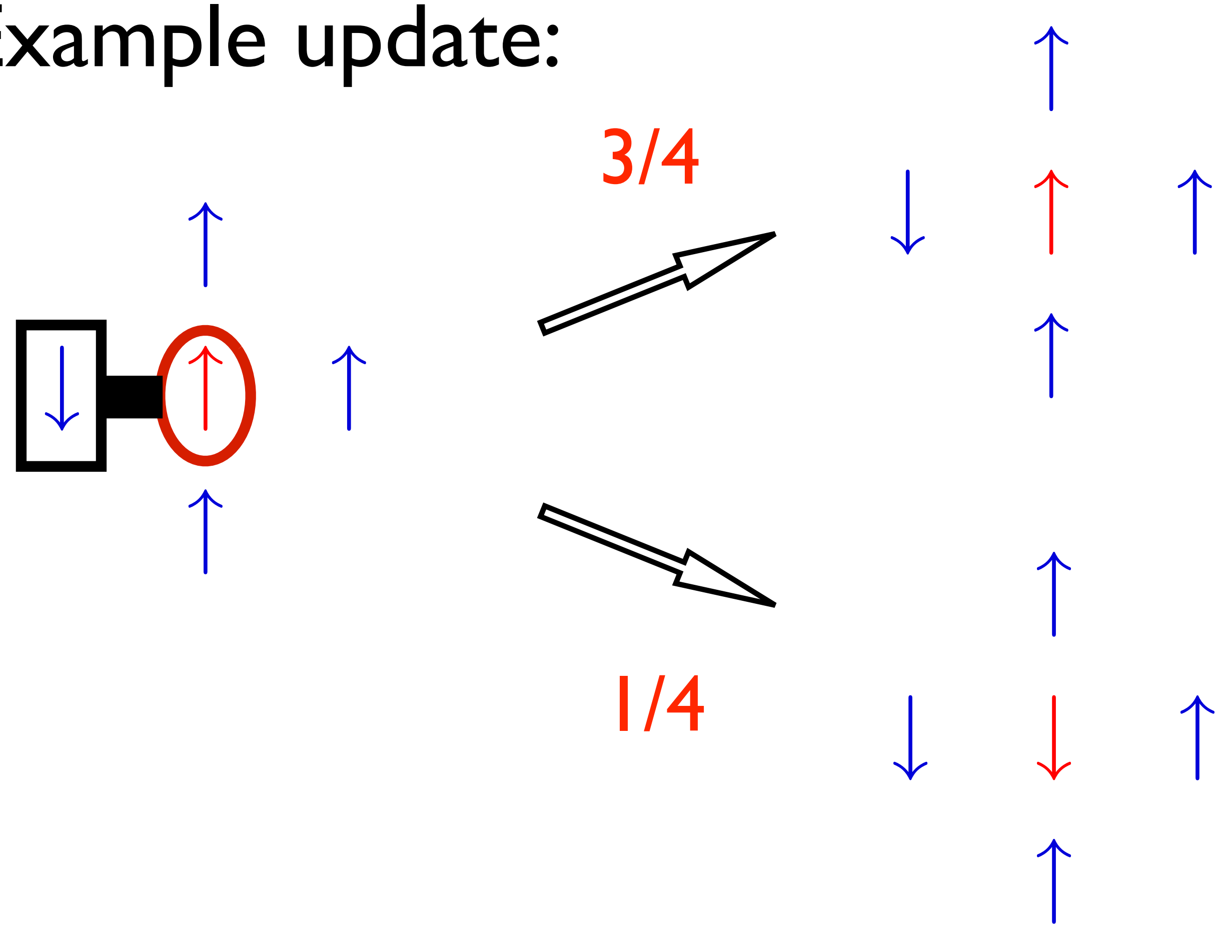

proportional rule

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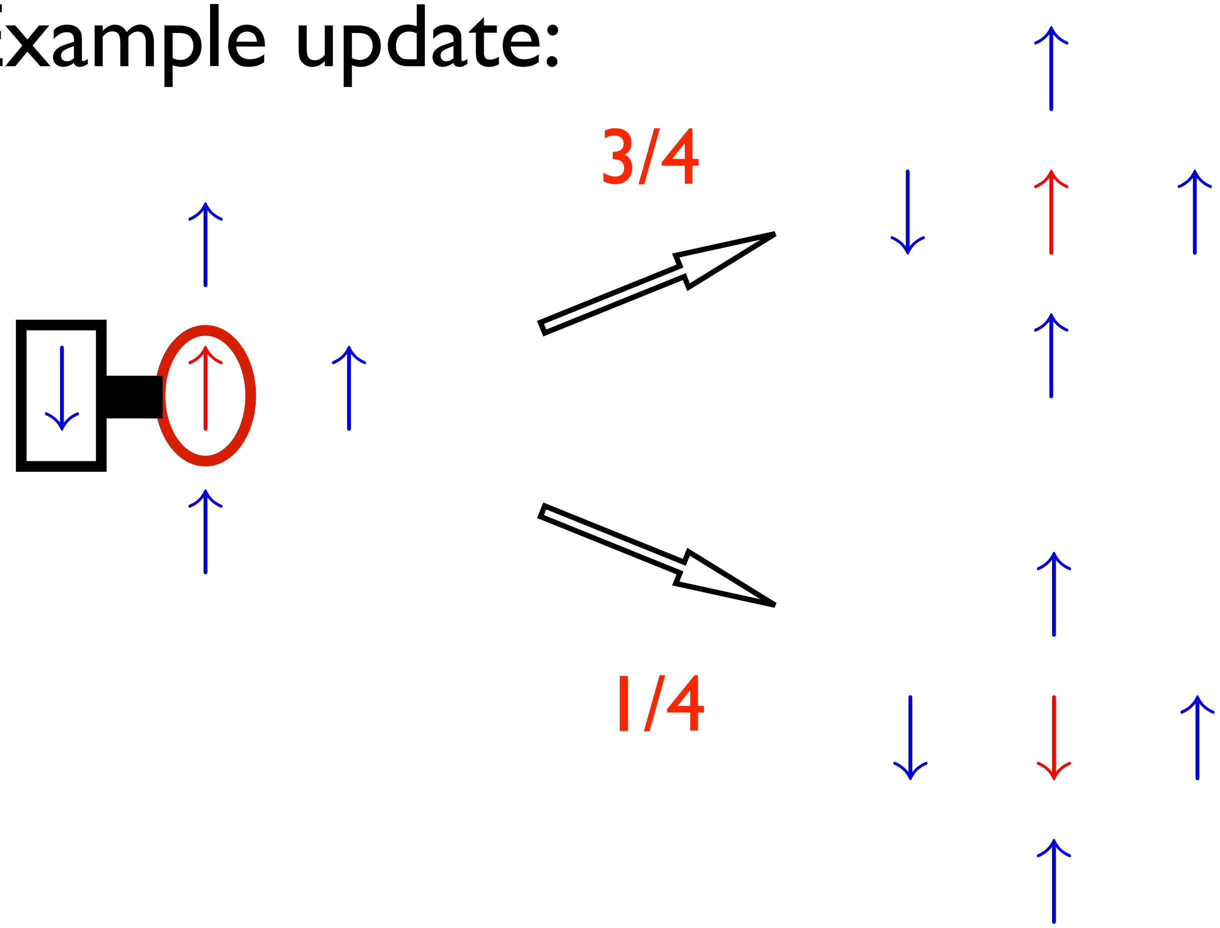


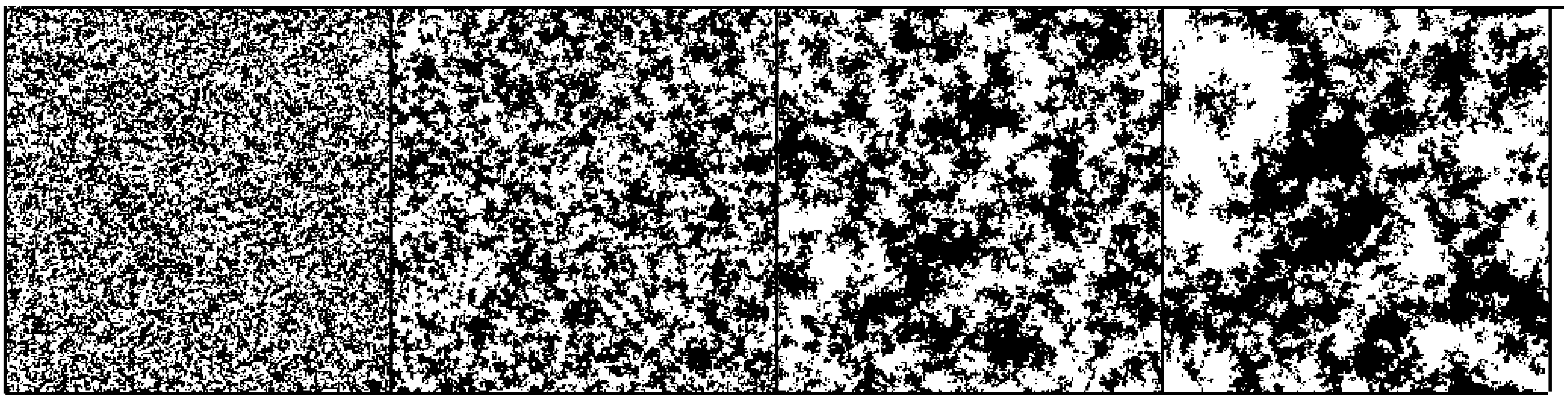
proportional rule

0. Binary voter variable at each site i
1. Pick a random voter
2. Assume state of randomly-selected neighbor
   individual has no self-confidence & adopts neighbor's state
3. Repeat 1 & 2 until consensus *necessarily* occurs in a finite system

Voter Model Evolution

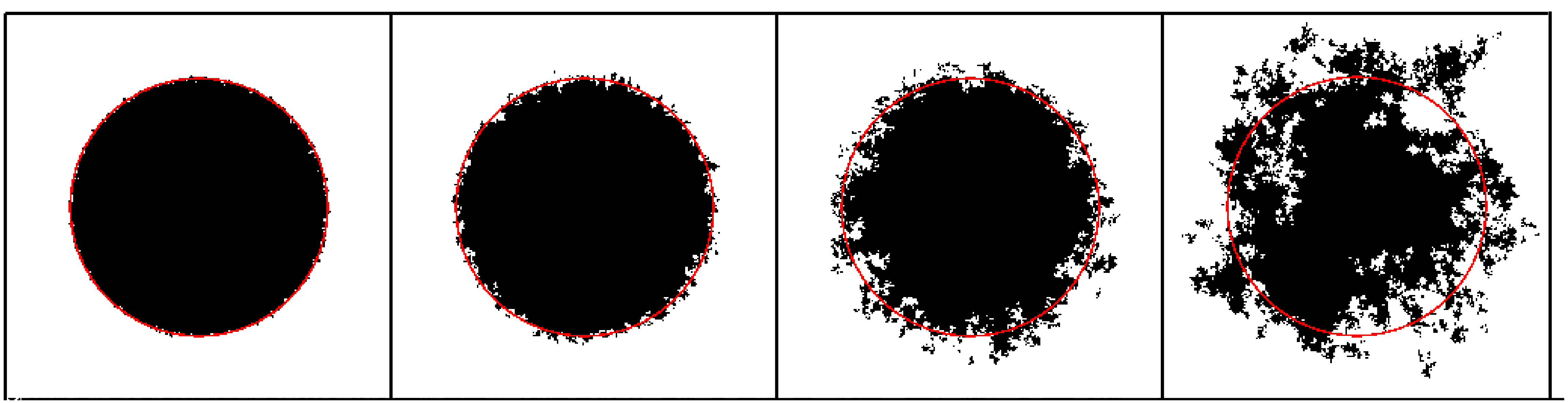
Dornic et al. (2001)

random initial condition:



t=4 t=16 t=64 t=256

droplet initial condition:



Lattice Voter Model: 3 Basic Properties

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1. Final State (Exit) Probability $\mathcal{E}(\rho_0)$

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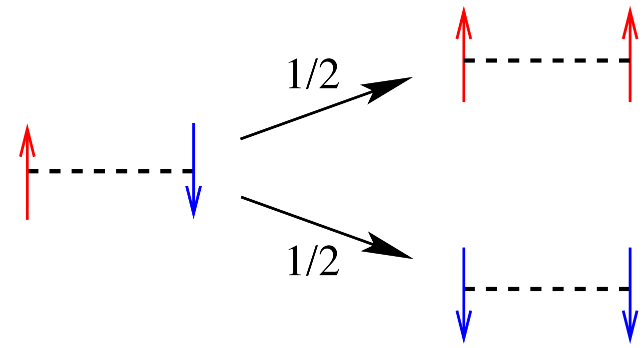
Evolution of a single active link:



average magnetization conserved

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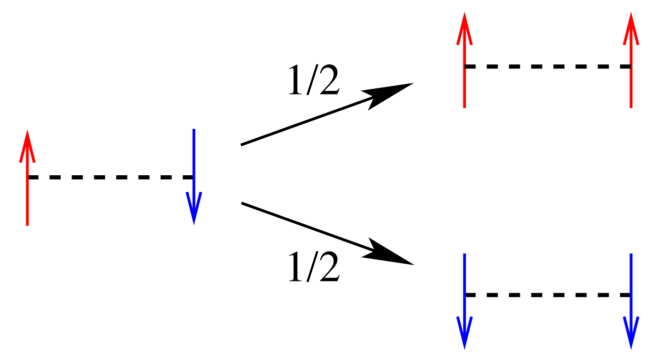

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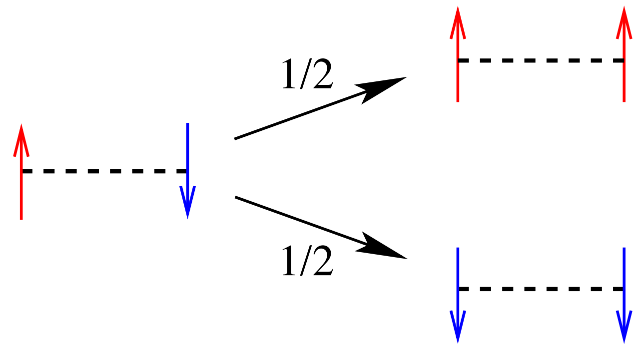

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2. Two-Spin Correlations

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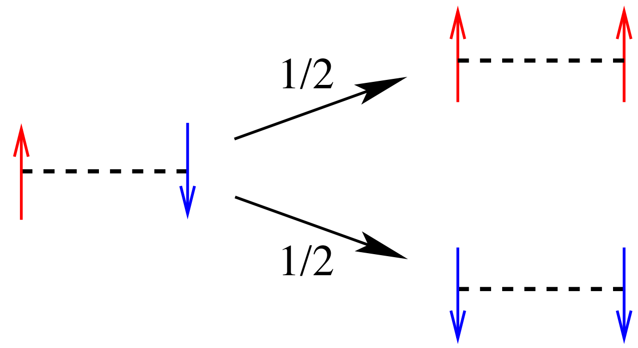

average magnetization conserved

2. Two-Spin Correlations

$$\frac{\partial c_2(\mathbf{r},t)}{\partial t} = \nabla^2 c_2(\mathbf{r},t)$$

$c_2(r=0,t) = 1$

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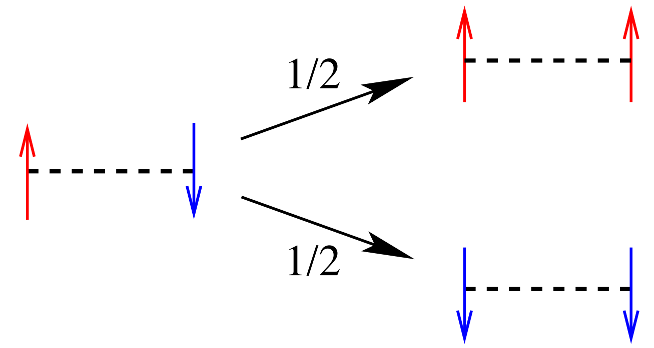

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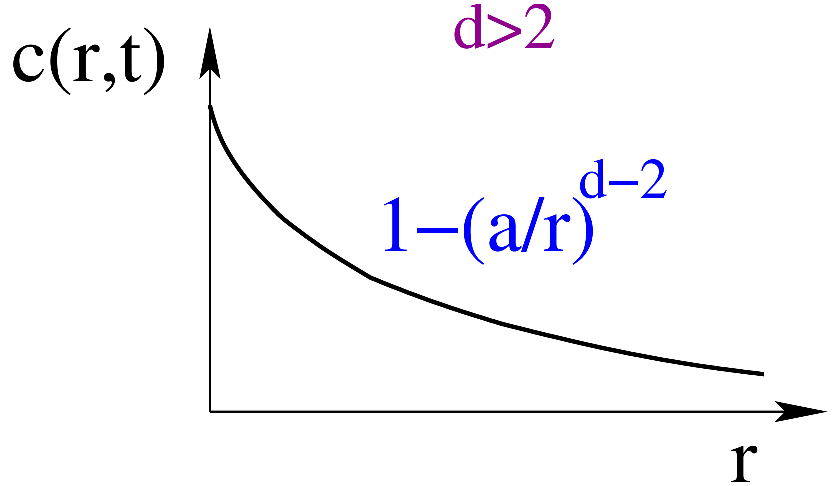

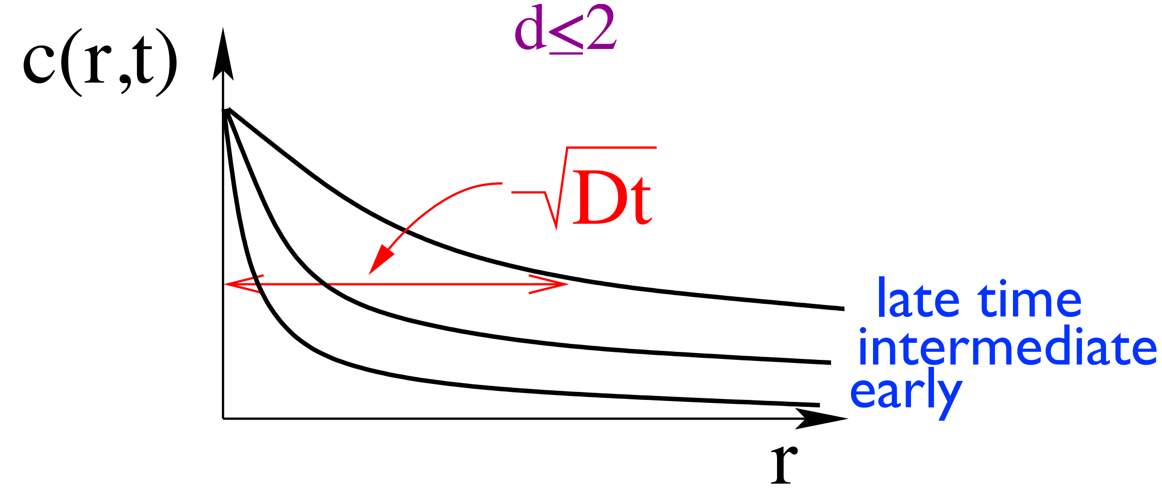

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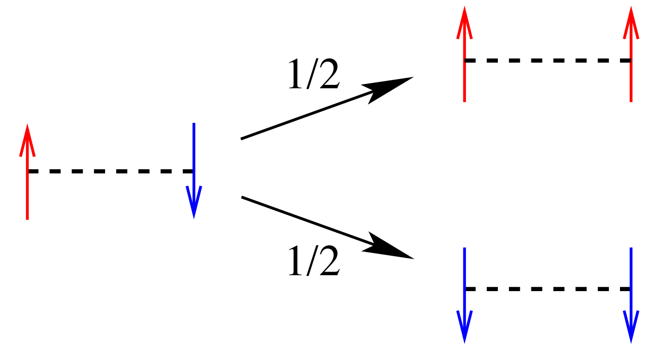

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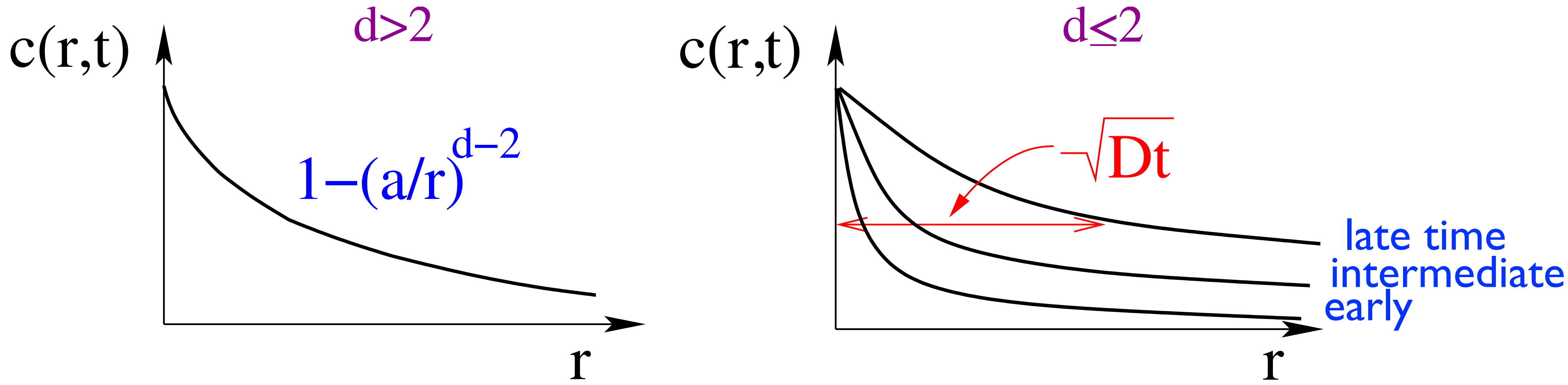

3. Consensus Time

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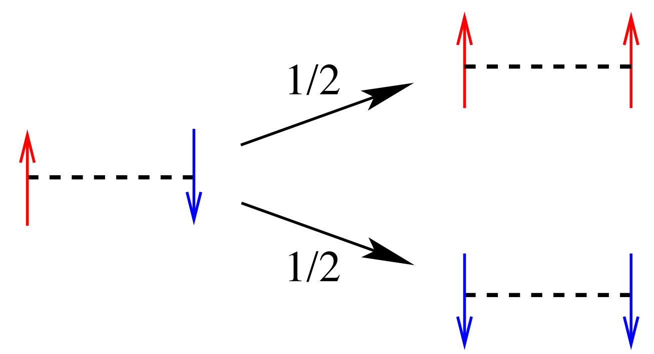

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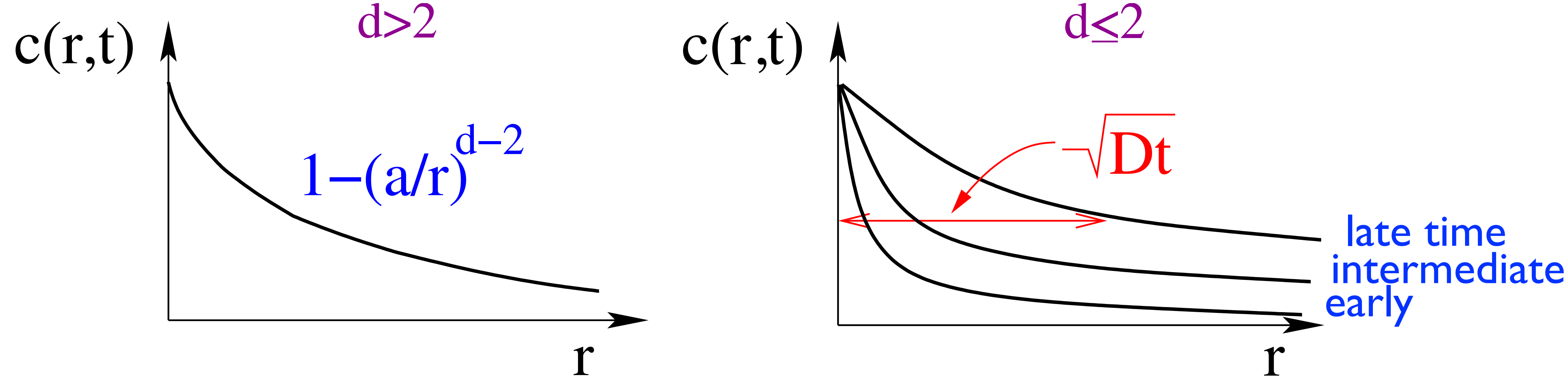

3. Consensus Time

dimension	consensus time
1	N^2
2	N ln N
>2	N

Voter Model on Complex Networks

C. Castellano, D. Vilon, A. Vespignani, EPL **63**, 153 (2003)
K. Suchecki, V. M. Eguiluz, M. San Miguel, EPL **69**, 228 (2005)
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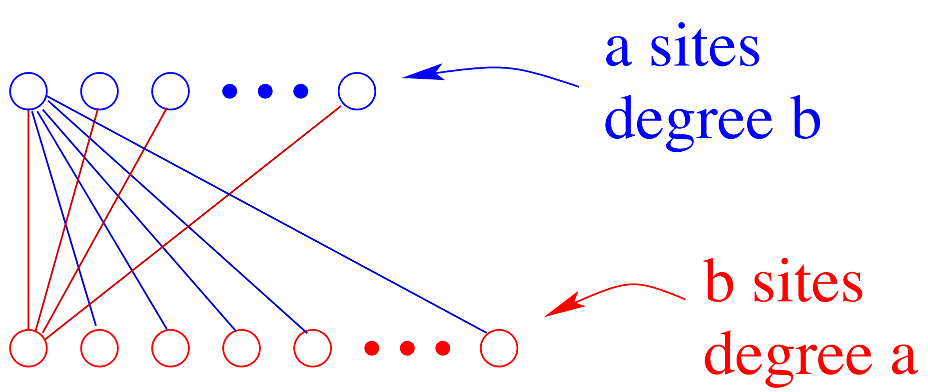
illustrative example:
complete bipartite graph



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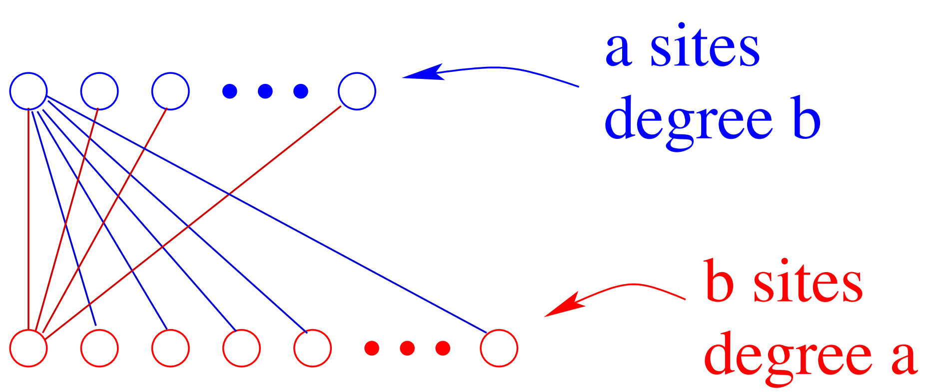

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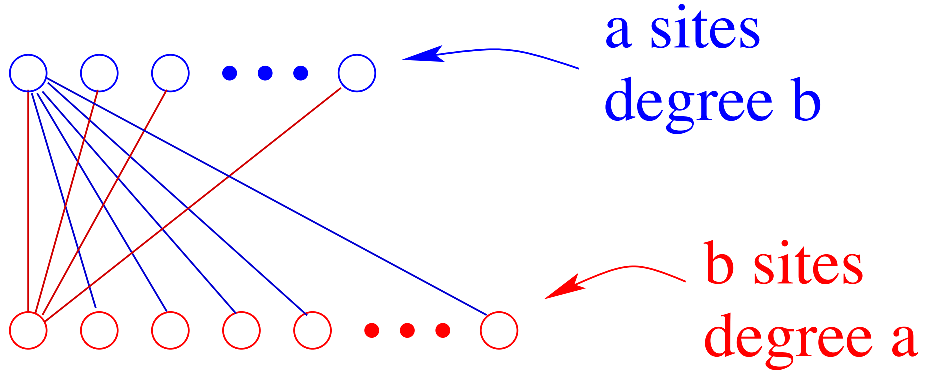

pick site on
a sublattice

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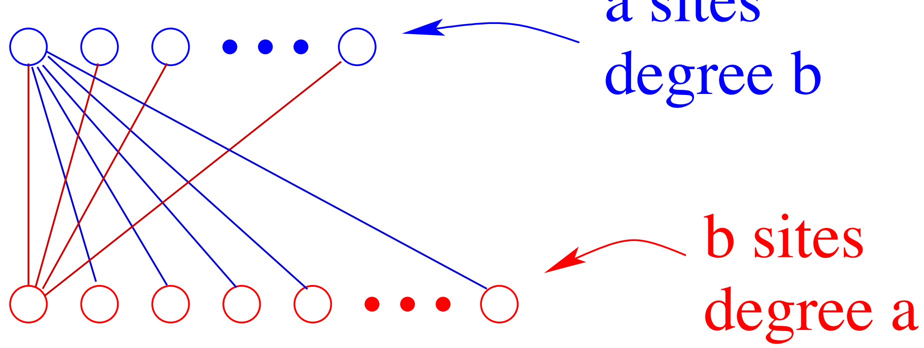

pick site on **a** sublattice ⇓ pick ↓ on **a** ⇓

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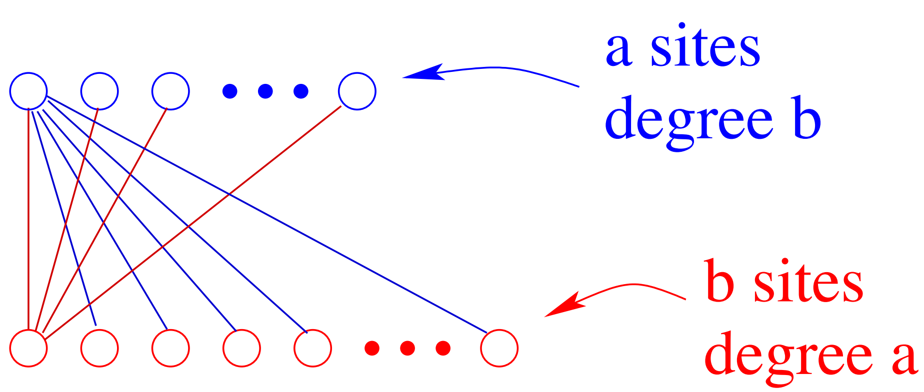

pick site on **a** sublattice

pick ↓ on **a**

pick ↑ on ***b*** sublattice

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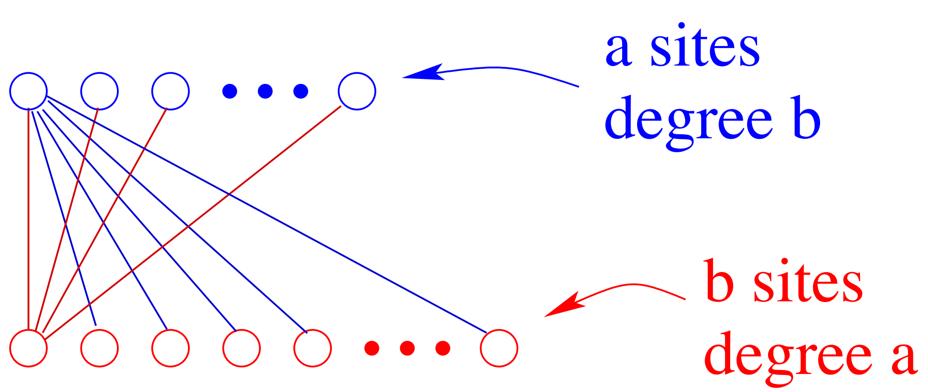

pick site on a sublattice — pick ↓ on a — pick ↑ on *b* sublattice

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Subgraph densities: $\rho_a = N_a/a,\ \rho_b = N_b/b \quad dt = 1/(a+b)$

$$\rho_{a,b}(t) = \frac{1}{2}[\rho_{a,b}(0) - \rho_{b,a}(0)]\, e^{-2t} + \frac{1}{2}[\rho_a(0) + \rho_b(0)]$$

$$\rightarrow \frac{1}{2}[\rho_a(0) + \rho_b(0)]$$

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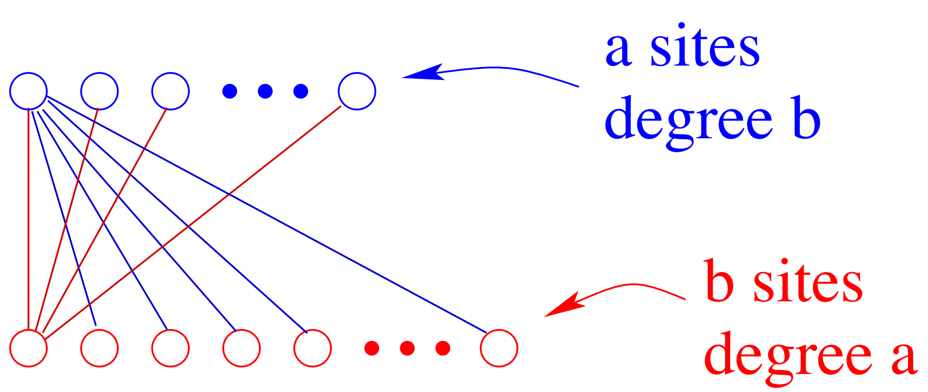

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magnetization ***not*** conserved

Voter Model on Complex Networks

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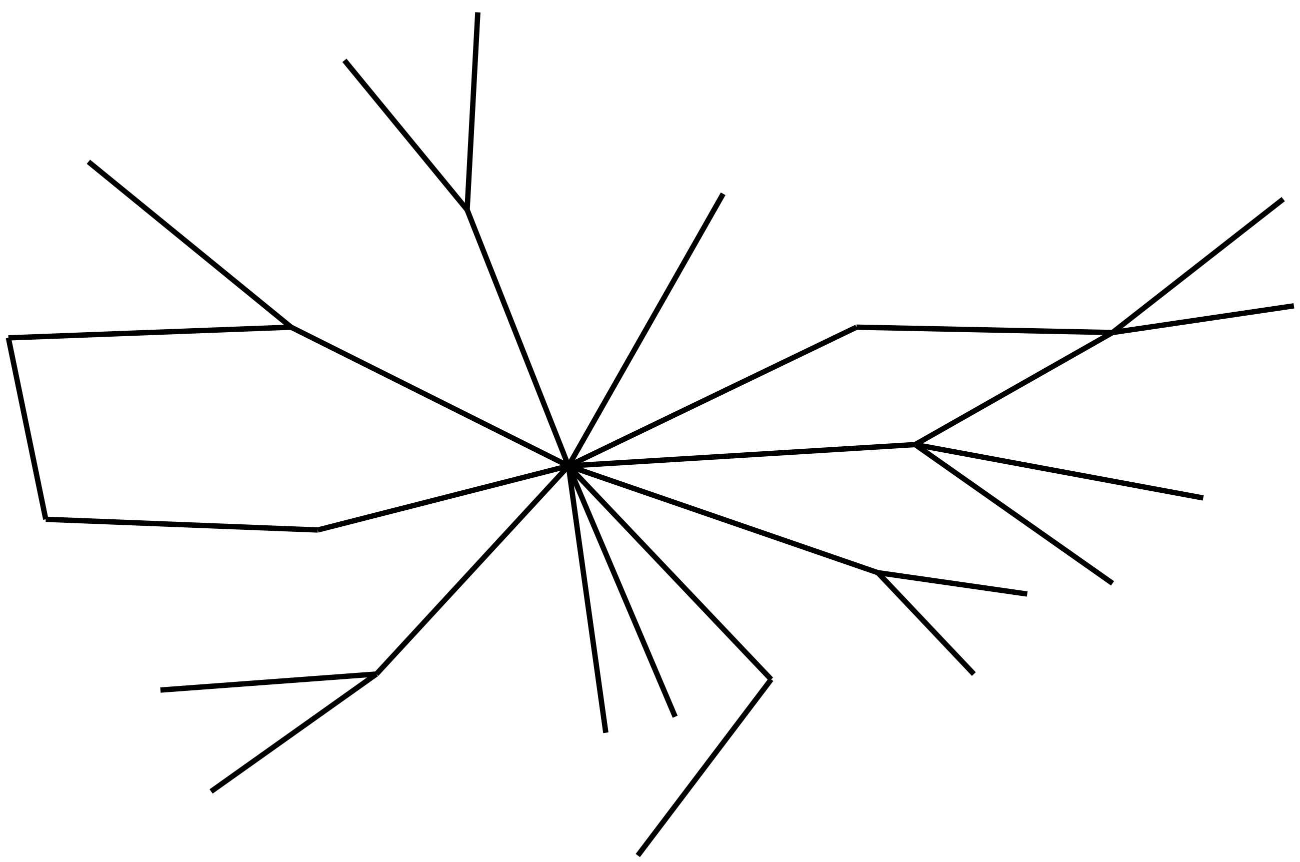

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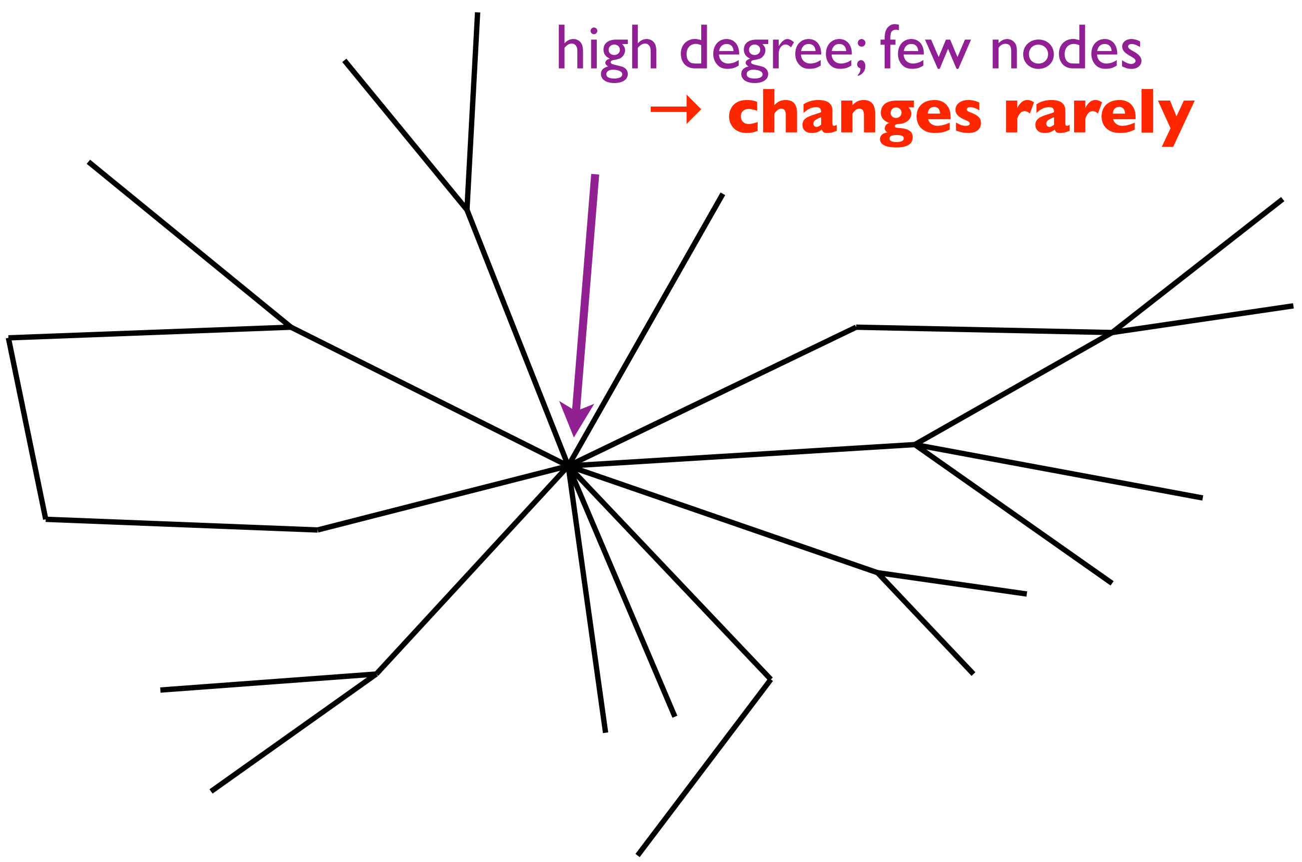

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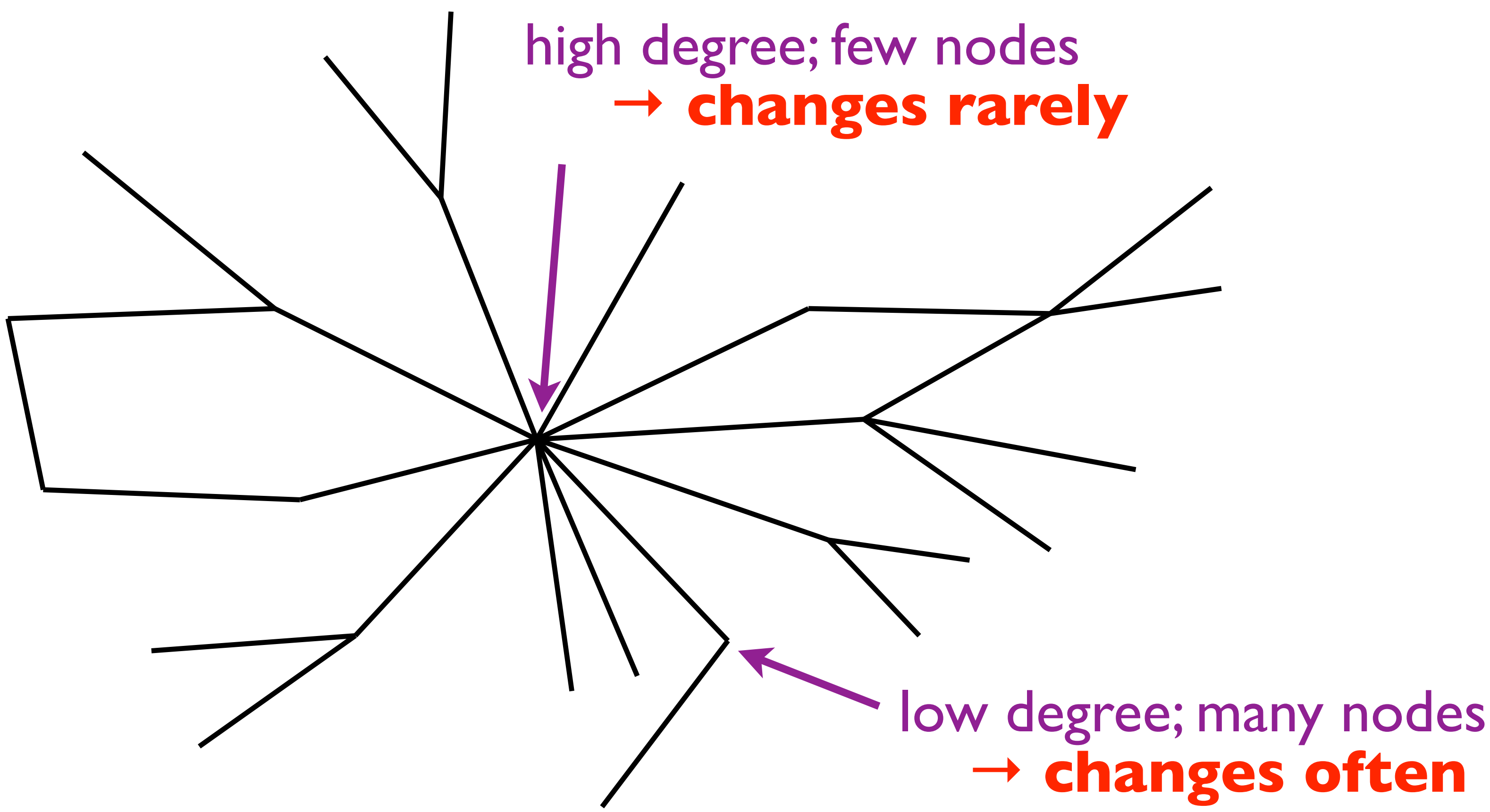

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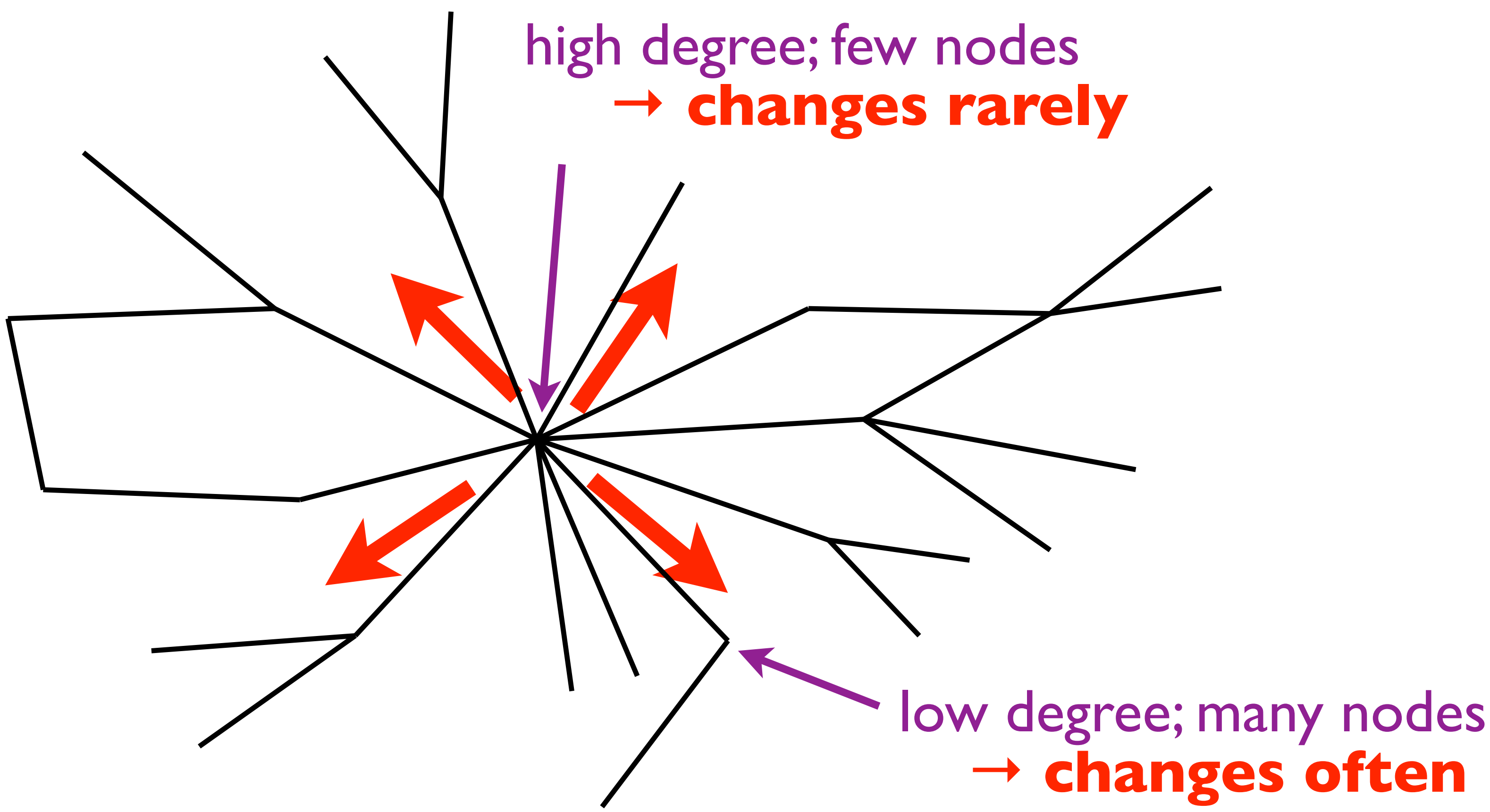
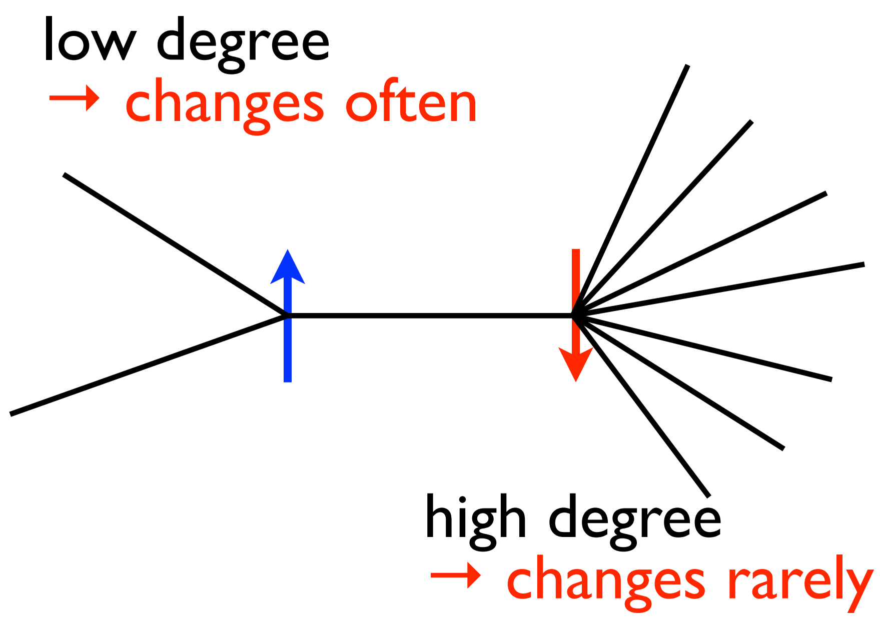


*"flow" from **high** degree to **low** degree*

New Conservation Law



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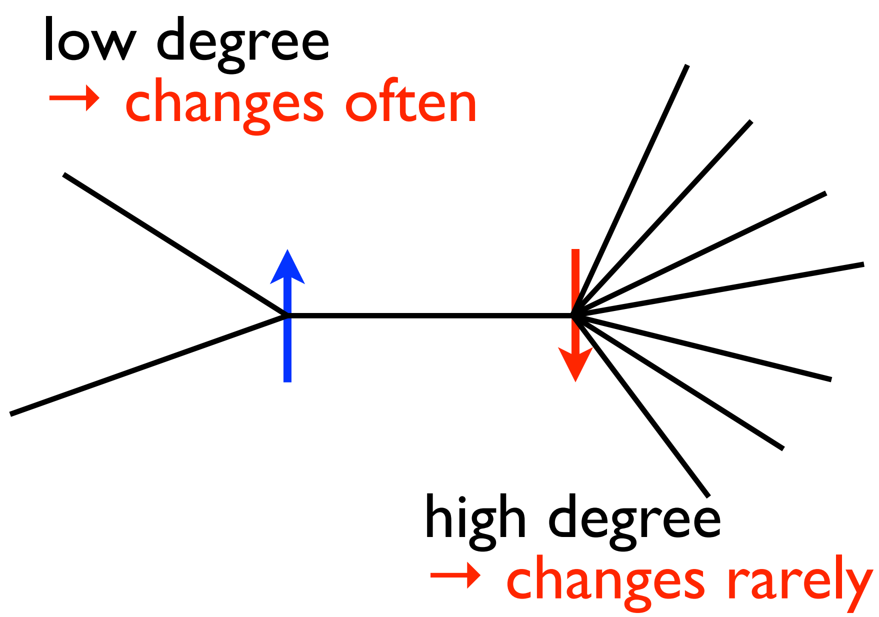


to compensate different rates, consider:

degree-weighted
1st moment:

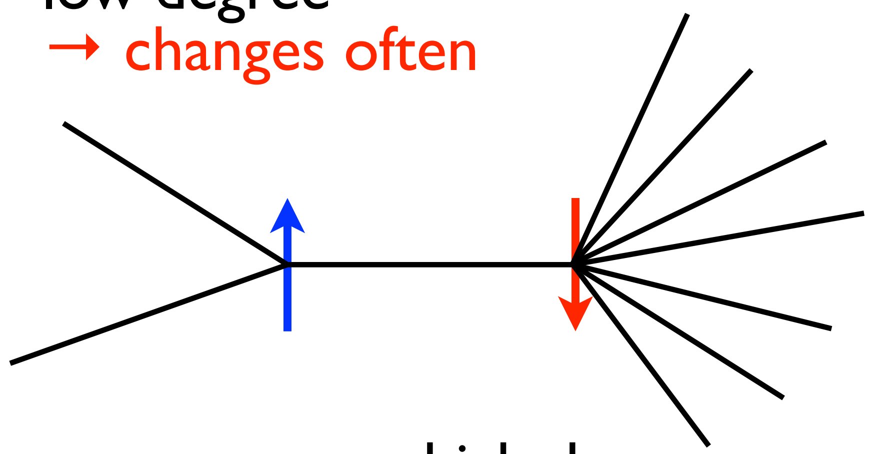
$$\omega = \frac{1}{\mu_1} \sum_k k \, n_k \rho_k$$

$\mu_1 = \text{av. degree}$

$n_k = \text{frac. nodes of degree } k$

$\rho_k = \text{frac. } \uparrow \text{ on nodes of degree } k$

New Conservation Law



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conserved!

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Exit Probability on Complex Graphs

$$\mathcal{E}(\omega) = \omega$$

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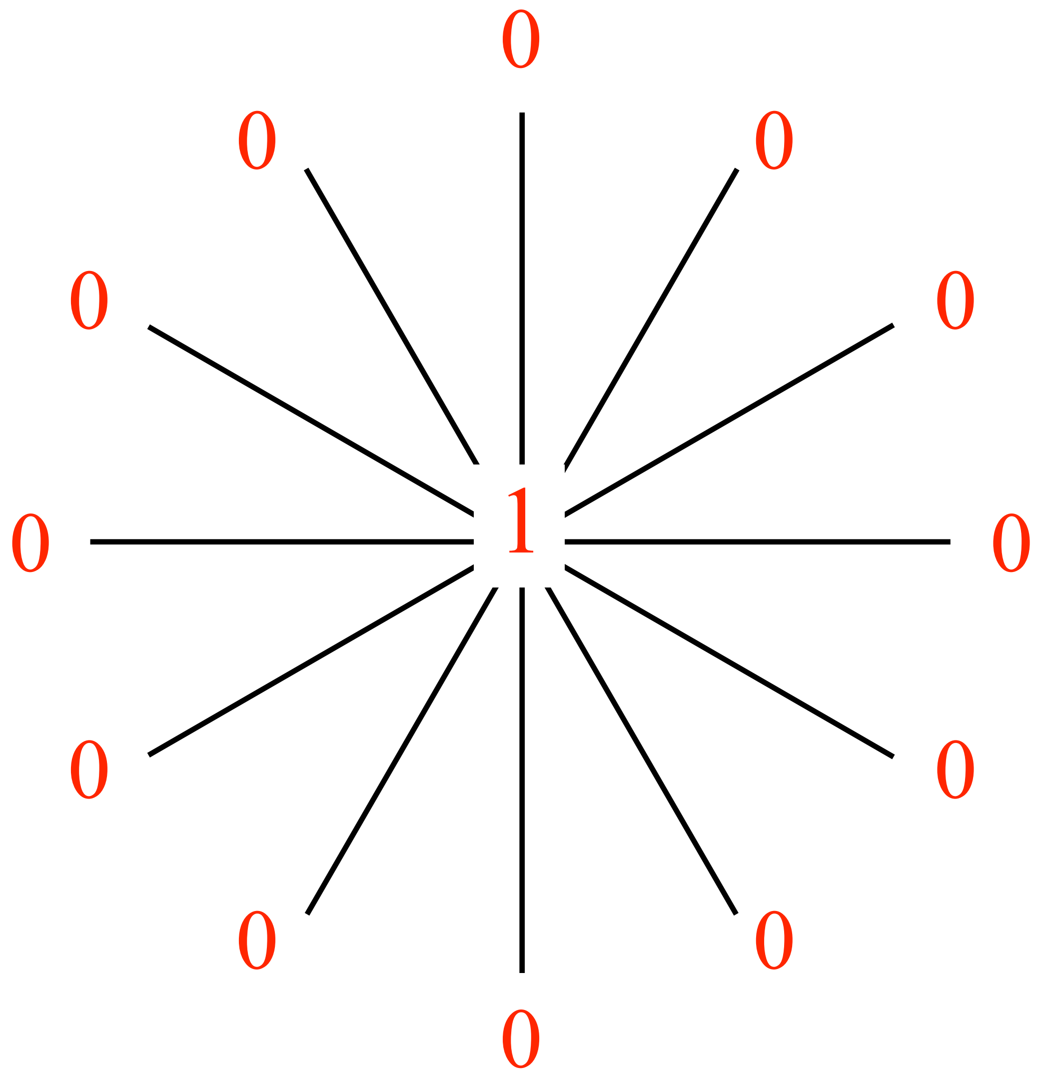
Extreme case: star graph

N nodes: degree 1
1 node: degree N



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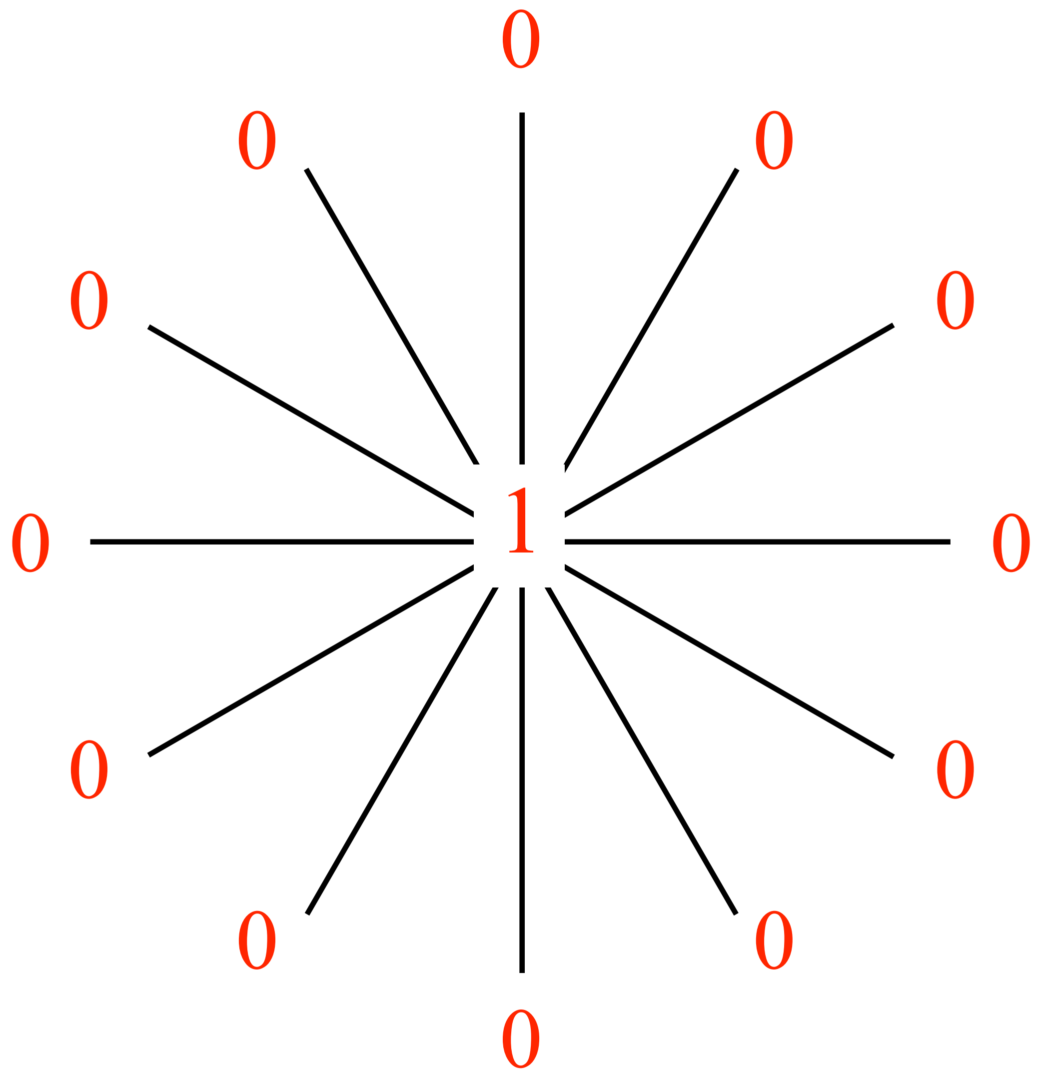

$$\omega = \frac{1}{\mu_1} \sum_k k\, n_k \rho_k = \frac{1}{2}$$

Final state: all 1 with prob. 1/2!

Route to Consensus on Complex Graphs

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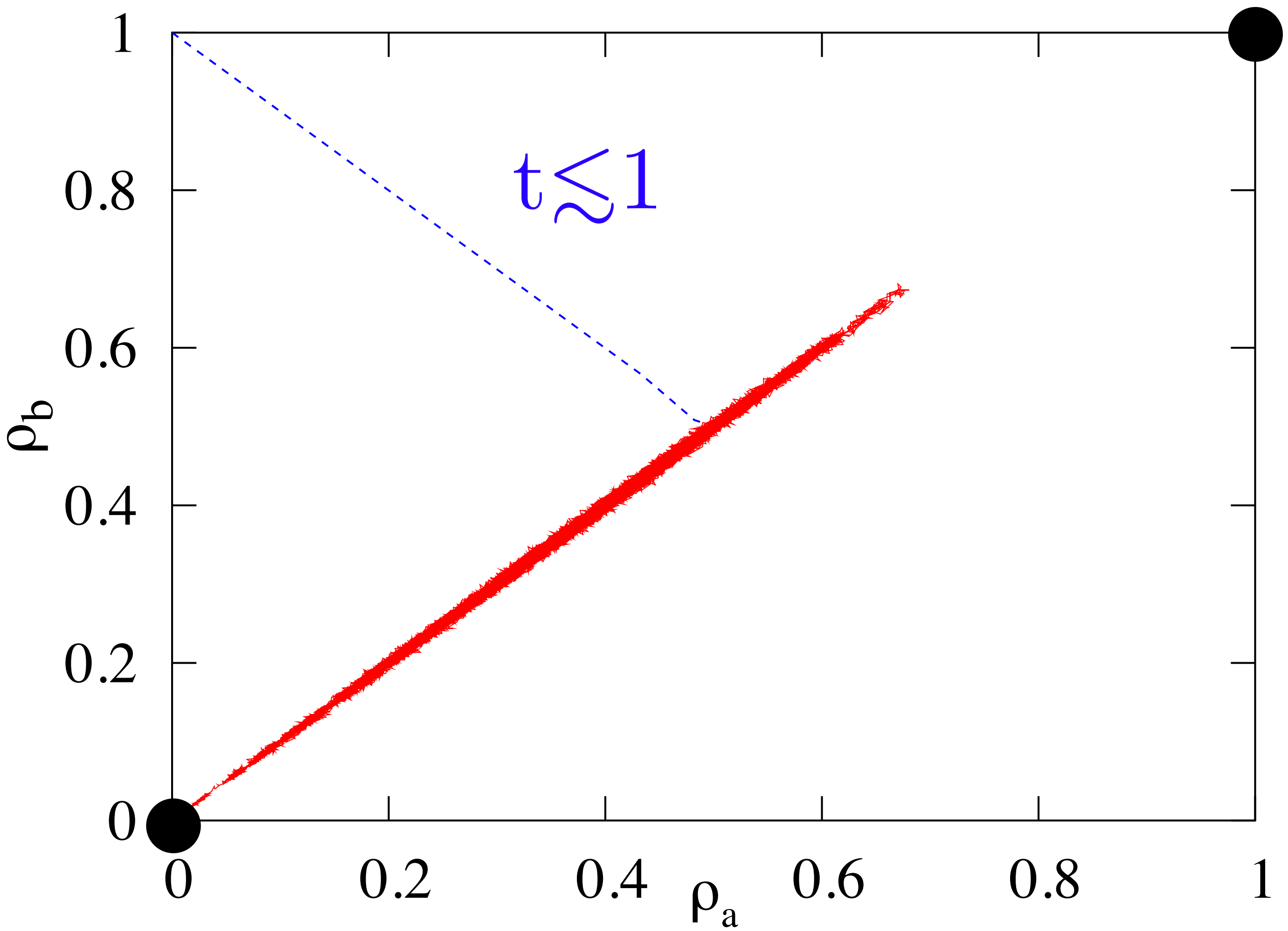
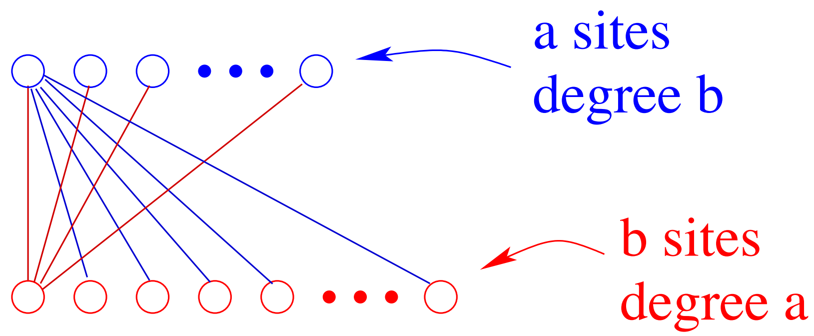
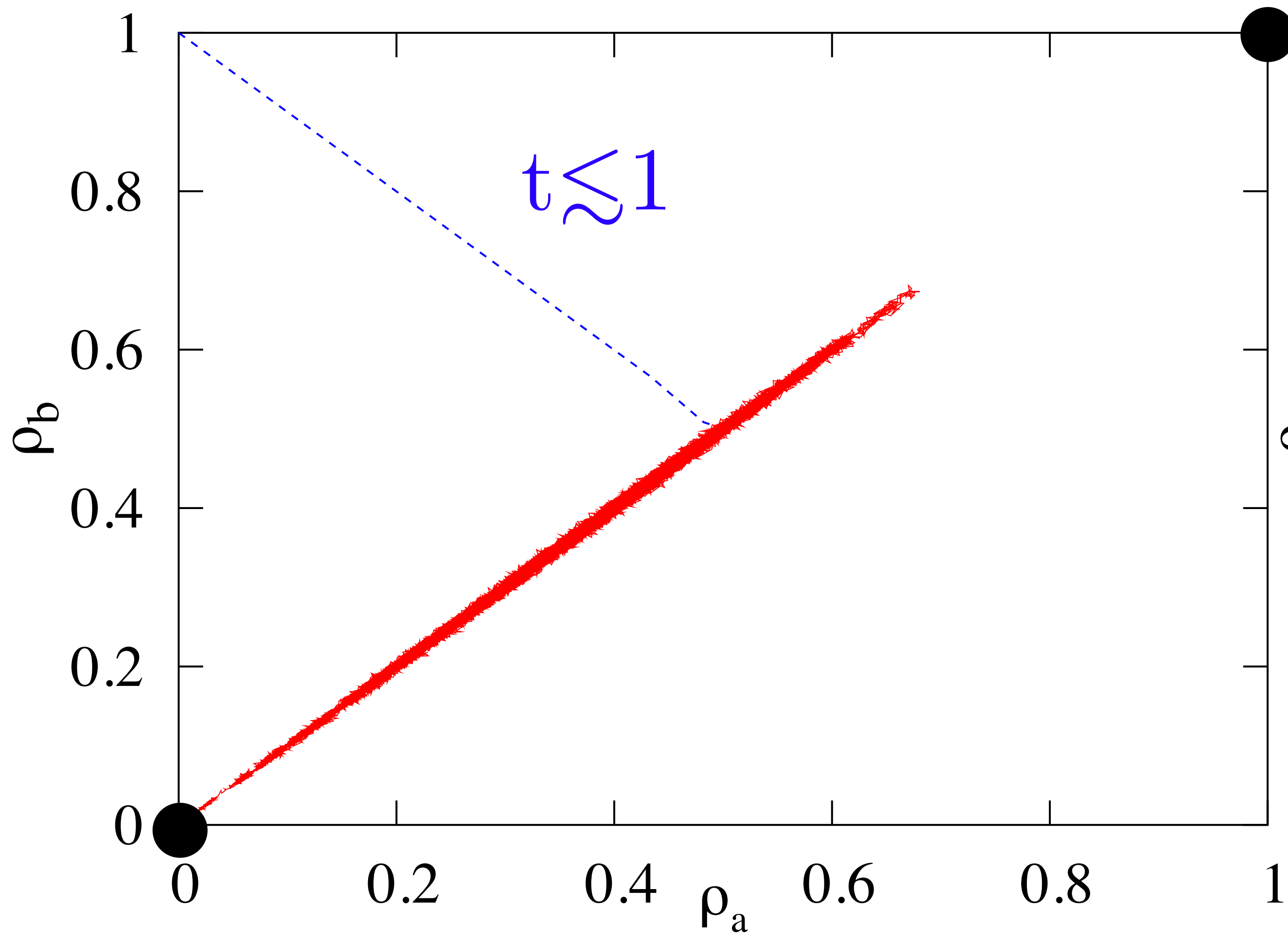
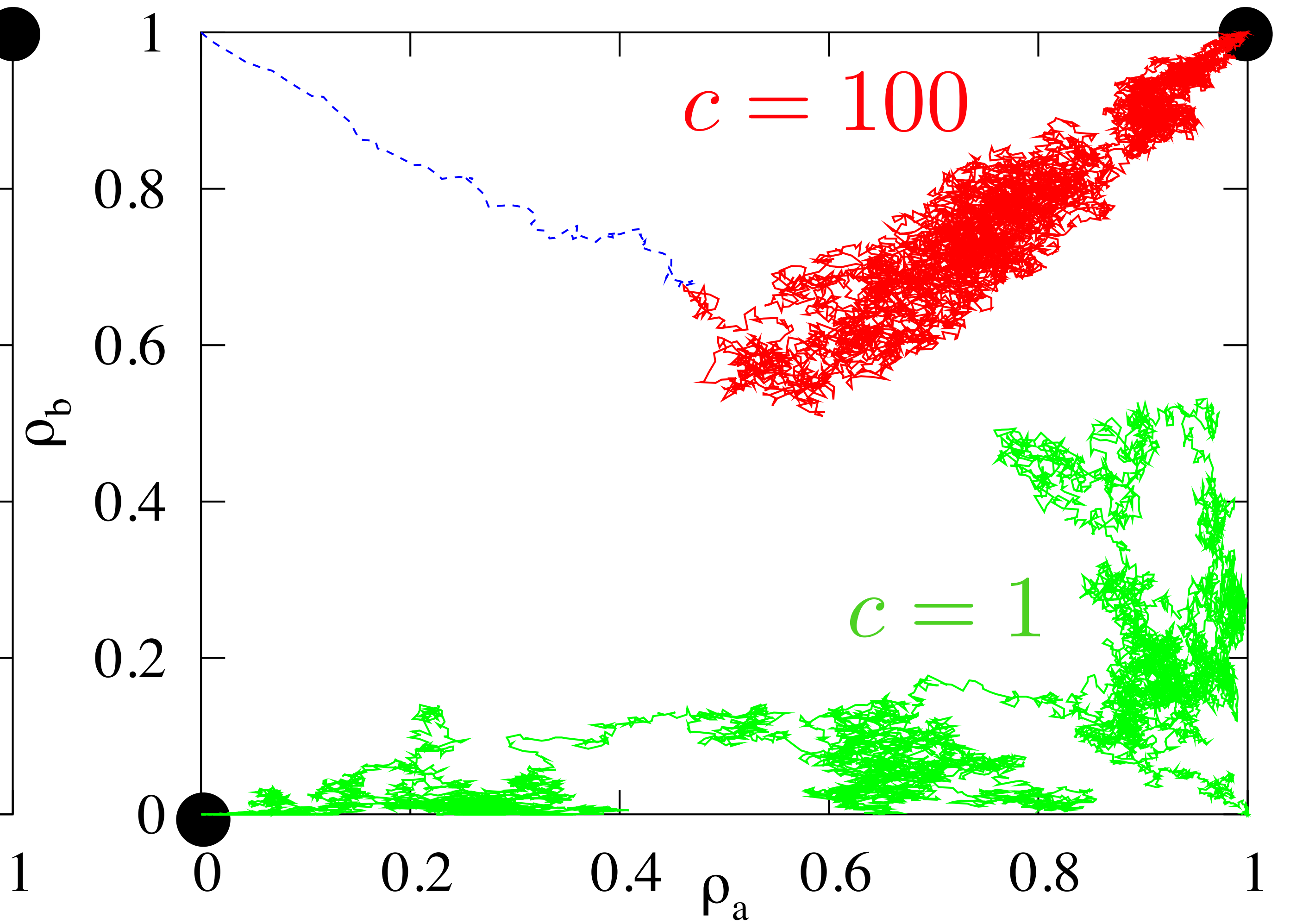


complete bipartite graph

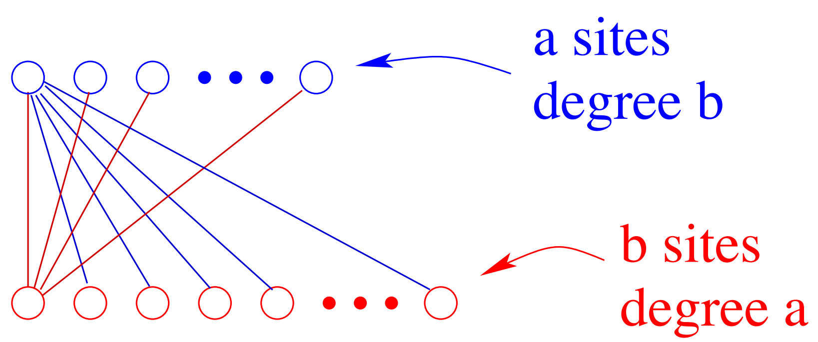


Route to Consensus on Complex Graphs

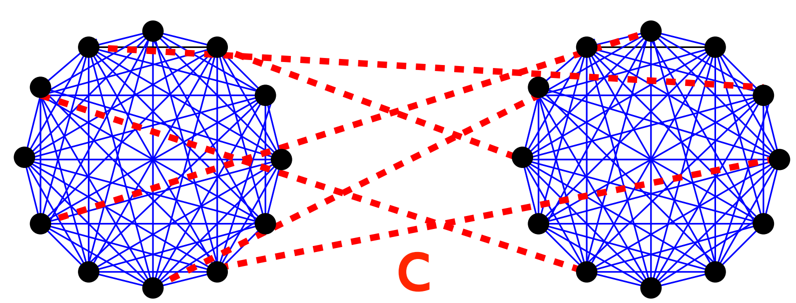




complete bipartite graph



two-clique graph



N=10000, C links/node

Consensus Time Evolution Equation

Consensus Time Evolution Equation

warmup: complete graph

$T(\rho) \equiv$ av. consensus time starting with density ρ

Consensus Time Evolution Equation

warmup: complete graph

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$$\begin{aligned} T(\rho) &= \mathcal{R}(\rho)[T(\rho+d\rho)+dt] \\ &\quad +\mathcal{L}(\rho)[T(\rho-d\rho)+dt] \\ &\quad +[1-\mathcal{R}(\rho)-\mathcal{L}(\rho)][T(\rho)+dt] \end{aligned}$$

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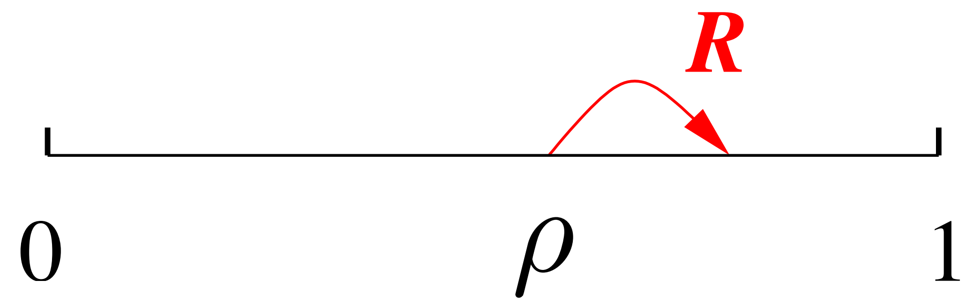

$$\begin{aligned} \mathcal{R}(\rho) &\equiv \text{prob}(\downarrow\uparrow\rightarrow\uparrow\uparrow) \\ &= \rho(1-\rho) \end{aligned}$$

Consensus Time Evolution Equation

warmup: complete graph

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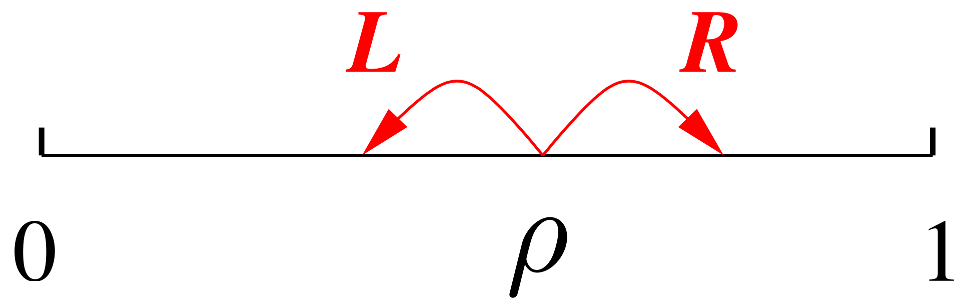

$$\begin{aligned} \mathcal{R}(\rho) &\equiv \text{prob}(\downarrow\uparrow\to\uparrow\uparrow) \\ \mathcal{L}(\rho) &\equiv \text{prob}(\uparrow\downarrow\to\downarrow\downarrow) \\ &= \rho(1-\rho) \end{aligned}$$

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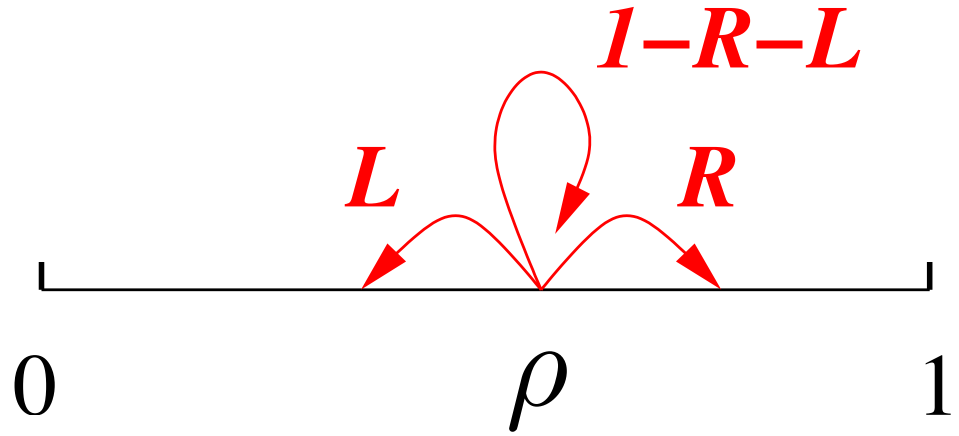

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Consensus Time on Complete Graph

$$
\begin{aligned}
T(\rho) \quad &= \quad \mathcal{R}(\rho)[T(\rho+d\rho)+dt] \\
&\quad +\mathcal{L}(\rho)[T(\rho-d\rho)+dt] \\
&\quad +[1-\mathcal{R}(\rho)-\mathcal{L}(\rho)][T(\rho)+dt]
\end{aligned}
$$

continuum limit:

$$T'' = -\frac{N}{\rho(1-\rho)}$$

Consensus Time on Complete Graph

$$\begin{aligned} T(\rho) &= \mathcal{R}(\rho)[T(\rho+d\rho)+dt] \\ &\quad +\mathcal{L}(\rho)[T(\rho-d\rho)+dt] \\ &\quad +[1-\mathcal{R}(\rho)-\mathcal{L}(\rho)][T(\rho)+dt] \end{aligned}$$

continuum limit: $$T'' = -\frac{N}{\rho(1-\rho)}$$

solution:

$$T(\rho) = -N\left[\rho\ln\rho + (1-\rho)\ln(1-\rho)\right]$$

Consensus Time on Heterogeneous Networks

$T(\{\rho_k\}) \equiv$ av. consensus time starting with density ρ_k on nodes of degree k

$$
\begin{aligned}
T(\{\rho_k\}) = &\sum_k \mathcal{R}_k(\{\rho_k\})[T(\{\rho_k^+\}) + dt] \\
&+ \sum_k \mathcal{L}_k(\{\rho_k\})[T(\{\rho_k^-\}) + dt] \\
&+ \Big[1 - \sum_k [\mathcal{R}_k(\{\rho_k\}) + \mathcal{L}_k(\{\rho_k\})]\Big][T(\{\rho_k\}) + dt]
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{R}_k(\{\rho_k\}) &= \text{prob}(\rho_k \to \rho_k^+) \\
&= \frac{1}{N} \sum_x{}' \frac{1}{k_x} \sum_y P(\downarrow, \text{---}, \uparrow) \\
&= n_k \omega (1 - \rho_k)
\end{aligned}
$$

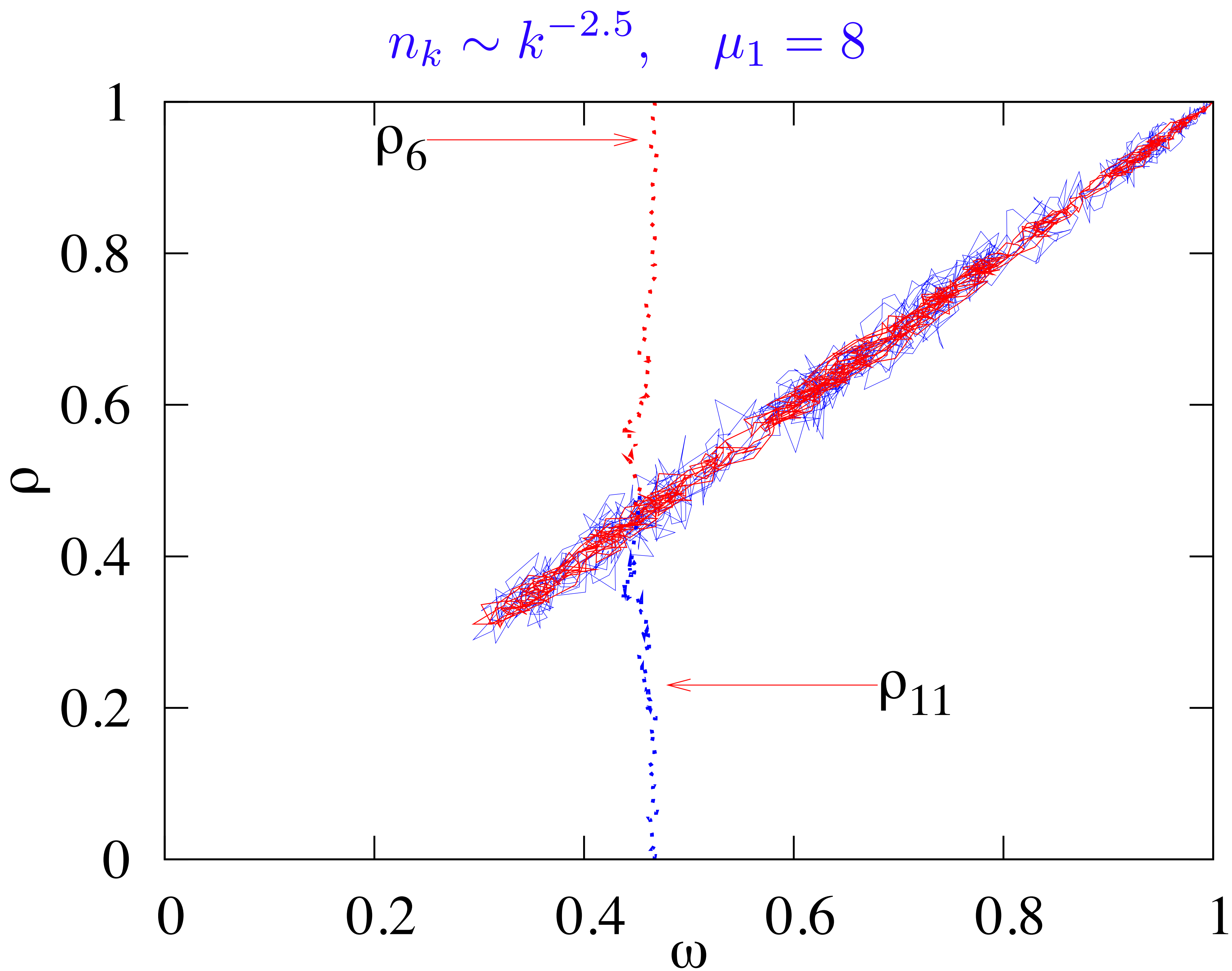
$$
\mathcal{L}_k(\{\rho_k\}) = n_k \rho_k (1 - \omega)
$$

Consensus Time on Heterogeneous Networks

continuum limit:

$$\sum_k \left[(\omega - \rho_k) \frac{\partial T}{\partial \rho_k} + \frac{\omega + \rho_k - 2\omega\rho_k}{2Nn_k} \frac{\partial^2 T}{\partial \rho_k^2} \right] = -1$$

Molloy-Reed Scale-Free Network



Consensus Time on Heterogeneous Networks

continuum limit:

$$\sum_k \left[(\omega - \rho_k) \frac{\partial T}{\partial \rho_k} + \frac{\omega + \rho_k - 2\omega\rho_k}{2Nn_k} \frac{\partial^2 T}{\partial \rho_k^2} \right] = -1$$

now use $\rho_k \rightarrow \omega \quad \forall k$

and $\frac{\partial}{\partial \rho_k} = \frac{\partial \omega}{\partial \rho_k} \frac{\partial}{\partial \omega} = \frac{k n_k}{\mu_1} \frac{\partial}{\partial \omega}$

Consensus Time on Heterogeneous Networks

continuum limit:

$$\sum_k \left[(\omega - \rho_k) \frac{\partial T}{\partial \rho_k} + \frac{\omega + \rho_k - 2\omega\rho_k}{2Nn_k} \frac{\partial^2 T}{\partial \rho_k^2} \right] = -1$$

now use $\rho_k \to \omega \quad \forall k$

and $\frac{\partial}{\partial \rho_k} = \frac{\partial \omega}{\partial \rho_k} \frac{\partial}{\partial \omega} = \frac{k n_k}{\mu_1} \frac{\partial}{\partial \omega}$

to give $\frac{\partial^2 T}{\partial \omega^2} = -\frac{N\mu_1^2/\mu_2}{\omega(1-\omega)}$

Consensus Time on Heterogeneous Networks

continuum limit:

$$\sum_k \left[(\omega - \rho_k) \frac{\partial T}{\partial \rho_k} + \frac{\omega + \rho_k - 2\omega\rho_k}{2Nn_k} \frac{\partial^2 T}{\partial \rho_k^2} \right] = -1$$

now use $\rho_k \longrightarrow \omega \quad \forall k$

and $\frac{\partial}{\partial \rho_k} = \frac{\partial \omega}{\partial \rho_k} \frac{\partial}{\partial \omega} = \frac{kn_k}{\mu_1} \frac{\partial}{\partial \omega}$

to give $\frac{\partial^2 T}{\partial \omega^2} = -\frac{N\mu_1^2/\mu_2}{\omega(1-\omega)}$ same as $T'' = -\frac{N}{\rho(1-\rho)}$

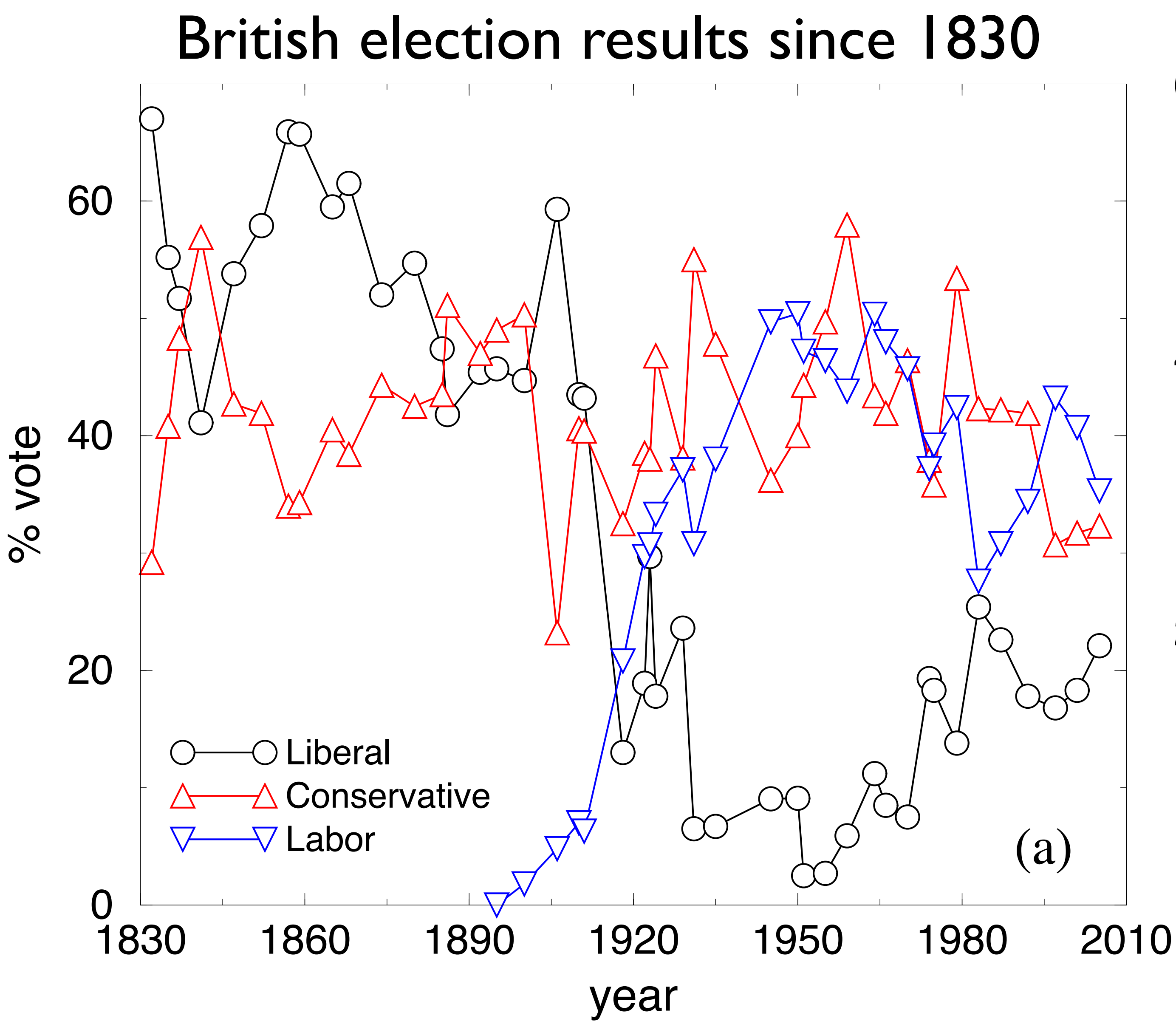
with *effective* size $N_{\rm eff} = N\,\mu_1^2/\mu_2$

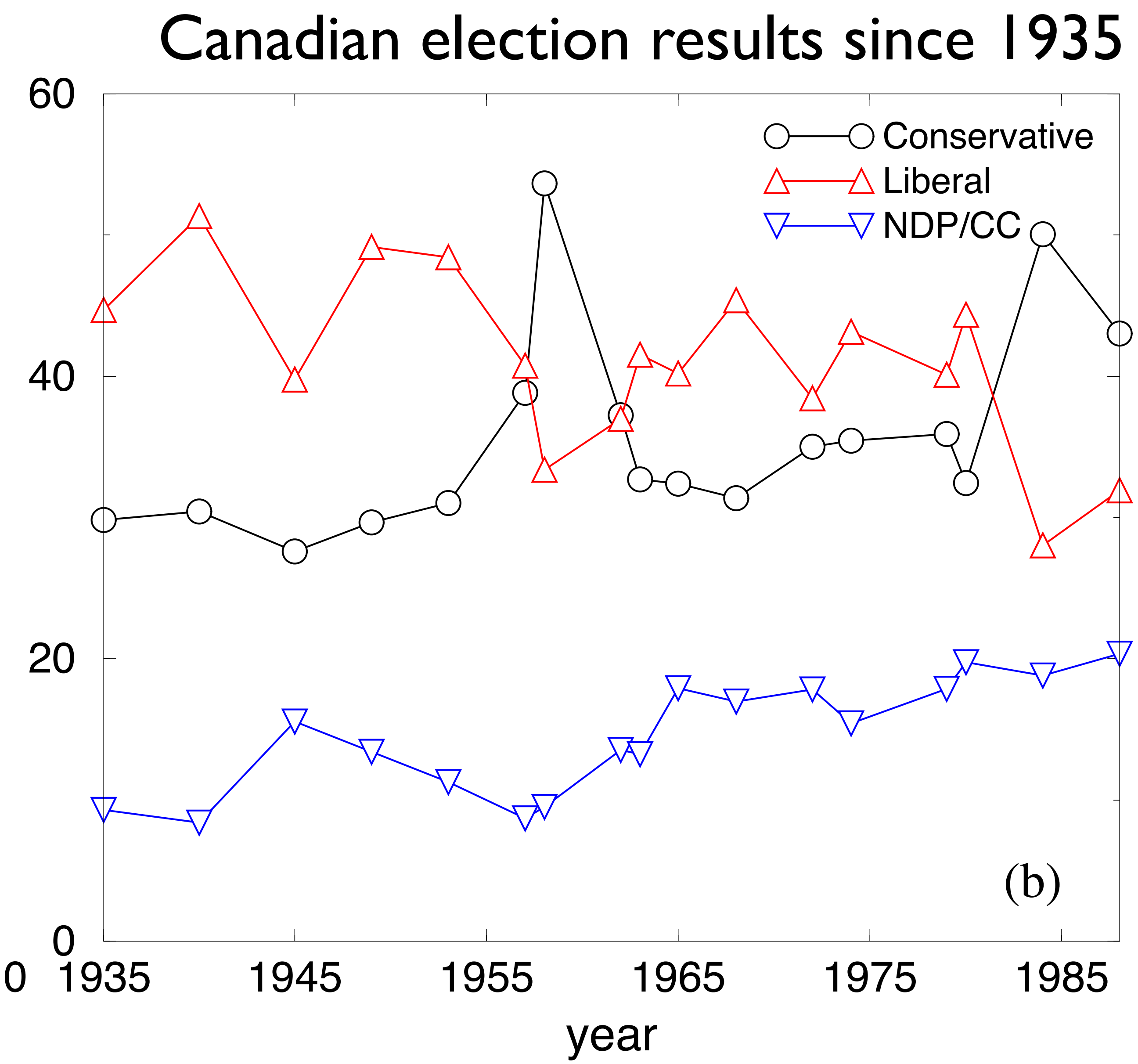
Consensus Time for Power-Law Degree Distribution $n_k \sim k^{-\nu}$

$$T_N \propto N_{\text{eff}} = N\frac{\mu_1^2}{\mu_2} \sim \begin{cases} N & \nu > 3 \\ N/\ln N & \nu = 3 \\ N^{2(\nu-2)/(\nu-1)} & 2 < \nu < 3 \\ (\ln N)^2 & \nu = 2 \\ \mathcal{O}(1) & \nu < 2 \end{cases}$$

fast (<N) consensus

Strategic Voting





Strategic Voter Model

D. Volovik, M. Mobilia, SR
EPL **85**, 48001 (2009)

randomly-selected voter changes to any other state equiprobably (rate T)

majority-minority interaction: minority *preferentially* changes to majority (rate r)

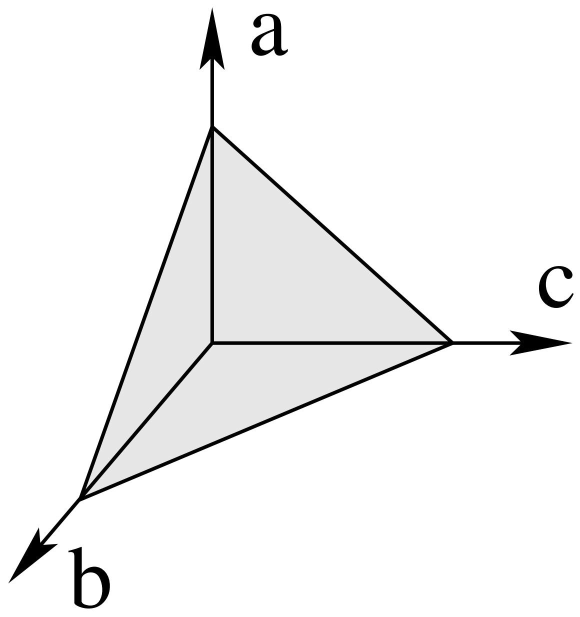
rate equations (A, B majority; c minority):

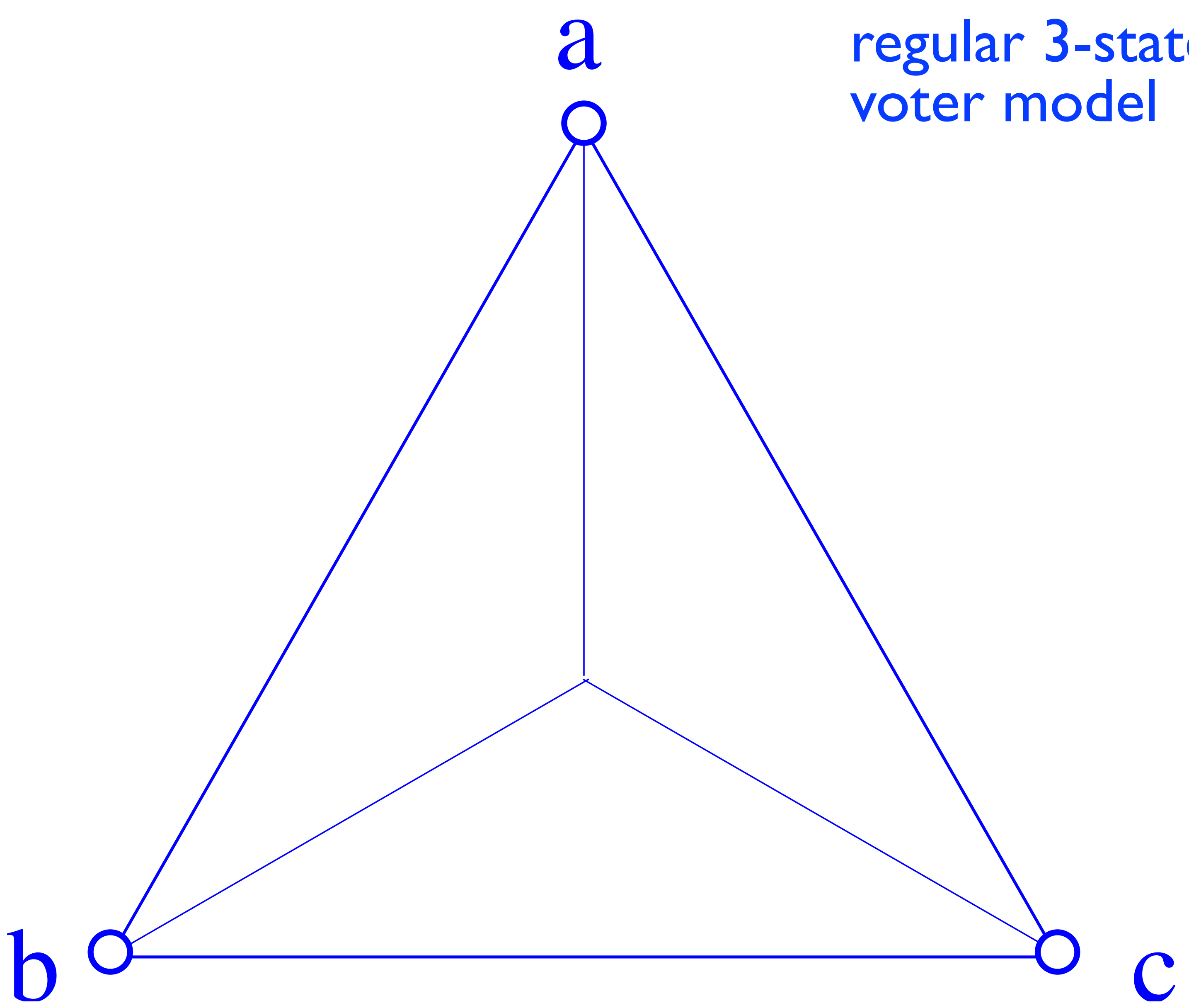
$$\dot{A} = T(B + c - 2A) + r\,Ac$$

$$\dot{B} = T(c + A - 2B) + r\,Bc$$

$$\dot{c} = T(A + B - 2c) - r\,(A + B)c$$

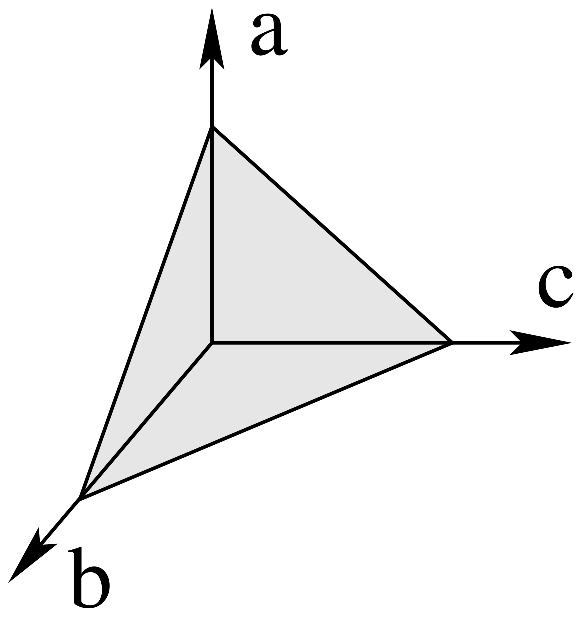
Phase Portrait

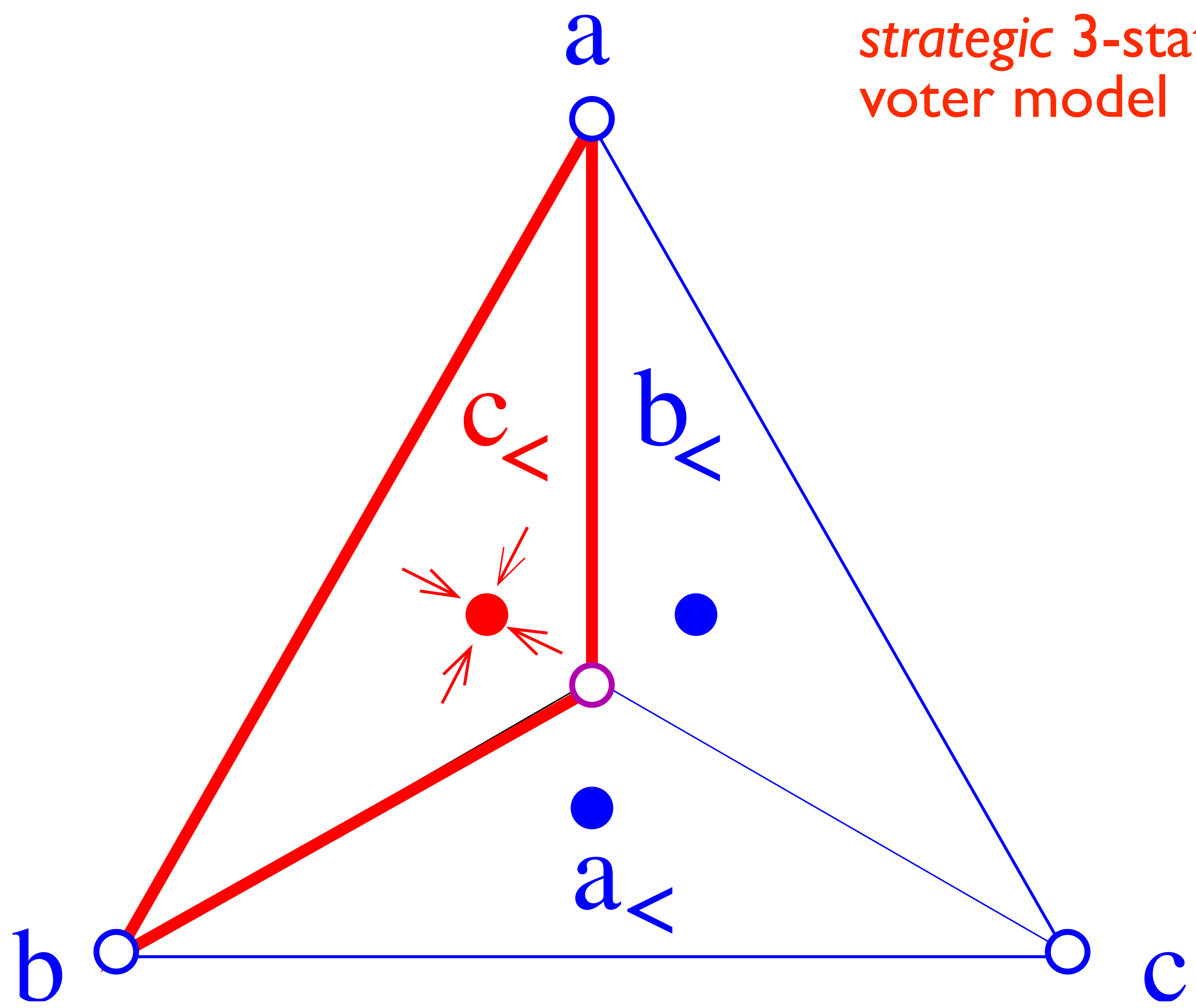




regular 3-state
voter model

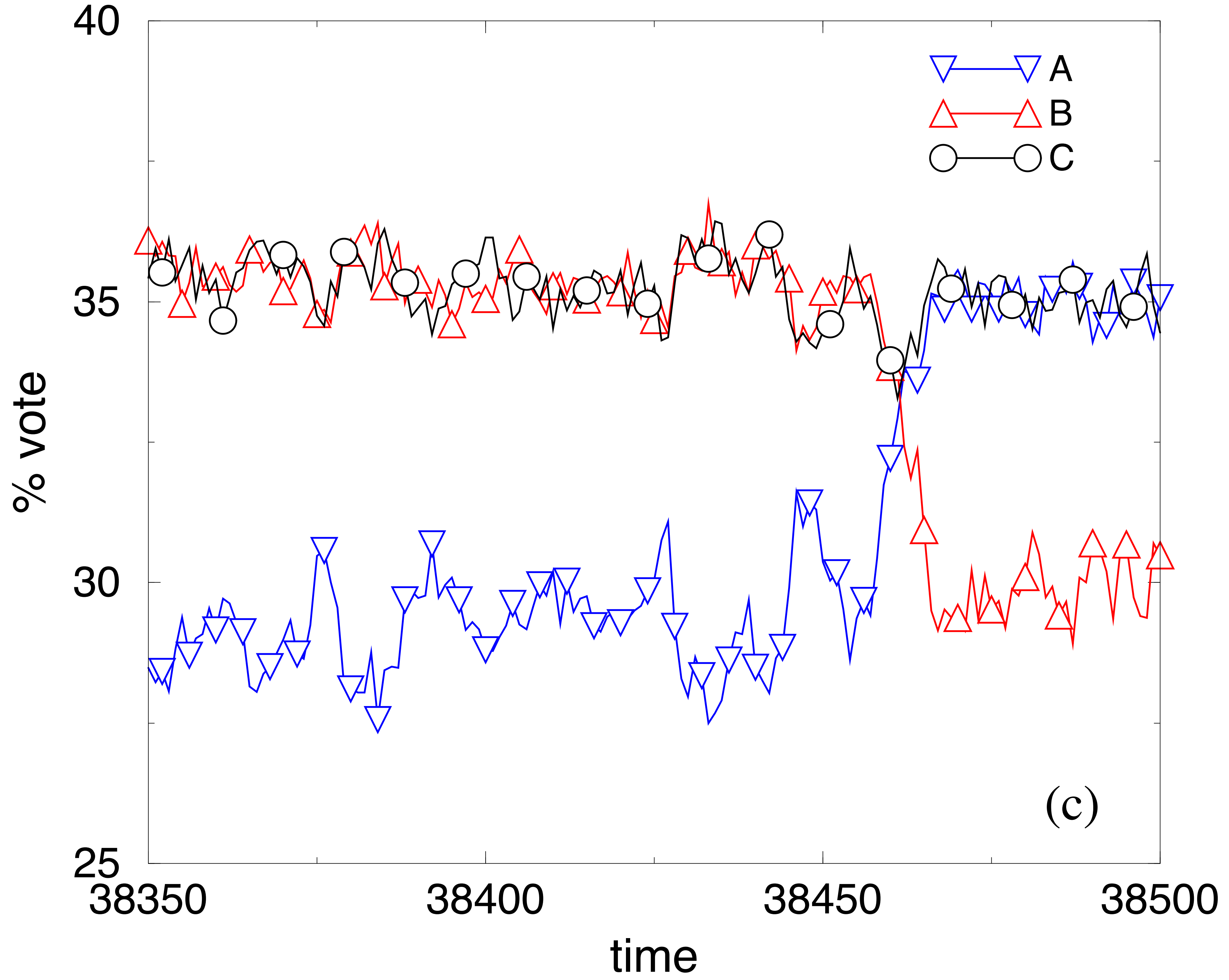
Phase Portrait

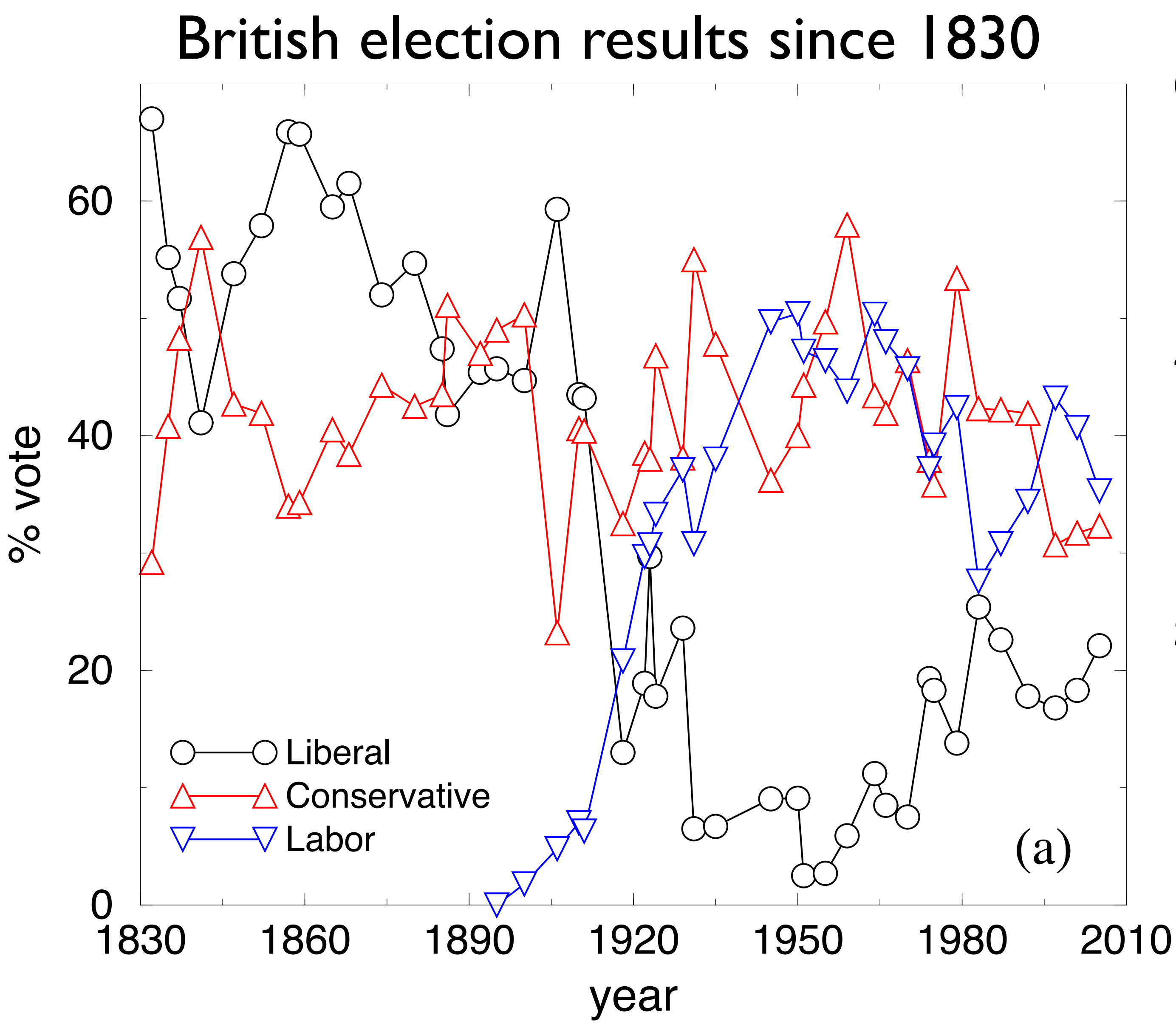


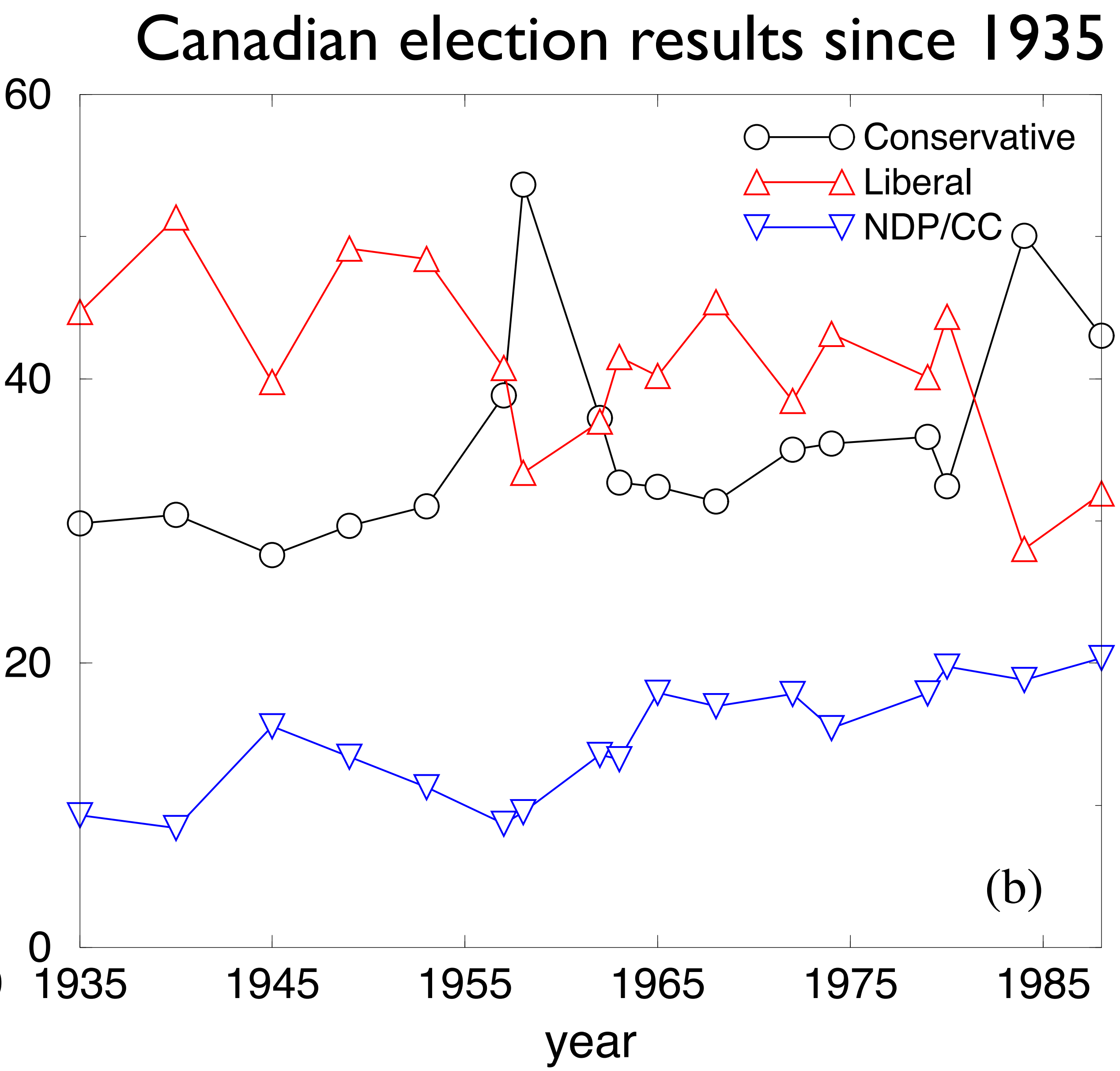


strategic 3-state
voter model

Slow Switching




British election results since 1830
% vote
60
40
20
0
1830
1860
1890
1920
1950
1980
2010
year
Liberal
Conservative
Labor
(a)


Canadian election results since 1935
60
40
20
0
1935
1945
1955
1965
1975
1985
year
Conservative
Liberal
NDP/CC
(b)

Partisan Voter Model

N. Masuda, N. Gibert, SR
arXiv:1003.0768

Partisan Voter Model

N. Masuda, N. Gibert, SR
arXiv:1003.0768

↑ happy democrat
density D_h

↓ sad democrat
density D_s

↑ sad republican
density R_s

↓ happy republican
density R_h

Partisan Voter Model

N. Masuda, N. Gibert, SR
arXiv:1003.0768

↑ happy democrat — density D_h

↓ sad democrat — density D_s

↑ sad republican — density R_s

↓ happy republican — density R_h

partisan voting update:

Partisan Voter Model

N. Masuda, N. Gibert, SR
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↑ happy democrat — density D_h

↓ sad democrat — density D_s

↑ sad republican — density R_s

↓ happy republican — density R_h

partisan voting update:

1. Pick voter, pick neighbor (as in usual voter model);

Partisan Voter Model

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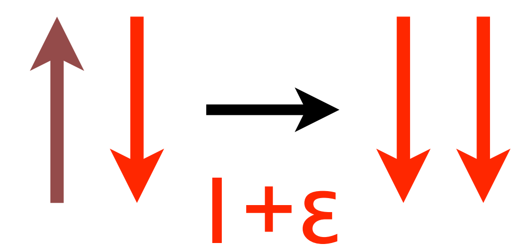
1. Pick voter, pick neighbor (as in usual voter model);

2a. If initial voter becomes *happy* by adopting neighboring state, change occurs with rate 1+ε;



Partisan Voter Model

N. Masuda, N. Gibert, SR
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↑ happy democrat — density D_h

↓ sad democrat — density D_s

↑ sad republican — density R_s

↓ happy republican — density R_h

partisan voting update:

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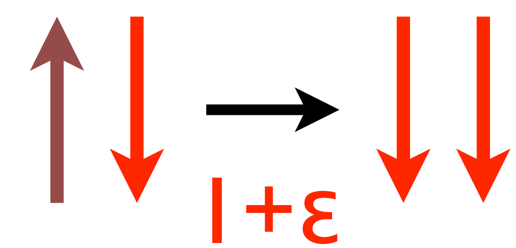
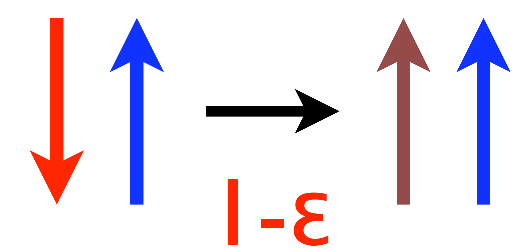
2a. If initial voter becomes *happy* by adopting neighboring state, change occurs with rate 1+ε;



2b. If initial voter becomes *unhappy* by adopting neighboring state, change occurs with rate 1-ε.



Partisan Voter Model: Mean-Field Limit

rate equations:

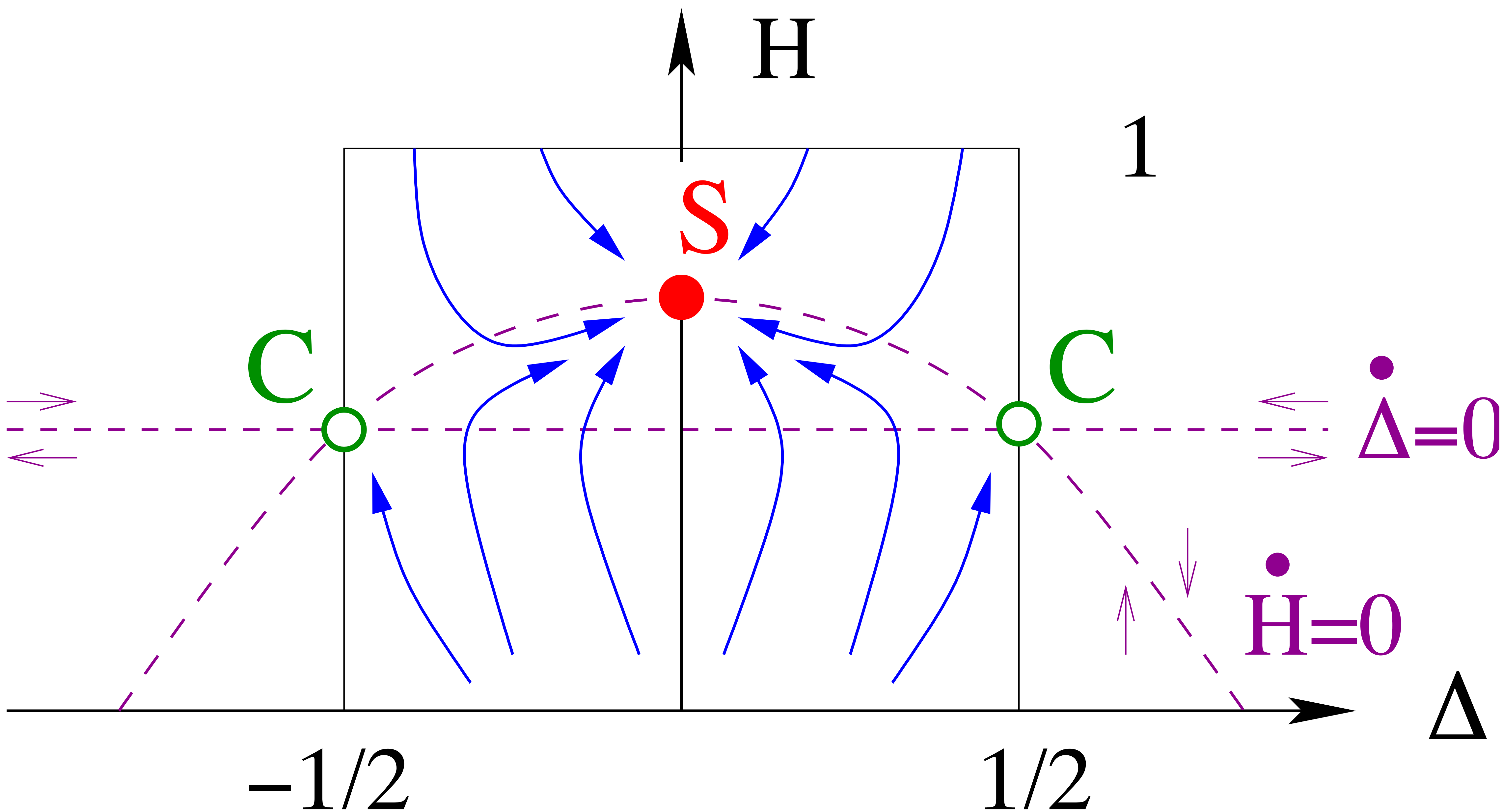
$$\dot{D}_h = \;\; 2\epsilon\, D_h D_s + (1+\epsilon)\, D_s R_s \; - (1-\epsilon)\, D_h R_h$$
$$\dot{D}_s = -2\epsilon\, D_h D_s + (1-\epsilon)\, D_h R_h - (1+\epsilon)\, D_s R_s$$

$$\text{and } R \leftrightarrow D$$

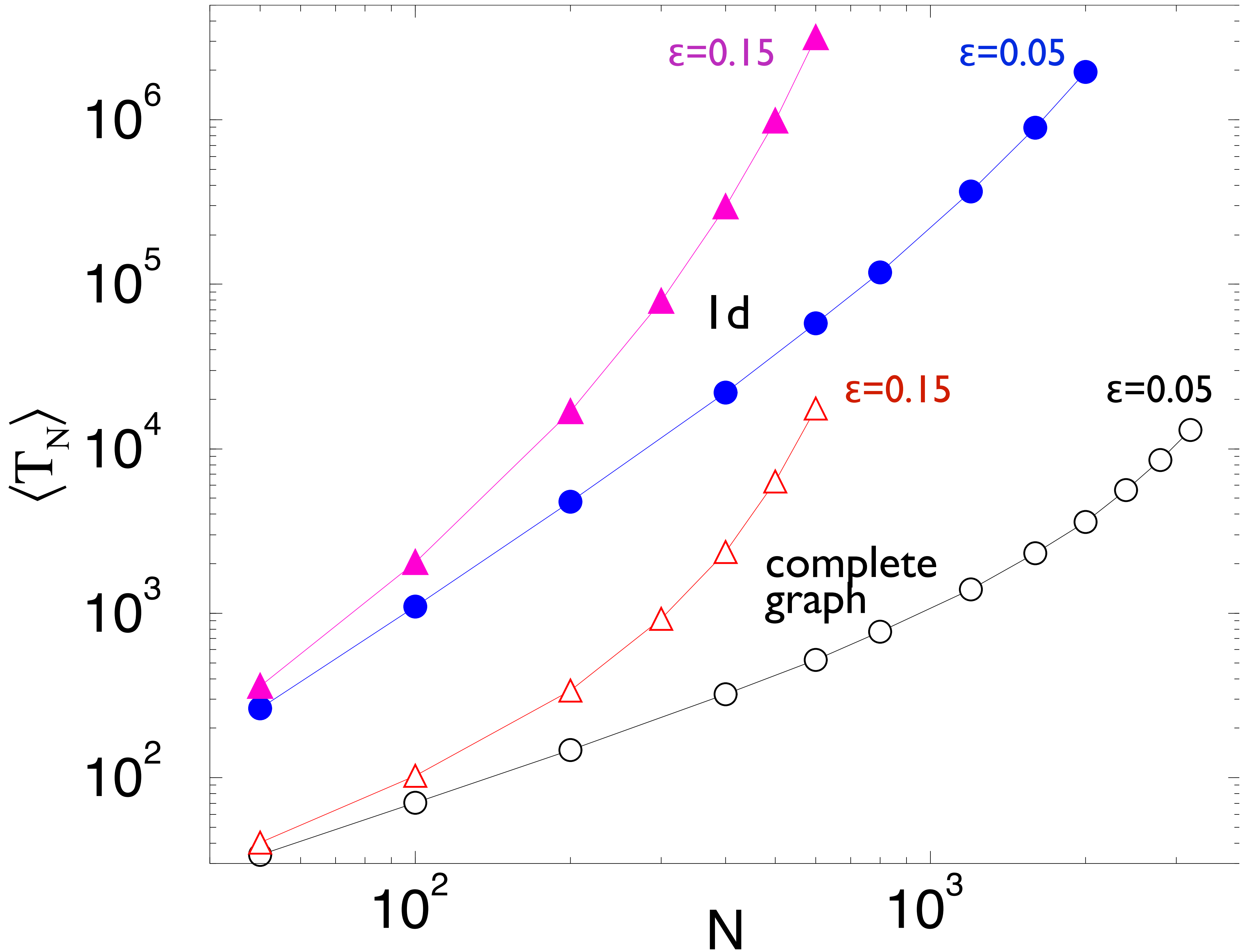
Symmetric Case: D=R=½

$$H \equiv D_h + R_h$$
$$= \text{density of happy voters}$$

$$\Delta \equiv D_h - R_h = D_h - (\tfrac{1}{2} - R_s) = \rho - \tfrac{1}{2}$$
$$= \text{density democratic voters} - \tfrac{1}{2}$$



Consensus Time on Finite Graphs



Summary & Outlook

Voter model:

paradigmatic, soluble, (but hopelessly naive)

Voter model on complex networks:

new conservation law
meandering route to consensus
fast consensus for broad degree distributions

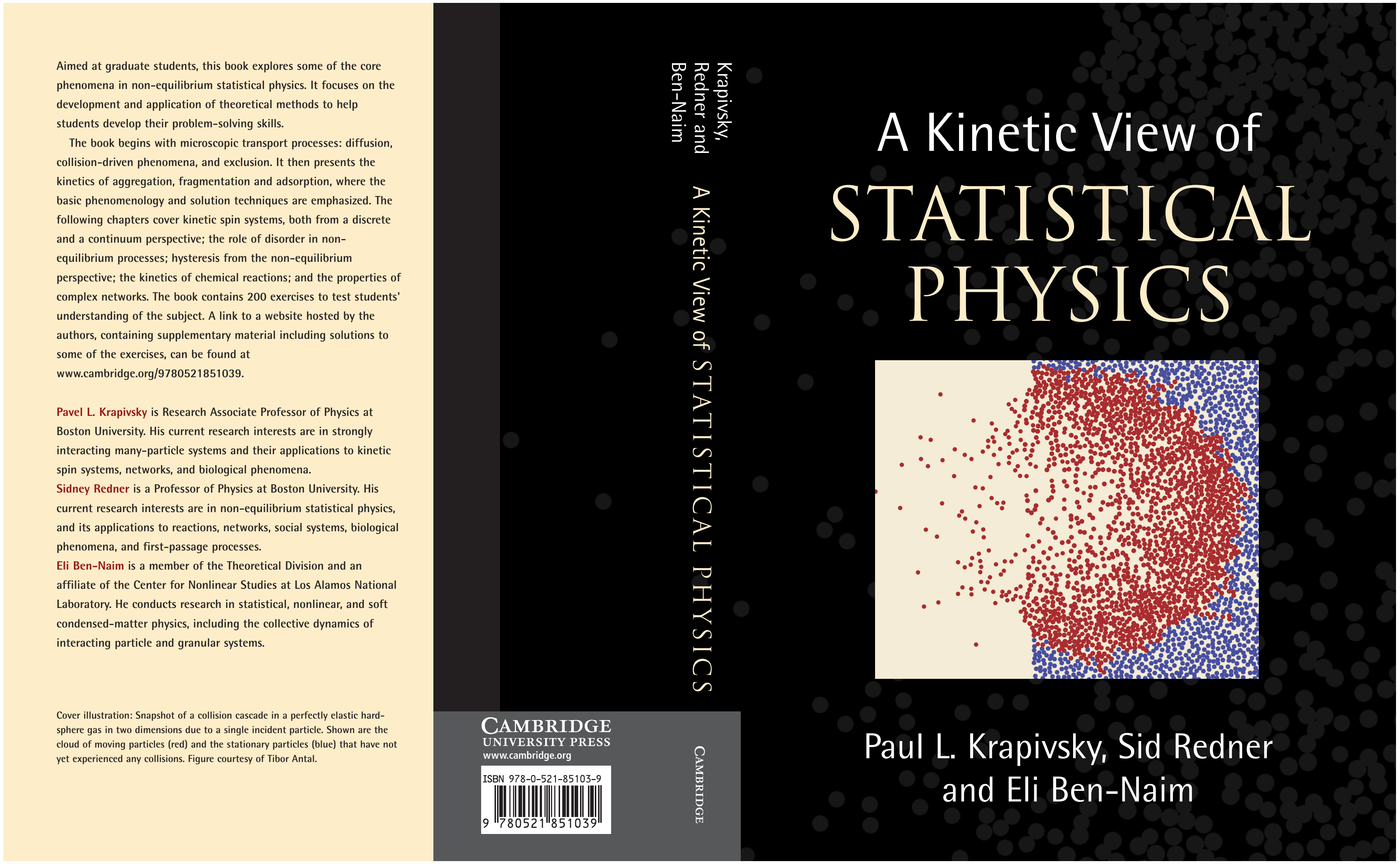
Extensions:

strategic voting → minority suppressed
partisan voting → selfishness forestalls consensus

Future:

"churn" rather than consensus
heterogeneity of real people
positive and negative social interactions → social balance

Crass Commercialism



to appear this October

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